Estimation of Per Unit Length Parameters of Multiconductor Lines by the Method of Rectangular Subareas

Saswati Ghosh and Ajay Chakrabarty
Kalpana Chawla Space Technology Cell, Dept. of E & ECE,
Indian Institute of Technology, Kharagpur-721302
India

1. Introduction

The high operating frequency and small dimensions of modern electronic systems causes strong electromagnetic interaction within electronic circuits. The accurate evaluation of the per unit length capacitance and inductance of multiconducting lines and PCB lands is an important step in the design and packaging of these high frequency electronic circuits. Considerable work was already performed by other researchers on the development of different wideband microstrip interconnects and determination of capacitance of microstrip transmission lines (Ruehli and Bernnan, 1973), (Rao et al., 1979), (Ponnapalli et al., 1993). In the work of Ruehli and Brennan, the basic equations for the potential coefficients of rectangular conducting element were derived and used for the evaluation of capacitance of square plate, cube via Method of Moments (Ruehli and Bernnan, 1973). However, the resulting equations for the potential coefficients are found to be complicated and also these were mainly used for two / three dimensional bodies with square / rectangular surfaces. The Method of Moments analysis with triangular and square subsections for the evaluation of capacitance of arbitrary-shaped conducting surface is available in other literatures (Rao et al., 1979), (Harrington, 1985), (Ponnapalli et al., 1993). Harrington evaluated data on the capacitance of a square conducting plate employing square subdomain regions, but did not present clearly the exact formulas of the matrix elements for the evaluation of capacitance (Harrington, 1985). The triangular subdomains have been used for more complex surfaces by Rao et al (Rao et al., 1979). Also some interesting work on capacitance evaluation of square, cube etc. using method of subareas was developed in Matlab by Bai (Bai and Lonngren, 2002, 2004). However the authors had not noticed any work on the evaluation of per unit length parameters of multiconductor lines with a more generalized and simple elemental shape which can be used for any planar surface and can be extended for three dimensional lines. In the present work, the per unit length parameters of multiconductor lines are evaluated using Method of Moments and rectangular subdomain modeling. The rectangular subsection is chosen because of its ability to conform easily to any geometrical surface or shape and at the same time to maintain the simplicity of approach compared to the triangular patch modelling. The exact formulation for the evaluation of the impedance matrix for rectangular subdomain is determined. The Method of Moments with Pulse basis function and Point Matching is used to evaluate the charge distribution and hence the capacitance and inductance of multiconducting bodies. The capacitances of different conducting structures such as square plates, circular disc are compared with other available data in literature (Harrington, 1985), (Higgins and Reitan, 1957), (Nishiyama and M. Nakamura, 1992). Next the same method is extended for multiconducting bodies e.g. parallel rectangular plates, parallel circular discs and later for three dimensional structures e.g. circular coaxial conducting structures. The per unit length capacitance and inductance of circular coaxial structures is presented and compared with the analytical results.

2. Theory

We consider a perfectly conducting surface is charged to a potential V. The unknown surface charge density distribution σ (r′) can be determined by solving the following integral equation (Harrington, 1985)

$$V = \iint_S \frac{\sigma(r')}{4\pi\varepsilon|r - r'|} ds' \tag{1}$$

Here r and r' are the position vectors corresponding to observation and charge source points respectively, ds' is an element of surface S and ε is the permittivity of free space. The conducting bodies are approximated by planar rectangular subdomains (Figure 1). The Method of Moments with pulse basis function and point matching is then used to determine the approximate charge distribution (Harrington, 1985). On each subdomain, a pulse expansion function Pn(r) is chosen such that Pn(r) is equal to 1 when r is in the n-th rectangle and Pn(r) is equal to 0 when r is not in the n-th rectangle. With the above definition of expansion function, the charge density, $\sigma(r')$ may be approximated as follows

$$\sigma(r') = \sum_{n=1}^{N} \sigma_n P_n(r') \quad where \quad P_n = \begin{matrix} 1 & for\ n-th\ subsection \\ 0 & elsewhere \end{matrix} \tag{2}$$

Here N is the number of rectangles modelling the surface and σ_n's are the unknown weights (charge density).

Substitution of charge expansion (2) in (1) and point matching the resulting functional equation, yields an N x N system of linear equations which may be written in the following form

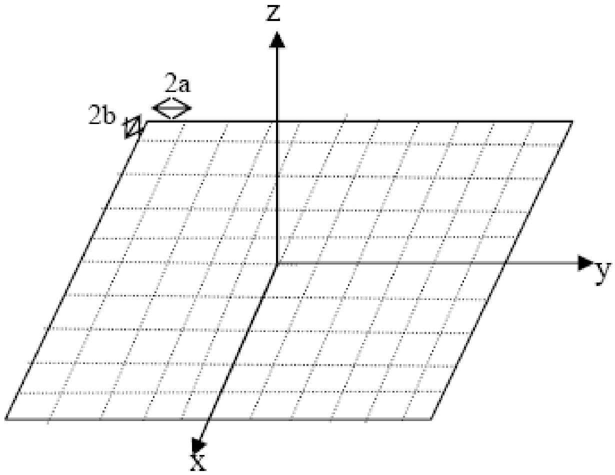

Fig. 1. Square plate divided into rectangular subsections.

$$[V] = [K][Q] \tag{3}$$

Here [K] is an N × N matrix and [Q] and [V] are column vectors of length N. The elements of [K], [Q] and [V] are given as follows

$$K_{mn} = \iint_{rectangle} \frac{1}{4\pi\varepsilon |r_m - r'|} dA' \tag{4}$$

$Q_n = \sigma_n$ =unknown charge density in subdomain n
Vn= V
r_m denotes the position vector of the center of the mth rectangle. A' is the area of the source rectangle.

$$|r_m - r'| = \sqrt{(x_m - x')^2 + (y_m - y')^2}$$

Here we have considered the conducting surface at z=0 plane.

Since the numerical formulation of (1) via the Method of Moments is well-known [4], we consider only the evaluation of the element of the moment matrix as given by equation (4). Each element of the matrix corresponds to the potential at some point in space, r = (x, y, z), due to a rectangular patch of surface charge of unit charge density. In general, the patch is arbitrarily positioned and oriented in space.

The integration of equation (4) is quite tedious, but the final result is relatively simple (Ghosh and Chakrabarty, 2006).

For the diagonal elements of the matrix, the integration is evaluated as follows

$$K_{nn} = \frac{1}{\pi\varepsilon}\left(a\,ln\left(\frac{b}{a} + \sqrt{\frac{b^2}{a^2} + 1}\right) + b\,ln\left(\frac{a}{b} + \sqrt{\frac{a^2}{b^2} + 1}\right)\right) \tag{5}$$

Here 2a and 2b are the sides of each rectangular subsection.
Using the standard integral formula the non-diagonal elements are evaluated as follows

$$K_{mn} = \frac{1}{4\pi\varepsilon}\left(-\left[|x_m - x'|\, ln \frac{(|y_m - y_n + b| + \sqrt{(x_m - x')^2 + (y_m - y_n + b)^2})}{(|y_m - y_n - b| + \sqrt{(x_m - x')^2 + (y_m - y_n - b)^2})} \right]_{x_n - a}^{x_n + a} - \left[|y_m - y'|\, ln \frac{(|x_m - x_n + a| + \sqrt{(y_m - y')^2 + (x_m - x_n + a)^2})}{(|x_m - x_n - a| + \sqrt{(y_m - y')^2 + (x_m - x_n - a)^2})} \right]_{y_n - b}^{y_n + b} \right) \tag{6}$$

Here the source point is (x_n , y_n) and the field point is (x_m , y_m). The x′ and y′ of equation (6) are replaced by their respective limits. Solution of the matrix equation (3) yields values for the surface charge density at the centres of the subdomains. The capacitance, C, of the conducting surface is obtained from the following equation

$$C = Q/V = \frac{1}{V}\sum_{n=1}^{N} \sigma_n A_n \tag{7}$$

Here N is the total number of rectangular subsections.
The same method for a single conductor is extended for evaluating the capacitance of multiconducting bodies.

We consider two parallel rectangular conducting plates (2Lx2W) each divided into equal number of subsections (Figure 2).

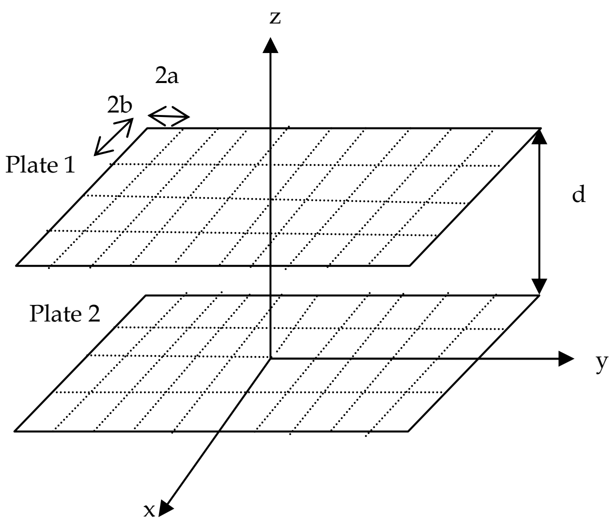

Fig. 2. Parallel plate divided into rectangular subsections.

The simplified formula achieved is as follows

$$V = \sum_{n=1}^{N} K_{mn}\sigma_n$$
$$\text{where } K_{mn} = \frac{1}{4\pi\varepsilon}\int_{x_n-a}^{x_n+a} dx' \int_{y_n-b}^{y_n+b} \frac{dy'}{\sqrt{(x_m-x')^2+(y_m-y')^2+(z_m-z')^2}} \tag{8}$$

In matrix form, equation (8) can be written as follows

$$[K_{mn}][\sigma_n] = [V_n] \tag{9}$$

Here

$$[K_{mn}] = \begin{bmatrix} [K_{mn}^{11}] & [K_{mn}^{12}] \\ [K_{mn}^{21}] & [K_{mn}^{22}] \end{bmatrix}; \qquad [V_n] = \begin{bmatrix} [V_n^1] \\ [V_n^2] \end{bmatrix}$$

The diagonal sub matrices represent the effect of the plate itself and the non diagonal sub matrices represent the mutual interaction between the plates.

The elements of the diagonal matrix remain same as the single element case. The elements of the non-diagonal matrix are evaluated following the same method. The diagonal and non-diagonal elements of the matrix is evaluated as follows

$$
\begin{aligned}
K_{nn}^{12} &= K_{nn}^{21} \\
&= \frac{1}{\pi\varepsilon}\left(a \ \ln \frac{b+\sqrt{a^2+b^2+d^2}}{\sqrt{a^2+d^2}} + b \ \ln \frac{a+\sqrt{a^2+b^2+d^2}}{\sqrt{b^2+d^2}} \right) \\
&+ \frac{2d}{\pi\varepsilon}\left(\tan^{-1}\left(\frac{\sqrt{b^2+d^2}+b}{d} \tan\left(\frac{1}{2}\tan^{-1} \frac{a}{\sqrt{b^2+d^2}} \right) - \frac{1}{2}\tan^{-1}\frac{a}{d} \right) \right)
\end{aligned} \tag{10}
$$

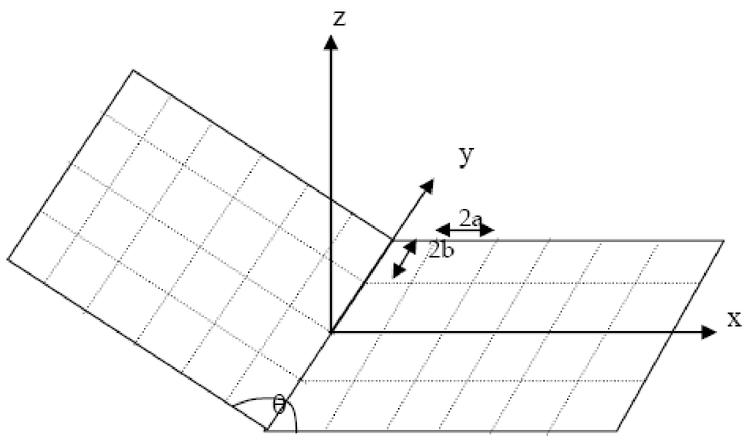

Fig. 3. Inclined plate divided into rectangular subsections.

$$K_{mn}^{12} = K^{21}_{mn}$$

$$= \frac{1}{4\pi\varepsilon}\left(\begin{array}{l} |x_m - x_n + a| \; ln\dfrac{\left(|y_m - y_n + b| + \sqrt{(x_m - x_n + a)^2 + (y_m - y_n + b)^2 + d^2}\right)}{\left(|y_m - y_n - b| + \sqrt{(x_m - x_n + a)^2 + (y_m - y_n - b)^2 + d^2}\right)} \\ - |x_m - x_n - a| \; ln\dfrac{\left(|y_m - y_n + b| + \sqrt{(x_m - x_n - a)^2 + (y_m - y_n + b)^2 + d^2}\right)}{\left(|y_m - y_n - b| + \sqrt{(x_m - x_n - a)^2 + (y_m - y_n - b)^2 + d^2}\right)} \\ - |y_m - y_n + b| \; ln\dfrac{\left(|x_m - x_n - a| + \sqrt{(x_m - x_n - a)^2 + (y_m - y_n + b)^2 + d^2}\right)}{\left(|x_m - x_n + a| + \sqrt{(x_m - x_n + a)^2 + (y_m - y_n + b)^2 + d^2}\right)} \\ + |y_m - y_n - b| \; ln\dfrac{\left(|x_m - x_n + a| + \sqrt{(x_m - x_n + a)^2 + (y_m - y_n - b)^2 + d^2}\right)}{\left(|x_m - x_n - a| + \sqrt{(x_m - x_n - a)^2 + (y_m - y_n - b)^2 + d^2}\right)} \end{array} \right) \tag{11}$$

$$+ \frac{d}{2\pi\varepsilon}\left(\begin{array}{l} tan^{-1}\left(\dfrac{\sqrt{(y_m - y_n + b)^2 + d^2} + |y_m - y_n + b|}{d} tan\left(\dfrac{1}{2} tan^{-1} \dfrac{|x_m - x_n + a|}{\sqrt{(y_m - y_n + b)^2 + d^2}} \right) \right) \\ - tan^{-1}\left(\dfrac{\sqrt{(y_m - y_n - b)^2 + d^2} + |y_m - y_n - b|}{d} tan\left(\dfrac{1}{2} tan^{-1} \dfrac{|x_m - x_n + a|}{\sqrt{(y_m - y_n - b)^2 + d^2}} \right) \right) \\ - tan^{-1}\left(\dfrac{\sqrt{(y_m - y_n + b)^2 + d^2} + |y_m - y_n + b|}{d} tan\left(\dfrac{1}{2} tan^{-1} \dfrac{|x_m - x_n - a|}{\sqrt{(y_m - y_n + b)^2 + d^2}} \right) \right) \\ + tan^{-1}\left(\dfrac{\sqrt{(y_m - y_n - b)^2 + d^2} + |y_m - y_n - b|}{d} tan\left(\dfrac{1}{2} tan^{-1} \dfrac{|x_m - x_n - a|}{\sqrt{(y_m - y_n - b)^2 + d^2}} \right) \right) \end{array} \right)$$

Similarly the exact expression for the elements of the non-diagonal sub matrices can be evaluated for two inclined plates (Figure 3). In this case, the expressions for the non diagonal elements remain almost same as for parallel plates, the only difference is that the value of d does not remain constant – it varies with the positions of the subsections.

For multiconductor lines surrounded by homogeneous medium the inductance of the line is evaluated from the following relation

$$C = \mu\varepsilon L^{-1} \tag{12}$$

The surrounding medium is characterized by μ and ε.

The characteristic impedance can be found out using the simple formula Z=1/vC where v=$3x10^8$ *m/sec*

3. Results and Discussions

A computer program based on the preceding formulation has been developed to determine the charge distribution and hence the capacitance of arbitrary shaped multiconducting bodies. The capacitance of different conducting surfaces e.g. rectangular plate, square plate, circular disc have been calculated (Figure 4 – 6). The capacitance data for a square and rectangular plate agrees with the available data in literature (Harrington, 1985; Higgins and Reitan, 1957; Nishiyama and M. Nakamura, 1992; Hariharan et al., 1998; Goto et al., 1992). Also the result for a circular disc (radius=1m, N=24, capacitance=68.36 pF) matches with the value available in literature (Nishiyama and M. Nakamura, 1992). Next the same method has been extended for the evaluation of per unit length parameters of multiconducting bodies e.g. parallel rectangular and circular plates, co-axial conductors with circular cross-section. The per-unit-length parameters for parallel square conductors, circular discs and co-axial conductors are presented and compared with other available data in Table 1 - 3.

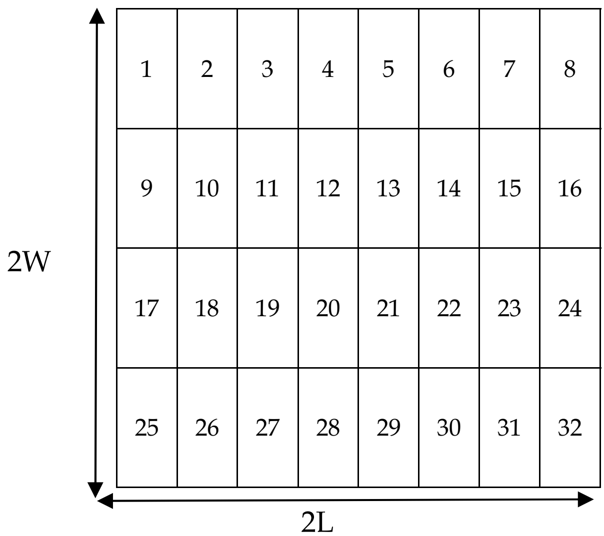

Fig. 4. Square plate (2L=1m; 2w=1m; V=1 volt) divided into N=32 subsections. Capacitance=38.69 pF.

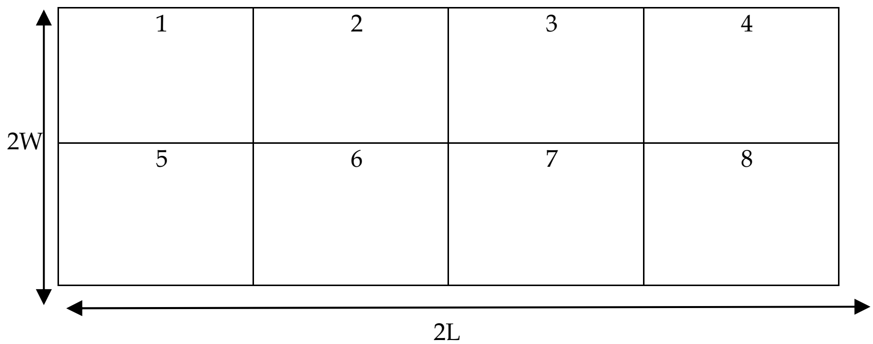

Fig. 5. Rectangular plate (2L=4m; 2w=1m; V=1 volt) divided into 4x2 subsections
Capacitance=54.73 pF.

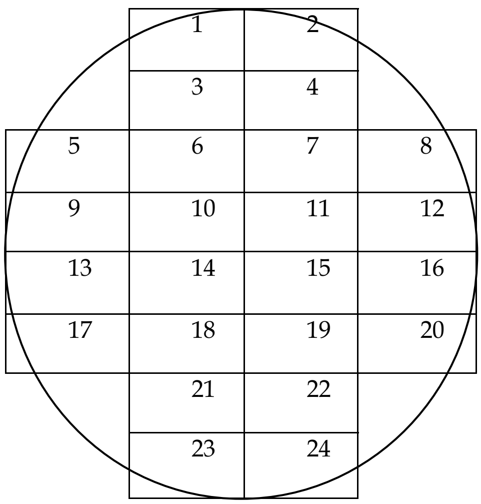

Fig. 6. Circular disc (radius=1m, N=24). Capacitance=68.36 pF agrees withthe value in literature =70.73 pF [9].

For circular coaxial conductor, each circular cylinder is replaced by a cylinder with octagonal structure of surface area equal to that of the circular cylinder (Figure 7). Each side of the octagonal cylinder is divided into rectangular subsections. For co-axial conductors of finite length, there is appreciable fringing effect. The per unit length capacitance of the circular coaxial line is found by evaluating the capacitance of various lengths and then subtracting the part due to the fringing effect. Also the characteristic impedance of the coaxial conductor is evaluated and compared with the analytical value (Figure 8).

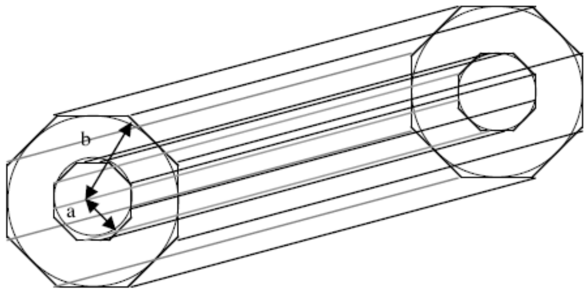

Fig. 7. Circular coaxial conductor approximated with octagonal cross-sectional coaxial structure.

d/2L	Capacitance in pF (calculated)	Capacitance in pF ($C_0=\varepsilon A/d$)	C/C_0	C/C_0 (Harrington, 1985)
0.01	904.48	884.14	1.023	1.024
0.025	378.43	353.67	1.07	1.05
0.05	203.35	176.83	1.15	1.15
0.10	105.92	88.41	1.198	1.2

Table 1. Capacitance of parallel square conducting plate (length=width=2L=1 m)

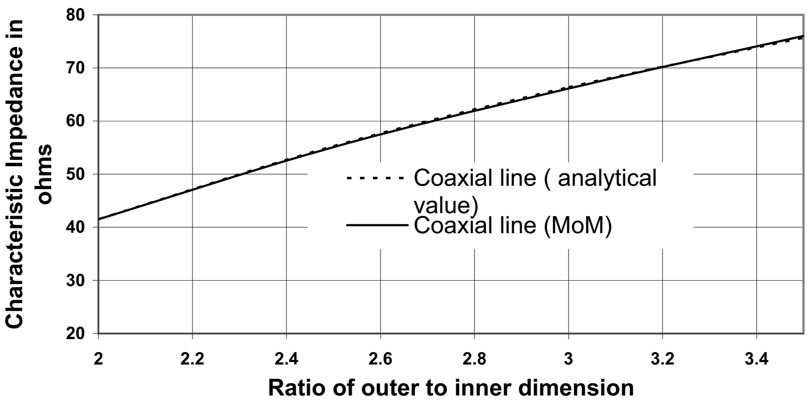

Fig. 8. Plot of the characteristic impedance versus ratio of outer to inner dimension of circular coaxial line.

d in meter	Capacitance in pF (MoM)	Capacitance in pF using analytical formula ($C_0=\varepsilon\pi r^2/d$)	C/C_0	C/C_0 (Jordan and Balmain, 1971)
0.02	1405.86	1388.8	1.015	
0.03	981.16	925.92	1.0597	1.062
0.05	617.64	555.56	1.11	
0.07	471.5	396.82	1.18	

Table 2. Capacitance of parallel circular conducting plates (radius=1m)

Ratio of outer to inner dimension	Capacitance in pF (including fringing effect)		Capacitance / unit length in pF/meter	Analytical value $C=2\pi\varepsilon/\ln(b/a)$ pF/meter	Inductance / unit length in μH / m
	Length =1m	Length =2 m			
2	109.44	189.75	80.21	80.15	0.1386

Table 3. Capacitance and inductance of circular coaxial lines (radius=1m)

4. Conclusion

A simple and efficient numerical procedure based on Method of Moments is presented for the evaluation of the per-unit length parameters of multiconducting bodies. The conducting structure is divided into rectangular sub areas. The data for capacitance of different planar and non-planar conducting structures show well agreement with their analytical value. This method can be used for the determination of equivalent circuit models of multiconductor or multiwire arrangements used in electronic systems.

5. References

Albert E. Ruehli, Pierce A. Bernnan, Efficient Capacitance Calculations for Three-Dimensional Multiconductor Systems, *IEEE Transactions on Microwave Theory and Techniques*, Vol. MTT-21, No. 2, February 1973.

E. Goto, Y. shi and N. Yoshida, Extrapolated Surface Charge method for Capacity Calculation of Polygons and Polyhedra, *Journal of Computational Physics,* Vol. 100, 1992.

Er-Wei Bai, Karl E. Lonngren, Capacitors and the method of moments, *Journal of Computers and Electrical Engineering*, Elsevier Publishers, Vol. 30 (2004) 223–229.

Er-Wei Bai, Karl E. Lonngren, On the capacitance of a cube, *Journal of Computers and Electrical Engineering*, Elsevier Publishers, Vol. 28 (2002) 317–321.

E. C. Jordan, K. G. Balmain, *Electromagnetic Waves and Radiating Systems*, Prentice Hall of India Private Limited, New Delhi, 1971.

N. Nishiyama and M. Nakamura, Capacitance of Disk Capacitors by the Boundary Element Method, *Proceedings of First European Conference on Numerical Methods in Engineering*, September 1992.

R. F. Harrington, Field Computation by Moment Method, Krieger Publishing Company, Florida, 1985.

Sadasiva M. Rao, Allen W. Glisson, Donald R. Wilton, B. Sarma Vidula, A Simple Numerical Solution Procedure for Statics Problems Involving Arbitrary-Shaped Surfaces, *IEEE Transactions on Antennas and Propagation,* Vol. AP-27, No. 5, September 1979.

S. Ghosh and A. Chakrabarty, Capacitance evaluation of Arbitrary-shaped Multiconducting Bodies using Rectangular Subareas, *Journal of Electromagnetic Waves and Applications,* Vol. 20, No. 14, pp. 2091-2102, 2006.

S. Ghosh and A. Chakrabarty, Estimation of Capacitance of Different Conducting Bodies by the Method of Rectangular Subareas, *Journal of Electrostatics, Elsevier Publication,* Vol. 66, Issues 3-4, March 2008, Pages 142 – 146.

Saila Ponnapalli, Alina Deutsch, Robert Bertin, A Package Analysis Tool Based on a Method of Moments Surface Formulation, *IEEE Transactions on Components, Hybrids, and Manufacturing Technology*, Vol. 16, No. 8, December 1993.

T. J. Higgins and D. K. Reitan, Accurate Determination of the Capacitance of a Thin Rectangular Plate, *AIEE Transactions* Vol.75, part-1, January 1957.

V. K. Hariharan, S. V. K. Shastry, Ajay Chakrabarty and V. R. Katti, Free Space Capacitance of Conducting Surfaces, *Journal of Spacecraft Technology*, Vol. 8, No. 1, pp. 61-73, January 1998.

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InTech Europe
University Campus STeP Ri
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