

# Design and Implementation of a Decision Support System for Analysing Ranking Auction Markets for Internet Search Services

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## 1. Introduction

Nowadays, Internet is the usual platform for people around the world to search for firms offering specific services. However, many Internet search engines provide useless lists due to the fact that they are extremely long or not very well organized. This has been the starting point for some Internet search service providers to create new systems for ranking firms according to different searching engines.

One outstanding example of these providers is the giant Google. Google has developed an auction mechanism (see, for example, (Krishna, 2002) or (Klemperer, 2004) for details on auction mechanisms) for firms to advertise their services on the Internet, known as Google Adwords system. Under this mechanism, when a consumer searches for firms offering specific services, the results for a particular keyword (or group of keywords) are ranked in descending order according to what previously the firms have bid. Then, when a consumer clicks on the name of the firm listed on the search site, this firm has to pay the provider an amount equal to the bid price regardless of whether the consumer finally purchases or not. This way of ranking firms has several benefits over other possibilities. On one hand, the provider offers pay-for performance service since firms pay only when a consumer clicks on their corresponding hyperlink. On the other hand, each firm is encouraged submitting a new bid anytime to change the order at which it appears on the list.

This issue has been studied previously in the literature. (Lim & Tang, 2006) introduced a one-stage game for two firms that captures the advertising mechanism of a search service provider. So, game theory allowed them to analyze the firm's optimal bidding strategy and assess the impact of several parameters on the provider's revenue. Nevertheless, it just was the first attempt to analyse bidding behaviour arising from this type of situations, since their model presents several limitations. First, Lim and Tang's model is limited to only two firms, each one with just three feasible bids. Secondly, Lim and Tang's model does not take into account the dynamic interactions among firms (e.g., fluctuating coalition structures) since they assume just one stage. For these reasons, it seems suitable to extend the analysis of (Lim & Tang, 2006) to other more complex situations.

In this chapter we describe software which could be used as a decision support system tool or framework for analysing, at least from an academic point of view, ranking auction markets for Internet search service providers. The software tool is based on the behaviour of

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the firms in a realistic market, thus many different parameters are considered. Taking into account that the problem is really complex from a mathematical point of view, the results are obtained by simulation. This kind of approach using computational tools to analyse a problem is very often in engineering problems because of their mathematical complexity and it has been also used to analyse economic problems as markets based on auction mechanisms. For instance, (Sancho et al., 2008) provide a simulation framework to analyse competitive electricity markets, (Atkins et al., 2007) provide an agent based computational framework to study large commodity markets or (Mehlenbacher, 2009) studies signal averaging in English auctions using a multi-agent system. Additionally, some computational experience is reported to illustrate what kind of results we could obtain.

Finally, we would like to point out that other related papers are (Feng et al., 2007), who focused their analysis on how to improve the seller's expected revenue by enforcing a reserve price in ranked items auctions, and (Sancho et al., 2009), who deal with auction situations arising from Internet search service providers but considering a cooperative approach.

The rest of the chapter is organised as follows. In Section 2 we provide a description of the Internet ranking auction situation and introduce the main parameters involved in the problem. In Section 3 we introduce software tool, describe its main elements and how it works. Furthermore we include some computational experience. Finally, Section 4 concludes.

## 2. Brief description of the Internet ranking auction

In this section we formally introduce the Internet ranking auction situation and the parameters we use in the developed software tool for analysing such situations. Our approach involves analysing the problem from a competitive point of view, i.e., we are considering that the firms will compete to obtain a better position on the list because that is profitable for them. The position of a firm on the list will depend on the money each firm agrees to pay per click and, hence, the strategies of the firms would be their possible bids and their goals being focused on maximizing their expected profit.

In particular, we consider a multi-stage situation in which an arbitrary number,  $n$ , of firms, each owns a homepage, are planning to list their names (links) under the same group of keywords in order to obtain as many visits as possible. Indeed, each visitor is a potential client to buy their products or to contract their professional services. To this end, they resort to an Internet search service provider, as could be Google or Yahoo. As the firms are interested in being on the top of the list they should pay some amount of money to the Internet search service provider in order to avoid the usual ordering provided by the searching engine used by the Internet search service provider. Furthermore the Internet search service provider could vary the order of the firms on the list from one period to another according to the paid money by them. In this sense, we are considering different periods and thereby the problem is dynamic.

We denote by  $T$  the total number of periods,  $t=0, \dots, T-1$ . And, we denote by  $N_t$  the total number of customers who use the aforementioned group of keywords to conduct a search in period  $t$  (day, hour, minute, etc.). Additionally, we denote by  $N_{dt}$  the total number of disloyal customers in period  $t$ , i.e., those who do not have a clear preference among all the firms on the list and therefore they can click any homepage link. Consequently,  $N_t - N_{dt}$

will be the number of loyal customers in period  $t$ . We assume that loyal customers always visit only the site of their preferred firm. In this context,  $l_i$  will represent the market share of firm  $i$  over the set of all loyal clients interested on that particular group of keywords. In other words,  $l_i$  is the proportion of visits that firm  $i$  receives from all loyal customers (it is obvious that  $\sum_{i=1}^n l_i = 1$ ).

On the other hand,  $p_j$  denotes the proportion of clicks from the disloyal customers that a firm in position  $j$  on the list will receive. In this way, if firm  $i$  is ranked in position  $j$  in period  $t$  then it will receive a total number of clicks in that period equals to the sum of the clicks received from its loyal customers and the clicks received from the disloyal customers, in formula

$$c_{it} = l_i (N_t - N_{dt}) + p_j N_{dt} \tag{1}$$

The unitary reward per customer of firm  $i$  in period  $t$ , when a customer clicks on the link to enter in its homepage  $i$ , is denoted here by  $\theta_{it}$ . In order to obtain their position in period  $t$ , the firms have to make a bid. These bids are the amount of money that firms agree to pay for each click received. Finally, they achieve the position in the ranking corresponding to their bids taking into account that all submitted bids are arranged in decreasing order. Therefore, firm  $i$  must only make a single bid and their final profit will be given by

$$(\theta_{it} - b_{it})(l_i (N_t - N_{dt}) + p_j N_{dt}) \tag{2}$$

Therefore, we assume that firms pay for all clicks from both loyal and disloyal customers. Whereas the revenue for the provider is given by

$$\sum_{i=1}^n b_{it} (l_i (N_t - N_{dt}) + p_j N_{dt}) \tag{3}$$

Other input which the Decision Support System uses is the average expense that any customer spends for each click to the firms' homepages. It is denoted here as  $e$ . However a customer not always spends that money when she enters in the homepage of a firm, therefore there is uncertainty about the expense happens or not and so we will denote by  $\pi$  the probability of such expense happens.

Since firm  $i$  does not have information on the proportion of clicks that a firm receives from the disloyal customers, we assume that each firm  $i$  has a private forecast,  $f_{ijt}$ , about  $p_j$  in period  $t$ . In practice, all these estimations can be obtained by each firm using whatever market information and statistical tool at its disposal. Additionally, we assume that the private forecast of each firm can be updated over time with the new information obtained from the previous periods. For this reason, we use the subscript  $t$  to highlight that  $f_{ijt}$  is the firm  $i$  estimation of  $p_j$  obtained with the information available for firm  $i$  until period  $t$ .

Finally, after receiving all the bids in period  $t$ , the Internet service provider announces the ranking and the bid of each firm for this period and so on. Since the bids are revealed in period  $t$ , each firm has incentives to submit a new bid for period  $t+1$  in order to try to change or keep its position in the ranking, i.e., the order at which the firm appears on the list.

### 3. Fundamentals and development of the Decision Support System for Internet ranking auctions

Based on the above description of the Internet ranking auction situation, we have implemented a software tool, using C++ Builder 6, which we have tried to reflect the reality of the firms' bids and carry out the ranking of these bids, considering also the dynamic component of the situation.

The uncertainty about the number of clicks for each position has been modelled through the private forecasts  $f_{ijt}$ . And both the forecasts and other parameters will be updated over time by means of specific algorithms that we will explain below.

On the other hand, we have resorted to simulation for analyzing the firms' behaviour. The reason is not hard to see. Considering more than two firms and three strategies leads to a mathematically intractable scenario and therefore the simulation approach seems suitable and reasonable to deal with.

#### 3.1 General outline of the application

The developed and implemented Decision Support System works taking into account the risk profiles defined for each firm which participates in the ranking auction, the total number of customers (loyal and disloyal), the average expense per click, the proportion of clicks per position, the average reward for each firm, the private forecasts and the number of periods to be simulated. Once all necessary inputs to start the simulation have been introduced, which constitute the initial working conditions, the implementation cycle is the following.

(a) *Storing inputs.* The system stores information about firms; customers; expenses; number of simulations; number of periods to be simulated; and bids.

(b) *Computation of the variables of interest.* When the parameters which are necessary to start the simulation have been introduced into the system, the number of total customers is obtained through simulation. Then, for each customer, the system determines whether she is loyal or disloyal. If she is loyal, the system determines to which firm. In any case, by simulation, the system obtains the values of all variables necessary to run on the simulation. If we are in the first period ( $t=0$ ), then the system simulates all the bids with the initial information introduced into the system and ranks the firms. All this information is then saved. If we are in a general intermediate period  $t$ , before simulating the firms' bids, the system updates  $\theta_{it}$  and  $f_{ijt}$  for all firms using the information obtained from previous periods and then it simulates the firms' bids and ranks them.

(c) *Presentation of results.* Once the system has simulated the target period, all obtained data are reported in a practical and friendly format such as spreadsheets, which allow us to store on the hard disk the results of all simulations carried out. The results are sorted in different spreadsheets showing with tables and figures the following information:  $b_{it}$ ,  $\theta_{it}$ ,  $e$ ,  $f_{ijt}$  and  $c_{it}$  for each period  $t$ ,  $t=0, \dots, T-1$ , which we consider relevant to analyse a particular Internet ranking auction situation. This way in which the simulation results are presented eases to analyse them using the different mathematical and statistical utilities that the most of spreadsheets usually have.

Figure 1 shows the flow chart which represents and summarises the operating model of the application we are describing. We can observe that the general structure of the application is very simple consisting basically of two consecutive cycles, one for the customers' behaviour and another for the firms' bids.

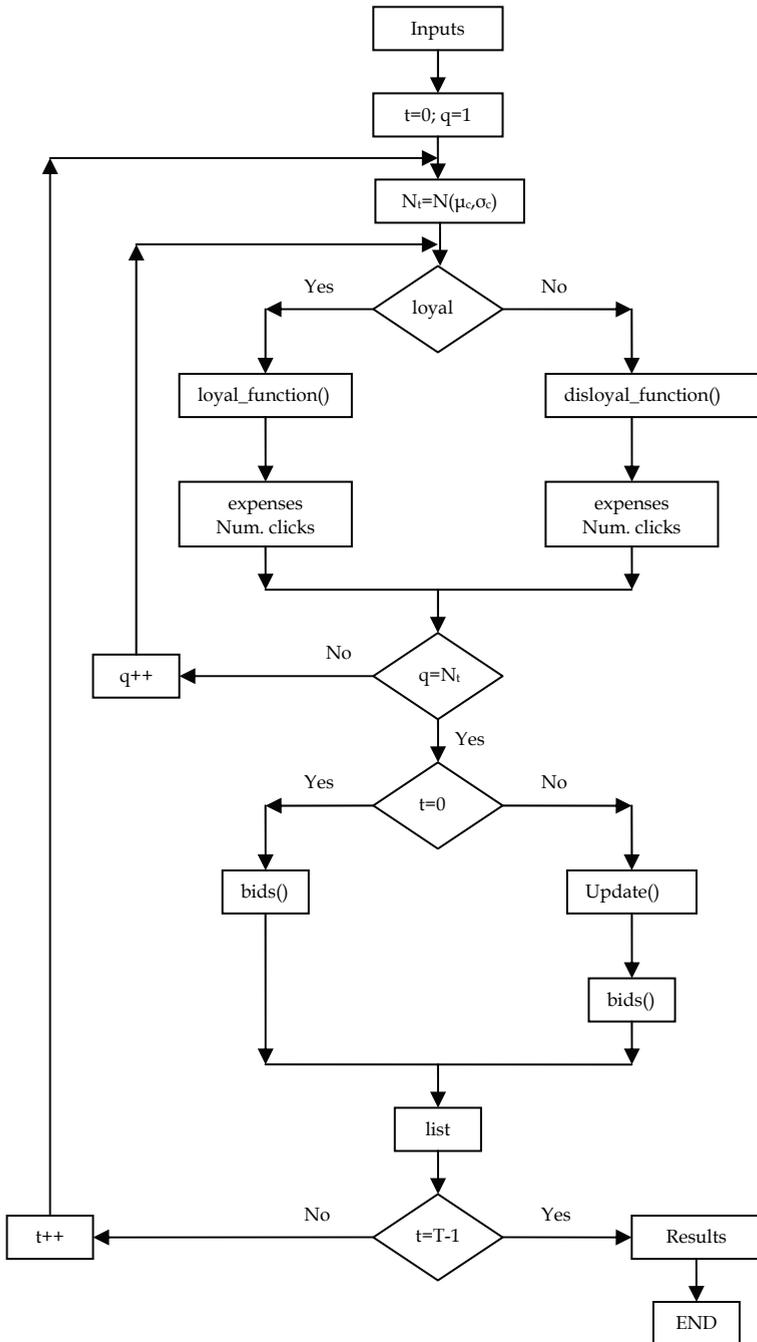


Fig. 1. The flow chart which represents the operating model of the application

We should also describe the main aspects of the software developed. One advantage of this system is the clarity which the different parameters are dealt with, and also its easy use. The software consists of a graphic and intuitive interface through which the user can introduce all the parameters necessary to carry out the simulation of an Internet ranking auction. This interface consists of only one window (see Figure 2) in which the main parts of the application with very different characteristics are shown. In the following subsections we will show and explain in detail each of these parts. Finally, another advantage of this software tool is the presentation of the results in spreadsheets, because, as we said before, it eases the posterior analysis of the obtained results from the simulation.

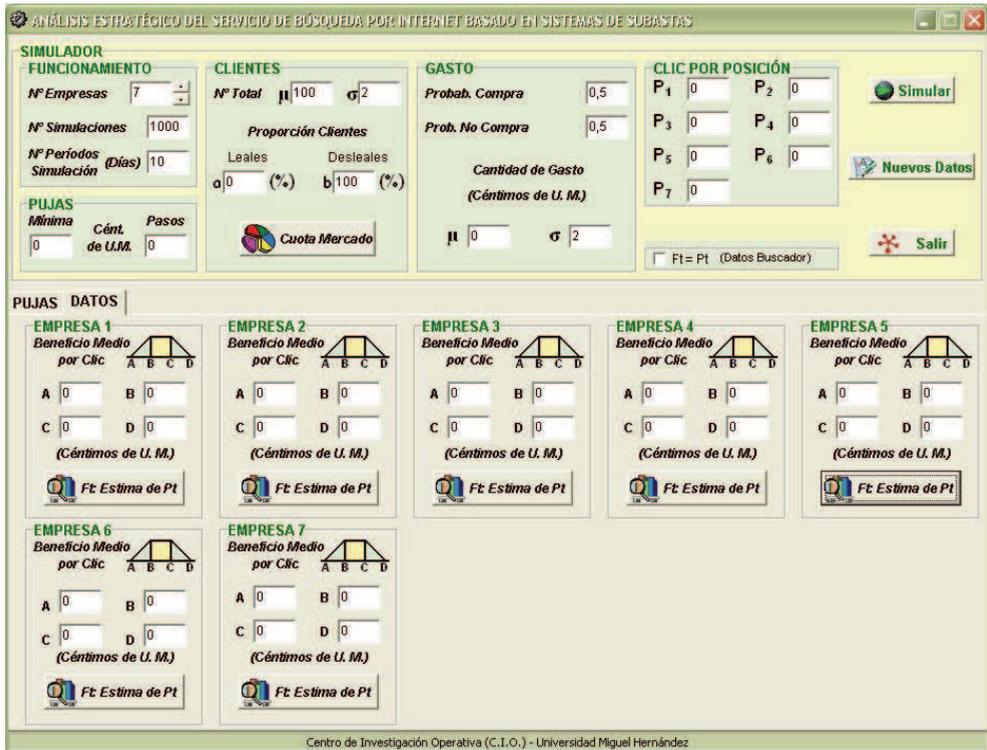


Fig. 2. The main application window. [Análisis estratégico del servicio de búsqueda por Internet basado en sistemas de subastas (Strategic analysis of the Internet search service based on auction systems), funcionamiento (starting conditions), empresas (firms), pujas (bids), leales (loyals), desleales (disloyals), gasto (expense), probab. compra (purchase probability), cuota mercado (market share), nuevos datos (new data), salir (exit), datos buscador (search service provider data), beneficio medio por clic (average profit per click), estima de (estimation of)]

### 3.2 Parts of the Decision Support System

#### 3.2.1 System variables

This part allows us to configure the internal operations of the Decision Support System. First, we have to enter as an input the number of firms (the limit depends on the features of

the computer in which we are running the software). Other necessary inputs are the number of simulations and the number of periods to simulate. The first input is related to the times in which the system repeats the calculations to determine suitable values for different parameters. Specifically, the system tries to estimate the probability of being in any position of the ranking conditioned to a particular submitted bid. Nevertheless, this process will be shown in detail later on.

### 3.2.2 Bids

In this part we need to introduce two parameters. The first one is related to the reserve price, which the provider can impose in the ranking auction. In many instances, providers (sellers, in general) reserve the right to not provide the service if the price determined in the auction is lower than some threshold. This threshold amount is called "the reserve price". The system takes into account this possibility. Obviously, if such reserve price does not exist, then we must only introduce a value zero for this parameter. The second parameter is the amount that must be added to the reserve price to build the set of feasible bids. Let us denote this amount as  $\Delta$ . Therefore the minimum feasible bid will be the reserve price,  $r$ , the maximum feasible bid will be the unitary reward,  $\theta_{it}$ , and the possible bids in between will be calculated as  $r+k\Delta$ ,  $k=1,2,\dots,K$  where  $K$  is an integer number such that  $K \leq (\theta_{it} - r)/\Delta$ .

### 3.2.3 Customers

We must also enter as an input the total number of customers who use the Internet service each period  $t$ . To this end, we introduce into the system this parameter modeled by a Gaussian distribution of mean  $\mu_c$  and standard deviation  $\sigma_c$ , therefore we are considering the possibility that the number of customers is not constant along the number of periods under consideration. For each period  $t$ , the system simulates an execution of a Gaussian distribution  $N(\mu_c, \sigma_c)$  and, afterwards, rounds off this value to obtain an integer number of customers. Additionally, it is necessary to introduce into the system the percentage of loyal and disloyal customers in the (simulated) market. We assume that this information is shared for all the participating firms in the auction.

Finally, we have a button to add information about the firms' market shares,  $l_i$ . All these inputs will be used for the system, by simulation, to determine the total number of customers, the disloyal clients and the number of loyal clients for each firm.

### 3.2.4 Expenses

The main aim of this part is to provide estimations of the customers' expenses, once they have clicked on a particular homepage. In order to obtain this information, we have to enter as an input the probability of a customer purchasing the product or contracting the service from any firm on the list. If, finally, a customer purchases the product, then it is necessary to know how much she spends. For this reason, we introduce into the system the amount of expense per client modeled again by a Gaussian distribution  $N(\mu_e, \sigma_e)$ . All these parameters will be used later in the simulation stage.

### 3.2.5 Clicks per position on the list

It is necessary to introduce into the system information on the proportion of clicks that a firm receives from the disloyal clients when a firm is ranked in position  $j$ ,  $p_j$ , for all  $j=1,\dots,n$ .

In this framework, we assume that these parameters depend solely on the ranking. In addition, we point out that  $p_j$ , for all  $j=1, \dots, n$ , is private information of the Internet search service provider. Consequently, the firms, through whatever market information gathering techniques at its disposal, need to have a private forecast,  $f_{ijt}$ , about the parameter  $p_j$ . These estimations help firms to make a decision about what bid to submit in each period  $t$ . Our system updates the private forecasts over time from the number of clicks obtained in the positions at which firms appear in each of the simulated periods.

The system also allows us to simulate that the Internet search service provider discloses information on  $p_j$ , for all  $j=1, \dots, n$ , to the firms so as to check whether this strategy encourages them to bid more aggressively or not. The question, in this case, is to verify whether reducing the uncertainty on the number of clicks per position on the list implies that firms bid more aggressively.

### 3.2.6 Information about firms

#### 3.2.6.1 Data

As for the part of the framework devoted to the firms, we need to enter as an input the unitary reward per customer and the private forecast of  $p_j$  for each firm in period  $t=0$ , i.e.,  $\theta_{i0}$  and  $f_{ij0}$ , respectively (see Figure 3).

In our system  $\theta_{i0}$  has been characterized as a trapezoidal fuzzy number (A,B,C,D). Therefore, it is necessary to introduce these four parameters. We use a trapezoidal fuzzy number to represent the (imperfect) knowledge of the firm about the unitary reward per customer.

In the first period,  $t=0$ , the system simulates a value for  $\theta_{i0}$  from the fuzzy number. Later, the system will update the reward per customer over time,  $\theta_{it}$ ,  $t=1, \dots, T-1$ , using information about the customer's expenses and other variables involved in the problem. We should also point out that  $\theta_{i0}$  could be defined as a crisp number. To this end, it would be enough to consider  $A=B=C=D$ .

The screenshot shows a software interface for 'EMPRESA 1'. At the top, it says 'Beneficio Medio por Clic' (Average profit per click). Below this is a trapezoidal fuzzy number diagram with vertices labeled A, B, C, and D. A yellow shaded area is shown within the trapezoid. Below the diagram are four input fields labeled A, B, C, and D, each containing the number '0'. The unit is specified as '(Céntimos de U. M.)' (cents of monetary unit). At the bottom, there is a button with a magnifying glass icon labeled 'Ft. Estima de Pt' (Estimation of percentage of clicks per position).

Fig. 3. Part of the application devoted to the firm 1. [Empresa (firm), beneficio medio por clic (average profit per click), centimos de u.m. (cents of monetary unit), Estima de (estimation of)]

On the other hand, the system needs information about the estimation of the percentage of clicks per position for each firm. In other words, we have to introduce into the system the

perceived percentage of disloyal clients who will visit the firm's homepage depending on the position at which it appears on the list, i.e.,  $f_{ij0}$ . We note that this parameter is defined only for the first period to simulate,  $t=0$ , because after that initial period, the system automatically updates it over time for each firm obtaining the value of  $f_{ijt}$  for all  $t=1, \dots, T-1$ . The strategies of the firms can vary from one period to another as a consequence of the updated perceived estimations of  $f_{ijt}$  since the firms will bid more aggressively to obtain positions with higher  $f_{ijt}$  i.e., to obtain more visits and hence a higher expected profit.

### 3.2.6.2 Risk aversion and behaviour

Other data we have to enter as an input is the risk profile of firm  $i$ ,  $i=1, \dots, n$ , and at the same time the risk profile of firm  $k$  ( $k=1, \dots, i-1, i+1, \dots, n$ ) following the perception of firm  $i$  about it. Obviously firms' risk profile is directly related to the shape of the utility function of each firm. We denote here the utility function of firm  $i$  by  $UF_i$ . Following von Neumann-Morgenstern tradition (see (von Neumann & Morgenstern, 1944)), a firm  $i$  is risk averse if  $UF_i$  is a convex function, risk neutral if  $UF_i$  is a linear function and risk loving if  $UF_i$  is a concave function. In particular, in this software we assume that  $UF_i$  is a square root function for the first case, the identity function for the second case and, finally, a square function for the third case. Particularly, in our system we have modeled a more general situation. We allow firms to behave in a different way over time. In other words, in a period  $t$  firm  $i$  could behave as a risk averse, risk neutral or risk loving player depending on a probability distribution. We have to enter into the system as an input this distribution for each firm  $i$ ,  $i=1, \dots, n$  (see Figure 4). In a similar manner, for running, the system needs the perception of firm  $i$  about the risk profile of each firm  $k$ ,  $k=1, \dots, i-1, i+1, \dots, n$  (see Figure 5). So, the system in each period will simulate a value from the risk profile to determine the type of the utility function of each firm. Therefore we are considering that a firm to make a decision not only takes into account its risk profile but also its perception about the risk profiles of its competitors.

### 3.3 Main algorithms implemented

In this section we show the main algorithms that have been implemented with the task of calculating the number of loyal and disloyal customers and their expenses, the firms' bids, the ranking in each period, the updates values for the private forecasts of the clicks per position, the parameters of the fuzzy numbers, etc.



Fig. 4. Risk profile of the firm. [Empresa (firm), perfil de riesgo (risk profile), alto (high), neutro (neutral), bajo (low), otras empresas (other firms)]

The screenshot shows a window titled "PERFIL DE RIESGO" with a subtitle "EVALUACIÓN DE LA EMPRESA 1 SOBRE LAS DEMÁS EMPRESAS". It contains two sections for evaluating competitors. The first section lists "Empresa 2" through "Empresa 5", and the second lists "Empresa 6" and "Empresa 7". Each section has three input fields: "Alto" (High), "Neutro" (Neutral), and "Bajo" (Low), each followed by a percentage sign. In the "Neutro" fields, the value "100" is entered. The "Alto" and "Bajo" fields are empty. At the bottom, there are two buttons: "Aceptar" (Accept) with a green checkmark icon and "Cancelar" (Cancel) with a red X icon.

Fig. 5. Perception of firm 1 about the risk profile of the rest of competitors. [Perfil de riesgo (risk profile), empresa (firm), evaluacion de la empresa 1 sobre las demas empresas (perception of firm 1 about the risk profile of the rest of firms), alto (high), neutro (neutral), bajo (low), aceptar (OK), cancelar (cancel)]

### 3.3.1 Loyal and disloyal customers

Here we are going only to show the algorithm used for a disloyal client because the case of a loyal client requires an easier algorithm.

For any period  $t$ , once the system has simulated a value for the total number of customers who are going to use the Internet search service in that period, we need to know whether each customer is loyal or disloyal. To this end, we make use of the information previously introduced into the system (see Section 3.2.3) about the percentage of loyal and disloyal clients in the market. So, we simulate a random variable, which simply follows a Bernoulli distribution  $B(p)$  where  $p$  is the probability to be loyal, that determines whether a customer  $q$  is or not loyal. If  $q$  is, finally, a loyal customer then, using the firms' market shares,  $l_i$ , we can determine, by simulating an execution of the multinomial distribution  $M(l_1, l_2, \dots, l_n)$ , which is her preferred firm among all the participants on the list.

Now, let us assume that a particular customer  $q$  is disloyal. Then, the system follows the algorithm described in Figure 6. In particular, for each position on the list  $j$ ,  $j=1, \dots, n$ , the system simulates an execution of a uniform random variable  $U_j[0,1]$ , we call this execution by  $u_j$ . After that, if  $u_j$  is lower than  $p_j$  (the proportion of clicks associated to the position  $j$ ) then the system understands that the disloyal customer  $q$  clicks on the homepage which appears in position  $j$ , otherwise the system considers that customer  $q$  does not click it. On the other hand, we assume that a disloyal customer could click on all the homepages on the list if she is willing to, unlike loyal customers which only click on their preferred firm's homepage. Therefore, the sum of all  $p_j$ ,  $j=1, 2, \dots, n$ , could be greater than 1.

When  $u_j$  is lower than  $p_j$  for a position  $j$ , then the system simulates the amount of money that customer  $q$  spends in the firm which appears at position  $j$ . We note that both the probability of clicking, purchasing and how much to spend in the site do not depend on which firm is but the first probability depends on the position while the others two are always the same for all customers, firms and positions. Therefore, in this sense, the position on the list plays a crucial role in our approach because it makes the difference in the expected revenues of the

firms. However, once a customer enters in a homepage her behaviour is not affected by the position, the firm or anything else. Consequently, the important question in this setting is whether a customer clicks or not the link to enter in a site.

Regarding the algorithm to simulate the expenses of customer  $q$ , let us suppose that  $u_j < p_j$ , i.e., customer  $q$  clicks on the homepage placed in position  $j$ . Then, the system simulates an execution of a uniform random variable over the interval  $[0,1]$ . If the obtained value is lower than the probability of purchasing (introduced previously into the system as an input), it means that the customer  $q$  will spend his money on the products of the firm which appears in position  $j$ . Afterwards, the system simulates the expenses by means of an execution of a Gaussian distribution  $N(\mu_e, \sigma_e)$  (see Section 3.2.3).

Finally, the system saves for each customer  $q$  the positions on the list she visited and the expenses she spent in each visited position.

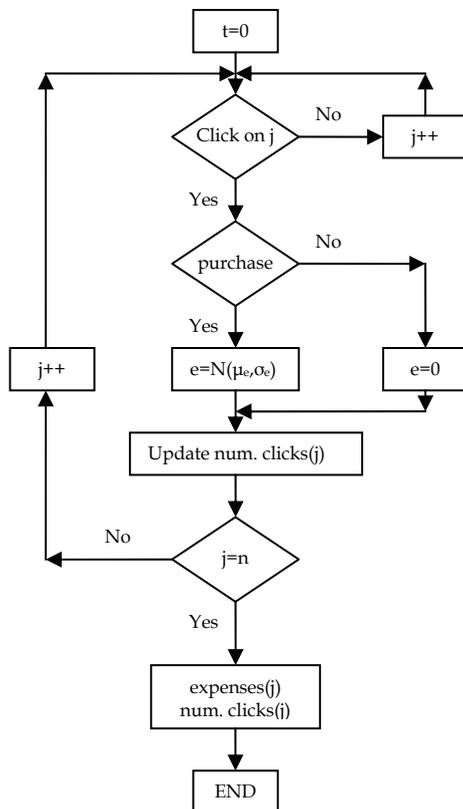


Fig. 6. The flow chart of the function for the disloyal customers

The algorithm for loyal customers only consists of the expenses part of the algorithm for disloyal customers for this reason it is omitted. Furthermore, we note loyal customers only spend money in their preferred firms' homepages while disloyal customers could spend money in several or all firms' homepages. On the other hand, it is nevertheless true that we could have considered that the probability of purchasing and/or spending change when a

previous purchases has been done but the present approach is enough for our purposes and that extension or modification is left for further versions of this software tool. In fact, this modification would stress more the role of the position in this kind of situations.

### 3.3.2 Updating process

This process involves modifying the value of the variables that change over time. The firms' strategies can vary depending on the value of the simulated parameters in previous periods, since it is important for them to improve their estimations on the parameters used for them to make a bid. Therefore firms update their information available incorporating the data obtained from the previous periods in order to improve their knowledge about some system parameters relevant for them. In this section, we briefly show which parameters are updated by the firms and, additionally, how this process is carried out.

The parameters that we consider relevant for the firms from a strategic point of view, and hence they will be modified period by period, are the following:  $f_{ijt}$ , the estimation of the percentage of clicks per position, and  $\theta_{it}$ , the unitary reward per customer who clicks on firm  $i$ 's homepage.

Regarding estimations  $f_{ijt}$ , the Internet search service provider knows the real value of the percentage of clicks per position on the list,  $p_j$ ,  $j=1, \dots, n$ . However, each firm at the end of period  $t$  only knows the number of received clicks on its homepage with absolute certainty. Therefore, firm  $i$  in position  $j$  at the end of period  $t$  can just update the estimation  $f_{ijt+1}$ . This new estimation can be calculated as the percentage of received clicks from the disloyal customers, i.e. we compute the ratio of the number of disloyal clients who click on firm  $i$ 's homepage to the total number of disloyal clients. In order to know the number of clicks due to disloyal customers, we calculate the total number of clicks (from loyal and disloyal clients) minus the number of clicks from loyal customers. Firm  $i$  knows the total number of clicks received after playing period  $t$ , denoted by  $c_{it}$ , because each firm has a counter on its homepage. And the number of clicks from loyal customers to firm  $i$  is calculated by multiplying the total number of customers who visited the Internet search service in period  $t$ ,  $N_t$ , times the proportion of loyal customers in the market and, finally, times the market share of firm  $i$ . It is worth to note that we consider  $N_t$  is common knowledge to all firms, because we assume that the search service provider publishes this information when the auction for period  $t$  is over. This assumption is not restrictive since it is important for the search service provider to advertise the number of customers using its search services in order to attract more firms over all when  $N_t$  is large enough.

As a consequence of the previous calculation, firm  $i$  has a first new estimation  $f_{ijt+1}$  on  $p_j$ , it will be denoted as  $f_{ijt+1}(1)$ . Nevertheless, we assume that each firm can improve the accuracy of the estimation  $f_{ijt+1}(1)$ . To do that, we will use the previous estimation  $f_{ijt}$  on  $p_j$ . First, we define a discrepancy index (DI) for measuring the difference between  $f_{ijt+1}(1)$  and, the previous estimation,  $f_{ijt}$ :

$$DI = |f_{ijt} - f_{ijt+1}(1)| / f_{ijt+1}(1) \quad (4)$$

Depending on the value of DI, the system will weight each one of these estimations on  $p_j$  to build a new compound forecast. We only distinguish three cases:

Case 1:  $DI \leq 0.25$

In this case, both  $f_{ijt}$  and  $f_{ijt+1}(1)$  have relevant information about the real percentage of clicks for position  $j$  on the list. Therefore we consider that both estimations on  $p_j$  are equally credible. So, we update the estimation of  $f_{ijt+1}$  by the following expression.

$$f_{ijt+1}=(f_{ijt} + f_{ijt+1}(1))/2 \tag{5}$$

Case 2:  $0.25 < DI \leq 0.5$

In this case,  $f_{ijt}$  and  $f_{ijt+1}(1)$  are a little different. In this case we consider more credible estimation  $f_{ijt+1}(1)$  than estimation  $f_{ijt}$ . Therefore, we use a weight of  $2/3$  for  $f_{ijt+1}(1)$  and a weight of  $1/3$  for  $f_{ijt}$  to capture this feeling on the estimations. In this way, we have to calculate:

$$f_{ijt+1}=(2f_{ijt} + f_{ijt+1}(1))/3 \tag{6}$$

Case 3:  $DI > 0.5$

This is the more extreme case. Here  $f_{ijt}$  and  $f_{ijt+1}(1)$  are clearly different. In this case,  $f_{ijt}$  is very far to the estimation obtained with the data of period  $t$ , i.e.,  $f_{ijt+1}(1)$ . Therefore we consider that estimation  $f_{ijt+1}(1)$  is much more credible than estimation  $f_{ijt}$ . Hence, we update firm  $i$ 's estimation on  $p_j$  exclusively with the value of  $f_{ijt+1}(1)$ .

$$f_{ijt+1}=f_{ijt+1}(1) \tag{7}$$

Overall, the updating process is carried out to update the firms' private forecasts of the percentage of clicks per position. Given the available information that firm  $i$  has at the end of period  $t$ , we exclusively update the estimation of  $f_{ijt+1}$ , where  $j$  is the position at which the firm  $i$  appears on the list during that period. It means that we will use the previous forecasts  $f_{ikt}$ , where  $k=1, \dots, j-1, j+1, \dots, n$ , to estimate  $f_{ikt+1}$  because firm  $i$  does not have new information about those positions. Therefore, each firm is able to update the information about only one position at the end of each period. On the other hand, it is not difficult to modify the software tool in order to consider another procedure or additional cases to update estimations  $f_{ijt}$ .

It is worth to note that the system allows us to simulate a situation where the search service provider publishes the real parameters, i.e.,  $p_j$ ,  $j=1, \dots, n$ . Due to that option, we can study the behaviour of the firms (regarding their bids) when they have more and better information about the relevance of the different positions for their interests.

On the other hand, as we said above, other parameters that will be modified over time are  $\theta_{it}$ 's, i.e., the unitary reward per customer who clicks on firms' homepages. Remember that  $\theta_{it}$  is modeled in this system as a trapezoidal fuzzy number  $(A_{it}, B_{it}, C_{it}, D_{it})$ . Regarding this, in order to update  $\theta_{it}$  period by period, the system will revise separately each one of the four parameters  $A_{it}$ ,  $B_{it}$ ,  $C_{it}$  and  $D_{it}$ . Obviously, this updating process depends on the money spent by the customers in the firms' homepages. We know that the gross revenue obtained by the firm  $i$  at the end of period  $t$  will coincide with the loyal and disloyal customers' expenses. Then, in order to obtain the reward per click, we calculate the ratio of the gross revenue to the total number of clicks. We will denote this value by  $\hat{\theta}_{it}$ . The system uses  $\hat{\theta}_{it}$  as a tool to update the four parameters defining the trapezoidal fuzzy number. Specifically, the system uses the following expressions:

$$A_{it+1}=0.9A_{it}+0.1 \hat{\theta}_{it} \tag{8}$$

$$B_{it+1}=0.9B_{it}+0.1 \hat{\theta}_{it} \tag{9}$$

$$C_{it+1}=0.9C_{it}+0.1 \hat{\theta}_{it} \tag{10}$$

$$D_{it+1} = 0.9D_{it} + 0.1\hat{\theta}_{it} \quad (11)$$

Therefore, the reward per click corresponding to firm  $i$  at the end of period  $t$  will be modeled as a trapezoidal fuzzy number  $(A_{it+1}, B_{it+1}, C_{it+1}, D_{it+1})$ . So, the system will simulate a value from  $(A_{it+1}, B_{it+1}, C_{it+1}, D_{it+1})$  to determine an estimation of the new reward per click  $\theta_{it+1}$ . We note that for the rewards per click we consider a position more conservative than for the percentage of clicks per position on the list, i.e., the firms update more slowly their estimations on the reward per click. However, as in the previous updating algorithm, it is not difficult to modify the software tool to consider another procedure of updating the rewards per click, for example, following the same idea as in the case of the percentage of clicks per position on the list. This kind of possibility provides certain flexibility to the software tool implemented with respect to consider other additional situations not included in the version presented in this chapter.

### 3.3.3 Bids

In this section we show how the system assesses a finite set of feasible bids for each firm in order to determine their optimal bid. The optimal bid will be the feasible bid which maximizes expected value of the utility function of the evaluated firm. In order to determine the optimal bid of firm  $i$ ,  $i=1, \dots, n$ , the system carries out several steps.

First of all, we need to define the risk profile for firm  $i$ . Also, we have to specify the number of bids that firm  $i$  will consider as feasible to submit to the Internet search service provider. The system will select one of these bids as the optimal one for firm  $i$  in period  $t$  according to its utility function  $UF_i$ .

Secondly, we have to determine the probability of appearing in each position  $j$  on the list, for all  $j=1, \dots, n$ , once the firm has submitted a particular bid. This process is carried out for each one of the feasible bids for firm  $i$ , for all  $i=1, \dots, n$ .

Third, we calculate the expected value of the utility function for each one of the feasible bids. In this set, a bid is chosen so as to maximize the expected utility, following our assumptions. Roughly speaking, this is the procedure to determine the optimal strategy for firm  $i$ .

Obviously, this process is carried out for each period  $t$ ,  $t=0, \dots, T-1$ , and for each firm  $i$ ,  $i=1, \dots, n$ , obtaining the simulated bids and the ranking list for each period.

Overall, the determination of the optimal bid depends on the risk profile, the average reward per click, the private forecast about the number of clicks per position, and the perception that each firm has about the rest of competitors, as we show next.

*Step 0.* The system executes a realization of the trapezoidal fuzzy number  $(A_{it}, B_{it}, C_{it}, D_{it})$  to obtain an average reward per unit  $\theta_{it}$  to be used in the following steps.

*Step 1.* Let us assume that we are working with firm  $i$  in period  $t$ . A similar process is performed for all periods and all participating firms in the ranking auction.

The system simulates the risk profile for the firm (see Section 3.2.6.2). So, we obtain an integer number, 0, 1 or 2, corresponding to a risk loving, a risk neutral or a risk averse player, respectively. Once the system has simulated the risk profile, we know how firm  $i$  will behave. Nevertheless, we need to know the set of feasible bids as well.

*Step 2.* Following the inputs introduced into the system and the values obtained in the previous steps, the number of feasible bids will be  $K+1$ , where  $K$  is an integer number such that  $K \leq (\theta_{it} - r)/\Delta$  (see Section 3.2.2). In particular, the set of feasible bids is obtained by

means of the simple expression  $r + k\Delta$ , where  $k = 0, 1, \dots, K$ . The number of bids to be evaluated can be as greater as one likes because the tool allows us to choose the minimum amount  $\Delta$  to be considered (see Section 3.2.2).

*Step 3.* Once we know the set of feasible bids and the risk profile, we have to determine the probability of appearing in each position on the list since a firm is not able to know its position on the list before submitting a particular bid. Therefore, in particular, we are interested in determining the corresponding probability distribution associated to each feasible bid  $b_{it}$ . We will use this probability distribution to calculate the expected value of the utility function given bid  $b_{it}$ .

Obviously, firm  $i$ 's profit not only depends on firm  $i$ 's bid but also on the bids of the rest of competitors. Therefore, it is also necessary that the firm assumes a certain kind of behaviour for each of its competitors.

In order to make clear the above point, we first show the expression of the expected value of the utility function given bid  $b_{it}$  for firm  $i$ :

$$E[UF_i] = \sum_{j=1}^n UF_i \left( (\theta_{it} - b_{it}) (1_i (N_t - N_{dt}) + f_{ijt} N_{dt}) \right) P(j/b_{it}), \quad (12)$$

where  $P(j/b_{it})$  denotes the probability of appearing in position  $j$  after submitting the feasible bid  $b_{it}$ .

Taking into account that computing mathematically the above probabilities is almost intractable, we will approximate them by simulation. To this end, we will use the ratio of the number of times that firm  $i$  has appeared in position  $j$ , after submitting  $b_{it}$ , to the total number of simulations. In other words, the system simulates an auction as many times as the user introduced into the system. In each auction, firm  $i$  will always submit the same bid  $b_{it}$ , and will keep fix the same risk profile. Regarding the competitors, the process is more sophisticated. First, the system simulates a value from the fuzzy number of firm  $i$   $\theta_{it}(k)$ , for all  $k=1, \dots, i-1, i+1, \dots, n$ , in order to obtain the average reward per click which each competitor use. Secondly, the system multiplies that realization of  $\theta_{it}(k)$  times a factor which depends on the perception of firm  $i$  about the risk profile of each firm  $k$ . This factor has been modeled in the system by a Beta( $a, b$ ) distribution. The parameters  $a$  and  $b$  have been defined in a different way depending on the kind of risk profile of the firm. If the firm is risk averse then we consider  $a=2$  and  $b=3$ . If the firm is risk neutral then we consider  $a=3$  and  $b=3$ . And if the firm is risk loving then  $a=3$  and  $b=2$  (other values for parameters  $a$  and  $b$  could be easily considered just modifying the corresponding part of the code). In this way, the probability density function is asymmetric with a high left tail, symmetric, and asymmetric with a high right tail, respectively, following the natural bidding behaviour of the firms. Finally, the system builds the bid of competitor  $k$ ,  $k=1, \dots, i-1, i+1, \dots, n$ , by means of the following expressions

$$b_{kt} = \theta_{it}(k) \cdot \text{Exc}(\text{Beta}(a_k, b_k)), \quad (13)$$

where  $\text{Exc}(\text{Beta}(a_k, b_k))$  is an execution of the distribution  $\text{Beta}(a_k, b_k)$ .

We note that we consider that each firm knows neither the average reward per unit of the others nor a particular estimation on them, therefore they use their own knowledge about the average reward per unit to evaluate the possible averages reward per unit which can be used by their competitors. In some sense, each firm considers that its knowledge on the

average reward per unit is good enough and the other firms have the same (or very similar) information about that.

*Step 4.* The system calculates the expected value of the utility function, (12), for the bid  $b_{it}$ . This procedure is repeated for each feasible bid for firm  $i$ . In this way, the system is able to select the optimal bid, i.e., the feasible bid with the highest expected value of the utility function.

To end this section, it is worth noting that all firms that participate in the auction submit their optimal bids. So, the system ranks the firms in descending order according to all these bids. Consequently, the system is able to build a list for each period  $t$ ,  $t=0, \dots, T-1$ .

### 3.4 Some computational experience

In order to show how the system works we use two stylized and simple examples one with only two firms and another with five firms. In the first case, the example has the following characteristics:

Inputs	Value
Num. of firms	2
Num. Simulations	1000
Num. of periods	10
Num. Customers	$N(1000,5)$
% disloyals	100%
Prob. of purchasing	0.2
Expenses	1 €
Reserve price	5 cent/€
$\Delta$	0.1
$p_1$	0.9
$p_2$	0.2

Table 1. Example with two firms

In this first example, we work with only two risk neutral firms, during a period of 10 days, assuming that all the customers which visit the Internet search engine are disloyal, the probability of purchasing is 0.2 and when a customer makes the decision of purchasing her expenses is constant and equals to 1€. On the other hand, the proportion of clicks received if the firm is ranked first clearly higher than if the firm is ranked second. Also, we assume that both firms are symmetric. In other words, both present the same features. In particular, the starting average reward per click is modelled by means of the trapezoidal fuzzy number (17, 17.5, 19.5, 20.0). Regarding the private forecasts about the number of clicks per position on the list, we consider that the firms' estimations deviate significantly from the actual value of the parameters,  $p_1$  and  $p_2$ . In particular, we consider that  $f_{110}=f_{120}=f_{210}=f_{220}=0.5$ , therefore, the firms evaluate that the position is not relevant to obtain more clicks and hence a higher expected revenue. Therefore, one could expect that the firms bid for the first day the reserve price to appear on the list in whatever position. Finally, we assume that each firm believes that its rival is risk neutral as well.

In Figure 7 the bidding strategy for each firm over time is shown. As it can be seen, on the first day, both firms submit the reserve price as optimal bids as one could expect without any analysis. As it was noted before, it is due to the fact that the number of clicks the firms

will receive if they are ranked first or second is little sensitive. In other words, the firms think that they will receive the same number of visits independently on the position at which they appear. Therefore, they have little incentives to bid aggressively. However, since the private forecasts about these parameters change over time (see Figure 8, for Firm 2 we obtained a similar figure) showing increasingly the importance to be ranked first on the list, the firms bid more aggressively. In some way, they are having additional information about the real number of visits per position and it allows them to improve the forecast accuracy of  $p_j, j=1,2$ . Since  $p_1 \gg p_2$  the firms have incentives to bid higher.

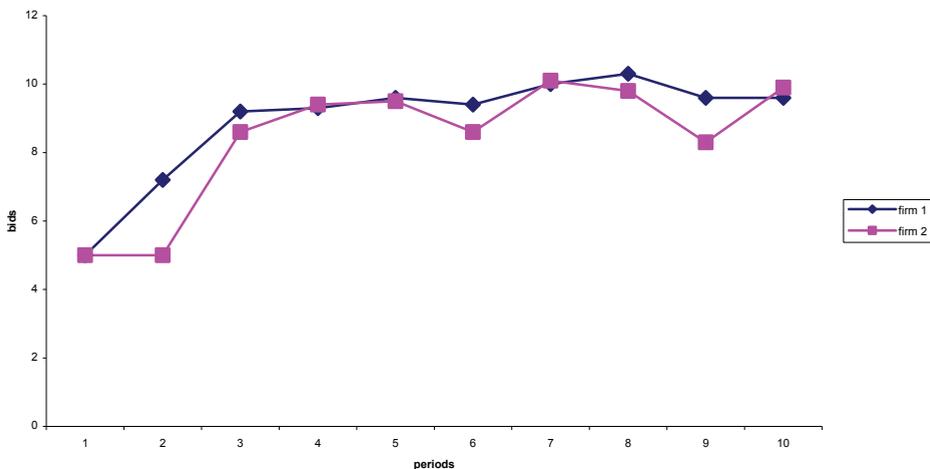


Fig. 7. Bids of the two firms for the studied period

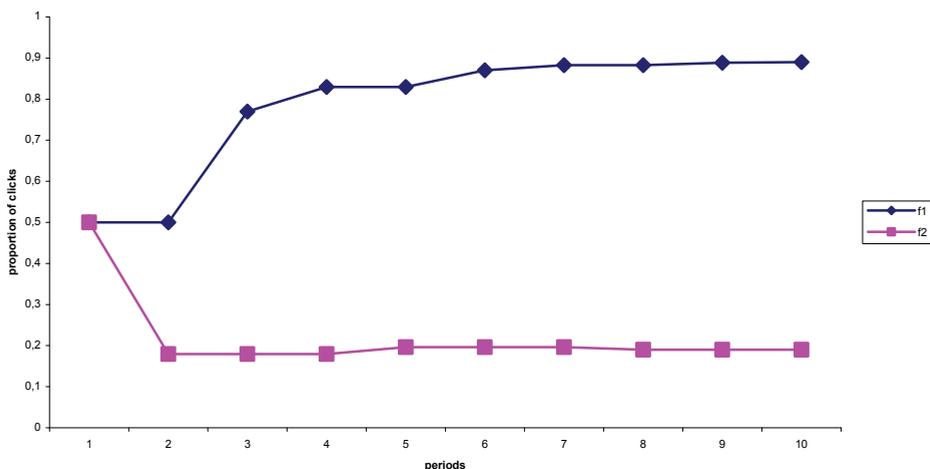


Fig. 8. Evolution of Firm 1's private forecasts about  $p_j$

Finally, from Figure 7 and Figure 8 we can observe that the optimal bid for both firms after the ten periods is approximately 10 cents and in few periods the firms have a reasonable good estimation on the parameters  $p_1$  and  $p_2$ .

In order to show how the system works under more competition, we consider a second example with five firms. The particular characteristics of this numerical example are shown in the following table:

Inputs	Value
Num. of firms	5
Num. Simulations	1000
Num. of periods	10
Num. Customers	$N(1000,5)$
% disloyals	100%
Prob. of purchasing	0.2
Expenses	1 €
Reserve price	5 cent/€
$\Delta$	0.1
$p_1$	0.9
$p_2$	0.7
$p_3$	0.3
$p_4$	0.2
$p_5$	0.1

Table 2. Example with five firms

Apart from the characteristics given in Table 2, we also assume that the five firms are risk neutral and symmetric again. In particular, we modelled  $\theta_{kt}$  by the trapezoidal fuzzy number (17, 17.5, 19.5, 20). On the other hand, the private forecasts about  $p_j$ ,  $j=1, \dots, 5$ , will be 0.8, 0.7, 0.6, 0.3 and 0.1, respectively. It is worth noting that in this second example the starting estimations are more realistic than in the previous example. It should imply that firms use a more aggressive bidding strategy from the first period,  $t=0$ .

Next in Figure 9 we show the information about the bids submitted to the system for each firm period by period. We observe that the optimal bid for firms after the ten periods analysed is close to 12.

In this case, the value of the parameters encourages aggressive bidding even in the first period. Unlike the previous example, the firms submitted a bid strictly greater than the reserve price in period  $t=0$ . Note also that in the final periods the average bid with five firms is greater than the average bid with two firms. Obviously, it is consequence of the intrinsic competition of both examples.

Regarding the estimation of  $p_j$ , each firm learnt over time about the real proportion of clicks per position. In this way, at the last period, the private forecasts about these parameters are very close to the actual values (see Figure 10).

It is worth noting that the system generates more information about the auction over time than the above presented. For example, number of clicks received, expenses related to the customers, the evolution of the reward per click, etc. Once the simulation has been completed, all the data obtained are presented in a practical format such as spreadsheets, which allows us to store on the hard disk the results of all the simulations carried out. Nevertheless, we only wanted to show briefly some of the results that the developed software is able to yield.

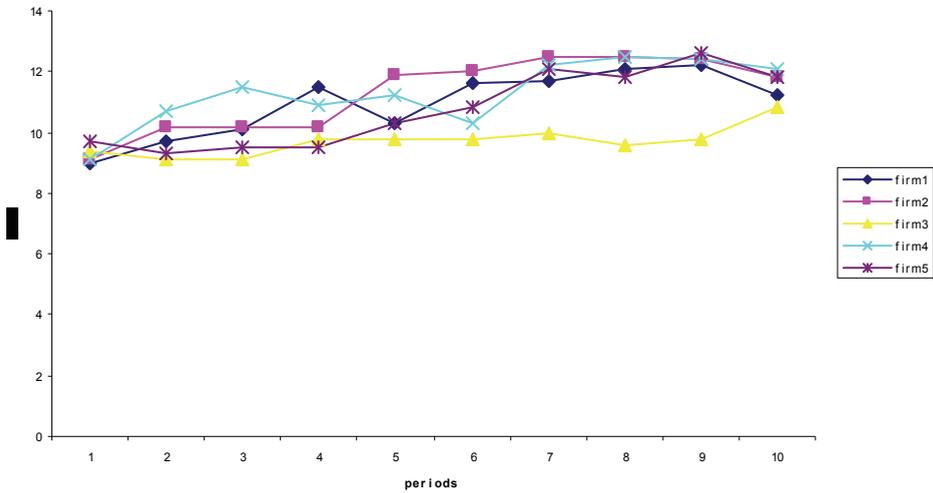


Fig. 9. Bids of the five firms for the studied period

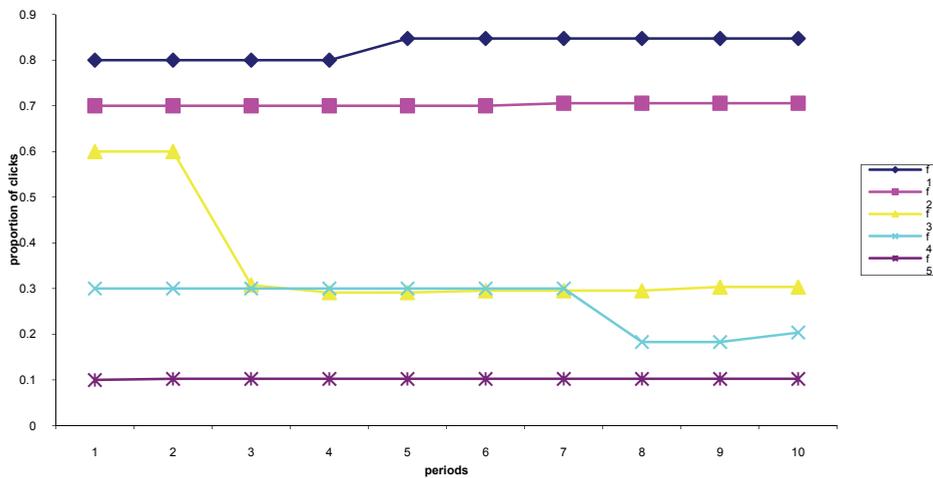


Fig. 10. Evolution of Firm 1's private forecasts about  $p_j$

#### 4. Conclusions and further research

The main objective of this work has been to develop a simple software tool in the form of a Decision Support System or computational framework for analysing ranking auction markets for Internet search service providers which could be useful for economic scholars or practitioners. The particular features of this tool make possible to simulate in a clear and

simple way the bidding behaviour of a set of firms when facing ranking auctions situation on the Internet.

This tool could be interesting for analysing different aspects of Internet search engines. In particular, the tool provides information about how the Internet search engine could induce firms to bid more aggressively, or whether it is beneficial for the provider to disclose more information regarding the number of clicks per position to the firms, the effect of collusion or coordination, etc. On the other hand, the software tool has been implemented to give the possibility to modify easily some parts (in particular some algorithms) in order to consider other situations not included in the present version.

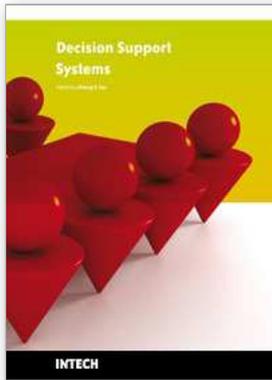
We would like to finish mentioning some additional topics for further research on the Decision Support System considered in this chapter. First, we could use the software to check whether the results obtained by (Lim & Tang, 2006) for only two firms are correct for a greater number of firms. Secondly, we could analyse how a multiple period auction affects firms' bidding strategies. Third, we could also study how collusion over time can distort the final results of the auction. Overall, we view our approach as a building block or framework for developing further analysis.

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Decision support systems (DSS) have evolved over the past four decades from theoretical concepts into real world computerized applications. DSS architecture contains three key components: knowledge base, computerized model, and user interface. DSS simulate cognitive decision-making functions of humans based on artificial intelligence methodologies (including expert systems, data mining, machine learning, connectionism, logistical reasoning, etc.) in order to perform decision support functions. The applications of DSS cover many domains, ranging from aviation monitoring, transportation safety, clinical diagnosis, weather forecast, business management to internet search strategy. By combining knowledge bases with inference rules, DSS are able to provide suggestions to end users to improve decisions and outcomes. This book is written as a textbook so that it can be used in formal courses examining decision support systems. It may be used by both undergraduate and graduate students from diverse computer-related fields. It will also be of value to established professionals as a text for self-study or for reference.

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