Optimization Design Method of IIR Digital Filters for Robot Force Position Sensors

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1. Introduction

Digital filtering plays an important role in sensors' signal processing of robots. Not like analog system, it is not limited by parameters of electronic components, so it can process signals of rather low frequency, which is one of its advantages. According to different structure, digital filters can be divided into finite impulse response (FIR) digital filters and infinite impulse response (IIR) digital filters. The output of FIR digital filters only relates with the previous and the present input. Whereas the output of IIR digital filters relates not only with the input but also the previous output. It is to say that IIR digital filters have their feedback. Seen from signal-processing, IIR digital filters have great advantages over FIR digital filters, but they also have their disadvantages at design. The coefficient of IIR digital filters is highly nonlinear, whereas the coefficient of FIR digital filters is linear.

2. Signal processing system of the robot joint force/position sensor

2.1 Configure of the signal processing system

There are two kinds of design methods for IIR digital filters: 1) Frequency translation method, this method has two design routes: one route is first get analog lowpass filter, analog highpass filter, analog bandpass filter and analog band elimination filter by doing frequency band transform to the analog normalized prototype, and then get digital lowpass filter, digital highpass filter, digital bandpass filter and digital band elimination filter by digitization; the other route is first get digital lowpass filter by digitizing the analog normalized prototype, and then get digital highpass filter, digital bandpass filter and digital band elimination filter by frequency band transform in digital domain. 2) Optimization algorithm, it is to design digital filters under certain optimization criterions to get the best performance. Now, there are minimum P-error method, least mean square error (LMSE) method, linear programming method and model-fitting frequency response method etc.

In recent years, some scholars have already applied such intelligent algorithms as genetic algorithm, artificial immune algorithm and particle swarm optimization (PSO) algorithm etc into the design of IIR digital filters and achieved better result. Commonly speaking, filters' capacity is often shown by the permissible error of amplitude characteristic of its frequency response. When designing a filter, we should consider such main technical index as passband cutoff frequency ω_c , stopband cutoff frequency ω_c , passband tolerancea1, stopband tolerance a_2 and passband maximum ripple δ_1, stopband minimum attenuation δ_2,etc. At present, both traditional and optimized design methods need to consider the above mentioned capability index. The author will put forward an optimized design method based on the prior knowledge. According to the method, people only need to know the structure of a filter and to master an intelligent optimization algorithm before finishing the filter's design.

For the signal frequency of the robot joint force/position sensors is rather low, their signal fits to be processed by lowpass filters. There are two kinds of filters: analog filters and digital filters. Here, both analog filters and digital filters are used to process the signals of the robot joint force/position sensors. The configuration of the filters sees Fig. 1.

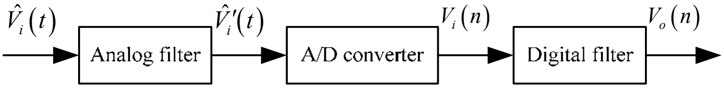


Fig. 1. Configuration of the filters

The output signals of the robot joint force/position sensor are analog input signals $\hat{V}_i(t)$ of the signal processing system. After analog filtering $\hat{V}_i(t)$ were converted to $\hat{V}_i'(t)$, and then $\hat{V}_i'(t)$ were sampled and discretized into input sequence $V_i(n)$ by A/D converter.

2.2 Realization of the signal processing system

(a) Realization of the analog filter

In this research, the sensor signal is magnified by instrument magnifier AD623, and the filter method by double capacitors is adopted which recommended by AD623 user's manual. The schematic of the analog filter is shown in Fig.2.

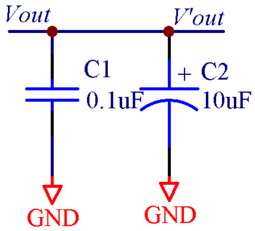


Fig. 2. Schematic of the analog filter

(b) Realization of the digital filter

Generally, the system function of N-order digital filter is

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_M z^{-M}}{1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_N z^{-N}} \tag{1}$$

Translating it to difference equation

$$\begin{aligned} y(n) &= b_0 x(n) + b_1 x(n-1) + b_2 x(n-2) + \cdots + a_M x(n-M) \\ &\quad - a_1 y(n-1) - a_2 y(n-2) - \cdots - a_N y(n-N) \end{aligned} \tag{2}$$

Then the digital filter can be realized via Eq. (2).

3. Optimization model of the IIR digital filter of robot joint force/position sensor

The system of IIR digital filter can be shown as Fig. 3

Fig. 3. Schematic diagram of the IIR digital filter

Suppose that the system function of N-order IIR digital filter is

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_M z^{-M}}{1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_N z^{-N}} \tag{3}$$

If Eq. (3) is adopted to design IIR digital filter, the number of parameter required optimize is $M + N + 1$, and it is difficult to choose the value range of every parameter. Generally, the system function of IIR digital filter is expressed as

$$H(z) = A \prod_{k=1}^{N} \frac{1 + a_k z^{-1} + b_k z^{-2}}{1 + c_k z^{-1} + d_k z^{-2}} \tag{4}$$

Both Butterworth filter and Chebyshev filter can be denoted as the cascade structural form with second-order unit shown as Eq. (4). When IIR digital filter is denoted by this structural form, the sensitivity of its frequency response to coefficient change is lower. And it is convenient to confirm the value range of every parameter with this structure form.

For robot force/position sensors, its measurement signal is low frequency, generally below 10Hz. And the disturbance is white noise mostly. If power supply is mains supply, the disturbance of 50Hz power frequency would exist. Generally, the analog lowpass filter is used to deal with these kinds of signals. However, it is very difficult to filtering the disturbance of 50Hz power frequency and the low-frequency white noise. If the digital filter is adopted and its cutoff frequency is set rather low, the filter can remove the disturbance of 50Hz power frequency and white noise mostly. From practical experience, it was known that the satisfying effect can be obtained when adopting a second-order lowpass.

The system function of the second-order Butterworth filter can be simplified as

$$H(z) = A\frac{1+2z^{-1}+z^{-2}}{1-cz^{-1}+dz^{-2}} \tag{5}$$

IIR digital filter as a system, the ideal signal after filtered could repeat the input signal perfectly, and with definite system delay. Then the similarity of the actual and ideal output signal would be considered as the evaluation index of filter performance. If the maximum frequency of robot force/position sensor signal is f, and the input signal is simulated with sine function, the function of input signal can be denoted as

$$x(n) = c_{\mathrm{s}} \cdot \sin(2\pi f \cdot n \cdot T_{\mathrm{s}}) + c_{\mathrm{dis}} \cdot \sin(2\pi \cdot 50 \cdot n \cdot T_{\mathrm{s}}) + c_{\mathrm{w}} \cdot \mathrm{randn}(1) \tag{6}$$

where c_{s} is the coefficient of sensor signal, c_{dis} is the coefficient of disturbance of 50Hz power frequency, c_{w} is the coefficient of white noise, T_{s} is the sample time of system. randn( ) is the normal distribution random number which represents white noise.
The ideal output after digital filter is

$$y_{\mathrm{d}}(n) = c_{\mathrm{s}} \cdot \sin(2\pi f \cdot n \cdot T_{\mathrm{s}}) \tag{7}$$

The actual output of filter is

$$y(n) = A \cdot x(n) + 2A \cdot x(n-1) + A \cdot x(n-2) - c \cdot y(n-1) - d \cdot y(n-2) \tag{8}$$

Suppose that the delay time is T, the sample size is n, the mean square error (MSE) of IIR digital filter at every sample points can be shown as

$$E = \frac{1}{n-T-1}\sum_{i=1}^{n-T}\sqrt{\left(y(T+i) - y_{\mathrm{d}}(i)\right)^2} \tag{9}$$

The mathematical model of optimization design of IIR digital filter is

$$\begin{cases} \min E = f(X) \\ s.t. X \in S = \left\{X \middle| g_j(X) \le 0 \; j = 1,\cdots,m\right\} \end{cases} \tag{10}$$

where X is optimization variable, $X = \{A, c, d\}$, $g_i(X)$ is restriction function, S restriction region, E optimization aim function, $E = f(X)$.

4. Optimize the parameters of the IIR digital filter using the particle swarm optimization algorithm

4.1 Introduction of particle swarm optimization

Particle Swarm Optimization (PSO) algorithm is a new global optimization evolutionary algorithm invented by Doctor Eberhart and Doctor Kennedy. This algorithm simulates preying of bird. It is outstanding to solve nonlinear optimization problem, and it is simply to realize this algorithm. So it has become an important optimization tool.

In PSO algorithm, the position of particle represents the possible solution. The superiority-inferiority of particle is measured by particle fitness. Firstly, a flock of random particles are initialized. Then the optimal solution is found out via multiple iterating. During every iteration, particle is updated by tracking the two optimal solutions which one optimal solution is found by this particle, namely, individual optimal solution, the other is found by the whole particle swarm presently, namely, global optimal solution.

After found two above-mentioned extremums, particle's velocity and position are updated according to two equations as follow.

$$\boldsymbol{V}_i^{k+1} = w \cdot \boldsymbol{V}_i^k + c_1 \cdot r_1 \cdot \left(\boldsymbol{p}_i^k - \boldsymbol{x}_i^k\right) + c_2 \cdot r_2 \cdot \left(\boldsymbol{p}_g^k - \boldsymbol{x}_i^k\right) \tag{11}$$

$$\boldsymbol{x}_i^{k+1} = \boldsymbol{x}_i^k + \boldsymbol{V}_i^{k+1} \tag{12}$$

where k is generation number, i is serial number of the particle, $\boldsymbol{V}$ is velocity of the particle, $\boldsymbol{x}$ is current position of particle, $\boldsymbol{p}$ is vector form of $\boldsymbol{p}_{\mathrm{Best}}$, $\boldsymbol{g}$ is vector form of $\boldsymbol{g}_{\mathrm{Best}}$, r_1 and r_2 are random numbers from 0 to 1, c_1 and c_2 are learning factors, generally, c_1=c_2=2, w is weighting factor, its value from 0.1 to 0.9.

4.2 Improved algorithm

(a) During the basic PSO algorithm search in solution space, the particle would oscillate round global optimal solution at later period. To solve this problem, the improvement can be done as follows: with iteration going, to let velocity update the weighting factor, the weighting factor decreases from maximum $w_{\max}$ to minimum $w_{\min}$ linearly, namely

$$w = w_{\max} - \frac{w_{\max} - w_{\min}}{G_{\max}} \cdot G \tag{13}$$

where G is generation number of current iteration, $G_{\max}$ is total generation number of iteration.

(b) To make the PSO algorithm search in solution space and to ensure convergence rate, the position space and velocity space of particle need to be limited. Eq. (5) will to be

$$x_{\mathrm{r}} = \left(x_{1\min}, x_{1\max}, x_{2\min}, x_{2\max}, x_{3\min}, x_{3\max}\right) \tag{14}$$

$$v_{\mathrm{r}} = \left(v_{1\min}, v_{1\max}, v_{2\min}, v_{2\max}, v_{3\min}, v_{3\max}\right) \tag{15}$$

4.3 Parameters coding and the choice of the fitness function

To solve X using PSO algorithm, the optimization variable X should be coded, to become particle of PSO algorithm. According to characteristics of PSO algorithm, parameters can be denoted with real number. To every particle of filter shown in Eq. (5), if the current position of particle is denoted with $\boldsymbol{X} = \left\{A, c, d\right\}$, and the velocity is denoted with $\boldsymbol{V} = \left\{V_1, V_2, V_3\right\}$, then the coding structure would be adopted as follows:

A, c, d	V_1, V_2, V_3
Position of the particles	Velocity of the particles

PSO algorithm confirms the superiority-inferiority of particle's current position via fitness. So the fitness function must be chosen according to the practical demand. Here Eq. (9) is chosen as the fitness function of IIR digital filter design. Evidently, the less the value of E is, the less the mean square error of filter parameter $\boldsymbol{X}$ corresponding this particle is. Then this particle is corresponding to the better coefficient of filter. When the algorithm finished, the particle with the minimum fitness during the whole running period is the optimal solution obtained by this algorithm, namely, filter parameter.

4.4 Optimization steps

(a) Set parameters of PSO algorithm, including population size S_{pop}, dimension S_{dim}, weighting factor w, position space x_{r}, velocity space v_{r}.
(b) Set parameters of IIR digital filter, such as c_{s}, c_{dis}, c_{w}, T_{s}, *etc*.
(c) Initialize the particle swarm, to randomly initialize every particle's position and velocity in parameter space.
(d) Solve the system delay time, and to calculate the particle's fitness according to Eq. (9).
(e) Initialize the current particle's position as individual extremum $\boldsymbol{p}_{\mathrm{Best}}$, and the position of particle with minimum fitness among all individual extremum as $\boldsymbol{g}_{\mathrm{Best}}$.
(f) Update the particle's position and velocity according to Eq. (14) and Eq. (15).
(g) Solve the system delay time, and to calculate the particle's fitness again.
(h) Judge whether to update the particle's individual extremum $\boldsymbol{p}_{\mathrm{Best}}$ and the global extremum $\boldsymbol{g}_{\mathrm{Best}}$ of particle swarm.
(i) Repeat step (f) to (h), till meeting precision demand or reaching iteration times pre-set.
(j) Output gBest, and to obtain the coefficient of IIR digital filter.

5. Results and analysis

To prove the validity of optimization design method of IIR digital filter for robot force/position sensor presented in this paper, an optimization program was completed in the Matlab 6.5 circumstance and several simulation experiments were made. In these simulation experiments, Parameters of the PSO algorithm were set as: swarm size $S_{\mathrm{pop}} = 50$, parameter dimension $S_{\mathrm{dim}} = 3$, minimum weighting factor $w_{\mathrm{min}} = 0.1$, maximum weighting factor $w_{\mathrm{max}} = 0.9$. According to former design experience, the value range of every filter coefficient are as: $x_{\mathrm{r}} = [0.00001, 0.01, 1, 2, 0.00001, 1]$, maximum velocity $v_{\mathrm{r}} = [-0.005, 0.005, -0.5, 0.5, -0.5, 0.5]$, sample time $T_{\mathrm{s}} = 0.02$ s, maximum frequency of sensor signal $f = 0.05$ Hz, signal amplitude $c_{\mathrm{s}} = 1.5$, disturbance amplitude of 50 Hz power frequency $c_{\mathrm{dis}} = 0.1$, amplitude coefficient of white noise $c_{\mathrm{w}} = 0.15$, simulation time $t = 50$ s.
Fig. 4 is the variety course of every generation's fitness of PSO algorithm.

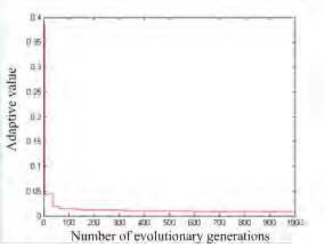
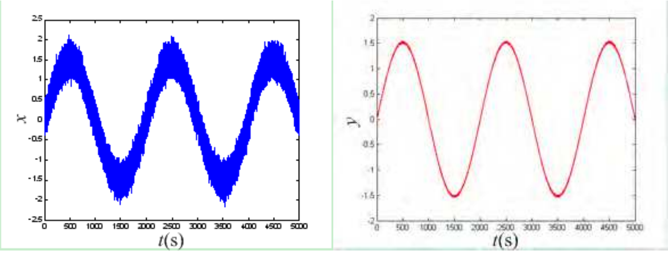


Fig. 4. Curves of unfiltered and filtered signals of the robot force/position sensors

Fig. 5 a) and b) are the signal curve of robot force/position sensor before and after filtering respectively. The last parameters of IIR digital filter are set as $A = 0.0064$, $c = 1.5186$, $d = 0.5439$, delay time $T = 0.36$ s, MSE $E = 0.0091$. From Fig. 4 b), we can know that the filter effect is perfect for sensor signal of $f = 0.05$ Hz.



(a) (b)

Fig. 5. Curves of unfiltered and filtered signals of the robot force/position sensors, (a) is curve of unfiltered signals, (b) is curve of filtered signals.

The frequency response optimized is

$$H\left(e^{T_s j\omega}\right) = 0.0064 \times \frac{1 + 2e^{-0.02 j\omega} + \left(e^{-0.02 j\omega}\right)^2}{1 - 1.5186e^{-0.02 j\omega} + 0.5439\left(e^{-0.02 j\omega}\right)^2} \tag{16}$$

From Eq. (16), the amplitude-frequency characteristics curve of filter designed in this research can be obtained, as shown in Fig. 6.

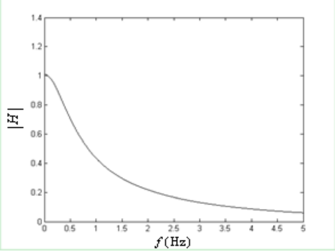


Fig. 6. Magnitude-frequency characteristics curve of the filter

From Fig. 6, it can be seen that signal is attenuated under 0.1, when its frequency decreased to 5 Hz. So the filter can remove the disturbance of 50 Hz power frequency and white noise mostly.

6. Conclusion

Aimed at the design of IIR digital filters of robot force/position sensors, a design method is put forward. Its optimization principle is the minimum MSE between ideal and actual output signal at time-domain. And the mathematics model aiming at second-order Butterworth lowpass filter was set up. This method needn't understand the complicated design theory and method for digital filter and the characteristic of filter, such as passband frequency, cutoff frequency, passband attenuation, ripple, etc. This method only requires the understanding of the structure characteristic of filter and the maximum frequency of sensor signal. Thus the parameters of the filter can be optimized in a suitable intelligent optimization method. An optimization program of the PSO algorithm was developed in the Matlab circumstance. The result of simulation experiment proves the validity of this method, and to be strongly practicable.

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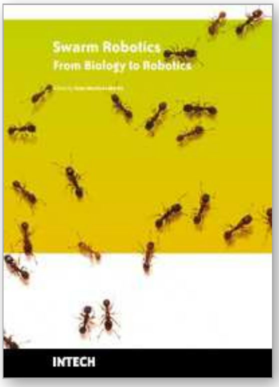

Swarm Robotics from Biology to Robotics

Edited by Ester Martinez Martin

ISBN 978-953-307-075-9
Hard cover, 102 pages
Publisher InTech

Published in print edition March, 2010

In nature, it is possible to observe a cooperative behaviour in all animals, since, according to Charles Darwin's theory, every being, from ants to human beings, form groups in which most individuals work for the common good. However, although study of dozens of social species has been done for a century, details of how and why cooperation evolved remain to be worked out. Actually, cooperative behaviour has been studied from different points of view. Swarm robotics is a new approach that emerged on the field of artificial swarm intelligence, as well as the biological studies of insects (i.e. ants and other fields in nature) which coordinate their actions to accomplish tasks that are beyond the capabilities of a single individual. In particular, swarm robotics is focused on the coordination of decentralised, self-organised multi-robot systems in order to describe such a collective behaviour as a consequence of local interactions with one another and with their environment. This book has only provided a partial picture of the field of swarm robotics by focusing on practical applications. The global assessment of the contributions contained in this book is reasonably positive since they highlighted that it is necessary to adapt and remodel biological strategies to cope with the added complexity and problems that arise when robot individuals are considered.

Fuxiang Zhang (2010). Optimization Design Method of IIR Digital Filters for Robot Force Position Sensors, Swarm Robotics from Biology to Robotics, Ester Martinez Martin (Ed.), ISBN: 978-953-307-075-9, InTech, Available from: http://www.intechopen.com/books/swarm-robotics-from-biology-to-robotics/optimization-design-method-of-iir-digital-filters-for-robot-force-position-sensors