Chapter

Mathematics Dictionary: Enhancing Students' Geometrical Vocabulary and Terminology

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Abstract

Students' understanding of geometric vocabulary and terminology is still an area of concern when teaching and learning geometry. The chapter addresses the benefits of integrating mathematics dictionary and polygon pieces into the teaching and learning in order for students to be proficient in geometry. However, there is little evidence in the literature regarding teachers' integration of mathematics dictionary and polygon pieces in the teaching and learning of geometry with an aim to support students' geometrical vocabulary and terminology. Consequently, the aim of the chapter is to provide an overview of how the integration of mathematics dictionary into the teaching and learning can be promoted. Also it provides the empirical and theoretical evidence of how mathematics dictionary influences students' understandings of geometrical vocabulary and terminology. The uses of mathematics dictionary and polygon pieces modify how learners learn, from passive to hands-on, and promote visualisation, respectively. The chapter recommends that mathematics teachers integrate mathematics dictionary and polygon pieces into the teaching and learning of geometry to all students to promote independent learning.

Keywords: geometry, terminology, dictionary, integration, self-depended

1. Introduction

This chapter presents theoretically and empirically how the integration of mathematics dictionary during mathematics instruction needs to be made a reality. The emphasis of the chapter is to demystify the myth that dictionaries are for language lessons. Hence, it is for all mathematics teachers to integrate mathematics dictionaries into their lessons. At a deeper level, the chapter highlights how to integrate mathematics dictionary into the instruction of geometry to promote critical thinking and the skill of information seeking.

However, there is an existing research on the benefits of integrating mathematics dictionary into mathematics instruction [1, 2]; however, most teachers do not bother themselves integrating the dictionary into their lessons. Most teachers stick to the same traditional methods of teaching where no resources are used to enhance students' geometrical understanding that promotes critical thinking [3].

There is an increase concern that most students underperform in geometry due to vocabulary and terminology that are not well established, but teachers still give it little attention [4].

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One of the most important highlights in research is that dictionaries develop in students the use of vocabulary and terminology with their true or multiple meanings [2]. The chapter uses both theoretical and empirical evidence to demonstrate how mathematics dictionary can influence students' understanding of geometrical vocabulary and terminology which is a challenge in geometry mostly. Research shows that the reason why most students fail geometry is due to lack of well-grounded knowledge on geometrical vocabulary and terminology and its abstractness [5]. Misunderstandings and alternative conceptions which are a result of lack of proper geometrical terminology are addressed when mathematics dictionary is integrated into the lesson. However, with the integration of mathematics dictionary in the lesson students are made to be focussed, this leads to enrichment and enhancement of the mathematical success of students now and in the future [2]. Students become focused since they get empowered with mathematical vocabulary and terminology which are the essential elements for understanding geometrical concepts. The integration of mathematics dictionary into the lesson also promotes the skill of information seeking which enhances self-dependent learning.

Research has noted that one of the factors that act as a barrier to learning of mathematics by deaf or hard-of-hearing students is difficulties with language [6]. On the other hand, [7–9] argue that mathematics language and vocabulary also pose challenges to all students. The challenges are different from ordinary reading situations for the reason that they are more of mathematical terminological challenges. In support of the recent statement [10], a research study found out that the highest percentage of errors students committed emanated from the use of technical words in mathematics. According to [11–13], the improvement of mathematical vocabulary enhances students' mathematical proficiency.

However, research reveals that mathematical proficiency rests on a constant growth and balance of sophisticated components of critical element skills such as concepts, procedures, algorithms, computation, problem solving and language [14].

In order to explore the influence of mathematics dictionary on students' learning of geometry, the study was underpinned by [15] a model of geometric thinking. The model is described as follows:

> *Level 0 of geometry thinking: visualisation*. At this level, polygons are judged according to their visual characteristics where students may, for example, judge a square as not being a parallelogram.
>
> *Level 1 of geometry thinking: analysis*. At this level, students through reflection and testing of geometric shapes' characteristics gradually develop and then they use the identified characteristics to define the given shapes.
>
> *Level 2 of geometry thinking: abstraction*. At this level, the learner has an ability to order figures and interpret them one from another using properties that are arranged chronologically.
>
> *Level 3 of geometry thinking*: formal deduction. At this level a learner is considered to be at an advanced level of making meaning out of the given figures. For instance, the learner can prove situations with valid reasons.
>
> *Level 4 of geometry thinking: rigour*. At this level, students have the ability to compare systems based on diverse axioms and can study geometric concepts abstractly (p. 311).

Clements and Battista [16] suggest that beyond the levels of van Hiele, there is a pre-recognition level (level 0) of geometry thinking. The argument is that students who cannot differentiate a shape from a cluster of shapes should be considered not yet operating at the visual level of van Hiele's theory but to be considered at the pre-recognition level [16]. This contribution adds up the levels of geometric thinking to five.

Van Hiele [15] proposes that to allow the sequential transition of students' ability of geometric thinking from one level to the next, teaching and learning must be guided by the five-phase structure, namely:

Phase 1: Inquiry phase. In this phase, resources lead students to discover and realise definite features of geometric figures.

Phase 2: Direct orientation. In this phase, activities are presented in such a way that their features appear steadily to the students, i.e., through brainteasers that disclose symmetrical sections.

Phase 3: Explication. The terms are introduced, and students are encouraged to use them in their discussion and written geometry exercises.

Phase 4: Free orientation. The teacher presents a variety of activities to be done using different approaches, and this instils in students capabilities to become more skilled in what they already know.

Phase 5: Integration. Students are given opportunities to summarise what they have acquired during instruction, possibly by creating their personal activities.

2. The integration of mathematics dictionary into teaching and learning for students' geometric proficiency

2.1 Methodology

The main research study was informed by the mixed method paradigm defined by [17] as the unification of quantitative and qualitative data analysis. The mixed method approach has been utilised for the following reasons:

i. To ensure that the outcomes are instructive, comprehensive, composed and convectional [18]

ii. For triangulation which is aimed at validation, deepening and widening the understanding of the viewpoint being studied [19]

The emphasis is that the mixed method approach gives a wide range of opportunities to analyse the collected data.

The cohorts of 56 eighth grade volunteers wrote the diagnostic test with an aim:

i. To find more on students' alternative conceptions and misunderstandings regarding geometric concepts

ii. To capture and explore the students' conceptual understanding of geometry before employing the intervention

iii. To help in designing appropriate intervention strategy that focuses on addressing the identified alternative conceptions among eighth grade students in learning geometry

2.2 Diagnostic and post-test content

Both the diagnostic and post-test questions 1.1–1.5 each with its three sub-questions were aligned to different levels of the van Hiele theory of geometric thinking as presented below.

Seven questions, 1.1(i), 1.2(i), 1.3(i) and (iii), 1.4(i) and (iii) and 1.5(i), were aligned to level 0 visualisation of the van Hiele theory. The questions at level 0 visualisation provided students with opportunities to use visual skills to determine the properties of triangles and also allowed them to recognise various triangles based on their unique properties.

Question 1.3(ii) considered to be at level 1 analysis of the van Hiele theory of geometric thinking focused on students' ability to identify a geometric shape's properties given all the symbols to describe it.

Six questions, 1.1(ii) and (iii), 1.2(ii) and (iii) and 1.4 (ii) and 1.5(ii) pitched at level 2 abstraction of the van Hiele theory of geometric thinking, required students to solve problems where properties of figures and interrelationships were significant [20].

Question 1.5(iii) was set at level 3 formal deduction of the van Hiele theory for students to think logically in order to provide the properties of the given triangle. The question at this level was set to assess whether how well the learner could give sufficient conditions of a triangle without the use of polygon pieces and mathematics dictionary. For details of the diagnostic and post-test content, refer to Appendix 1.

2.3 Intervention and results

After the diagnostic test was marked, purposeful sampling of nine students was employed according to individual students' performance: three with a high percentage, three with an average percentage and three with a below average percentage. Purposeful sampling was done to target a small number of individuals in order to maximise opportunities for eliciting more in-depth data [21] about the influence of mathematics dictionary in the teaching and learning of geometry, which was the spectacle under study [22]. **Figure 1** below presents how the selected nine students performed in both tests (diagnostic and post-test).

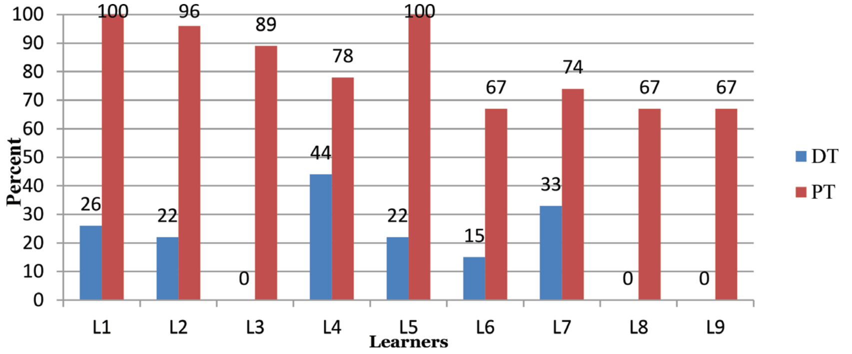

Figure 1.
The comparison of diagnostic test and the post-tests results. DT, diagnostic test; PT, post-test; L, learner.

Figure 1 shows that in the diagnostic test, all nine students involved in the research scored marks that were below 50%. However, after being engaged in the intervention activities that made use mathematics dictionary, all the students scored above 60% in the post-test.

The results of the diagnostic test helped in the designing of the nine intervention tasks which were aimed at addressing the alternative conceptions and misunderstandings that students demonstrated in geometry. In each of the planned intervention activities, students were supposed to answer each one of the questions after measuring and comparing angles and sides of the given triangles using cut polygon pieces of the same triangle.

However, to be engaged in the activities, students were to use A4 paper where triangle ABC was drawn; alongside the A4 paper, the two copies of triangles ABC were provided to every learner. **Figure 2** clarifies how the process of using the original triangle and its copies was done.

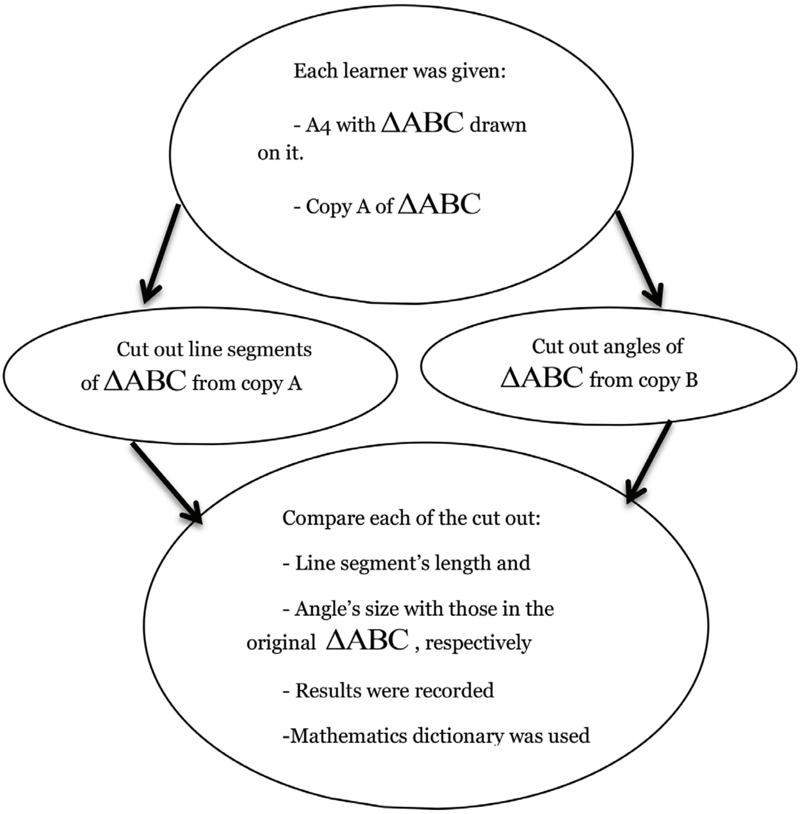

Figure 2.
The process of how the learners cut out polygon pieces during the intervention process.

In order to do the intervention activities as planned, students were supposed to answer each and every question after measuring and comparing angles and sides of the given triangles using the cutout polygon pieces. Mathematics dictionary was made available for the students to use during the process to support with vocabulary, spellings and terminology. After the intervention activities, all the nine students had to write a post-test. **Table 1** shows how students improved their responses in the post-test as compared to the diagnostic test.

Learner code	Question number	Diagnostic test students' responses	Post-test students' responses
L1	1.1(ii)	AB, parallel; AC, horizontal; BC, horizontal	The lines are not equal
L1	1.4(i)	Equal triangle	Equilateral triangle
L2	1.3(iii)	The name of a triangle is DEF	Isosceles triangle
L2	1.4(iii)	XY is bigger than YZ and XZ is bigger than XY	XY is equal to YZ and XZ
L3	1.1(ii)	It is small side and A bigger side AC small bigger from A to another A deduce about size	$\hat{A}$ is smaller than $\hat{C}$
L4	1.1(iii)	A and B are equal and C is less than A and B	It is a scalene triangle
L4	1.2(iii)	Triangular prism	It is a scalene triangle
L4	1.5(i)	It is an equilateral triangle because all sides are equal	Isosceles triangle
L5	1.1(i)	$\hat{A}$ is longer than $\hat{C}$ A is bigger than C	$\hat{A}$ is smaller than $\hat{C}$
L5	1.2(iii)	2 dimensional shapes	Scalene triangle
L5	1.3(iii)	2 dimensional shapes	It is a scalene triangle
L5	1.4(i)	2D shape	Equilateral triangle
L5	1.5(i)	2D shape	Isosceles triangle
L5	1.5(ii)	They were not equal, and they are used to make a shape	They are equal in size
L6	1.1(iii)	AB are associated, AC are the convection and BC are associated	It is the scalene triangle
L6	1.4(i)	It is triangular	It is the equilateral triangle
L7	14(i)	It is an equal triangle	Equilateral triangle
L8	1.4(i)	Triangular	Equilateral triangle
L9	1.3(iii)	D is 4 cm and F is 3 cm	Two angles are equal
L9	1.4(i)	Rectangle	Equilateral Δ

Table 1.
Students' responses to questions in both diagnostic test and post-test.

Table 1 shows how the use of mathematics dictionary enhanced students' comprehension of geometric concepts in the post-test as compared to the diagnostic test responses. To cite few examples, L1 and L7 when responding to questions 1.4(i) in a diagnostic test incorrectly answered, equal triangle, but after being engaged in the intervention activities that integrated mathematics dictionary the response was now correct, equilateral triangle.

After the intervention activities, all nine students were involved in a semi-structure interview just to find out their views on the use of mathematics dictionary in learning geometry; below are some few direct quotations:

> L3: *I like the programme that made use of mathematics dictionary because it helped me to understand mathematics concepts that I did not understand before.*
>
> L4: *Yes, sir, I got a clear picture because now I clearly understand the concepts of triangles and their properties.*
>
> L7: *I have learnt that: in a scalene triangle all side are not equal, in an isosceles two opposite side are equal and there are two angles equal, an equilateral, all the angles are equal in size the same as the sides, they are equal in length and also how to spell correctly the names of triangles*.

> L8: *I did not understand the properties of an obtuse angled triangle. Even the slashes that are used to show that two opposite sides of an isosceles triangle are equal. I did not know the meaning of such slashes, but now after your programme it is clear to me, the dictionary and shape cutting helped me a lot.*

The four students cited above echo the sentiments that the use of mathematics dictionary in the learning of geometry has empowered them to understand some of the concepts that were a problem to them as they moved throughout the grades. They have emphasised that the definitions, symbols and properties of different triangles were also addressed in the process of engaging with the dictionary, issues that were identified by [10] as challenges for deaf students only.

The citation by L8 shows how the integration of mathematics dictionary and polygon piece into their learning helped them to experience hands-on learning and

Question number in both DT and PT	van Hiele's levels of geometric thinking for each of the questions	Students' codes	Did students achieve questions in the DT at a given van Hiele level of geometric thinking?	Did students achieve questions in the PT at a given van Hiele level of geometric thinking?
1.1 (i) (ii) (iii)	Level 0	L: 3, 4, 5, 6 and 8	No	Yes
	Level 2	L: 1, 3, 6, 7 and 9	No	Yes
	Level 2	L: 1, 2, 3, 4, 5, 6, 7 and 8	No	Yes
1.2 (i) (ii) (iii)	Level 0	L: 1, 3, 4, 5, 6, 7 and 8	No	Yes
	Level 2	L: 2, 3, 4, 5, 6, 7 and 8	No	Yes
	Level 2	L: 1, 3, 5, 6, 7 and 9	No	Yes
1.3(i) (ii) (iii)	Levels 0 and 3	L: 2, 5, 8 and 9	No	Yes
	Level 1	L: 2, 3, 7, 8 and 9	No	Yes
	Level 0	L: 1, 3, 5, 6 and 9	No	Yes
1.4(i) (ii) (iii)	Level 0	L: 3, 5, 6, 7, 8 and 9	No	Yes
	Levels 2 and 3	L: 1, 2, 3, 5, 6, 8 and 9	No	Yes
	Level 0	L: 1, 2, 3, 6, 8 and 9	No	Yes
1.5(i) (ii) (iii)	Level 0	L: 1, 2, 3, 4 and 5	No	Yes
	Level 2	L: 1, 2, 3, 5, 8 and 9	No	Yes
	Levels 0 and 3	L: 1, 2, 3, 5, 6, 7 and 8	No	Yes

Table 2.
A summary of the students' responses to diagnostic test and post-test.

visualisation of geometric concepts before the answer was established. However, it is apparent that students gained other skills in the process of using dictionary and cutting polygon pieces, for example, the skills of looking up for a word in the dictionary, observation skills, psychomotor skills (established as they were cutting the angles and line segments) and measurement skills (as they were comparing one angle to another and one line segment to another). The intervention activities students engaged in made them to realise that learning is about active participation. In the process students gained the knowledge of how to use mathematics dictionary and polygon pieces to learn about geometry, which is a component of metacognition. However, students came to realise that properties of triangles are established through hands-on and self-dependent learning mediated by mathematics dictionary and polygon pieces.

A summarised version of students' results per question is presented in **Table 2**.

Table 2 presented shows how each of the students managed to achieve questions at various levels of van Hiele theory of geometric thinking after utilising mathematics dictionary in their learning. The questions were set to test students' geometrical proficiency which includes geometric vocabulary, terminology and conceptual understanding. As shown in **Table 2**, no learner managed to perform at the set level in the diagnostic test; these results are in agreement with those of [13] who found out that the comprehension of mathematical language enhances mathematical proficiency in students. This implies that if a learner's geometrical vocabulary is not clear, he/she cannot be able to perform well in geometry.

To improve students' understanding of geometry, it calls for teachers to integrate mathematics dictionary into teaching and learning. The integration of dictionary into the teaching and learning of geometry can help to deal with the five common problem areas identified by [10] as prevalent cases in deaf students: "words with multiple meanings, technical vocabulary, words with specialized importance in mathematics, varied but related forms; and abbreviations and specialized symbols" (p. 419). Yet, in reality the challenges are also dominant in all other students.

3. Conclusion

This chapter has given an empirical and theoretical account of and the reasons for the need to integrate mathematics dictionary into the teaching and learning of geometry. The study has established that students' independent learning and understanding of geometrical vocabulary and terminology were heavily influenced by the use of mathematics dictionary and polygon pieces mentioned in **Figure 2**. The integration of the two, mathematics dictionary and polygon pieces, into teaching and learning helped students to modify their way of learning geometrical vocabulary and terminology unlike in a traditional classroom where no hands-on learning tasks are executed. However, the availability of the dictionary in the lesson made students aware that once they were stuck with understanding of geometric vocabulary and terminology, mathematics dictionary had the answers. During the lesson students would make use of mathematics dictionary without being told what to do with it.

As highlighted in **Figure 2**, the use of polygon pieces also promoted independent learning in students in the sense that they were engaged in the cutting of line segments and angles of a particular triangle in order to establish its properties. The cutting and measuring of line segments and angles promote independent learning for the reason that no student was told how an isosceles or a scalene triangle looks like. No student was told how to do the cutting of polygon pieces; each one independently devised a way of cutting line segments, so that they were not half-way cut. Half-way cut line segment could not give students opportunity to execute the task of finding properties of triangles in terms of line dimensions. Each and every response was established through practical investigation and solution written down; hence, this promoted independent learning.

How the use of mathematics dictionary and polygon pieces nurtured the students' independent learning is reflected in how most students responded to the post-test. **Figure 2** attests to this. The errors that students committed in the diagnostics test, for example, spellings and failing to name triangles properly, after they were engaged in the use of the mathematics dictionary and polygon pieces were corrected in the post-test (refer to **Table 1**). This chapter contributes additional evidence that suggested how the dictionary and polygon pieces can be integrated into teaching and learning to promote independent learning. However, I highly recommend that mathematics teachers integrate the use of mathematics dictionary and polygon pieces into their lessons.

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Conflict of interest

I declare that no financial or personal relationships have inappropriately influenced me in writing this book chapter.

Disclaimer

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Appendix 1. Diagnostic and post-test content

Instructions

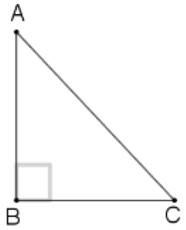
i. Answer all the questions.

ii. Write neatly.

iii. Provide your answers on the spaces provided under each question.

Question 1.
Study the 2D figures below and then answer the questions that.
follow:

1.1



i. What can you deduce about the sizes of $\hat{A}$ and $\hat{C}$?

ii. What are the properties of triangle ABC in terms of $\overline{AB}$, $\overline{AC}$ and $\overline{BC}$?

iii. According to answers in 1.1. (i) and (ii), what specific name is given to a shape with the properties mentioned above?

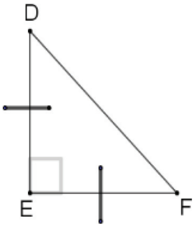
1.2

i. What are the properties of the triangle GHI in terms of $\hat{G}$, $\hat{H}$ and $\hat{I}$?

ii. Determine the properties of triangle GHI in terms of $\overline{GH}$, $\overline{HI}$ and$\overline{GI}$.

iii. What name is given to a triangle with such properties?

1.3

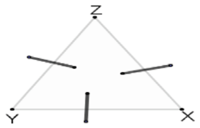


i. Write down the size of each of the following angles $\hat{D}$ and $\hat{F}$.

ii. Determine the length of $\overline{EF}$, $\overline{DE}$ and $\overline{DF}$; use terms like shorter, longer than, equal and the longest of all.

iii. What name is given to triangle DEF?

1.4

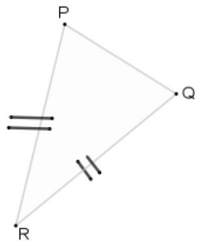


i. What type of a triangle is drawn above?

ii. Determine the size of $\hat{X}$, $\hat{Y}$ and $\hat{Z}$.

iii. Write down the length of $\overline{XY}$, $\overline{YZ}$ and $\overline{XZ}$; use terms like shorter, longer than, equal and the longest of all.

1.5



i. What name is given to triangle PQR?

ii. What is the relationship between $\hat{Q}$ and $\hat{P}$?

iii. What can you conclude about the properties of triangle PQR?

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