On the Accuracy of a Stewart Platform: Modelling and Experimental Validation

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1. Introduction

The accuracy modelling of parallel-type robot structures is based on finding out the position and orientation errors of the end-effector in relation with the modelled error-sources. In the case of small (infinitesimal) deviations, the error model is expressed through a *linear function* using special matrices called *error Jacobeans* (Neagoe, 2001). Hence, the main objective of accuracy modelling is to express the *error Jacobeans*.

The accuracy modelling of the parallel robots represents a real challenge for the researchers, due to the high level difficulties implied in the direct modelling of the errors and, as a result, in the error-Jacobean description. Unlike the case of the serial robots, the analytical expression of the direct error-Jacobean for parallel structures is generally inaccessible or very complex; contrary, the inverse Jacobean can be obtained without major difficulties.

The studies referring to the precision modelling of parallel structures are scarcely found in literature, the most of the papers dealing especially with the inverse kinematics problem of parallel structures. There can not be found important results concerning the direct Jacobean modelling, neither a generalization of modelling, due to the severe difficulties of modelling. Many papers present specific solutions for different specific parallel structures: Tau parallel robot (Cui et al., 2008), six-dof parallel kinematic machine Linapod (Pott & Hiller, 2008), a 4-DOF parallel manipulator H4 (Wu & Yin, 2008), 3-DOF planar parallel robots (Briot and Bonev, 2008), etc. Currently, the problem of the parallel robots kinematics is reduced to the expression of the inverse kinematic Jacobean.

Three representative methods applied in kinematic modelling of parallel robots were identified:

1. *The partial derivatives method*, which consists firstly in identifying the geometric relations for the modelled parallel structure and, than, the partial derivation relative to the independent parameters (Merlet, 1990; Merlet & Gosselin, 1991). Generally, the obtained model can be expressed by the relation $\mathbf{A} \cdot d\mathbf{q} = \mathbf{B} \cdot d\mathbf{X}$, used for the joint velocities $d\mathbf{q}/\mathbf{dt}$ calculus in relation with the operational velocities $d\mathbf{X}/\mathbf{dt}$. In this case, the inverse matrix $\mathbf{A}^{-1}$ is easily obtained, **A** being a square matrix for Stewart platforms. On the other hand, to express

the operational deviations $d\mathbf{X}$ and, implicitly, the direct Jacobean assumes to inverse the matrix **B**, which has the dimension equal to the number of independent modelling parameters.

2. *The vectorial method*, which expresses the articular velocities through the vectorial relations of the kinematic modelling of velocities (Benea, 1996; Merlet, 1990).
3. *The kinematic screws method*, which obtains the kinematic model by vectorial transformations applied to the plückerien coordinates of a line in space (Ficher, 1986; Lee et al., 1999; Toyama & Hatae, 1989).

Starting from the previous revealed aspects, the authors propose in the paper a general method, based on the use of homogenous operators, useful for accuracy modelling of parallel structures with any configuration and complexity, with application for a Stewart-Deltalab parallel platform.

The chapter is structured in four main sections. The first section introduces the theoretical background on the proposed method meant for accuracy modelling of parallel robots and presents the steps and mathematical support of a general algorithm derived from this method.

In the second part, the proposed modelling is concretely applied to derive the accuracy model of the Stewart–Deltalab platform, considering the independent kinematic parameters (independent joint variables) as error–sources. Numerical examples will be presented, based on the analytical closed-form accuracy model previously obtained.

The third section will contains the authors' contribution to the modelling of the Stewart–Deltalab platform accuracy, applying the same proposed modelling method and algorithm, considering a set of geometric parameters as error–sources. Also, numerical examples will be done.

In the last section, relevant aspects regarding the experimental testing of the error models used in accuracy studies of spatial parallel–structures will be presented. The theoretical accuracy models of the Stewart–Deltalab platform are verified by experimental testing, in conformity with a concrete experimental research program and a specific mathematical support.

2. Theoretical background

The method proposed for parallel robot accuracy modelling includes three main steps:

1. *Breaking* of parallel structure into *open kinematic chains* (OKC) and description of *the error models* for the obtained OKC, *considered as independent chains,* by applying the specific error modelling of serial robots (Gogu, 1995; Gogu et al., 1997).
2. *Recovering* the parallel structure by assembling the error models of OKC and finally expressing the *dependent errors*.
3. *Description of the end-effector errors* related to the independent errors, by replacing the dependent errors in the error model derived for one of the OKC (step 1) with their corresponding expressions (identified at step 2).

   Explanatory notes:

— Structurally, a parallel robot includes:

- *Active* (*actuated*) *joints*; the relative displacements in the actuated joints are the *robot generalized variables (independent joint variable)* of the structure. Thus, the deviations of these variables become input errors (*independent errors*) in the accuracy modelling.

- *Passive* (*non-actuated*) *joints;* these joints are included in any parallel structure in order to obtain parallel-type links. The relative displacements in passive joints are functions of independent joint variables and, hence, displacement errors in passive joints, called *dependent errors,* depend on the independent errors.

— Modelling the influence of geometrical parameters on the end-effector accuracy is done on an *equivalent structure*, obtained from the initial structure by associating *fictive joints* to the geometrical parameters affected by errors: a prismatic joint is introduced for each linear error and a revolute joint – for each angular error.

— The proposed modelling requires only the inverse geometrical model of the parallel structure and the direct geometrical models of OKC. All these models are expressed, generally, without significant difficulty.

The general case of a parallel robot structure (Fig. 1,a) is considered; it consists of a mobile platform (*m*) connected to the fixed one (*f*) by *p* kinematic chains (legs) A_iB_i, in a parallel layout. The spatial guiding of the mobile platform is obtained by actuating *n* joints nominated as *active* joints. In practice, generally $p = n$, each leg including only one active joint. For the sake of clarity, the following *explanations* have to be mentioned:

— For the simplicity, but without reducing the generality, Figure 1 shows only symmetrical legs in a parallel layout and their connecting joints to the base and moving platforms. In a general case, the robot legs can have different structures and usually include intermediate joints.

— The breakage of the parallel structure can be done in different ways: to the characteristic point of the end-effector, to the base joints or to any intermediate joints.

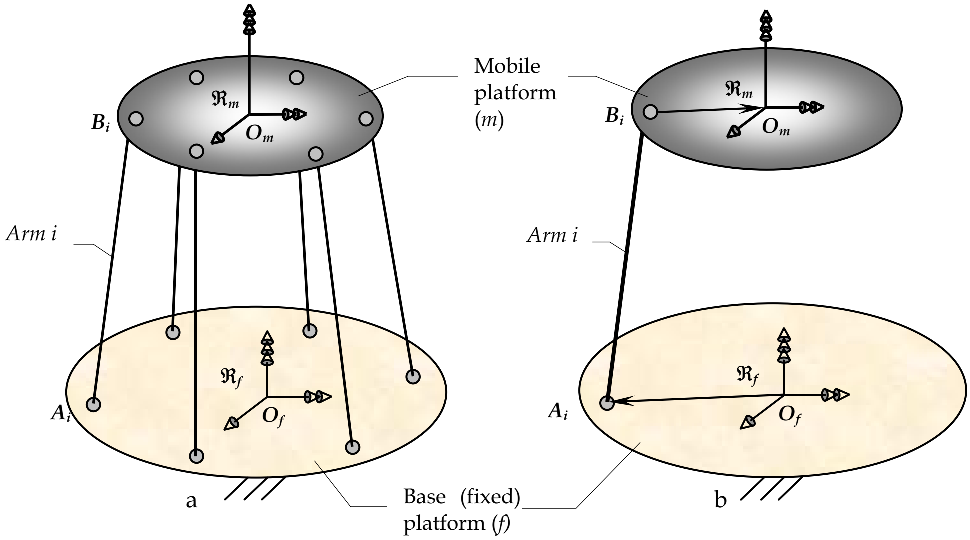


Fig. 1. Parallel robot structure in a general configuration (a), and the open chain *i* obtained by "disassembling" the parallel structure (b)

— In the following modelling, the independent parameters will be designated by notation $\mathbf{p}_{ind}$, while the dependent parameters by $\mathbf{p}_{dep}$; $\mathbf{p}^k_{ind}$ and $\mathbf{p}^k_{dep}$ designate independent parameters and respectively dependent parameters for the leg *k* (OKC *k*) of the parallel structure.

— *The independent parameters* $\mathbf{p}_{ind}$ designate *active joint variables,* in the case of joint error modelling, or *geometrical parameters* when the error model for geometrical deviations has to be established. In the global error modelling, both types of kinematic and geometrical parameters are considered independent.

— The end-effector errors can be reduced in the base, end-effector or other intermediate reference frame. In order to simplify the notations, the reference frame where the errors are reduced is not specified.

The proposed modelling begins with a preliminary step: *breaking* the parallel structure into open kinematic chains (OKC); there are many splitting variants that can be applied, each of them giving specific features to the modelling algorithm (Neagoe, 2001). The variant used in the paper is based on breaking the parallel structure in the origin O_m of the mobile frame $\Re_m$, obtaining the open chains $O_f A_i B_i O_m$, $i = 1..p$ (Fig. 1,b); all the open chains have the mobile platform (end-effector) as the final element. The obtained *p* independent OKC have the same property: *their extremity points are permanently coincident* and, consequently, the end-effector's errors for all the *p* OKC *are identically*!

Further on, the steps of the proposed modelling algorithm and its specific mathematical aspects are briefly presented.

Step I: deriving the end-effector errors for the p OKC

By applying the well known relations for open kinematic chains accuracy modelling (Gogu, 1995; Gogu et al., 1997 ; Paul, 1981), in the case of OKC *i* will be obtained:

$$ {}^{i}\begin{bmatrix} d_x \\ d_y \\ d_z \\ \delta_x \\ \delta_y \\ \delta_z \end{bmatrix} = \left[\mathbf{J}^i_{ind}\right]\left[d\mathbf{p}^i_{ind}\right] + \left[\mathbf{J}^i_{dep}\right]\left[\mathbf{J}^i_{dep}\right]\left[d\mathbf{p}^i_{dep}\right], \tag{1} $$

where $\mathbf{J}^i_{ind}$ is the error Jacobean for the independent errors and $\mathbf{J}^i_{dep}$ – the error Jacobean for the dependent errors of the parallel structure, $i = 1..p$. In this step, having only independent open kinematic chain (OKC), all the modelling parameters are characterized by independent errors.

Step II: identification of the dependent errors

In the parallel structure, the end-effector's errors are the same for all the *p* OKC (*the existence condition of a parallel structure*). As a result, the following *p*-1 independent matrix equations are obtained:

$$ \left[\mathbf{J}^j_{ind}\right]\left[d\mathbf{p}^j_{ind}\right] + \left[\mathbf{J}^j_{dep}\right]\left[d\mathbf{p}^j_{dep}\right] = \left[\mathbf{J}^k_{ind}\right]\left[d\mathbf{p}^k_{ind}\right] + \left[\mathbf{J}^k_{dep}\right]\left[d\mathbf{p}^k_{dep}\right], \quad j \neq k \ . \tag{2} $$

Without reducing generality, the following assumptions can be accepted: j = 1 and k = 2..p or j = 1..p–1 and k = j+1. Grouping together only the dependent terms from the left member of the equation (2), the relation (3) will be obtained:

$$\left[\mathbf{J}_{dep}^{1}\right]\left[d\mathbf{p}_{dep}^{1}\right]-\left[\mathbf{J}_{dep}^{k}\right]\left[d\mathbf{p}_{dep}^{k}\right] = \left[\mathbf{J}_{ind}^{k}\right]\left[d\mathbf{p}_{ind}^{k}\right]-\left[\mathbf{J}_{ind}^{1}\right]\left[d\mathbf{p}_{ind}^{1}\right]. \tag{3}$$

The p-1 equations (3) are grouped into a matrix system (4):

$$\left[\mathbf{J}_{dep}\right]\left[d\mathbf{p}_{dep}\right]= \left[\mathbf{J}_{ind}\right]\left[d\mathbf{p}_{ind}\right]. \tag{4}$$

where:

$$\left[\mathbf{J}_{dep}\right]=\begin{bmatrix} \mathbf{J}_{dep}^{1} & -\mathbf{J}_{dep}^{2} & 0 & \cdots & 0 \\ \mathbf{J}_{dep}^{1} & 0 & -\mathbf{J}_{dep}^{3} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & 0 \\ \mathbf{J}_{dep}^{1} & 0 & 0 & \cdots & -\mathbf{J}_{dep}^{n} \end{bmatrix},\ \left[\mathbf{J}_{ind}\right]=\begin{bmatrix} -\mathbf{J}_{ind}^{1} & \mathbf{J}_{ind}^{2} & 0 & \cdots & 0 \\ -\mathbf{J}_{ind}^{1} & 0 & \mathbf{J}_{ind}^{3} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & 0 \\ -\mathbf{J}_{ind}^{1} & 0 & 0 & \cdots & \mathbf{J}_{ind}^{n} \end{bmatrix},$$

$$\left[d\mathbf{p}_{dep}\right]= \left[d\mathbf{p}_{dep}^{1} \quad d\mathbf{p}_{dep}^{2} \quad \cdots \quad d\mathbf{p}_{dep}^{p}\right],\quad \left[d\mathbf{p}_{ind}\right]= \left[d\mathbf{p}_{ind}^{1} \quad d\mathbf{p}_{ind}^{2} \quad \cdots \quad d\mathbf{p}_{ind}^{p}\right].$$

Finally, the dependent errors can be expressed through the following relation:

$$\left[d\mathbf{p}_{dep}\right] = \left[\mathbf{J}_{dep}\right]^{-1}\cdot\left[\mathbf{J}_{ind}\right]\cdot\left[d\mathbf{p}_{ind}\right]=\left[\mathbf{J}_{dep}^{*}\right]\cdot\left[d\mathbf{p}_{ind}\right]. \tag{5}$$

The central problem of this modelling is to reverse the matrix $\mathbf{J}_{dep}$. For kinematically determinate structures, matrix $\mathbf{J}_{dep}$ is always a square matrix of s×s dimension; s is equal to the number of dependent parameters and does not depend on the number of independent parameters considered in modelling. For the structures with a reduced complexity, the reversion can be obtained analytically; for the other cases, a numerical approach is recommended.

Step III: end-effector errors establishment

The end-effector errors can be expressed by introducing the dependent error expressions (rel. 5) into (rel. 1), particularised for each arm. Considering the chain i, the end-effector errors become:

$$^{i}\begin{bmatrix} d_x \\ d_y \\ d_z \\ \delta_x \\ \delta_y \\ \delta_z \end{bmatrix}=\left[\mathbf{J}_{ind}^{i}\right]\left[d\mathbf{p}_{ind}^{i}\right]+\left[\mathbf{J}_{dep}^{i}\right]\left[\mathbf{J}_{dep}^{i*}\right]\left[d\mathbf{p}_{ind}\right]=\left[\mathbf{J}\right]\left[d\mathbf{p}_{ind}\right]. \tag{6}$$

Introducing the notation

$$\left[\mathbf{J}_{dep}^{i}\right]\left[\mathbf{J}_{dep}^{i*}\right] = \left[\mathbf{J}^{*}\right] = \left[\mathbf{J}_{1}^{*}\ \mathbf{J}_{2}^{*}\ \cdots\ \mathbf{J}_{i}^{*}\ \cdots\ \mathbf{J}_{p}^{*}\right], \tag{7}$$

the error-Jacobean **J** of the parallel structure can be described as:

$$[\mathbf{J}] = \left[\mathbf{J}_1^* \;\; \mathbf{J}_2^* \;\; \cdots \;\; \mathbf{J}_i^* + \mathbf{J}_{ind}^i \;\; \cdots \;\; \mathbf{J}_p^*\right]. \tag{8}$$

The Jacobean matrix **J** given by relation (8) describes the linear transformation of independent errors into operational errors associated to the end-effector. The complexity of the Jacobean **J** depends on the reference frame used for reducing the errors. Most frequently in practice, the Jacobean **J** is reduced in the final reference frame $\Re_m$ or in the fixed frame $\Re_f$ (Fig. 1).

3. The Stewart platform presentation

The Stewart DELTALAB platform (Fig. 2) is a parallel manipulator, composed by a moving platform connected to the base through 6 telescopic legs (of variable length). The links between the six legs and the two platforms are materialized by spherical joints.
The parallel structure geometry is completely defined by the coordinates of the points A_i şi B_i (the centres of spherical joints, Fig. 3), which can be established by means of parameters r_f = 270 mm; r_m = 195 mm; α = 4.25°; β = 5.885° (Fig. 2).
As a result, the analyzed Stewart platform can be geometrically defined as follows (Fig. 2):

The fixed platform:
— The fixed coordinate system attached to the fixed platform: $\Re_f(O_f x_f y_f z_f)$, is placed in the plate's centre (the centre of the circle of radius r_f).
— The centres of the spherical joints, formed of the six cylinders (legs) with the fixed platform, are placed in points A_i, distributed on the circle of radius r_f.
— The points A_i are organized in equidistant groups formed of two appropriate adjacent points separated by the angle 2α.

The moving platform:
— The mobile coordinate system: $\Re_m(O_m x_m y_m z_m)$, placed in the plate's centre (the centre of the circle of radius r_m).
— The centre points B_i of the spherical joints, distributed on a circle of radius r_m.
— The points B_i are also organized in equidistant groups with the centre angle 2β.
The platform initial position (at minimum high) is characterized through the position of point $O_{m'}$ in $\Re_f$ (see Fig. 2), the moving platform being parallel to the base. The distance between $O_{m'}$ and O_f in this position is defined by the parameter $h = O_f O_m$ = 326.679 mm, for which the minimum length of the cylinders (active joints) is $L_i = A_i B_i$ = 387 mm.
The spatial guidance of the mobile platform related to the fixed one, is described by a set of 6 parameters:

- 3 position (displacement) parameters, given by the coordinates of the point O_m in relation to the reference frame $\Re_{m'}$ attached to the moving platform, expressed in the fixed reference frame $\Re_f$,

$$\overrightarrow{O_{m'}O_m} = x_m \vec{i}_f + y_m \vec{j}_f + z_m \vec{k}_f \,. \tag{9}$$

- 3 orienting parameters: the angles θ_1, θ_2 şi θ_3, which characterize the reference frame $\Re_m$ orienting in relation to the reference frame $\Re_{m'}$; the associated rotational matrix is described by the following relation:

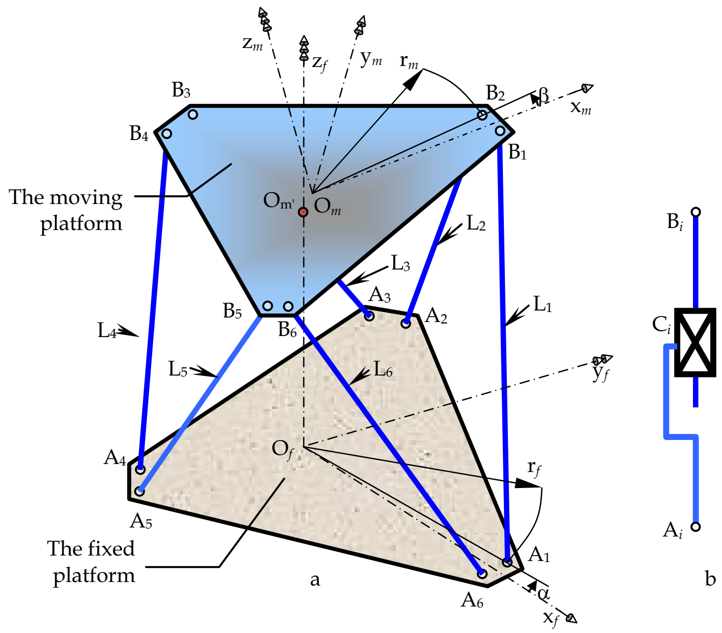


Fig. 2. The structure of the Stewart–DELTALAB platform (a) and of the leg *i* (b)

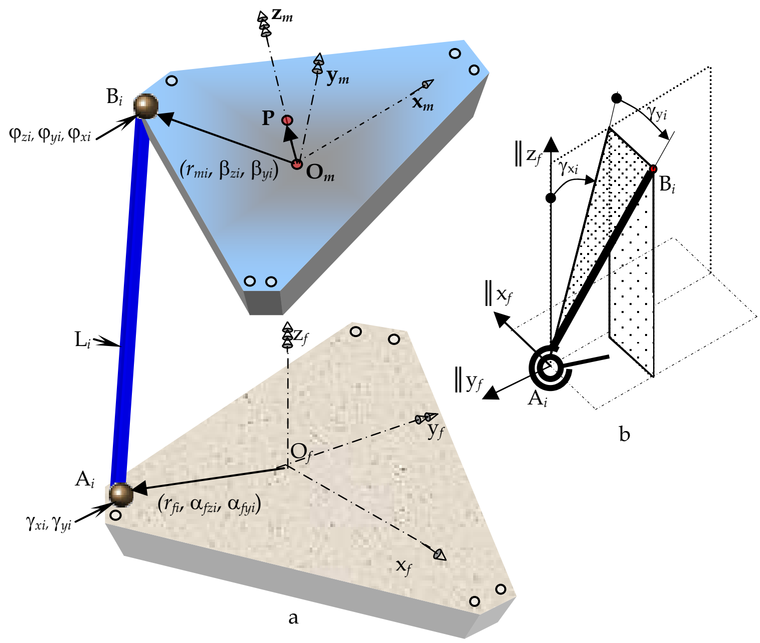


Fig. 3. Decomposition of the parallel structure in open chains and their parameterization

$$R_{m'm} = R_z(\theta_1)\cdot R_x(\theta_2)\cdot R_y(\theta_3) = \begin{bmatrix} c\theta_1 c\theta_3 - s\theta_1 s\theta_2 s\theta_3 & -s\theta_1 c\theta_2 & c\theta_1 s\theta_3 + s\theta_1 s\theta_2 c\theta_3 \\ s\theta_1 c\theta_3 + c\theta_1 s\theta_2 s\theta_3 & c\theta_1 c\theta_2 & s\theta_1 s\theta_3 - c\theta_1 s\theta_2 c\theta_3 \\ -c\theta_2 s\theta_3 & s\theta_2 & c\theta_2 c\theta_3 \end{bmatrix}, \quad (10)$$

where $c\theta_i = \cos(\theta_i)$ and $s\theta_i = \sin(\theta_i)$.

The position of the points A_i in the fixed reference frame $\Re_f$ is defined through a set of spherical coordinates (α_{zfi}, α_{yfi}, r_{fi}), and of the points B_i through coordinates (β_{zmi}, β_{ymi}, r_{mi}) defined in $\Re_m$ (Fig. 3). The relative angular displacements from joints A_i are modelled by means of angles γ_{xi} and γ_{yi} (Fig. 3,b) and the displacements from the spherical joints B_i by means of angles (φ_{zi}, φ_{yi}, φ_{xi}).

4. Direct error modelling

The objective of the *direct error modelling* is to establish the operational errors of the end-effector related to the values of the source errors; in the paper both active joint variable errors and geometric errors are considered.

4.1 Joint variable errors modelling

In this section, the error sources considered in the accuracy modelling are nominated by the *relative displacements* from the active joints (the prismatic joints C_i, Fig. 2,b). In the infinitesimal errors hypothesis, the error model becomes a *linear model,* where an *error Jacobean* $\mathbf{J_L}$ describes the influence of the independent kinematical parameters (L_1,..., L_6) on the end-effector accuracy. This modelling is based on the following assumptions: a) the modelling of I order errors is used; b) the geometrical parameters of both moving and fixed platforms have no errors (ideal geometry); c) the *passive joints* (joints A_i and B_i) are ideal joints; d) the error model expresses the end-effector errors for the characteristic point P (Fig. 3,a); e) The errors are reduced in the end-effector reference frame $\Re_p$.

In these assumptions, the linear error model is expressed by the following relation:

$$[d\mathbf{X}]_p = [\mathbf{J}_\mathbf{L}]_p \cdot [d\mathbf{L}], \quad (11)$$

where $[d\mathbf{X}]_p = [d_x \;\; d_y \;\; d_z \;\; \delta_x \;\; \delta_y \;\; \delta_z]_p^T$ is the 6 dimensions vector of the operational errors of the end-effector, for point P, reduced in $\Re_p$, while $[d\mathbf{L}] = [dL_1 \;\; dL_2 \;\; dL_3 \;\; dL_4 \;\; dL_5 \;\; dL_6]^T$ is the vector of the active joint variable errors.

According to the error model (rel. 11), the *central objective* of this modelling is to establish the *error Jacobean* $\mathbf{J_L}$ (through a similar approach, the error Jacobean $\mathbf{J_L}$ can be expressed also in any other frame).

In order to describe the Jacobean $\mathbf{J_L}$, the former algorithm (section 2) is proposed further on.

Step 1. Description of the end-effector errors for the six open chains (OKC)

Due to the fact that the effector errors are described in the reference frame $\Re_p$, the direct kinematic modelling for finite displacements of OKC must be done with the homogenous operators of *D–F type* (*type K*) (Gogu, 1995; Gogu et al., 1997). The kinematic model will also include, in this step, the relative displacements from the passive joints A_i and B_i as *independent parameters.*

The operational errors of each OKC can be now described easily by applying the well-known relations for open chains (Gogu, 1995; Gogu et al., 1997):

$$ {}^{i}\begin{bmatrix} d_{xi} \\ d_{yi} \\ d_{zi} \\ \delta_{xi} \\ \delta_{yi} \\ \delta_{zi} \end{bmatrix}_P = [\mathbf{J}_i] \begin{bmatrix} \delta\gamma_{xi} \\ \delta\gamma_{yi} \\ dL_i \\ \delta\varphi_{zi} \\ \delta\varphi_{yi} \\ \delta\varphi_{xi} \end{bmatrix}, \ [\mathbf{J}_i] = \begin{bmatrix} J_{\gamma_{xi}} & J_{\gamma_{yi}} & J_{L_i} & J_{\varphi_{zi}} & J_{\varphi_{yi}} & J_{\varphi_{xi}} \end{bmatrix}, \ i = 1..6, \tag{12} $$

where the column vectors from matrix $\mathbf{J}_i$ describe the influence of the errors of the modelling parameters.

Step 2.Identification of dependent errors

By splitting the parallel structure into open chains, the passive joints A_i and B_i became fictively actuated and, so, the displacements errors from these joints became independent too. In the parallel structure, all these errors are *dependent* of the independent error sources L_i. That's why, to express the dependent errors related to the independent ones represents the objective of this step; this desideratum becomes possible by modelling the recovering of the parallel connections of the Stewart platform, for which the following condition is used: *the effector errors are the same for all the six OKC;* analytically, the condition is expressed through the following equalities:

$$ \begin{bmatrix} d_{xi} \\ d_{yi} \\ d_{zi} \\ \delta_{xi} \\ \delta_{yi} \\ \delta_{zi} \end{bmatrix}_P = [\mathbf{J}_1] \begin{bmatrix} \delta\gamma_{x1} \\ \delta\gamma_{y1} \\ dL_1 \\ \delta\varphi_{z1} \\ \delta\varphi_{y1} \\ \delta\varphi_{x1} \end{bmatrix} = \cdots = [\mathbf{J}_6] \begin{bmatrix} \delta\gamma_{x6} \\ \delta\gamma_{y6} \\ dL_6 \\ \delta\varphi_{z6} \\ \delta\varphi_{y6} \\ \delta\varphi_{x6} \end{bmatrix}, \tag{13} $$

which lead to 5 independent matrix equations. Applying separation of dependent terms from the independent ones, the following relation is obtained:

$$ \begin{bmatrix} J_{\gamma_{x1}} & J_{\gamma_{y1}} & J_{\varphi_{z1}} & J_{\varphi_{y1}} & J_{\varphi_{x1}} \end{bmatrix} \begin{bmatrix} \delta\gamma_{x1} \\ \delta\gamma_{y1} \\ \delta\varphi_{z1} \\ \delta\varphi_{y1} \\ \delta\varphi_{x1} \end{bmatrix} - \begin{bmatrix} J_{\gamma_{xk}} & J_{\gamma_{yk}} & J_{\varphi_{zk}} & J_{\varphi_{yk}} & J_{\varphi_{xk}} \end{bmatrix} \begin{bmatrix} \delta\gamma_{xk} \\ \delta\gamma_{yk} \\ \delta\varphi_{zk} \\ \delta\varphi_{yk} \\ \delta\varphi_{xk} \end{bmatrix} = J_{L_k} dL_k - J_{L_1} dL_1 , \tag{14} $$

for k = 2..6.

The systems (14) are assembled into one matrix equation:

$$ [\mathbf{J}_\Phi^*] \cdot [\delta\Phi] = [\mathbf{J}_L^*] \cdot [d\mathbf{L}], \tag{15} $$

where $[\delta\Phi] = \begin{bmatrix} \delta\gamma_{x1} & \delta\gamma_{x2} & \delta\varphi_{z1} & \delta\varphi_{y1} & \delta\varphi_{x1} & \cdots & \delta\varphi_{y6} & \delta\varphi_{x6} \end{bmatrix}^T$ is the global vector of *dependent errors*, while

$$\left[\mathbf{J}_{\Phi}^{*}\right]=\begin{bmatrix} \mathbf{J}_{\Phi 1} & -\mathbf{J}_{\Phi 2} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{J}_{\Phi 1} & \mathbf{0} & -\mathbf{J}_{\Phi 3} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{J}_{\Phi 1} & \mathbf{0} & \mathbf{0} & -\mathbf{J}_{\Phi 4} & \mathbf{0} & \mathbf{0} \\ \mathbf{J}_{\Phi 1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{J}_{\Phi 5} & \mathbf{0} \\ \mathbf{J}_{\Phi 1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{J}_{\Phi 6} \end{bmatrix}, \quad \left[\mathbf{J}_{L}^{*}\right]=\begin{bmatrix} -J_{L1} & J_{L2} & 0 & 0 & 0 & 0 \\ -J_{L1} & 0 & J_{L3} & 0 & 0 & 0 \\ -J_{L1} & 0 & 0 & J_{L4} & 0 & 0 \\ -J_{L1} & 0 & 0 & 0 & J_{L5} & 0 \\ -J_{L1} & 0 & 0 & 0 & 0 & J_{L6} \end{bmatrix}, \quad (16)$$

$$\mathbf{J}_{\Phi k}=\left[J_{\gamma_{xk}} \quad J_{\gamma_{yk}} \quad J_{\varphi_{zk}} \quad J_{\varphi_{yk}} \quad J_{\varphi_{xk}}\right], \; k = 1..6 \text{ and } \left[d\mathbf{L}\right]=\left[dL_1 \quad dL_2 \quad \cdots \quad dL_6\right]^T .$$

Finally, the dependent errors can be expressed with relation (17):

$$[\delta\Phi]=\left[\mathbf{J}_{\Phi}^{*}\right]^{-1}\left[\mathbf{J}_{L}^{*}\right]\cdot[d\mathbf{L}]=\left[\mathbf{J}^{*}\right]\cdot[d\mathbf{L}]. \quad (17)$$

The main problem for expressing analytically the error model consists in reversing the square matrix $\left[\mathbf{J}_{\Phi}^{*}\right]$ of 30×30 dimension. In the case of Stewart platform, the reverse matrix $\left[\mathbf{J}_{\Phi}^{*}\right]$ was obtained numerically.

Step 3.Establishment of the and-effector errors

For each open kinematic chain, a set of 5 dependent parameters was used in modelling; in matrix $\mathbf{J}^*$ (relations 17), for each set correspond 5 lines: the first 5 for leg 1, the following 5 for leg 2 a.s.o.

The operational errors of the end-effector can be expressed by replacing the dependent errors expressions (rel. 17) into (rel. 12), with particularization for one of legs. Considering the chain 1, the end-effector errors expressed in the frame $\Re_p$ become:

$$\begin{bmatrix} d_x \\ d_y \\ d_z \\ \delta_x \\ \delta_y \\ \delta_z \end{bmatrix}_P=\left[\mathbf{J}_1\right]\begin{bmatrix} \delta\gamma_{x1} \\ \delta\gamma_{y1} \\ dL_1 \\ \delta\varphi_{z1} \\ \delta\varphi_{y1} \\ \delta\varphi_{x1} \end{bmatrix}=\left[\mathbf{J}_1\right]\left[\mathbf{J}_{\Phi 1}^{*}\right]\begin{bmatrix} dL_1 \\ dL_2 \\ dL_3 \\ dL_4 \\ dL_5 \\ dL_6 \end{bmatrix}, \quad \left[\mathbf{J}_{\Phi 1}^{*}\right]=\begin{bmatrix} J_{\gamma_{x1}}^{L_1} & J_{\gamma_{x1}}^{L_2} & J_{\gamma_{x1}}^{L_3} & J_{\gamma_{x1}}^{L_4} & J_{\gamma_{x1}}^{L_5} & J_{\gamma_{x1}}^{L_6} \\ J_{\gamma_{y1}}^{L_1} & J_{\gamma_{y1}}^{L_2} & J_{\gamma_{y1}}^{L_3} & J_{\gamma_{y1}}^{L_4} & J_{\gamma_{y1}}^{L_5} & J_{\gamma_{y1}}^{L_6} \\ 1 & 0 & 0 & 0 & 0 & 0 \\ J_{\varphi_{z1}}^{L_1} & J_{\varphi_{z1}}^{L_2} & J_{\varphi_{z1}}^{L_3} & J_{\varphi_{z1}}^{L_4} & J_{\varphi_{z1}}^{L_5} & J_{\varphi_{z1}}^{L_6} \\ J_{\varphi_{y1}}^{L_1} & J_{\varphi_{y1}}^{L_2} & J_{\varphi_{y1}}^{L_3} & J_{\varphi_{y1}}^{L_4} & J_{\varphi_{y1}}^{L_5} & J_{\varphi_{y1}}^{L_6} \\ J_{\varphi_{x1}}^{L_1} & J_{\varphi_{x1}}^{L_2} & J_{\varphi_{x1}}^{L_3} & J_{\varphi_{x1}}^{L_4} & J_{\varphi_{x1}}^{L_5} & J_{\varphi_{x1}}^{L_6} \end{bmatrix}, \quad (18)$$

where $J_{t_1}^{L_i}$ is an element of matrix $\mathbf{J}^*$ (relations 17) and represent the influence factor of deviation dL_i on the dependent parameter t_1.

By generalizing the relations 18, the error Jacobean for deviations of active joint variables can be expressed through any of the following relations:

$$\left[\mathbf{J}_{\mathrm{L}}\right]=\left[\mathbf{J}_1\right]\left[\mathbf{J}_{\Phi 1}^{*}\right]=\left[\mathbf{J}_2\right]\left[\mathbf{J}_{\Phi 2}^{*}\right]=\cdots=\left[\mathbf{J}_6\right]\left[\mathbf{J}_{\Phi 6}^{*}\right]. \quad (19)$$

4.2 Geometrical parameter errors modelling

The influence of deviations of the parameters which define the parallel structure geometry on the end-effector errors is described through a linear model, where the *error Jacobean* $\mathbf{J}_{\mathrm{G}}$ represents the system matrix:

$$[d\mathbf{X}]_p = [\mathbf{J}_G]_p \cdot [d\mathbf{G}], \tag{20}$$

where $[d\mathbf{X}]_p = \lfloor d_x \quad d_y \quad d_z \quad \delta_x \quad \delta_y \quad \delta_z \rfloor_p^T$ is the 6 dimensions vector of the operational errors of the end-effector, corresponding to the characteristic point P (Fig. 3), reduced in $\Re_p$;

$[d\mathbf{G}] = [\delta\alpha_{z1} \quad \delta\alpha_{y_1} \quad dr_{f1} \quad dr_{m1} \quad \delta\beta_{y_1} \quad \delta\beta_{z1} \cdots \delta\alpha_{z6} \quad \delta\alpha_{y_6} \quad dr_{f6} \quad dr_{m6} \quad \delta\beta_{y_6} \quad \delta\beta_{z6}]^T$ is the vector of geometric errors.

This modelling is based on the following assumptions:

- Platform's command is considered to be ideal and so the joint errors $d\mathbf{L}$ are null.
- Structure geometry is affected by known and constant in time errors.

In this case, the parallel structure is fictitious splitted into 6 open kinematic chains (OKC), one for each leg: $O_f A_i B_i O_m P$ (see Fig. 3). Further on, the following algorithm, consisting of three main steps, is applied.

Step 1. Description of the end-effector errors for the 6 OKC

In order to model the influence of the geometrical deviations, in the proposed method is used an *equivalent structure*, in which for each geometrical modelling parameter is associated one degree of freedom fictitious joint (prismatic or revolute). Each OKC associated to the Stewart platform becomes a serial structure with 12 one degree of freedom joints (therefore 12 independent parameters – see. Fig. 2): **RRTRRTRRRTRR**. Because the end-effector errors will be expressed in the final frame $\Re_p$, the direct kinematical modelling for finite displacements of OKC will be done with the following homogenous operators of *D–F type* (*type K*) (Gogu, 1995; Gogu et al., 1997):

$A_{01} = \mathbf{R_z}(\alpha_{zi})$; $A_{12} = \mathbf{R_y}(\alpha_{yi})$; $A_{23} = \mathbf{T_x}(r_{fi})$; $A_{34} = \mathbf{R_x}(\gamma_{xi})$; $A_{45} = \mathbf{R_y}(\gamma_{yi}) \cdot \mathbf{T_z}(L_i)$; $A_{56} = \mathbf{R_z}(\varphi_{zi})$; $A_{67} = \mathbf{R_y}(\varphi_{yi})$; $A_{78} = \mathbf{R_x}(\varphi_{xi})$; $A_{89} = \mathbf{T_x}(-r_{mi})$; $A_{9\_10} = \mathbf{R_y}(-\beta_{yi})$; $A_{10\_11} = \mathbf{R_z}(-\beta_{zi}) \cdot \mathbf{T_z}(h_{mp})$.

The operational errors for each OKC are described using the general relations for open chains:

$$\begin{aligned} &[d_{xi} \quad d_{y_i} \quad d_{zi} \quad \delta_{xi} \quad \delta_{y_i} \quad \delta_{zi}]_P^T = [\mathbf{J}_i][\delta\alpha_{zi} \quad \delta\alpha_{yi} \quad dr_{fi} \quad \delta\gamma_{xi} \quad \delta\gamma_{yi} \quad \delta\varphi_{zi} \quad \delta\varphi_{y_i} \quad \delta\varphi_{xi} \quad dr_{mi} \quad \delta\beta_{yi} \quad \delta\beta_{zi}]^T \\ &= [\mathbf{J}_i][d\mathbf{G}_i], \quad [\mathbf{J}_i] = [J_{\alpha_{zi}} \quad J_{\alpha_{yi}} \quad J_{r_{fi}} \quad J_{\gamma_{xi}} \quad J_{\gamma_{yi}} \quad J_{\varphi_{zi}} \quad J_{\varphi_{y_i}} \quad J_{\varphi_{xi}} \quad J_{r_{mi}} \quad J_{\beta_{yi}} \quad J_{\beta_{zi}}]^T, \qquad i = 1..6, \end{aligned} \tag{21}$$

where the column vectors from matrix $\mathbf{J}_i$ describe the influence of the modelling parameters errors, used in kinematical description of OKC.

Step 2. Identification of dependent errors

Even if there are considered independent in the equivalent structure, the relative displacements from the joints which are not commanded A_i and B_i are dependent displacements and, thus, the displacements errors are also dependent; there expressions can be identified by remodelling the parallel structure through the following condition: *the effector errors are identical for all the 6 OKC*:

$$[d_{xi} \quad d_{y_i} \quad d_{zi} \quad \delta_{xi} \quad \delta_{y_i} \quad \delta_{zi}]_P^T = [\mathbf{J}_1][d\mathbf{G}_1] = \cdots = [\mathbf{J}_6][d\mathbf{G}_6]. \tag{22}$$

Five independent matrix equations are derived from (rel. 22); separating the dependent terms from the independent ones, it results:

$$\begin{bmatrix} J_{\gamma_{x1}} & J_{\gamma_{y1}} & J_{\varphi_{z1}} & J_{\varphi_{y1}} & J_{\varphi_{x1}} \end{bmatrix} \begin{bmatrix} \delta\gamma_{x1} \\ \delta\gamma_{y1} \\ \delta\varphi_{z1} \\ \delta\varphi_{y1} \\ \delta\varphi_{x1} \end{bmatrix} - \begin{bmatrix} J_{\gamma_{xk}} & J_{\gamma_{yk}} & J_{\varphi_{zk}} & J_{\varphi_{yk}} & J_{\varphi_{xk}} \end{bmatrix} \begin{bmatrix} \delta\gamma_{xk} \\ \delta\gamma_{yk} \\ \delta\varphi_{zk} \\ \delta\varphi_{yk} \\ \delta\varphi_{xk} \end{bmatrix} =$$

$$= \begin{bmatrix} J_{\alpha_{zk}} & J_{\alpha_{yk}} & J_{r_{fk}} & J_{r_{mk}} & J_{\beta_{yk}} & J_{\beta_{zk}} \end{bmatrix} \begin{bmatrix} \delta\alpha_{zk} \\ \delta\alpha_{yk} \\ dr_{fk} \\ dr_{mk} \\ \delta\beta_{yk} \\ \delta\beta_{zk} \end{bmatrix} - \begin{bmatrix} J_{\alpha_{z1}} & J_{\alpha_{y1}} & J_{r_{f1}} & J_{r_{m1}} & J_{\beta_{y1}} & J_{\beta_{z1}} \end{bmatrix} \begin{bmatrix} \delta\alpha_{z1} \\ \delta\alpha_{y1} \\ dr_{f1} \\ dr_{m1} \\ \delta\beta_{y1} \\ \delta\beta_{z1} \end{bmatrix}, \quad k = 2..6. \tag{23}$$

The five systems (23) are assembled into one matrix equation:

$$\left[\mathbf{J}_{\Phi}^{*}\right] \cdot [\delta\Phi] = \left[\mathbf{J}_{G}^{*}\right] \cdot [d\mathbf{G}], \tag{24}$$

where $[\delta\Phi] = \begin{bmatrix} \delta\gamma_{x1} & \delta\gamma_{x2} & \delta\varphi_{z1} & \delta\varphi_{y1} & \delta\varphi_{x1} & \cdots & \delta\varphi_{y6} & \delta\varphi_{x6} \end{bmatrix}^{T}$ is the global vector of dependent errors and

$$\left[\mathbf{J}_{\Phi}^{*}\right] = \begin{bmatrix} \mathbf{J}_{\Phi1} & -\mathbf{J}_{\Phi2} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{J}_{\Phi1} & \mathbf{0} & -\mathbf{J}_{\Phi3} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{J}_{\Phi1} & \mathbf{0} & \mathbf{0} & -\mathbf{J}_{\Phi4} & \mathbf{0} & \mathbf{0} \\ \mathbf{J}_{\Phi1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{J}_{\Phi5} & \mathbf{0} \\ \mathbf{J}_{\Phi1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{J}_{\Phi6} \end{bmatrix}, \quad \left[\mathbf{J}_{G}^{*}\right] = \begin{bmatrix} -\mathbf{J}_{G1} & \mathbf{J}_{G2} & 0 & 0 & 0 & 0 \\ -\mathbf{J}_{G1} & 0 & \mathbf{J}_{G3} & 0 & 0 & 0 \\ -\mathbf{J}_{G1} & 0 & 0 & \mathbf{J}_{G4} & 0 & 0 \\ -\mathbf{J}_{G1} & 0 & 0 & 0 & \mathbf{J}_{G5} & 0 \\ -\mathbf{J}_{G1} & 0 & 0 & 0 & 0 & \mathbf{J}_{G6} \end{bmatrix}, \tag{25}$$

$$\mathbf{J}_{\Phi k} = \begin{bmatrix} J_{\gamma_{xk}} & J_{\gamma_{yk}} & J_{\varphi_{zk}} & J_{\varphi_{yk}} & J_{\varphi_{xk}} \end{bmatrix}, \quad \mathbf{J}_{Gk} = \begin{bmatrix} J_{\alpha_{zk}} & J_{\alpha_{yk}} & J_{r_{fk}} & J_{r_{mk}} & J_{\beta_{yk}} & J_{\beta_{zk}} \end{bmatrix}, \; k = 1..6. \tag{26}$$

Finally, the dependent errors expressions can be expressed by relation:

$$[\delta\Phi] = \left[\mathbf{J}_{\Phi}^{*}\right]^{-1} \left[\mathbf{J}_{G}^{*}\right] \cdot [d\mathbf{G}] = \left[\mathbf{J}^{*}\right] \cdot [d\mathbf{G}], \quad \left[\mathbf{J}^{*}\right] = \begin{bmatrix} \mathbf{J}_{1}^{*} \\ \vdots \\ \mathbf{J}_{6}^{*} \end{bmatrix}. \tag{27}$$

The matrix $\left[\mathbf{J}_{\Phi}^{*}\right]$ is a square matrix, of 30×30 dimension, and, therefore, reversible.

Step 3. Establishment of effector errors

Matrix $\mathbf{J}^{*}$ (relations 27) has the dimension 30×36 and it can be split into 6 submatrix $\mathbf{J}_{i}^{*}$ of 5 lines, representing the error Jacobean of the dependent deviations from leg *i* related to deviations $d\mathbf{G}$. Particularized for one of the chains, for instance OKC 1, the end-effector errors expressed in $\Re_p$ become:

$$\begin{bmatrix} d_{xi} & d_{y_i} & d_{zi} & \delta_{xi} & \delta_{y_i} & \delta_{zi} \end{bmatrix}_p^T = [\mathbf{J}_1]\begin{bmatrix} \delta\alpha_{z1} & \delta\alpha_{y1} & dr_{f1} & \delta\gamma_{x1} & \delta\gamma_{y1} & \delta\varphi_{z1} & \delta\varphi_{y_1} & \delta\varphi_{x1} & dr_{m1} & \delta\beta_{y1} & \delta\beta_{z1} \end{bmatrix}^T =$$
$$= [\mathbf{J}_1]\,[\mathbf{J}^*_{\Phi 1}]\,[d\mathbf{G}], \tag{28}$$

$$[\mathbf{J}^*_{\Phi 1}] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ J^{\alpha_{z1}}_{\gamma_{x1}} & J^{\alpha_{y1}}_{\gamma_{x1}} & J^{r_{f1}}_{\gamma_{x1}} & J^{r_{m1}}_{\gamma_{x1}} & J^{\beta_{y1}}_{\gamma_{x1}} & J^{\beta_{z1}}_{\gamma_{x1}} & \cdots & J^{\beta_{y6}}_{\gamma_{x1}} & J^{\beta_{z6}}_{\gamma_{x1}} \\ J^{\alpha_{z1}}_{\gamma_{y1}} & J^{\alpha_{y1}}_{\gamma_{y1}} & J^{r_{f1}}_{\gamma_{y1}} & J^{r_{m1}}_{\gamma_{y1}} & J^{\beta_{y1}}_{\gamma_{y1}} & J^{\beta_{z1}}_{\gamma_{y1}} & \cdots & J^{\beta_{y6}}_{\gamma_{y1}} & J^{\beta_{z6}}_{\gamma_{y1}} \\ J^{\alpha_{z1}}_{\varphi_{z1}} & J^{\alpha_{y1}}_{\varphi_{z1}} & J^{r_{f1}}_{\varphi_{z1}} & J^{r_{m1}}_{\varphi_{z1}} & J^{\beta_{y1}}_{\varphi_{z1}} & J^{\beta_{z1}}_{\varphi_{z1}} & \cdots & J^{\beta_{y6}}_{\varphi_{z1}} & J^{\beta_{z6}}_{\varphi_{z1}} \\ J^{\alpha_{z1}}_{\varphi_{y1}} & J^{\alpha_{y1}}_{\varphi_{y1}} & J^{r_{f1}}_{\varphi_{y1}} & J^{r_{m1}}_{\varphi_{y1}} & J^{\beta_{y1}}_{\varphi_{y1}} & J^{\beta_{z1}}_{\varphi_{y1}} & \cdots & J^{\beta_{y6}}_{\varphi_{y1}} & J^{\beta_{z6}}_{\varphi_{y1}} \\ J^{\alpha_{z1}}_{\varphi_{x1}} & J^{\alpha_{y1}}_{\varphi_{x1}} & J^{r_{f1}}_{\varphi_{x1}} & J^{r_{m1}}_{\varphi_{x1}} & J^{\beta_{y1}}_{\varphi_{x1}} & J^{\beta_{z1}}_{\varphi_{x1}} & \cdots & J^{\beta_{y6}}_{\varphi_{x1}} & J^{\beta_{z6}}_{\varphi_{x1}} \\ 0 & 0 & 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & \cdots & 0 & 0 \end{bmatrix}, \tag{29}$$

where $J^{p_i}_{t_1}$ represents the influence factor of deviation dp_i on the dependent parameter t_1 which is a component part of matrix $\mathbf{J}^*$ (rel. 27).
Generalizing, the error Jacobean for the deviations of joint variables can be expressed through any of the following relations:

$$[\mathbf{J}_\mathbf{G}] = [\mathbf{J}_1][\mathbf{J}^*_{\Phi 1}] = [\mathbf{J}_2][\mathbf{J}^*_{\Phi 2}] = \cdots = [\mathbf{J}_6][\mathbf{J}^*_{\Phi 6}] \,. \tag{30}$$

4.3 The error model for geometrical and kinematical parameters deviations

The complete kinematical error model, which includes both the influences of the active joint variables L_i and of the geometrical parameters can be deduced through a similar approach; the only changing in the modelling prerequisites is referring to the fact that the displacements deviations from the commanded joints have to be included between the error sources of Stewart platform. Therefore, the number of error source parameters is changing from 36 to 42.
In the first step of the modelling algorithm is included also deviation dL_i generated when the relative displacements from the actuated joints C_i are commanded. In step 2 the 30 dependent deviations are identified in relations with the 42 independent variables. In this case is also necessary to reverse the matrix of 30×30 dimension.
Finally, the end-effector errors expressed in reference frame $\Re_p$ can be deduced by replacing the expressions of the dependent errors in the error model of one of the OKC.
In conclusion, the complete error model can be obtained by assembling the previous partial models:

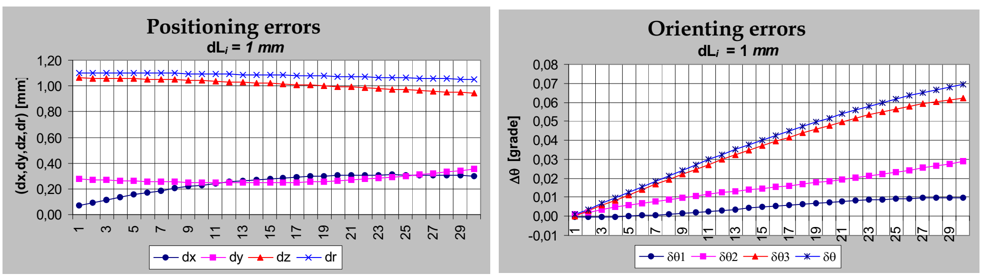
$$\begin{bmatrix} d_{xi} & d_{y_i} & d_{zi} & \delta_{xi} & \delta_{y_i} & \delta_{zi} \end{bmatrix}_p^T = [\mathbf{J}_{1G}][\mathbf{J}^*_{\Phi 1G}]\,[d\mathbf{G}] + [\mathbf{J}_{1L}][\mathbf{J}^*_{\Phi 1L}]\,[d\mathbf{L}] = [\mathbf{J}_\mathbf{G}][d\mathbf{G}] + [\mathbf{J}_\mathbf{L}][d\mathbf{L}]. \tag{31}$$

5. Accuracy numerical simulations

The numerical simulation of the error model for Stewart-DELTALAB platform had the following objectives:

1. *Validation of the error model* by verifying the results obtained in the numerical and graphical simulation. Thus, for a set of representative configurations of Stewart platform were applied the known deviations of source-parameters and the end-effector errors were calculated (rel. 18). The configurations tested with error, were generated with a graphical tool (in this case AutoCAD) and were established the effective values of the modeling independent and dependent parameters, using specific functions. In all the tested variants, the simulation showed the correctness of the elaborated models.
2. *Identification of the end-effector errors* on a given trajectory, for specified values of the source errors. Using numerical simulation, the global effect of error sources and the importance of each error parameter on the positioning and orienting precision of end-effector can be calculated. In this way, can be identified the factors with a maximum influence and thus recommendations for constructive and functional design can be elaborated.

The results of numerical simulation of the precision model for the case of a linear trajectory, given through the start configuration (x_m, y_m, z_m, θ_1, θ_2, θ_3) = (-100mm, 100mm, 100mm, 0°,0°,0°) and –final (100mm, -100mm, 200mm, 45°, 30°, -30°), are presented in Figure 4. The represented end-effector errors were generated considering that all active joint errors ΔL_i = 1 mm. Thus, the positioning on axe *z* is achieved with the biggest deviations (Fig. 4,a), while the maximum angular deviations are registered on axis *y* (Fig. 4,b).



a b

Fig. 4. Numerical simulation of the joint error model: end-effector positioning errors (a) and orienting errors (b) obtained for joint errors ΔL_i = 1mm

The objective of the numerical simulation of the geometric error model was to identify the end-effector errors on a given trajectory, for the specified values of source errors (Fig. 5). Thus, it can be established both the global effect of all the error sources and the importance of each error parameter on the positioning and orienting precision of the end-effector; therefore, the factors with a maximum influence can be identified and the recommendations for constructive and functional design can be elaborated.

Considering the former trajectory, the end-effector errors were generated in the assumption that all the linear geometrical parameters have 1 mm deviations, while the angular ones have 1° deviation. The following conclusions and recommendations can be formulated analyzing the graphical representations of the end-effector errors (Fig. 5):

- Referring to the positioning precision it can be noticed the superior influence of the parameters α_{yi} (Fig. 5,a) and β_{yi} (Fig. 5,b) similar to the relatively reduced effects of parameters r_{fi} (Fig. 5,c) and r_{mi} (Fig. 5,d).
- The orienting precision depends in a small measure of the deviations of parameters r_{fi} (Fig. 5,e) and r_{mi} (Fig. 5,f). On the other hand, the orienting precision is more dependent on the deviations of parameters α_{zi} and β_{zi}.

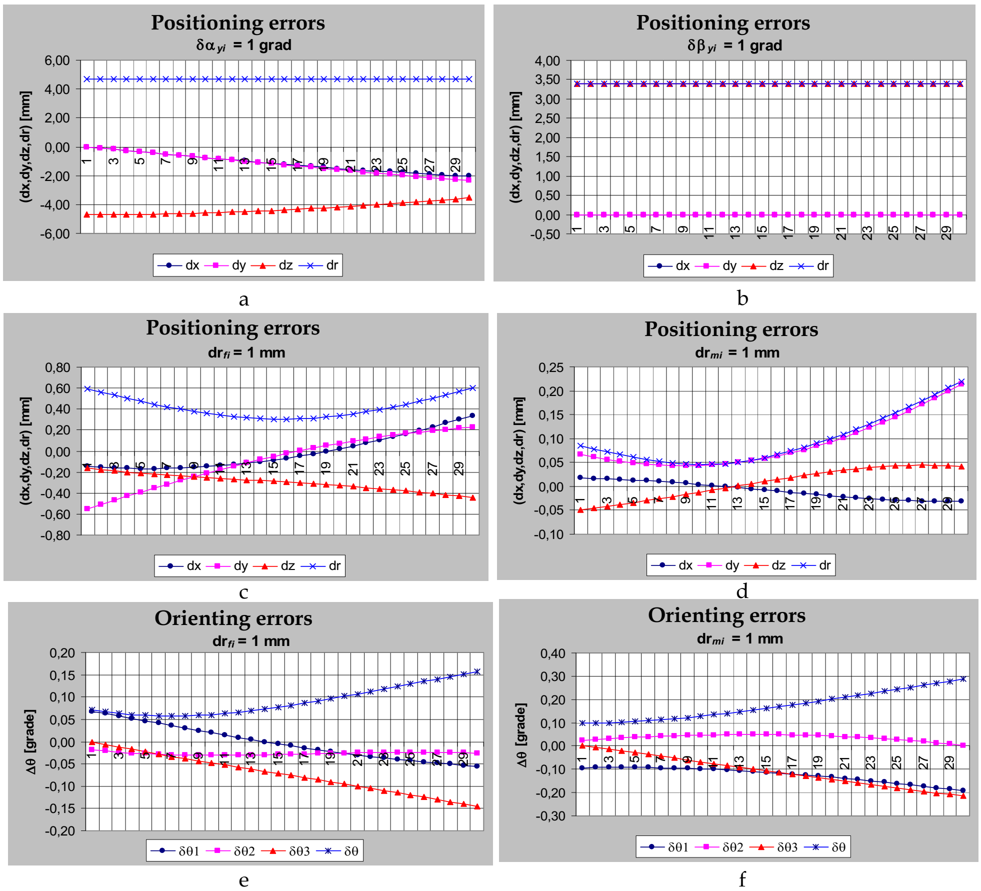


Fig. 5. The influence of some geometric parameter errors on the trajectory end-effector errors

- From the numerical study of the precision of Stewart platform on the mentioned trajectory, it can be formulated the recommendation that the maximum precision for the angular parameters (α_{yi}, β_{yi}, α_{zi}, β_{zi}) has to be assured.

6. Accuracy experimental validation

Concerning the Stewart DELTALAB platform error modelling, some explanatory notes and the necessary notations in the experimental testing are presented further on:

— two categories of reference frames are associated to the tested platform: *theoretical reference frames* (used in the platform command program) – associated to the theoretical plane given by centres of the spherical joints of mobile platform (points B_i), and *measure frames* (used in the measure process) – associated to a real surface of the mobile platform;

— the taken measurements were of *relative type*, in relation to a reference frame defined by the 3D measurement machine, on the base of the measure frame for the reference position of the platform;

- $\Re_{m\exp}\left(O_{m\exp}x_{m\exp}y_{m\exp}z_{m\exp}\right)$ — the *theoretic* frame associated to the mobile platform in an experimentally specified position;
- $\Re_{pref}\left(O_{pref}x_{pref}y_{pref}z_{pref}\right)$ — the *measurement* frame associated to the mobile platform in a reference position;
- $\Re_{p\exp}\left(O_{p\exp}x_{p\exp}y_{p\exp}z_{p\exp}\right)$ — the *measurement* frame associated to the mobile platform in an experimentally established position.

In order to become possible and to offer complete data on the accuracy, a first step in experimental testing is to identify the constructive elements of the platform and the accessories. In this context, the following explanatory notes are made:

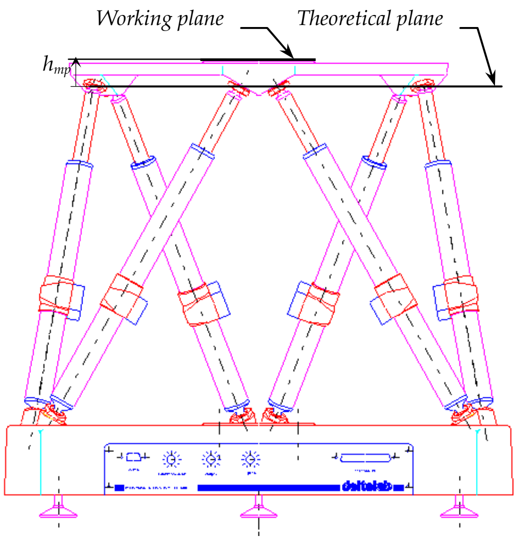


Fig. 6. Theoretical and working planes of Stewart platform

1°. Plane $x_mO_my_m$ of frame $\Re_m$ is identical to points B_i plane; being a fictive plane, in experimental research was used frame $\Re_p$ for which plane $x_pO_py_p$ is materialized by a plane finished surface (*working plane* – see Fig. 6). The frame $\Re_p$ is parallel to $\Re_m$ and is obtained through a translation with the distance $h_{mp} = O_mO_p = 40.55$ mm on the O_mz_m axis.

2°. The Stewart platform includes, as additional accessories, two cylindrical finished bolts, assembled on the working plate in point O_p and, respectively, in a point on the axis O_px_p (Fig. 3 and 4). In this way, frame $\Re_p$ is materialized as follows:

— axis O_pz_p through the normal to the working plane;
— the origin O_p as the intersection point of working plane and the axis of the bolt which is assembled in this point;
— axis O_px_p as the line described by 2 points: O_p and the intersection point of the second bolt axis and the working plane;
— axis O_py_p with, implicitly, the unit vector $\vec{j}_p = \vec{k}_p \times \vec{i}_p$.

6.1 The 3D measurement machine TEMPO

The *Tri-Mesures* machine (Fig. 7) is metrology equipment with high performances. From the structural point of view, the machine is assimilated to an orthogonal robot of portal type, with three independent axes. The final element, which performs a translation on vertical (z axle), has an orientation measurement head at it extremity, that has a sensing head system (Fig. 7).
The machine is characterized by a high rigidity, its mobile parts moving on an aerostatic cushion and a high geometrical precision.
The logistic administration of measurement machine functioning is obtained through the program METROSOFT 3D. The program allows to measure parts with plane, spherical, cylindrical or conical surfaces. The program is handling almost exclusively through a control desk, with specialized keys for different categories and command types.

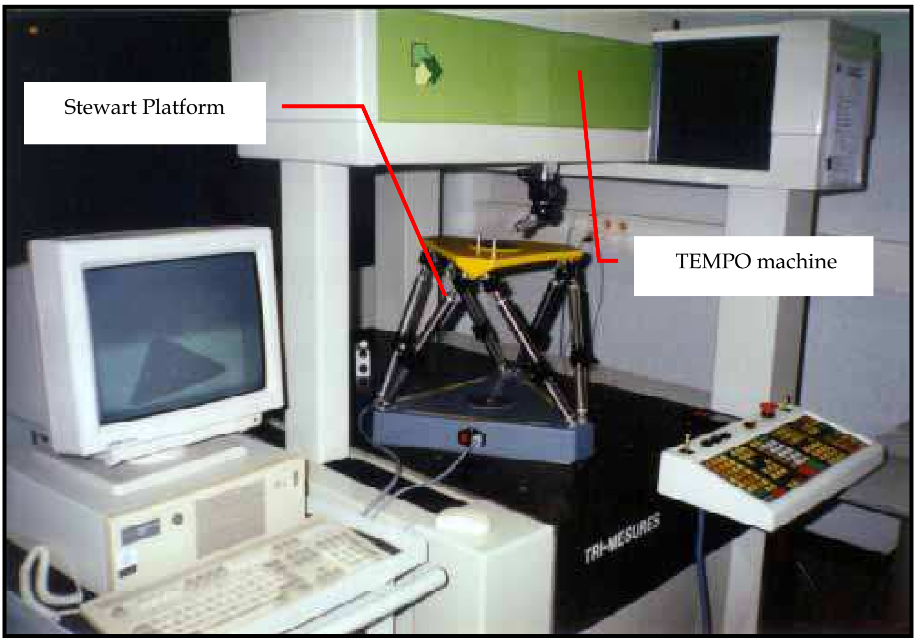


Fig. 7. The 3D Measurement TEMPO machine and the Stewart-DELTALAB platform

6.2 The experimental research program

The experimental research had in view to fulfil a program based on the following objectives:

1. Establishment of Stewart platform *repeatability*.
2. Establishment of absolute precision (*accuracy*).
3. The experimental testing for proposed precision models validation.

The experimental testing was preceded by identification of a minimum number of parameters (points and vectors) which have to be measured and which allow the analytical establishment of real position of mobile platform (of the reference frame $\Re_p$).

Starting from the experimental and command values, the actual operational errors of the Stewart platform are established, with their help being quantified the accuracy and repeatability.

Therefore, the main steps in the measurement process of Stewart platform accuracy are presented:

1. The Stewart platform is placed on the measurement machine working table and runs the platform command program.
2. The platform is put under tension and is commanded to move to the initial position (*zero position*).
3. After starting up the measurement machine, the necessary configuration of the measurement head is calibrated through filling a standard sphere and their memorization.
4. A *frame part* is defined, materializing the reference frame $\Re_{p'}$. This is achieved in 3 phases:
   a. Establishment of a *primary direction,* which gives one of the frame part axes.
   b. Establishment of a *secondary direction*, which materializes the second axle of the part frame.
   c. Specification of the origin of the frame part.
5. The platform is moved in the testing pose.
6. Are taken measurements in order to establish the frame $\Re_{pexp}$ position (Fig. 8):
   a. *Measurement of working plane*. The procedure imposes a *plane* command selection and filing of 4 points (4 is the implicit value, which can be modified and represents the minimum accepted number of points). There are supplied to the user the components of the unit vector normal to the plane.
   b. Fulfil of the two cylinders, placed in the two points P_1 and P_2, through *cylinder* command initialization and fulfil of minimum 9 points per cylinder. These cylinders were materialized through calibrated bolts with Ø8×25 dimensions.
   c. The two points P_1 and P_2 are obtained as intersections of the two cylinders with working plane. The coordinates of the two points are displayed in the part system declared active.
   d. Identification of Ox axle as a line which passes through points P_1 and P_2. It is used the *connex* command and it is given the axle by the unit vector.

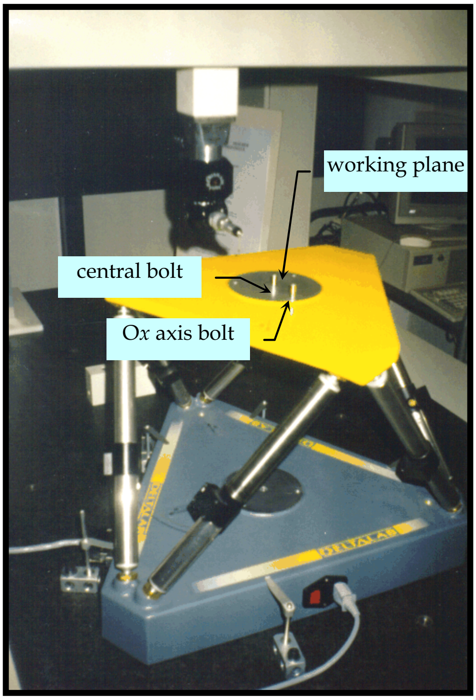


Fig. 8. Geometric elements used in the experimental research

Explanatory notes:

- The machine can report measured values and can make calculus both in the machine frame and in the frame defined by the user. The machine reference frame is defined with the axis parallel to the displacement directions of mobile elements (x axle – the longitudinal axle, y – transversal and z – vertical, see Fig. 7) and the origin in the standard sphere centre. A user frame is defined in accordance to point 4 and is associated to the geometrical form of a part; this will be named further on as *frame part*.
- At the intersection of a plane with a cylinder, the machine program obtains a point and not an ellipse (circle), considering the intersection between the plane and the cylinder axle.

  The algorithm for actual operational errors calculus, corresponding to the relative measurements, is based on scheme from Figure 9.

The final purpose of the mathematical processing of experimental values is to identify the 6 dimensions vector of the actual operational errors as a measure of the difference between the real position (experimentally established) and the commanded one (theoretical). So, first are established the expression of the homogenous operator $\mathbf{A}_{p\to pexp}$ for each measurement; in phase 2 are identified the real errors $\mathbf{A}_{p\to pexp}$, related to frame $\Re_p$.

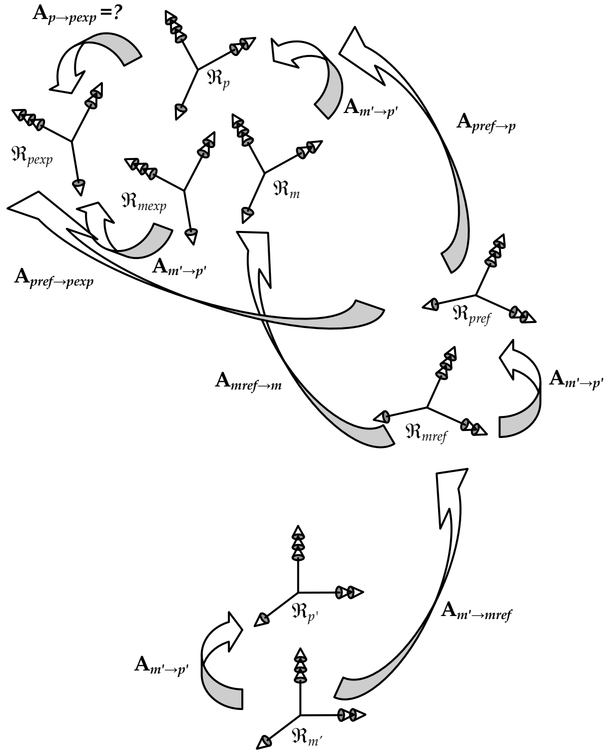


Fig. 9. Reference frames associates to the Stewart platform

The selection of frame $\Re_p$ as a reference frame (and not of $\Re_m$) is justified through the physical existence of working plane (as a measurement plane); this interferes as a location element in the Stewart platform tasks, unlike frame $\Re_m$, which is fictive.
In the relative measurements case, the measured values are reported to a part frame defined in a reference position $\Re_{pref}$.

– There are known (Fig. 9):

○ $\mathbf{A}_{m'\to p'}$, $\mathbf{A}_{m'\to m}$ şi $\mathbf{A}_{m'\to mref}$,

○ $\mathbf{A}_{p'\to pref} = (\mathbf{A}_{m'\to p'})^{-1} \cdot \mathbf{A}_{m'\to mref} \cdot \mathbf{A}_{m'\to p'}$;

○ $\mathbf{A}_{pref\to pexp}$, established by means of experimental values, related to $\Re_{pref}$;

– There is identified the homogenous operator $\mathbf{A}_{p\to pexp}$ from the following equality (see Fig. 9):

$$\mathbf{A}_{pref\to pexp} = \mathbf{A}_{pref\to p} \cdot \mathbf{A}_{p\to pexp} \Leftrightarrow \mathbf{A}_{p\to pexp} = (\mathbf{A}_{pref\to p})^{-1} \cdot \mathbf{A}_{pref\to pexp},$$

where $\mathbf{A}_{m'\to p'} \cdot \mathbf{A}_{pref\to p} = \mathbf{A}_{mref\to m} \cdot \mathbf{A}_{m'\to p'} \Rightarrow \mathbf{A}_{pref\to p} = (\mathbf{A}_{m'\to p'})^{-1} \cdot \mathbf{A}_{mref\to m} \cdot \mathbf{A}_{m'\to p'}$,

$$\mathbf{A}_{mref\to m} = (\mathbf{A}_{m'\to mref})^{-1} \cdot \mathbf{A}_{m'\to m}.$$

6.3 The experimental validation of the error models

The accuracy model represents the mathematical expression of the dependencies between the operational errors and the source-errors. From the tests on Stewart platform it was pursued the accuracy model validation, considering as source deviations the relative displacements from the actuated joints.

Validation of the precision model consists of:

— The platform is moved in the tested reference positions;

— A part frame is defined in $\Re_{pref}$ (the relative measurements case);

— The platform is moved in adjacent positions, by commanding displacements L_i bordered to those corresponding to $\Re_{pref}$;

— The real position of the platform is measured and identified;

— The theoretical and real deviations are established (with the precision model).

For the experimental validation of precision model were selected several representative test-configurations. For one of them, the expression of the error Jacobean, in accordance to (rel. 19), is:

$$J := \begin{bmatrix} -.08237 & .05989 & -.5176 & .6115 & .6305 & -.5668 \\ .7215 & -.7201 & -.3781 & .4234 & -.2856 & .2719 \\ .1949 & .1699 & .2631 & .1183 & .1485 & .2468 \\ -.0002606 & .0002159 & .002222 & .001055 & -.001426 & -.001923 \\ -.001962 & -.001920 & .0007583 & .001220 & .001377 & .0006450 \\ .001861 & -.001910 & .001614 & -.001863 & .001675 & -.001583 \end{bmatrix}.$$

As it can be seen in Figure 10, there is a good concordance between the values given by the precision model and the finite displacements, established on the theoretical model (by numerical simulation) and the experimental values.

The deviations from Figure 10 have the following meanings:

- the "*exact*" deviations were established with the direct kinematical model for finite displacements, in which were included the final values (corrected) of the kinematical variables $L_i = L_{iref} + \Delta L_i$. The "*exact*" deviations are theoretical deviations, defined by finite displacements from the reference-test configuration $\Re_{pref}$ to the commanded configuration with errors $\Re_p$;
- the *experimental deviations* express the difference between the measured configuration $\Re_{pexp}$ and the reference one $\Re_{pref}$;
- the *calculated deviations* are obtained applying the linear error model, in which the error Jacobean *J* corresponds to the test-configuration $\Re_{pref}$.

The differences between the results obtained through the proposed model and the "exact" model are explained by the linear nature of the precision model (the infinitesimal displacements are level 1 approximations of the finite displacements); these differences tend to zero only for the small values (infinitesimal) of input parameters (source errors) and, respectively, increase for values from the finite domain of inputs in the model.

7. Conclusion

— The proposed modelling method allows deriving the error model through a systemic and algorithmic approach and it is applied for parallel structures of any complexity.

— The analytical error model of Stewart–DELTALAB platform has a relatively high complexity, due to the fact that a matrix of 30×30 dimension has to be reversed; the problem was solved numerically.

— The Jacobean $\mathbf{J_L}$ can be numerically expressed. Numerical simulation was used for checking the correctness of the algorithm and of modeling. The error model was also verified through graphical simulation, using AutoCAD.
— The relations for the Jacobean $\mathbf{J_G}$ were used, through numerical simulation, to verify the correctness of the algorithm and of the modelling.
— The results, offered by the precision models are in a good concordance both with the values given by numerical simulation and, also, with the experimental values (Fig. 10).

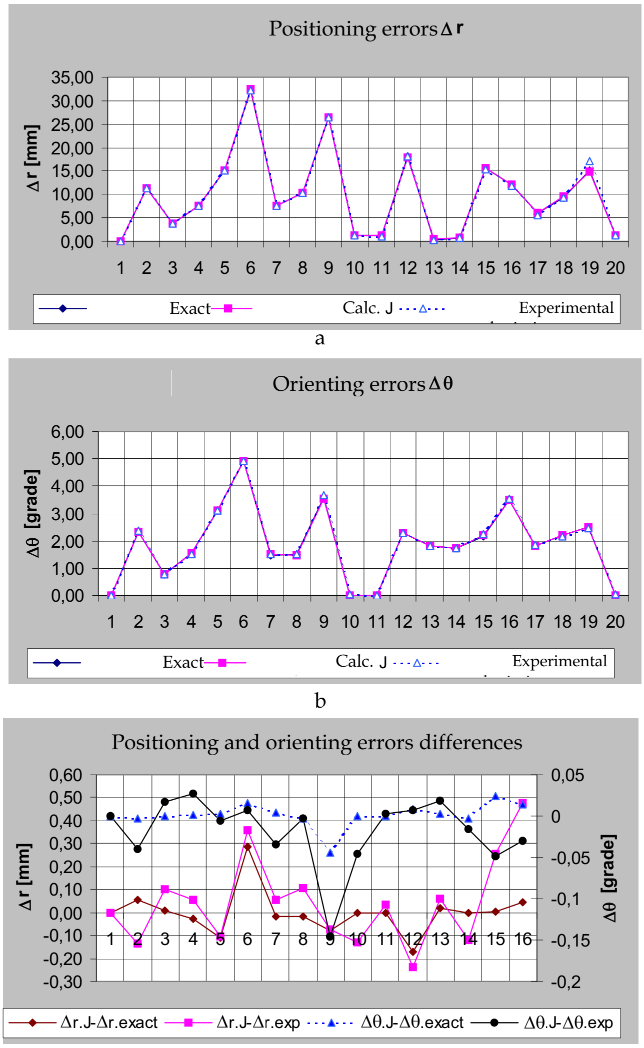


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Fig. 10. Experimental validation of theoretical models

— The differences between the results obtained with the proposed model and the experimental values are relatively small (of 10^{-1} mm, respectively of 10^{-1} degrees) and are enclosed in the error limit given by the accuracy error of tested Stewart platform.
— The experimental testing validates the precision models; the conclusion is that the proposed models are correct and the algorithm and the numerical implementation of the models are, also, correct.

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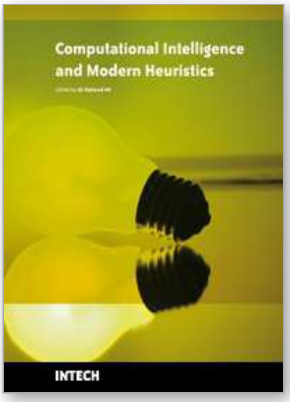

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