

Probabilistic Broadcasting in Wireless Ad Hoc Networks

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1. Introduction

In the next generation of wireless communication systems, there will be a need to deploy independent mobile users. Significant examples include establishing survivable, efficient, dynamic communication for emergency/rescue operations, disaster relief efforts, and military networks. Such network scenarios cannot rely on centralized and organized connectivity, and can be conceived as applications of wireless ad hoc networks. A wireless ad hoc network is an autonomous collection of static or mobile users that communicate over relatively bandwidth constrained wireless links. The fact that these nodes could be mobile changes the network topology rapidly and unpredictably over time. The network is decentralized, where all network activity including discovering the topology and delivering messages must be executed by the nodes themselves.

The set of applications for MANETs¹ is diverse, ranging from small, static networks that are constrained by power sources, to large-scale, mobile, highly dynamic networks. The design of network protocols for these networks is a complex issue. Regardless of the application, MANETs need efficient distributed algorithms to determine network organization, link scheduling, and routing. However, determining viable routing paths and delivering messages in a decentralized environment where network topology fluctuates is not a well-defined problem. While the shortest path (based on a given cost function) from a source to a destination in a static network is usually the optimal route, this idea is not easily extended to MANETs. Factors such as variable wireless link quality, propagation path loss, fading, multiuser interference, power expended, and topological changes, become relevant issues. The network should be able to adaptively alter the routing paths to alleviate any of these effects. Moreover, in a military environment, preservation of security, latency, reliability, intentional jamming, and recovery from failure are significant concerns. Military networks are designed to maintain a low probability of intercept and/or a low probability of detection. Hence, nodes prefer to radiate as little power as necessary and transmit as infrequently as possible, thus decreasing the probability of detection or interception. A lapse in any of these requirements may degrade the performance and dependability of the network.

Wireless ad hoc networks, ever since its inception in the *packet radio* networks during the 1970s, has been a topic of extensive research because it meets the demands of the next generation communication systems. With the number of possibilities it opens up, it also presents to us

¹ Mobile Ad hoc Networks

formidable constraints such as network organization, link scheduling, power management, security and routing are a few to mention.

This chapter would present an overview of Broadcasting, a very prominent issue existing in design and deployment of wireless ad hoc networks today. Broadcasting plays a major role in successful communication in wireless networks such as manets, sensor networks etc. essentially because nodes in these networks do not have information about the topology of the network instead have to discover it.

Although broadcasting happens to be a very useful mechanism, it also presents to us a lot of challenges. Some of these are grouped under what is popularly known as the Broadcast Storm problems (Sze-Yao et al., 1999) . As one would imagine, optimizing broadcasting operation would indeed bring down the energy consumption of the entire network. This however does not have very straight forward answers. A lot of algorithms such as counter based, location based, area based etc (Williams & Camp, 2002). have been proposed in the past and each of them incorporate a different approach to optimize this operation.

This chapter would provide an introduction to some of these broadcasting schemes. In particular, the probability based scheme (Sasson et al., 2002) would be talked about in more detail. To understand the probability scheme better, understanding the key concepts in percolation theory would be necessary. Although this theory forces us to develop a very theoretical perspective of the topic at hand, it is useful as it gives us an idea of the bounds. However, we will verify these bounds by discussing some of the results obtained from simulations.

2. Broadcasting Schemes

Most routing protocols(reactive, pro active, hybrid etc.) use broadcasting in their route discovery scheme. A comprehensive classification of these broadcasting schemes is provided in (Williams & Camp, 2002). Simple Flooding requires each node to rebroadcast all packets. Probability Based Methods use some basic understanding of the network topology to assign a probability to a node to rebroadcast. Area Based Methods assume nodes have common transmission distances; a node will rebroadcast only if the rebroadcast will reach sufficient additional coverage area. Neighbor Knowledge Methods maintain state on their neighborhood, via *Hello* packets, which issued in the decision to rebroadcast.

The robustness of a broadcasting technique is based on how well it can handle network partitioning, highly mobile nodes, power sensitivity, collisions etc. These are essentially the basis of optimization of broadcasting operations. Most challenges raised by broadcasting and packet forwarding in adhoc networks are essentially due to their unconstrained mobility characteristics. Links form and break at a rapid rate. Like mentioned above, each class of the optimization technique deals with these issues in a particular fashion, which is our next topic of discussion:

2.1 Flooding

The classical Flooding algorithm is by far the simplest way to broadcast a message in the network. The process starts with a node which intends to broadcast a message in the network. Upon receiving the message each of the nodes rebroadcasts it exactly once and this continues until all the nodes which are reachable receive the message. An implementation of this is presented in (Ho et al., 1999). When a node receives a packet, it waits a uniformly distributed time interval between 0 and flooding-interval before it broadcasts the packet.

That time interval which is known as the *Random Assessment Delay* serves two purposes, firstly it allows the nodes sufficient time to receive redundant packets and assess whether to rebroadcast. Secondly, the randomized scheduling prevents the collisions.

2.1.1 The Broadcast Storm Problem (Sze-Yao et al., 1999)

Broadcasting is a common operation in many applications, e.g. graphs related problems and distributed computing problems. It is also very widely used to resolve many network layer problems. In MANET in particular, due to mobility, broadcastings are expected to be more often. One straightforward solution is like we have discussed so far is the blind flooding. In a CSMA/CD² network the drawbacks of flooding include:

- Redundant rebroadcasts: When a mobile host decides to rebroadcast a broadcast message to its neighbors, all its neighbors already have the message.

The following analysis shows that rebroadcasts are very expensive and should be used with caution. Consider a simple scenario (Fig. 1) in which there are two nodes *A* and *B*. Node *A* sends a broadcast message to node *B*. *B* upon receiving this message rebroadcasts the message. A simple calculation can tell us the usefulness of the rebroadcast by *B* i.e., how much area does it cover that *A* with the first message could not reach. Consider S_A to be the area that can be covered by *A* and S_B the area that can be covered by *B*. The area that can be shaded by *B* has been shaded and can be represented as S_{B-A} . Let r be the radii of S_A and S_B , and d the distance between *A* and *B*.

$$|S_{B-A}| = |S_B| - |S_{A \cap B}| = \pi r^2 - \text{INTC}(d)$$

where $\text{INTC}(d)$ is the interesection area of the two circles centered at two points distanced by d .

$$\text{INTC}(d) = 4 \int_{d/2}^r \sqrt{r^2 - x^2} dx$$

When $d = r$, teh coverage area $|S_{B-A}|$ is the largest which equals $\pi r^2 - \text{INTC}(r) = r^2(\frac{\pi}{3} + \frac{\sqrt{3}}{2}) \approx 0.61\pi r^2$. This shows that only 61% can be additional coverage over that already covered by the previous transmission.

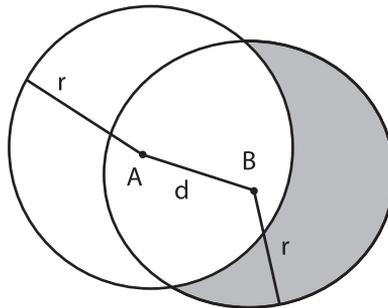


Fig. 1. The shaded area represents the additional coverage of node *B*

Supposing that *B* can be randomly located in any of *A*'s tranmission range, the average value can be obtained by integrating the above value over circle of radius x centered at *A* for x in $[0, r]$:

$$\int_0^r \frac{2\pi x [\pi r^2 - \text{INTC}(x)]}{\pi r^2} dx \approx 0.41\pi r^2$$

² Carrier Sense Multiple Access With Collision Detection

Thus, after the rebroadcast can only cover an additional 41% area in average.

In general, the benefit of a host rebroadcasting a message after having heard the message k times has been obtained. For $k \geq 4$, the additional coverage is below 0.05%.

- **Contention:** After a mobile host broadcasts a message, if many of its neighbors decide to rebroadcast the message, these transmission may severely contend with each other.

We now consider the situation where host A transmits a broadcast message and there are n hosts hearing message. If all these hosts try to rebroadcast the message, contention may occur because two or more hosts around A are likely to be close and thus contend with each other on the wireless medium.

Let's analyze the simpler case of $n = 2$. Let hosts B and C be the two receiving hosts. Let B randomly locate A 's transmission range. In order for C to contend with B , it must locate in the area $S_{A \cap B}$. So the probability of contention is $\frac{|S_{A \cap B}|}{\pi r^2}$. Let x be the distance between A and B .

$$\int_0^r \frac{2\pi x \text{INTC}(x) / \pi r^2}{\pi r^2} dx \approx 59\%$$

This value of contention has been shown to rise over 80% as $n \geq 6$.

- **Collision:** Because of the deficiency of backoff mechanism, the lack of RTS/CTS³ dialogue, and the absence of CD, collisions are more likely to occur and cause more damage.

Now consider the scenario where several neighbor hosts hear a broadcast from host X . There are several reasons for collisions to occur. First, if the surrounding medium of X has been quiet for enough long, all X 's neighbors may have passed their backoff procedures. Thus, after hearing the broadcast message, they may all start rebroadcasting at around the same time. This is especially true if carriers can not be sensed immediately due to RF delays and transmission latency. Second, because the RTS/CTS forewarning dialogue is not used in a broadcast transmission, the damage of collision is more serious. Third, once collision occurs, without collision detection (CD), a host will keep transmitting the packet even if some of foregoing bits have been garbled. And the longer the packet is, the more the waste.

One approach to alleviate the broadcast storm problem is to inhibit some hosts from rebroadcasting to reduce the redundancy, and thus contention and collision. In the following, we will discuss three broad classes of broadcasting schemes (Williams & Camp, 2002). They are namely Probability based methods; Area based methods, Neighbor knowledge methods.

2.2 Probabilistic Flooding

Probabilistic flooding is a slight modification over the flooding technique. In probabilistic flooding, a node upon receiving a broadcasted message rebroadcasts it with a probability $p < 1$. In dense networks, having some nodes rebroadcast probabilistically does not harm the coverage. Determining value of the broadcast probability can be a very interesting topic of research. Although there have been some attempts at it (Kadiyala & Sunitha, 2008; Sasson et al., 2002), it is still an open field. The basic understanding is that the broadcast probability could be determined by the node density. For example, a sparse network must have higher broadcast probability compared to a dense network. A detailed discussion of this is presented later. This scheme is identical to flooding when $p = 1$.

³ Request to Send/ Clear to Send

2.2.1 Counter Based Scheme

The following steps summarize counter based scheme:

- Random Assessment Delay⁴ is used.
- The counter is incremented by one for each redundant packet received.
- If the counter is less than a threshold value when the RAD expires, the packet is re-broadcast. Otherwise, it is simply dropped.

2.2.2 Phase Transitions in Wireless Ad Hoc Networks (Krishnamachari et al., 2001)

We have so far looked at a straightforward model of probabilistic flooding i.e. nodes trying to rebroadcast with a probability $p < 1$. The analysis of this scheme presents to us a very new direction. The idea of phase transitions in wireless ad hoc networks was first presented by B. Krishnamachari et al. in (Krishnamachari et al., 2001). Phase transitions are characterized by an abrupt emergence or disappearance of a property beyond a critical value of a parameter. In (Krishnamachari et al., 2001), the authors have shown that some properties of a wireless ad hoc network (node reachability with probabilistic flooding, ad hoc network connectivity and sensor network coordination) exhibit this phase transition and the critical behaviour.

Though this idea of phase transitions happen to be new to the field of wireless communications, such behavior has been known to mathematicians for several decades in the form of *zero-one* laws in Random Graphs⁵. The idea is that for randomly generated graphs, monotone properties such as connectivity, as we vary the average density of the graph transitions sharply from zero to one at a threshold value.

The basic idea is that for certain monotone graph properties such as connectivity, as we vary the average density of the graph, the probability that a randomly generated graph exhibits this property asymptotically transitions sharply from zero to one at some critical threshold value.

In probabilistic flooding each node decides to rebroadcast with a probability p . In (Krishnamachari et al., 2001) authors present that reachability in such probabilistic flooding schemes show a phase transition. The parameter that this property depends upon is the broadcast probability p . Thus for a $p > p_c$ ⁶,

$$\Pr \{ \text{reachability to all nodes in probabilistic flooding} \} \rightarrow 1$$

i.e., the property of reachability in probabilistic schemes takes birth, thus exhibiting the phase transition. The authors also present an interesting problem to ponder upon- As the number of neighbors that each node has increases the critical value p_c decreases, as is to be expected. Thus there is an interesting trade-off in this situation: if the transmission radius R is large, more power is expended, but the query traffic is minimized, whereas if the transmission radius is small then less power is expended by each node, but the number of route query packets will increase as the critical value p_c increases. Towards the end of this section we extend this particular discussion further.

⁴ Many of the broadcasting protocols require a node to keep track of redundant packets received over a short time interval in order to determine whether to rebroadcast. That time interval known as Random Assessment Delay (Williams & Camp, 2002) (RAD), is randomly chosen from a uniform distribution between 0 and T_{max} seconds, where T_{max} is the highest possible delay interval. This delay allows nodes sufficient time to receive redundant packets and assess whether to rebroadcast. Also the randomized scheduling prevents the collisions.

⁵ Random graphs were first defined by Paul Erdos and Alfréd Rényi in 1959.

⁶ p_c is the critical/threshold value of the parameter

The authors in (Krishnamachari et al., 2001) modeled the problem using Fixed Radius random graph model⁷.

These results however theoretical in their nature are very useful and gives us a new dimension to analyze probabilistic flooding. The next section on Percolation Theory (Broadbent & Hammersley, 1957) and its applications to our problem of probabilistic flooding would be an extension to this.

2.2.3 Percolation theory and its application to probabilistic flooding

In (Sasson et al., 2002) explore the phase transition phenomenon observed in percolation theory and random graphs as a basis for defining probabilistic flooding algorithms. The ever-changing topology of a MANET could be another good reason for implementing a probabilistic flooding scheme. The idea like we earlier discussed is that a node upon receiving a broadcast message rebroadcasts it with a probability $p < 1$. The existence of such a p_c (Fig. 2)

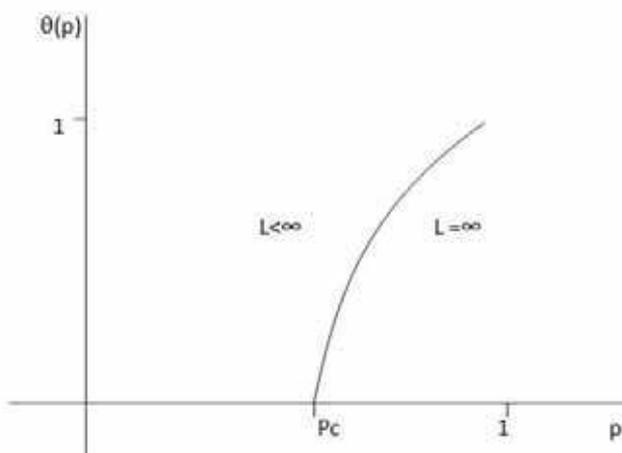


Fig. 2. On the Y-axis is $\theta(p)$ - Probability a vertex is part of an ∞ cluster and on the X-axis the Probability of a parameter. Note p_c , where the phase transition takes place. L denotes the length of the largest cluster which is ∞ when $p > p_c$

beyond which the probabilistic flooding reaches all nodes can be for the moment assumed to exist. However, the real question here is to be able to determine the value of p_c , which would eventually improve the implementation of wireless ad hoc networks. In (Sasson et al., 2002) the authors have tried to answer this question by attempting to apply a theory well studied in the context of percolation theory, *phase transition*, to determine the value of p_c .

For $p > p_c$ (percolation threshold) an infinite cluster which spans the entire network exists and for $p < p_c$ there only exist large finite clusters that run through the infinite lattice. In the latter

⁷ $G = G(n, R)$, given n points placed randomly accordingly to some distribution in the Euclidean plane, construct G with n vertices corresponding to these points in such a way that there is an edge between two vertices v and w if and only if the corresponding points are within a distance R of each other

case there is definitely more than one component in the lattice. This infinite spanning cluster translates to a path existing between any two nodes of a MANET in probabilistic flooding.

A lot of analysis in percolation theory is performed on lattice structures of different geometry such as square, triangular, simple cubic, body centered, face centered, honeycomb etc. Percolation theory in (Stauffer & Aharony, 1992) is defined as:

Every site of a very large lattice is occupied randomly with probability p , independent of its neighbors. Percolation theory deals with the clusters thus formed, in other words with the groups of neighbouring occupied sites.

One of the classical examples discussed in an introductory theory to percolation is the stone and water example (Grimmett, 1999). The question goes as follows, when a stone is immersed in water what is the probability that the center of the stone gets wet? The stone ofcourse is considered porous(very fine). The porous stone can be modelled as a lattice in Z^d . We consider a cross section of the stone($d = 2$) and claim that if there is a path for the water travel from the surface to the center of the stone, the center indeed would get wet. These paths are equivalent to the edges in the lattice and are modeled stochastically i.e., an edge is open(or closed) with a probability p (or $1 - p$). Only an open edge lets water pass from one vertex to another. Thus for the center to get wet, we need to study the existence of such large open cluters(connected path) which connect the bottom of the stone to the center. Percolation is primarily concerned with study of such *open clusters/paths*.

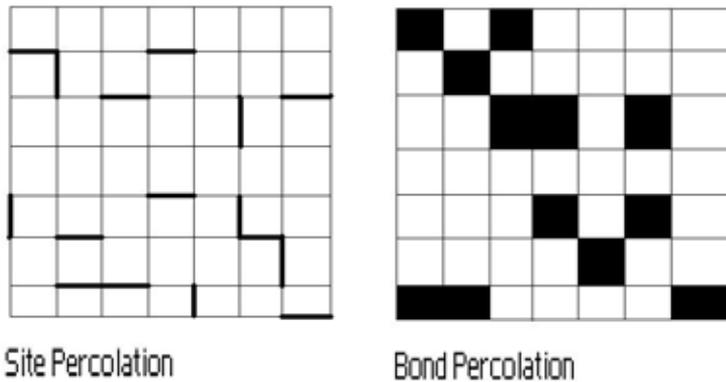


Fig. 3. Site and Bond percolation

The kind of percolation described in the above example is known as the *bond percolation*(Fig. 3). As the name suggests the uncertainty in bond percolation is existent in the bonds or the edges of the lattice. The other kind of percolation widely studied is the *site percolation*. Once again, as the name suggests the uncertainty here is implemented in the sites or the vertices of the lattice i.e., each site is open or closed with a probability p or $1 - p$. Paths within the lattice in the case of site percolation are restricted to between two neighboring open sites.

A lot of research has gone into determining the value of p_c for lattices of different dimensions and structures. In some cases there exists an analytic proof and in rest of the cases, computer simulation is an option. Most of these predictions are conjectural except when the number d of dimensions satisfies either $d = 2$ or $d \geq 19$

Like many other problems in physics, the percolation problem can be solved exactly in one dimension and some aspects of that solution seem to be valid also for higher dimensions. The value of $p_c = 1$ for the one dimension case (Stauffer & Aharony, 1992). The other interesting case is that of *Bethe lattice* which has ∞ dimensionality (Stauffer & Aharony, 1992). Every vertex in the Bethe lattice has the same number of neighbors, say z . The threshold probability for the bond percolation in this case

$$p_c = \frac{1}{z-1}$$

A simple derivation of this is presented in (Stauffer & Aharony, 1992). Critical probabilities for a few cases are presented in the Table 1.

Lattice	Site	Bond
Honeycomb	0.6962	0.65271
Square	0.592746	0.5000
Triangular	0.5000	0.34729
Diamond	0.43	0.388
BCC	0.246	0.1803
FCC	0.198	0.119
Simple Cubic	0.3116	0.2488

Table 1. Selected percolation thresholds for various lattices (Stauffer & Aharony, 1992)

2.2.4 The Critical Phenomenon

The principal quantity of interest is the *percolation probability* $\theta(p)$, being the probability that a given vertex belongs to an infinite open cluster. Thus we can define:

$$\theta(p) = P_p(|C| = \infty); \quad \text{where } |C| \text{ represents the length of the largest open cluster}$$

Alternatively, we may write,

$$\theta(p) = 1 - \sum_{n=1}^{\infty} P_p(|C| = n)$$

It is fundamental to percolation theory that there exists a critical value of $p_c = p_c(d)$ of p such that

$$\theta(p) \begin{cases} = 0 & \text{if } p < p_c \\ > 0 & \text{if } p > p_c \end{cases}$$

$p_c(d)$ is called the critical probability and is defined formally by

$$p_c(d) = \sup \{ p : \theta(p) = 0 \} \quad \text{where } d \text{ is the dimension of the lattice}$$

Case of one dimension is of no interest since, if $p < 1$, $\theta(p) = 0$ if $p < 1$, thus $p_c(1) = 1$. Following are some important results involving the critical probability and the dimension of the lattice in the form of theorem.

Theorem 1. $\theta(p) = \theta_d(p)$ is non-decreasing in d , which implies that

$$p_c(d+1) \leq p_c(d); \quad \text{for } d \geq 1$$

Theorem 2. If $d \geq 2$ then $0 \leq p_c(d) \leq 1$

Theorem 3. *The probability $\psi(p)$ that there exists an infinite open cluster satisfies*

$$\psi(p) = \begin{cases} 0 & \text{if } \theta(p) = 0 \\ 1 & \text{if } \theta(p) > 0 \end{cases}$$

There are exactly three phases in zero-one transition of the property. The *subcritical* when $p < p_c$, *supercritical* when $p > p_c$ and the *critical* phase when $p = p_c$. We are mostly concerned with the behavior of the largest open cluster in the lattice in these three regions. We will discuss very briefly in the following (Fig. 4):

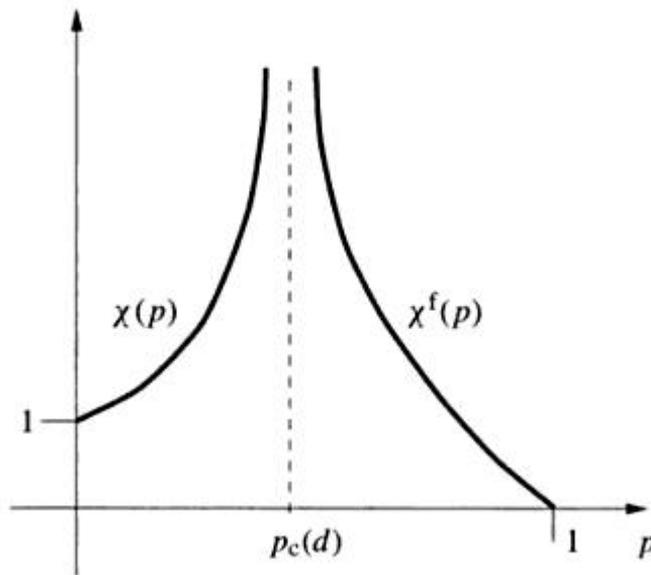


Fig. 4. On the Y-axis is $\chi(p)$ - Mean value of the open clusters is finite for $p < p_c$ since no infinite clusters exist in this region. $\chi^f(p)$ is the mean value of finite clusters and as is expected tends to 0 as $p \rightarrow 1$

Let us define a term $\chi(p)$ as the mean cluster size at some probability p . Similarly $\chi^f(p)$ refers to the mean size of a finite open cluster when $p > p_c$. We can note here that $\chi(p) = \chi^f(p)$ if $p < p_c(d)$. This is because in the subcritical phase, there exist no infinite clusters, so all the clusters that form are finite clusters.

(i) Subcritical phase:

$$P_p(|C| = n) \approx e^{-n\alpha(p)}; \quad \text{as } n \rightarrow \infty$$

(ii) Supercritical phase: When $p > p_c$, there exist infinite open clusters almost surely. So all the cluster with $|C| < \infty$ will decay very fast. As one would expect when $p = 1$, there would be no finite clusters in the lattice, thus $E[\chi^f(1)] = 0$ i.e.the expected value is 0.

- (iii) At the critical point: Firstly, does there exist an infinite open cluster when $p = p_c$? The answer is known to be negative when $d = 2$ or $d \geq 19$, and is generally believed to be negative for all $d \geq 2$. Therefore we would have

$$P_{p_c}(|C| \geq n) \approx n^{-\frac{1}{d}}; \quad \text{as } n \rightarrow \infty$$

Near the critical point as $p \rightarrow p_c$ from above or beneath, quantities such as $\theta(p)$ and $\chi(p)$ behave as powers of $|p - p_c|$. In (Grimmett, 1999) it is conjectured that the following limits exist:

$$\gamma = -\lim_{p \uparrow p_c} \frac{\log \chi(p)}{\log |p - p_c|}$$

$$\beta = \lim_{p \downarrow p_c} \frac{\log \theta(p)}{\log (p - p_c)}$$

To conclude this section on percolation we would present one last theorem which relates threshold probabilities in bond and site percolations.

Theorem 4. Let $G=(V,E)$ be an infinite connected graph with countably many edges, origin 0, and maximum vertex degree $\Delta(< \infty)$. The critical probabilities of G satisfy

$$\frac{1}{\Delta - 1} \leq p_c^{\text{bond}} \leq p_c^{\text{site}} \leq 1 - (1 - p_c^{\text{bond}})^\Delta$$

2.2.5 In the context of probabilistic flooding

Having gone through some preliminary results in percolation theory, it is all the more important to understand what they mean in the context of probabilistic flooding. Like we mentioned earlier, (Sasson et al., 2002) talks at length about application of percolation theory to the probabilistic flooding scheme. The first step would be to develop a model for the probabilistic flooding. As described in (Sasson et al., 2002), given a broadcast source node S , let G_B be the connected subgraph of G representing all nodes that will receive the broadcasted message by flooding ($S \in G_B$). G_B may be thought of as an infinite open cluster as defined in the earlier section. The key is to have the connectivity but getting rid of some edges which only increase redundancy. So essentially, operating above the percolation threshold p_c of G_B , we can ensure connectivity while reducing the number of edges at the same time.

In (Sasson et al., 2002), the authors have considered two models:

- (1) 2 X 2 grid (Fig. 5), where nodes are placed at every intersection. This model assumes that the nodes are immobile and have the same fixed radius of transmission R . Limitation of this approach are - lack of mobility in the network, restriction on the number of nodes in the neighborhood of a particular node to 4 and ofcourse ideal network conditions are assumed. Despite the fact that there exists these drawbacks, it still helps to answer a few questions given such idealistic network conditions.
- (2) The second model is that of a fixed radius model as described in (Krishnamachari et al., 2001). This takes into account more realistic network conditions like mobility and non-constant number of neighbors.

In the scenario 1, we can model the 2 X 2 grid as a square lattice in 2 dimensions. Results obtained in percolation theory on 2-D square lattices can be applied to this model. Since we are considering omni-directional flooding here, we can think of it as site percolation, where as if the broadcast was directional (Shen et al., 2006) it could be thought of as a bond percolation

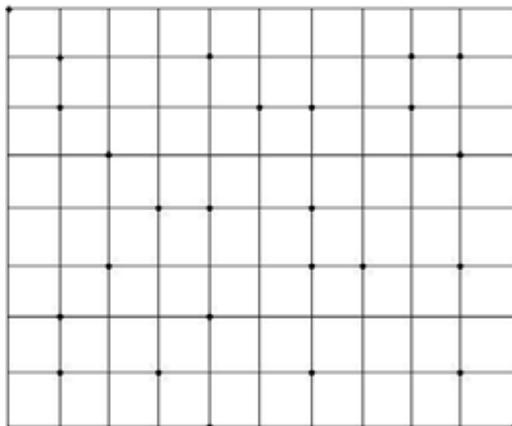


Fig. 5. Grid of wireless ad hoc network nodes

problem on square lattice. Infact, as we know from the above section on percolation that the critical probabilities in bond percolation is lesser than that of critical probabilities in site percolation. Thus it could optimize flooding even further. The scenario 2 has been simulated on the NS2 and critical probabilities have been estimated to be far lesser than the percolation threshold.

Having presented the application of percolation theory to probabilistic flooding, we need to relook at the assumptions we have made while applying them to probabilistic flooding. One of the fundamental aspects of percolation theory is that it is studied on infinite lattices and the results can not entirely hold true for finite lattices which is the case in the MANETs. So the questions we earlier posed need to be modified a little now. For example, we would no longer be concerned about existence of infinite open cluster but we would want to understand how the size of the largest cluster(as opposed infinite cluster) varies with changing probability and the size of the network.

2.2.6 Finite size scaling (Stauffer & Aharony, 1992)

What happens to the various quantities of interest near the percolation threshold in a large but *finite* lattice? Of course, even for p far below p_c the system has a largest cluster. But only for $p > p_c$ is the size of the largest cluster of the order of the system size. The question as phrased in (Stauffer & Aharony, 1992)- How does the size of the largest cluster increase with L in a system with L^d sites?

To be able to understand these questions we define a couple of other terms:

Cluster Radius 1. *It is very similar to the radius of gyration, which is defined as the*

$$R_s^2 = \sum_{i=1}^s \frac{|r_i - r_0|^2}{s}$$

where

$$r_0 = \sum_{i=1}^s \frac{r_i}{s}$$

is the center of mass of the cluster, where r_i is the position of the i^{th} occupied site in the cluster. If we average all clusters having a size of s , the average of the squared radii is denoted as R_s^2 .

At $p = p_c$ the radius (R_s) of the largest cluster will be of the order of the system length (Stauffer & Aharony, 1992) L ,

$$R_s \propto L$$

Also at $p = p_c$, we have,

$$R_s \propto s^{\frac{1}{D}}$$

Thus the condition for the largest cluster is:

$$L \propto s^{\frac{1}{D}}$$

where D is called the *fractal* dimension. Above p_c , the mass s of the largest cluster increases as L^d which means that it is no longer a fractal but $D = d$ as for finite clusters.

Another view of finite scaling shifts the focus from the critical probability to the *crossover length* ($\xi \propto (p - p_c)^{-\nu}$). The mass of the largest cluster s is proportional to L^D , as long as L is much smaller than the crossover length ξ . On the other hand if $L > \xi$, the mass of the largest cluster is proportional to PL^d , where $P \propto (p - p_c)^\beta$.

Also the theoretical value of D is computed as:

$$D = d - \frac{\beta}{\nu} \quad \text{where } d \text{ is the dimension of the lattice and this hyperscaling works for } d < 6$$

Kapitulnik et al. (1984) showed that the values of $\nu = 1.33$ and $\beta = 0.14$. Therefore in 2 dimensions, $D = 1.896$ and in 3 dimensions, $D = 2.5$ (Stauffer & Aharony, 1992). From this section on finite size scaling we can conclude,

$$M(\text{mass of the largest cluster}) \propto \begin{cases} L^D & \text{when } L < \xi \\ PL^d & \text{when } L > \xi \end{cases}$$

for $d = 2$, we have $D = 1.896$.

2.2.7 Finite size scaling in the context of probabilistic flooding

Once we get back to our original problem of determining how well probabilistic flooding performs, we try to answer at what values of broadcast probability would all the nodes in the network be reachable. Since our networks are finite, the analysis provided in the previous section would be more apt. To model the wireless ad hoc network, we consider the same 2×2 square lattice (scenario (i)) discussed earlier.

We aspire to have the number of nodes that get the broadcasted message to be proportional to the total number of nodes in the network. Given the size of the network in 1 dimension is L , and since it's a 2-dimensional square, total nodes in the network are L^2 . Basing on what we presented in the finite scaling section, we can claim that,

nodes that rebroadcasted message

$$\propto \begin{cases} \text{\#of nodes in 1 dimension}^{1.9} & \text{when \#of nodes in 1 dimension} < \xi \\ P \times \text{\#of nodes in 1 dimension}^2 & \text{when \#of nodes in 1 dimension} > \xi \end{cases}$$

Also,

$$\begin{aligned} & \text{Total \# of nodes that received the broadcasted message}(T_{br}) \\ & = \text{\# nodes that rebroadcasted message +} \\ & \quad \text{its perimeter}(t) \end{aligned}$$

Perimeter : is defined as the total number of nodes(non-rebroadcasting) that are immediate neighbors of nodes that rebroadcast.

The above relation holds because, even if a node decides not to rebroadcast, it still can receive the broadcasted message if any of its neighbors had rebroadcasted it.

This result is quite different from what we have seen earlier. It basically tells us that # of nodes that receive broadcasted message will be propotional to the total # of nodes in the network if the size of the network in 1 dimension is greater than the crossover length. So its not sufficient to operate with $p > p_c$ but we need to ensure $L > \xi$ for successful broadcast. We can infer from the proportionalities that the closer (but lesser) the value of ξ we choose to L the lesser will be the required broadcast probability p to broadcast the entire network. For a given lattice of size $L_1 \times L_1$, we need to check what the closest value of crossover length(ξ) is and then determine the corresponding p . Although we mostly have proportionalities a more accurate value can be estimated using simulations.

Another interesting point to note here is, when operating at $p = p_c$, the value of crossover length(ξ) is infinite, hence no matter what the size of the lattice is, the mass of the largest cluster would vary as $L^{1.9}$ and not L^2 . So it is definitely advisable to operate above p_c . In order to verify our conclusions, we check the results obtained from simulation. Since we know $p_c = 0.59$ (Stauffer & Aharony, 1992) for a 2D square lattice, using the relation($\xi \propto (p - p_c)^{-1}$) we can generate a table corresponding to p and ξ (Table 2).

p	$\xi \propto$
0.59	∞
0.6	457
0.65	42
0.7	18
0.75	12
0.8	8
0.85	6
0.9	5
0.95	4

Table 2. p and corresponding crossover length ξ value (Kadiyala & Sunitha, 2008)

2.2.8 Simulated Results

There are two sets of simulations that would help us verify our results so far. Firstly, to verify our equations we can simulate a 2 dimensional square lattice on MATLAB and study site percolation on it. In (Kadiyala & Sunitha, 2008), we have studied the simulation and it involves:

- (i) Generating random numbers and depending on the value of connection probability either place an open or closed site.
- (ii) Using the Hoshmen Kopelman (Babaliievski, 1998) algorithm, percolating labels⁸ through the lattice.
- (iii) Estimated the size of the largest cluster in the lattice for varying sizes of lattice for a particular value of connection probability p .

The second set of simulations would involve simulation of a wireless ad hoc network on a 2×2 grid. In (Kadiyala & Sunitha, 2008) such a study has been done. The simulation involved the following:

- (i) Decided with a probability p if each node rebroadcasts or not to its neighbours in the vertical and horizontal direction.
- (ii) Broadcasting a message from a different node every time, we calculated the number of nodes which are actually receiving it.

The Table 2 basically gives us the following information: Given a wireless ad hoc network with $L \times L$ number of nodes in it, we can select the value of crossover length(ζ) closest to L but lesser than L . Then depending on what the value of the crossover length is one can select the corresponding value of broadcast probability p from the above table. Also what can be inferred from above table is that, operating at $p = p_c = 0.59$, we cannot reach all nodes of the network(since $L \propto \infty$). For a more realistic scenario of the number of nodes (≈ 100 nodes) in the network, we could check that the broadcast probability(p) needs to be around 0.8

One of the major issues while applying percolation theory to wireless ad hoc networks is that, percolation is studied on infinite static systems where as realistic wireless ad hoc networks are finite and mobile. One way is to consider finite size scaling which deals with one of the problems.

Apart from the probabilistic flooding scheme we so far looked at, there are other classes of broadcasting techniques such as area based methods and neighbor based method (Williams & Camp, 2002). The following sections would briefly introduce those ideas.

2.3 Area Based Methods

The idea behind area based method is calculation of the additional distance that can be covered by a new rebroadcast. As shown earlier, the additional distance covered is highest when the receiving node is on the boundary of the hosts transmission range ($\approx 61\%$ (Sze-Yao et al., 1999)). A node using an area based method can evaluate additional coverage area based on all received redundant transmissions. This could determine whether the node rebroadcasts the packet into the network or not. Some of the area based methods suggested in (Williams & Camp, 2002) Distance Based and Location Based. The distance based is briefly explained in the following.

⁸ are equivalent of packets, open sites that are neighbors would share the same label.

2.3.1 Distance Based Scheme

- Using Random Assessment Delay
- Estimating the distance d between sender and receiver by signal strength
- Calculate the additional coverage by d (additional coverage = $\pi r^2 - \text{INTC}(d)$)
- If the additional coverage which is calculated by the minimum distance is more than a threshold value when the RAD expires, the packet is rebroadcast. Otherwise, it is simply dropped

2.4 Neighbor Knowledge Methods

The simplest of the simplest neighbor knowledge method suggest in (Lim & Kim, 2000) is referred to as the Self Pruning. This approach requires each node to have information about its 1-hop neighbors and this is made possible through *Hello* packets. Each broadcasted message has the list of neighbors of the node that sent the message. Upon receiving, a node compares its list of neighbors to the sender's list of neighbors. Depending on whether it would reach to any additional node, it decides either to rebroadcast or not. The Dominant Pruning extends this same logic to 2-hop apart nodes. This can be obtained by exchanging adjacent node lists with neighbors. This would certainly perform better than the self pruning scheme because of the additional information a node can get. Also in the case of dominant pruning, sending node selects the adjacent nodes that should relay the packet to complete broadcast unlike self pruning where every node decides for itself. Some of the neighbor knowledge based methods presented in (Williams & Camp, 2002) are:

- (1) Flooding with Self Pruning
- (2) Scalable Broadcast Algorithm
- (3) Dominant Pruning
- (4) Multipoint Relaying
- (5) Ad Hoc Broadcast Protocol
- (6) CDS-Based Broadcast Algorithm
- (7) LENWB

3. Conclusion

Broadcasting as mentioned in the introduction of the chapter is a very essential operation in wireless ad hoc networks. It helps not just in route discovery but also in emergency conditions. Because of the constraints that the network being wireless and ad hoc at the same time, puts forward, finding an optimum solution is a challenge. Bringing in uncertainty into the system would be one way of optimizing the broadcasting operation. A more effective way to use this probabilistic broadcasting operation is to introduce some intelligence into the system. Intelligence in the form of neighborhood information, transmission ranges would be very useful. In particular, if the network is very sparse it would make sense to use higher value of broadcast probability and vice versa. This idea has been explored in (Zhang & Agrawal, 2005).

Another way to optimize the broadcasting operation is by ensuring that instead of having a common optimum transmission range a variable transmission range (Member-Gomez & Member-Campbell, 2007) could be used to increase the capacity (Gupta & Kumar, 2000) of the network.

It is plausible to say that rebroadcast probability should be a function of the neighbour density, i.e., if the number of nodes in the neighbourhood is high, we could use a smaller broadcast probability and vice versa. This argument leads to another interesting point which is, the neighbour density of a node is a function of the transmission range, i.e. a node with larger transmission range is likely to have more number of neighbours purely by the virtue of its reachability. What we can hence conclude is that, there is an inherent relation between the rebroadcast probability and transmission range of the node. It would thus be very interesting to work on dynamically changing rebroadcast probability(p) and transmission range(R) simultaneously with a logic implemented with it. The logic could be as simple as, a node with large transmission range has lower rebroadcast probability and vice versa with,

$$Rp = k \quad (\text{constant})$$

The other interesting result that percolation theory gives us is the value of $p_c \approx 0.246$ for a BCC lattice. That is the broadcast operation in a 3D wireless ad hoc network can be optimized further. To be arrive at the exact values of broadcast probability for 3D wireless ad hoc networks, one has to do a similar research as we has been done for 2D wireless ad hoc networks.

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