Advanced Simulation for Semi-Autogenous Mill Systems: A Simplified Models Approach

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1. Introduction

Modelling and simulation of semi-autogenous (SAG) mills are valuable tools for helping to design control laws for a given application and subsequently to optimise its performance and process control. SAG mills (see Figure 1) are presently one of the most widely used alternatives in the field of mineral size reduction as a result of their advantages such as higher processing capacity, lower physical space requirements, and lower investment and maintenance costs, as compared to conventional circuits (Salazar, et al., 2009).

Due to the size of SAG mills, pilot plants are usually used for research purposes to improve the control strategies. In cases where a pilot-scale is not available for test, simulations using models based on data from a wide range of full-scale plants are helpful and can significantly reduce risks for process control purposes. Simulations also provide an additional and very valuable crosscheck against the pilot results (Morell, 2004).

Fig. 1. Typical semi-autogenous (SAG) mills

This chapter presents a dynamic simulator of a semi-autogenous grinding operation deduced from first principles coupled to an on-line parameter estimation scheme able to simulate industrial operations for future control purposes. The proposed procedure for simulation purposes is as follows: Model equations are based on a conventional non-stationary population balance approach to develop the necessary dynamic model of the semi-autogenous mill operation. The presented models are able to predict the time-evolution of key operating variables such as product flow rate, level charge, power-draw,

load position and others, as functions of other important variables such as mill rotational speed and fresh feed characteristics. The set of ordinary differential equations was solved using MATLAB/SIMULINK as a graphic programming platform, a useful tool for understanding the grinding process.

Additionally, this work presents results using dynamic simulations from a 1700 t/h copper–ore mill showing the effectiveness of the system to track the dynamic behaviour of the variables.

The remainder of this chapter is organised as follows. Theory about specific models for SAG mill processes is presented in section 2. Simulations for the prescribed application are presented in section including the results using MATLAB/SIMULINK. The main conclusions of the chapter are provided in the final section, as well as ideas about future industrial applications of this work.

2. Models for semi-autogenous mills

Essentially, the modelling exercise consists in formulating non steady-state material balances in the milling equipment, along with force conservation relations and hydraulic considerations. The methodology used in this study has already been established by Magne (Magne et al., 1995) and Morrell (Morrell, 2004) and involves formulating particle inventories for each particle size inside the mill. The input variables are: water flow rate, mineral flow rate and size distribution, grinding media flow rate and the mill critical speed. The model output variables are: power-draw, load level, ball load, mineral discharge rate and size distribution, water discharge rate, ball throughput, bearing pressure, pebble throughput, and toe and shoulder angles of the internal load.

2.1 SAG mill model

The particles fed to the mill are ground in the milling chamber and subsequently downloaded into the discharge zone, where, according to a classification probability, they are either returned to the milling chamber for further grinding or become part of the mill output stream. For modelling purposes the mill is divided in two zones according to the process taking place (Fig. 2). The first zone encompasses the milling chamber where the particle reduction process is identified and modelled. In the second, the output zone, the material is internally classified and the final product is discharged. To complete the system
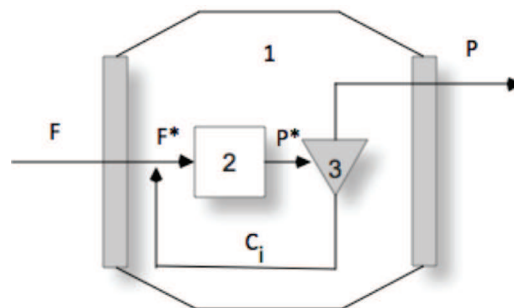

Fig. 2. Schematic representation of a SAG mill. (1) Mill. (2) Grinding Chamber. (3) Internal classifier

description it is necessary to consider the relationship between the feed stream and mill charge level. This relationship is known as the transport rate and is probably the least developed aspect in models proposed so far (Apelt et al., 2002 a,b).

2.2 Transport and water balance

The fictitious flow P* (Fig. 2.) that represents the amount of mineral in the internal charge that is handled by the classification grate or internal classification, is the representation of the mineral transport proposed by Magne (Magne et al., 1995). Several experimental studies have found the following rather unsatisfactory correlation of P* with the mass of the mineral retained in the mill W:

$$P^* = 29 \cdot W^{0,5} \tag{1}$$

Where W is in tonnes (t) and P* in t/h.

2.3 Water balance

The following equation represents the experimental variation of the internal water load, $W_w(t)$, as a result of changes in input and output water flow rates, F_w and P_w (t/h), the latter being estimated by $P_w = C_w \cdot W_w$ (Magne et al., 1995):

$$\frac{dW_w}{dt} = F_a - C_w \cdot W_w \tag{2}$$

The parameter C_w (h-1), water output, has been correlated to the mass of mineral in the mill, W, according to the following relation (Magne et al., 1995):

$$C_w = \exp\left(64.41 - 19.56\ln(W) + 1.55\left(\ln\left(W\right)\right)^2\right) \tag{3}$$

The proposition that the classification system always allows particles of a size less than X_m to pass (Fig. 3) is the basis for the development proposed by Morrell (Morrell, 2004), who, like Magne (Magne et al, 1995), considers that particles less than this size behave like water in the grinding chamber, i.e. all particles with less than a certain size pass through the grate with the same classification efficiency.
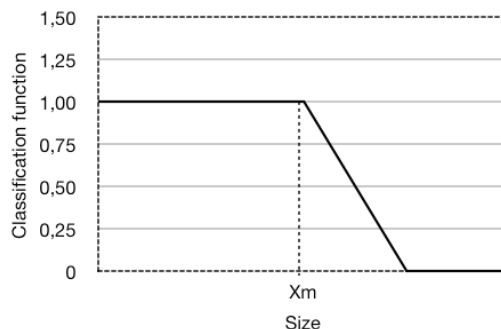

Fig. 3. Classification function against particle size

This discharge function, constant for sizes less than X_m and defined as d_m, is directly related to the flow of discharge of the pulp by the size of the mill, p_i, and the mass of the particles in the internal charge of the mill, w_i, according to:

$$d_m = \frac{\sum_m p_i}{\sum_m w_i} \tag{4}$$

In order to determine this discharge function, Morrell (Morrell, 2004) considers two effects. The first is the flow via grinding media interstices, and the second considers the flow via the slurry pool (where present). In addition, the contributions of Latchireddi (Latchireddi, 2002) have allowed this proposition to be studied in large-scale pilot models and to determine the influence of the design and the geometry of the mill pulp lifters. The results of the correlation between the fill level and discharge flow can be seen in the following general equation:

$$J = \eta \gamma^{n_1} A^{n_2} J_b^{n_3} \phi^{n_4} Q^{n_5} D^{n_6} \tag{5}$$

Where:
J is the net fractional slurry hold-up inside the mill;
A is the fractional open area;
J_b is the fractional grinding media volume;
ϕ is the fraction of critical speed;
Q is the slurry discharge flowrate;
γ is the mean relative radial position of the grate holes;
η is the coefficient of resistance, which varied depending on whether flow was via the grinding media interstices or the slurry pool (where present); and
n_1-n_6 are the models parameters.
The value of γ is a weighted radial position, which is expressed as a fraction of the mill radius and is calculated using the formula:

$$\gamma = \frac{\sum r_i a_i}{r_m \sum a_i} \tag{6}$$

Where a_i is the open area of all holes at a radial position r_i, and r_m is the radius of the mill inside the liners.
Latchireddi's (Latchireddi, 2002) contribution can be seen in the parameters n_i and η from equation (6) which shows the effect of the design of the pulp lifter. These were modeled according to:

$$n_i = n_g - k_i e^{(-k_j \lambda)} \tag{7}$$

Where:
n_g are the parameter values for the grate-only condition;
λ is the depth of the pulp lifter expressed as a fraction of mill diameter; and
k_i and k_j are constants.

For large-scale mills, the pulp discharge flow can be determined by combining equations (4) and (5) as follows:

$$d_m = \frac{Q}{J} \tag{8}$$

The calculation procedure can be transformed in an iterative numerical sequence.

A numerical approximation of the proposal by Gupta & Yan (Gupta & Yan, 2006) shows the product flow (m³/h) from equation (5) as separate from the flow of the fluid through the zone of grinding medium (equation 9) and the flow from the pool zone (equation 10).

$$Q_M = 6100\, \gamma^{2,5} A\, J_H^2\, \phi^{-1,38} D^{0,5} \qquad J_H < J_{MAX} \tag{9}$$

$$Q_t = 935 \gamma^2 A\, J_S D^{0,5} \qquad J_S = J_P - J_{MAX}, J_P > J_{MAX} \tag{10}$$

Where:

γ is the mean relative radial position of the grate apertures;

A is the total area of all apertures (m²);

ϕ is the fraction of the critical speed of the mill;

D is the mill diameter (m);

Q_M is the volume flow rate through the grinding media zone (m³/h);

Q_t is the volume flow rate of slurry through the pool zone, (m³/h);

J_H is the net fraction of slurry hold-up within the interstitial spaces of the grinding media;

J_S is the net fractional volume of slurry in the slurry pool;

J_{MAX} is the maximum net fraction of slurry in the grinding zone; and

J_P is the net fraction of the mill volume occupied by pulp.

2.4 Internal classification and power-draw

For the internal classifier (Fig. 2.), the balance is carried out by defining a classification efficiency vector, c_i (fraction), which includes two effects: one produced by the mill's internal grate and the other by the pulp evacuation system (Magne et al., 1995). Thus, c_i is defined by:

$$1 - c_i = \frac{p_i}{p_i^*} \tag{11}$$

Where:

p_i is the product flow rate from the mill and p_i^* is the product flow rate from the grinding chamber (fictitious flow).

For each size class i, the mill chamber feed flow rate, f_i^* (t/h), is obtained by adding the mill feed flow rate, f_i (t/h), to the internal recirculation flow rate (Fig. 2.):

$$f_i^* = f_i + c_i p_i^* \tag{12}$$

Under experimental considerations (Magne et al., 1995) it is possible to find the following expression of the classification efficiency vector, where x_i is the size of particle, c_f is the solid pulp percentage and β is a parameter.

$$c_i = \psi\beta(x_iM)^{\beta-1}\exp\left(-\psi(x_iM)^{\beta}\right) + \dfrac{1}{1+\left(\dfrac{x_i}{x_{50}}\right)^z} \tag{13}$$

$$\psi = \exp\left(-13.12\ln(c_f) - 6.61\right) \tag{14}$$

$$M = \exp\left(16.53\ln(c_f) + 5.54\right) \tag{15}$$

For each size class i, Magne's (Magne et al, 1995) proposed model relates the mass variation in the milling chamber (Fig. 2) to the feed flow rate to the grinding chamber, f_i^* (t/h), to the product flow rate from the grinding chamber, p_i^* (t/h), and to the comminution kinetics, as follows:

$$\frac{dw_i}{dt} = f_i^* - p_i^* - K_iw_i - (K_i - K_{i-1})\sum_{l=1}^{i-1}w_l \tag{16}$$

Where K_i (h⁻¹) denotes the effective parameter (corresponding to S_i in conventional grinding) and w_i is the weight of size i particles in the mill charge (t).
The effective parameter, K_i is defined as the fraction of specific power supplied to the mill:

$$K_i = K_i^E \frac{M_p}{W} \tag{17}$$

Where K_i^E is defined as the specific grinding rate constant (t/kWh), M_p is the power-draw (kW) and W the total ore weight in the chamber (t). The equation used to predict the power consumed by the mill (power-draw), M_p, is based on a modification of Bond's Law (Austin, 1990):

$$M_p = K_pD^{2.5}L(1 - A \cdot J)\left(\frac{W}{V}\right)\phi_c\left[1 - \frac{0.1}{2^{9-10\phi_c}}\right] \tag{18}$$

Where D (m) and L (m) are the mill dimensions, V (m³) is the mill effective volume, and K_p and A are parameters. The ratio between the internal load mass and the mill volume, (W/V), is related to the percentage of mill capacity by the following equation:

$$\frac{W}{V} = (1 - \varepsilon_b)J\rho_s(1 + w_c) + 0.6J_b(\rho_b - \rho_s(1 + w_c)) \tag{19}$$

Where ε_b is the porosity of the mill internal load (void fraction), ρ_s (t/m³) and ρ_b (t/m³) are the density of mineral and balls respectively, w_c is the mill water/mineral mass ratio, and J_b (fraction) is the ball weight fraction.
Assuming that the mill chamber behaves like a perfectly mixed reactor (Whiten, 1974), p_i^* can be related to particle size i mill charge by:

$$p_i^* = w_i\left(\frac{P^*}{W}\right) \tag{20}$$

Where P^* is the contribution of the total internal flow rate to the product stream (t/h). The relation between P^* and W can be obtained assuming that there is no recycling of fines from the internal classifier. This assumption simplifies the mass balance equation and allows the calculation of P^* on the basis of the product flow rate of fine particles, p_n (t/h), and the mass of fines in the internal charge, $w_n(t)$, as shown in equation (21):

$$P^* = W\left(\frac{p_n}{w_n}\right) \tag{21}$$

From equations (12), (16), and (21), the following expression is then obtained for the dynamic mass balance of size i particles in the milling chamber:

$$\frac{dw_i}{dt} = -\left(\frac{P^*}{W}\right)(1-c_i)w_i - K_i w_i - (K_i - K_{i-1})\sum_{l=1}^{i-1} w_l + f_i \tag{22}$$

As in equation (16), Morrell's (2004) proposal for the comminution process gives a similar relationship as follows:

$$\frac{dw_i}{dt} = f_i - p_i + \sum_{j=1}^{i} r_j w_j a_{ij} - r_i w_i \tag{23}$$

$$p_i = d_i w_i \tag{24}$$

Where r_i is a the breakage rate of particles of size i, d_i is the discharge rate of particles of size i and a_{ij} is the breakage distribution function.

The breakage rate function, r_i, can be obtained using data fitting techniques or full-scale mills with the general form being as follows:

$$Ln(r_i) = k_{i1} + k_{i2}J_b D_b + k_{i3}\varphi + k_{i4}J \tag{25}$$

Where D_b is make-up ball size, φ is the mill rotational rate and k_{i1-i4} are constants. The breakage distribution function, a_{ij}, is obtained via the specific comminution energy, E_{cs} (kWh/t) and the t_{10} parameters estimated, used to generate a size distribution. This equation is:

$$t_{10} = A\left(1 - e^{-b \cdot E_{cs}}\right) \tag{26}$$

Where A and b are parameters of rock breakage.

The mill power-draw studied by Morrell (Morrell, 2004) is similar to that used by Austin (Austin, 1990) and considers the individual power requirements for the cylindrical section and the conical sections. The mill power, P_m (kW), is then the sum of the net power, P_{net} (kW) and the no load power, P_{nl} (kW). Thus:

$$P_m = P_{net} + P_{nl} \tag{27}$$

$$P_{nl} = 1.68 D^{2.05}\left(\phi_c\left(0.667 L_{cone} + L_{cyl}\right)\right)^{0.82} \tag{28}$$

$$P_{net} = 7.98D^{2.5}L_e\rho_s J\left(\frac{5.97\phi_c - 4.43\phi_c^2 - 0.985 - J}{\left(5.97\phi_c - 4.43\phi_c^2 - 0.985\right)^2}\right)\phi_c\left(1 - \left(1 - 0.954 + 0.135J\right)e^{-19.52\left(0.954 + 0.135J - \phi_c\right)}\right) \quad (29)$$

$$L_e = L\left(1 + 2.28J\left(1 - J\right)\frac{L_d}{L}\right) \quad (30)$$

Where L_d (m) is the medium size of the final section of the conical zone, and L_{cone} and L_{cyl} are the sizes (m) of the conical and cylindrical sections of SAG mill.

2.5 Grinding media, bearing pressure and load position
The mass of grinding media inside the chamber is determined by a mass balance considering the ball replacement rate and the metal consumption rate; this latter parameter is proportional to the mass of mineral in the mill (Salazar et al., 2009):

$$\frac{dW_b}{dt} = F_b - \chi\left(W + W_b\right) \quad (31)$$

Where W_b is the ball mass (t) in the mill, F_b the ball replacement rate (t/h), χ a ball wear constant (h-1) and W the total internal mineral load (t).

The bearing pressure, P_b (psi) is estimated as a linear function of the total weight of the milling chamber (balls, water and mineral) as shown in equation (32) (Salazar et al., 2009), where α and λ are fitted parameters. The load position is expressed in terms of toe and shoulder angles, which are calculated by relations (33 to 35) (Apelt et al., 2001):

$$P_b = \alpha + \lambda\left(W + W_w + W_b\right) \quad (32)$$

$$\theta_T = 2.5307\left(1.2796 - J\right)\left(1 - e^{-19.42\left(\phi - \phi_c\right)}\right) + \frac{\pi}{2} \quad (33)$$

$$\theta_s = \frac{\pi}{2} - \left(\theta_T - \frac{\pi}{2}\right)\left(\left(0.3386 + 0.1041\phi_c\right) + \left(1.54 - 2.5673\phi_c\right)J\right) \quad (34)$$

$$\phi = 0.35\left(3.364 - J\right) \quad (35)$$

Where θ_T is the toe angle (radians), θ_S the shoulder angle (radians).

3. Simulation

3.1 SAG in Matlab-Simulink
The numerical solution of the set of algebraic–differential equations (model) described in the previous section, is obtained through a system in MATLAB/SIMULINKTM (Figure 3). Simulink is a programming system structured in blocks, which allows the solution of differential equations as well as the programming of user-blocks through S-functions. This feature, together with the possibility of using Matlab's specific toolboxes, makes it a powerful platform for the development of prototypes. The present model can be seen as a more complex simulation block compatible with this simulation strategy in (Salazar et al., 2009).
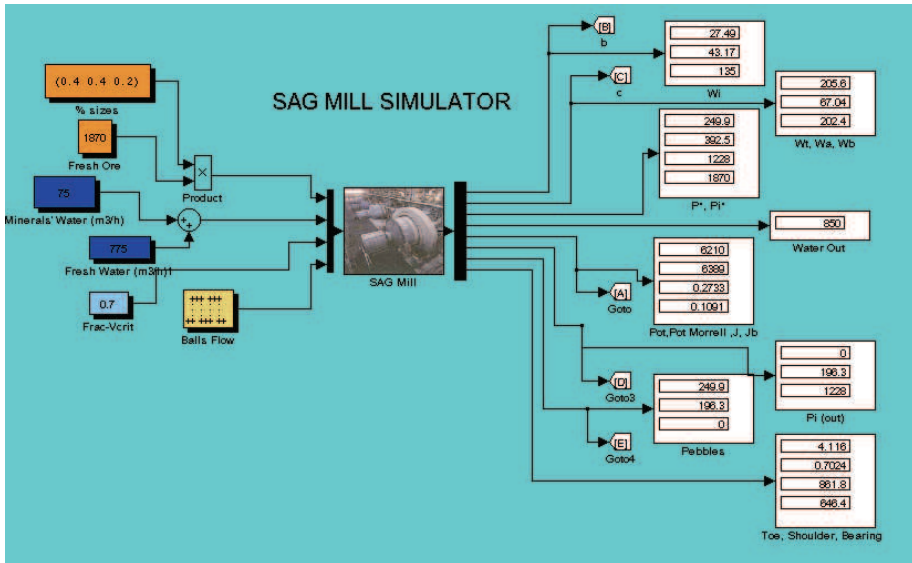
Fig. 3. SAG mill simulator in Matlab-Simulink

3.2 Results

An example of the simulation results is presented in Figs. 4 to 7. These figures respectively show the response of the power-draw and the fill level for the Magne approach (Magne et al., 1995) in Figures 4 and 5, and for the Morell approach (Morrell, 2004) in Figures 6 and 7. The results are the product of 10% flow change related to the nominal operation conditions (1700 t/h).
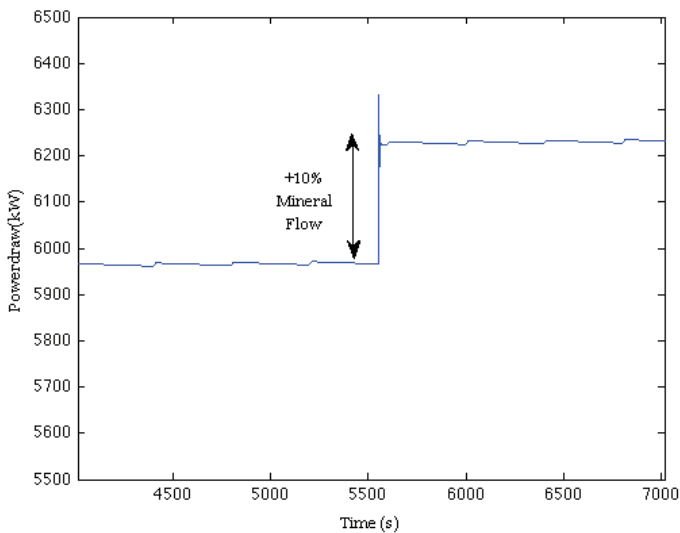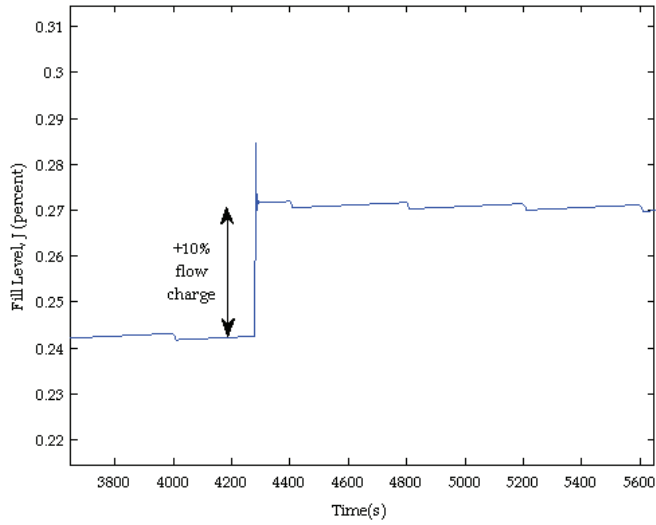

Fig. 4. Magne's model power-draw response

Fig. 5. Magne's model fill level response
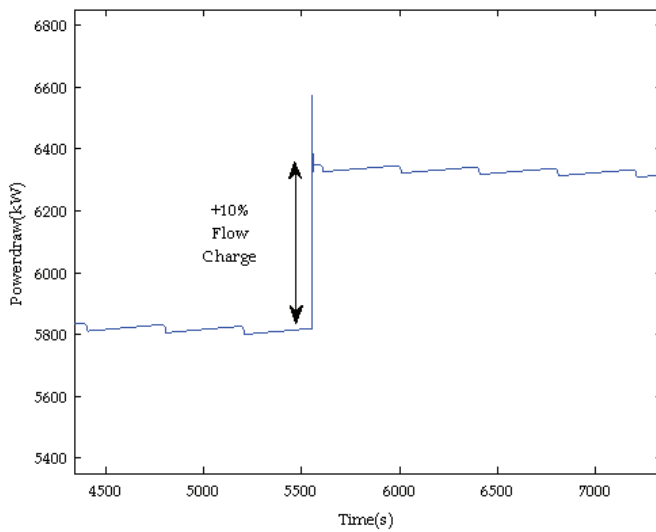
Fig. 6. Morell's model power-draw response
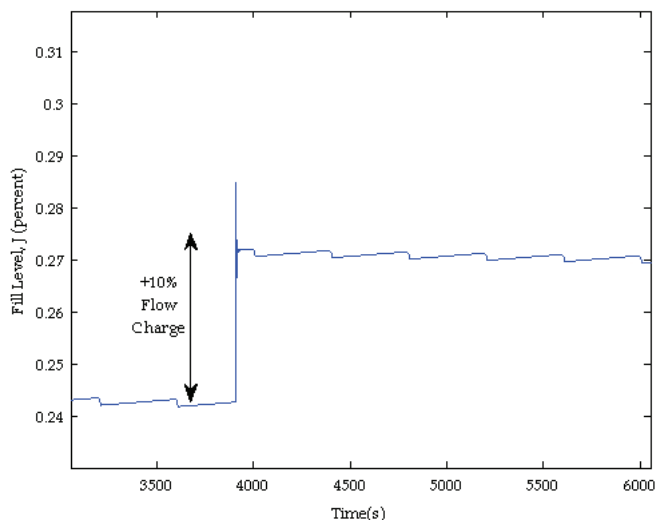
Fig. 7. Morell's model fill level response

4. Conclusion

Advanced simulation for semi-autogenous mill systems has been presented in the context of a simplified models approach that incorporated developments of (Magne et al., 1995) and Morrell (Morrell, 2004) among the others. A main focus has also been a comparison of these two models. This comparison showed that both models provided good predictive capability of two very important process variables, power draw and fill-level, especially under the same simulation conditions.

It is interesting to note that despite differences in the theoretical background for these approaches, the results of dynamic simulations under industrial operational conditions are similar. Thus, these results validate adequately the comminution process in the SAG mill, and in the future, these models could be combined for industrial purposes. With these results we believe that is possible to scale-up from pilot plant simulation and to optimise existing circuits for process control purposes using combinations of these models to reduce risks and improve performance.

5. Acknowledgment

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Dynamic Modelling

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When talking about modelling it is natural to talk about simulation. Simulation is the imitation of the operation of a real-world process or systems over time. The objective is to generate a history of the model and the observation of that history helps us understand how the real-world system works, not necessarily involving the real-world into this process. A system (or process) model takes the form of a set of assumptions concerning its operation. In a model mathematical and logical assumptions are considered, and entities and their relationship are delimited. The objective of a model – and its respective simulation – is to answer a vast number of "what-if" questions. Some questions answered in this book are: What if the power distribution system does not work as expected? What if the produced ships were not able to transport all the demanded containers through the Yangtze River in China? And, what if an installed wind farm does not produce the expected amount of energyt? Answering these questions without a dynamic simulation model could be extremely expensive or even impossible in some cases and this book aims to present possible solutions to these problems.

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