

Power Flow Analysis

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Abstract

Power flow, or load flow, is widely used in power system operation and planning. The power flow model of a power system is built using the relevant network, load, and generation data. Outputs of the power flow model include voltages at different buses, line flows in the network, and system losses. These outputs are obtained by solving nodal power balance equations. Since these equations are nonlinear, iterative techniques such as the Newton-Raphson, the Gauss-Seidel, and the fast-decoupled methods are commonly used to solve this problem. The problem is simplified as a linear problem in the DC power flow technique. This chapter will provide an overview of different techniques used to solve the power flow problem.

Keywords: power flow, load flow, iterative techniques, Newton-Raphson, Gauss-Seidel, fast-decoupled, DC power flow

1. Problem formulation

Power flow analysis is a fundamental study discussed in any power system analysis textbook such as [1–6]. The objective of a power flow study is to calculate the voltages (magnitude and angle) for a given load, generation, and network condition. Once voltages are known for all buses, line flows and losses can be calculated. The starting point of solving power flow problems is to identify the known and unknown variables in the system. Based on these variables, buses are classified into three types: slack, generation, and load buses as shown in **Table 1**.

The slack bus is required to provide the mismatch between scheduled generation the total system load including losses and total generation. The slack bus is commonly considered as the reference bus because both voltage magnitude and angles are specified; therefore, it is called the swing bus. The rest of generator buses are called regulated or PV buses because the net real power is specified and voltage magnitude is regulated. Most of the buses in practical power systems are load buses. Load buses are called PQ buses because both net real and reactive power loads are specified.

For PQ buses, both voltage magnitudes and angles are unknown, whereas for PV buses, only the voltage angle is unknown. As both voltage magnitudes and angles are specified for the Slack bus, there are no variables that must be solved for. In a system with n buses and g generators, there are $2(n-1)-(g-1)$ unknowns. To solve these unknowns, real and reactive power balance equations are used. To write these equations, the transmission network is modeled using the admittance matrix (Y-bus).

Bus type	Voltage ($ V_i \angle \delta_i$)		Real power			Reactive power		
	Magnitude	Angle	Generation	Load	Net (P_i)	Generation	Load	Net (Q_i)
Slack/ Swing	Specified	Specified	Unknown	Specified	Unknown	Unknown	Specified	Unknown
Generator/ Regulated/ PV	Specified	Unknown	Specified	Specified	Specified	Unknown	Specified	Unknown
Load/PQ	Unknown	Unknown	Specified	Specified	Specified	Specified	Specified	Specified

Table 1.
Type of buses in the power flow problem.

2. Admittance matrix and power flow equation

The admittance matrix of a power system is an abstract mathematical model of the system. It consists of admittance values of both lines and buses. The Y-bus is a square matrix with dimensions equal to the number of buses. This matrix is symmetrical along the diagonal.

$$Y = \begin{bmatrix} Y_{11} & \cdots & Y_{1n} \\ \vdots & \ddots & \vdots \\ Y_{n1} & \cdots & Y_{nn} \end{bmatrix} \quad (1)$$

The values of diagonal elements (Y_{ii}) are equal to the sum of the admittances connected to bus i . The off-diagonal elements (Y_{ij}) are equal to the negative of the admittance connecting the two buses i and j . It is worth noting that with large systems, Y-bus is a sparse matrix.

$$Y_{ii} = \sum_{\substack{j=0 \\ j \neq i}}^n y_{ij} \quad (2)$$

$$Y_{ij} = Y_{ji} = -y_{ij} \quad (3)$$

The net injected power at any bus can be calculated using the bus voltage (V_i), neighboring bus voltages (V_j), and admittances between the bus and its neighboring buses (y_{ij}) as shown in **Figure 1**.

$$I_i = V_i y_{i0} + (V_i - V_1) y_{i1} + (V_i - V_2) y_{i2} + \dots + (V_i - V_j) y_{ij}$$

Rearranging the elements as a function of voltages, the current equation becomes as follows:

$$I_i = V_i (y_{i0} + y_{i1} + y_{i2} + \dots + y_{ij}) - V_1 y_{i1} - V_2 y_{i2} - \dots - V_j y_{ij}$$

$$I_i = V_i \sum_{\substack{j=0 \\ j \neq i}} y_{ij} - \sum_{\substack{j=1 \\ j \neq i}} y_{ij} V_j = V_i Y_{ii} + \sum_{\substack{j=1 \\ j \neq i}} Y_{ij} V_j$$

The power equation at any bus can be written as follows:

$$S_i = P_i + jQ_i = V_i I_i^*$$

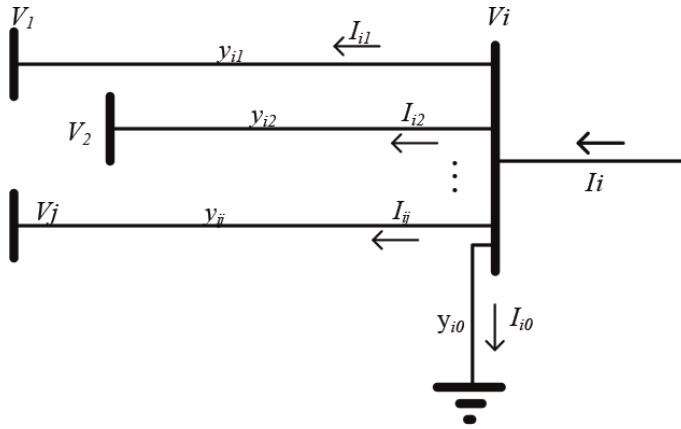


Figure 1.
 Net injected power.

Or

$$S_i^* = P_i - jQ_i = V_i^* I_i$$

Substituting the expression on the current in S_i^* equation results in the following formula:

$$S_i^* = V_i^* \left(V_i \sum_{\substack{j=0 \\ j \neq i}} y_{ij} - \sum_{\substack{j=1 \\ j \neq i}} y_{ij} V_j \right) = V_i^* \left(V_i Y_{ii} + \sum_{\substack{j=1 \\ j \neq i}} Y_{ij} V_j \right)$$

Real and reactive power can be calculated from the following equations:

$$P_i = \text{Re} \left\{ V_i^* \left(V_i \sum_{\substack{j=0 \\ j \neq i}} y_{ij} - \sum_{\substack{j=1 \\ j \neq i}} y_{ij} V_j \right) \right\} = \text{Re} \left\{ V_i^* \left(V_i Y_{ii} + \sum_{\substack{j=1 \\ j \neq i}} Y_{ij} V_j \right) \right\}$$

$$Q_i = -\text{Im} \left\{ V_i^* \left(V_i \sum_{\substack{j=0 \\ j \neq i}} y_{ij} - \sum_{\substack{j=1 \\ j \neq i}} y_{ij} V_j \right) \right\} = -\text{Im} \left\{ V_i^* \left(V_i Y_{ii} + \sum_{\substack{j=1 \\ j \neq i}} Y_{ij} V_j \right) \right\}$$

Or

$$P_i = \sum_{j=1}^n |V_i| |V_j| |Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) \quad (4)$$

$$Q_i = - \sum_{j=1}^n |V_i| |V_j| |Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \quad (5)$$

And the current (I_i) can be written as a function of the power as follows:

$$\frac{P_i - jQ_i}{V_i^*} = V_i \sum_{\substack{j=0 \\ j \neq i}} y_{ij} - \sum_{\substack{j=1 \\ j \neq i}} y_{ij} V_j = V_i Y_{ii} + \sum_{\substack{j=1 \\ j \neq i}} Y_{ij} V_j \quad (6)$$

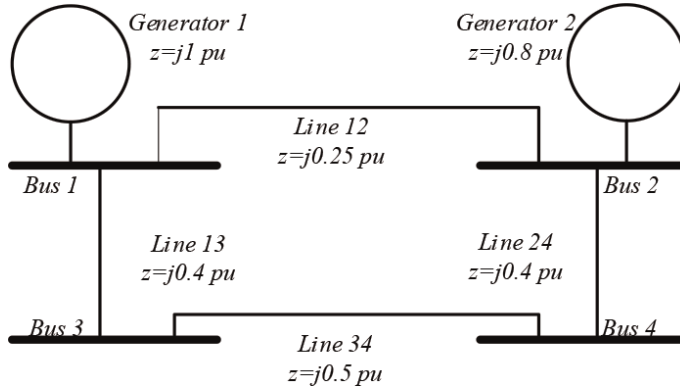


Figure 2.
Impedance diagram.

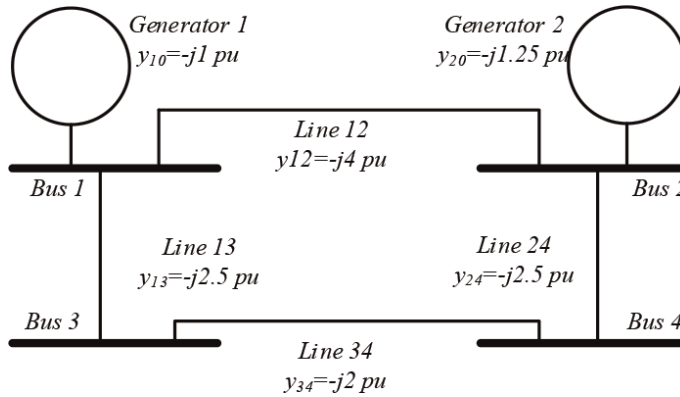


Figure 3.
Admittance diagram.

Example 1: Admittance matrix formation.

For the below 4-bus system in **Figure 2**, the admittance matrix is constructed by converting all impedances in the system into admittances as shown in **Figure 3**. Then, diagonal and off-diagonal elements are calculated using Eqs. (2) and (3).

$$Y = \begin{bmatrix} -j7.5 & j4 & j2.5 & 0 \\ j4 & -j7.75 & 0 & j2.5 \\ j2.5 & 0 & -j4.5 & j2 \\ 0 & j2.5 & j2 & -j4.5 \end{bmatrix} = \begin{bmatrix} 7.5 \angle -90^\circ & 4 \angle 90^\circ & 2.5 \angle 90^\circ & 0 \\ 4 \angle 90^\circ & 7.75 \angle -90^\circ & 0 & 2.5 \angle 90^\circ \\ 2.5 \angle 90^\circ & 0 & 4.5 \angle -90^\circ & 2 \angle 90^\circ \\ 0 & 2.5 \angle 90^\circ & 2 \angle 90^\circ & 4.5 \angle -90^\circ \end{bmatrix}$$

3. Gauss-Seidel technique

The Gauss-Seidel (GS) method, also known as the method of successive displacement, is the simplest iterative technique used to solve power flow problems. In general, GS method follows the following iterative steps to reach the solution for the function $f(x) = 0$:

- Rearrange the function into the form $x = g(x)$ to calculate the unknown variable.

- Calculate the value $g(x^{[0]})$ based on initial estimates $x^{[0]}$.
- Calculate the improved value $x^{[1]} = g(x^{[0]})$.
- Continue solving for improved values until the solution is within acceptable limits $|x^{[k+1]} - x^{[k]}| \leq \epsilon$.

The rate of convergence can be improved using acceleration factors by modifying the step size α .

$$x^{[k+1]} = x^{[k]} + \alpha [g(x^{[k+1]}) - x^{[k]}] \quad (7)$$

In the context of a power flow problem, the unknown variables are voltages at all buses, but the slack. Both voltage magnitudes and angles are unknown for load buses, whereas voltage angles are unknown for regulated/generation buses.

The voltage V_i at bus i can be calculated using either equations:

$$V_i = \frac{1}{\sum_{\substack{j=1 \\ j \neq i}} y_{ij}} \left(\frac{P_i^{sch} - jQ_i^{sch}}{V_i^*} + \sum_j y_{ij} V_j \right) \quad (8)$$

$$V_i = \frac{1}{Y_{ii}} \left(\frac{P_i^{sch} - jQ_i^{sch}}{V_i^*} - \sum_{\substack{j=1 \\ j \neq i}} Y_{ij} V_j \right) \quad (9)$$

where y_{ij} is the admittance between buses i and j , Y_{ij} is the Y-bus element, P_i^{sch} the net scheduled injected real power, Q_i^{sch} is the net scheduled injected reactive power, and V_i^* is the conjugate of V_i . The net injected quantities are the sum of the generation minus load. Typically, the initial estimates of $V_i = 1 \angle 0^\circ$.

The iterative voltage equation is as follows:

$$V_i^{[k+1]} = \frac{1}{Y_{ii}} \left(\frac{P_i^{sch} - jQ_i^{sch}}{V_i^{*[k]}} - \sum_{j \neq i} Y_{ij} V_j^{[k \text{ or } k+1]} \right) \quad (10)$$

Or

$$V_i^{[k+1]} = \frac{1}{\sum_{j=0} y_{ij}} \left(\frac{P_i^{sch} - jQ_i^{sch}}{V_i^*} + \sum_{j \neq i} y_{ij} V_j^{[k \text{ or } k+1]} \right) \quad (11)$$

Both real and reactive powers are scheduled for the load buses, and Eq. 6 is used to determine both voltage magnitudes and angles ($|V_i| \angle \delta$) for every iteration ($V_i^{[k+1]}$).

For regulated buses, only real power is scheduled. Therefore, net injected reactive power is calculated based on the iterative voltages ($V_i^{[k+1]}$) using either equations:

$$Q_i^{[k+1]} = -Im \left\{ V_i^{*[k]} \left(V_i^{[k]} \sum_{j=0}^n y_{ij} - \sum_{\substack{j=1 \\ j \neq i}}^n y_{ij} V_j^{[k \text{ or } k+1]} \right) \right\} \quad (12)$$

$$Q_i^{[k+1]} = - \sum_{j=1}^n |V_i|^{[k]} |V_j|^{[k \text{ or } k+1]} |Y_{ij}| \sin \left(\theta_{ij} - \delta_i^{[k]} + \delta_j^{[k]} \right) \quad (13)$$

where $|V_i|$ and $|V_j|$ are the magnitudes of the voltage at buses i and j , respectively. δ_i and δ_j are the associated angles. y_{ij} is the admittance between buses i and j . $|Y_{ij}|$ is the magnitude of the Y-bus element between the two buses; and θ_{ij} is the corresponding angle.

Since the voltage magnitude ($|V_i|$) is specified at regulated/PV buses, Eqs. (8) or (9) will be used to determine the voltage angles only. To achieve this, two options can be used:

1. When using the polar form ($|V_i| \angle \delta_i$), discard the iterative voltage magnitude and keep the iterative angle.
2. When using the rectangular form ($Re_i + j Im_i$), discard the real part (Re_i) and keep the imaginary part (Im_i) of the iterative voltage. The new real part (Re_i^{new}) can be calculated from the specified magnitude ($|V_i|$) and the iterative imaginary part.

$$Re_i^{new} = \sqrt{|V_i|^2 - Im_i^2} \quad (14)$$

The iterative process stops when the voltage improvement reaches acceptable limits: $|V_i^{[k+1]} - V_i^{[k]}| \leq \epsilon$.

Example 2: Gauss-Seidel power flow solution.

Figure 4 below shows a 3-bus system. Perform 2 iterations to obtain the voltage magnitude and angles at buses 2 and 3. Impedances are given on 100 MVA base.

Solution:

The admittance values of the transmission network and the injected power in per unit at buses 2 and 3 are calculated as shown in **Figure 5**. Note that net injected power at the load bus is negative while that of the PV bus is positive. Per units values are obtained by dividing actual values (MW and MVAR) by the base (100 MVA).

Iteration #1: assume $V_2^{[0]} = 1.00 \angle 0^\circ$ and $V_3^{[0]} = 1.03 \angle 0^\circ$.

$$\begin{aligned} V_2^{[1]} &= \frac{1}{y_{21} + y_{23}} \left(\frac{P_2^{sch} - jQ_2^{sch}}{V_2^{[0]*}} + y_{21}V_1 + y_{23}V_3^{[0]} \right) \\ &= \frac{1}{(5 - j15) + (15 - j50)} \left(\frac{-2 + j0.5}{1.00 \angle 0^\circ} + (5 - j15)1.02 \angle 0^\circ + (15 - j50)1.00 \angle 0^\circ \right) \\ V_2^{[1]} &= 1.0120 - j0.0260 = 1.0123 \angle -1.4717^\circ \end{aligned}$$

As Q_3 is not given, it is calculated based on the latest available information using Eqs. (12) or (13).

$$\begin{aligned} Q_3^{[1]} &= -Im \left\{ V_3^{*[0]} \left(V_3^{[0]} \sum_{j=0}^n y_{3j} - \sum_{\substack{j=1 \\ j \neq i}}^n y_{3j} V_j^{[k \text{ or } k+1]} \right) \right\} \\ Q_3^{[1]} &= -Im \left\{ V_3^{*[0]} \left(V_3^{[0]} (y_{31} + y_{32}) - [y_{31}V_1 + y_{32}V_2^{[1]}] \right) \right\} \end{aligned}$$

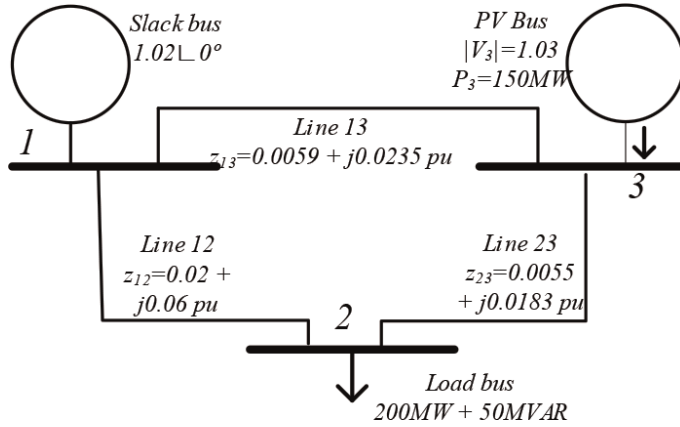


Figure 4.
3-Bus power system.

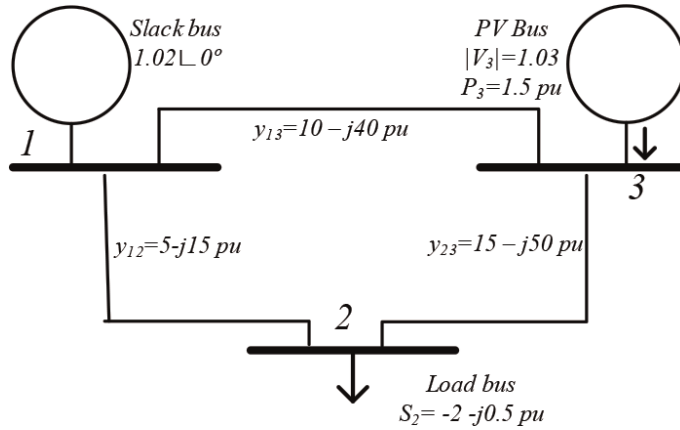


Figure 5.
Power flow input data.

$$Q_3^{[1]} = -\text{Im} \{ 1.03 \angle 0^\circ (1.03 \angle 0^\circ (10 - j40 + 15 - j50) - [(10 - j40)1.02 \angle 0^\circ + (15 - j50)(1.0120 - j0.0260)]) \}$$

$$= -\text{Im} \{ 1.7201 - j0.9373 \} = 0.9373 \text{ pu}$$

Now that $Q_3^{[1]}$ is calculated, the voltage $V_3^{[1]}$ can be calculated:

$$V_3^{[1]} = \frac{1}{y_{31} + y_{32}} \left(\frac{P_3^{sch} - jQ_3^{[1]}}{V_3^{[0]*}} + y_{31}V_1 + y_{32}V_2^{[1]} \right)$$

$$= \frac{1}{10 - j40 + 15 - j50} \left(\frac{1.5 - j0.9373}{1.03 \angle 0^\circ} + (10 - j40)1.02 \angle 0^\circ + (15 - j50)(1.0120 - j0.0260) \right)$$

$$= 1.0294 - j0.0022 \text{ pu}$$

Since the magnitude of V_3 is specified, we retain the imaginary part of $V_3^{[1]}$ and calculate the real part using Eq. (14).

$$Re_i^{new} = \sqrt{1.03^2 - 0.0022^2} = 1.03 \text{ pu}$$

Therefore,

$$V_3^{[1]} = V_3^{[1]} = 1.03 - j0.0022 = 1.03 \angle -0.1226^\circ \text{ pu}$$

Iteration #2: considering $V_2^{[1]} = 1.0120 - j0.0260$ and $V_3^{[1]} = 1.03 - j0.0022$.

$$\begin{aligned} V_2^{[2]} &= \frac{1}{y_{21} + y_{23}} \left(\frac{P_2^{sch} - jQ_2^{sch}}{V_2^{[1]*}} + y_{21}V_1 + y_{23}V_3^{[1]} \right) \\ &= \frac{1}{(5 - j15) + (15 - j50)} \left(\frac{-2 + j0.5}{1.0120 - j0.0260} + (5 - j15)1.02 \angle 0^\circ + (15 - j50)(1.03 - j0.0022) \right) V_2^{[2]} \\ &= 1.0115 - j0.0270 = 1.0119 \angle -1.5273^\circ \end{aligned}$$

$Q_3^{[2]}$ calculation is given below:

$$\begin{aligned} Q_3^{[2]} &= -Im \left\{ V_3^{*[1]} \left(V_3^{[1]} (y_{31} + y_{32}) - [y_{31}V_1 + y_{32}V_2^{[2]}] \right) \right\} \\ Q_3^{[2]} &= -Im \left\{ (1.03 - j0.0022) \left((1.03 - j0.0022)(10 - j40 + 15 - j50) \right. \right. \\ &\quad \left. \left. - [(10 - j40)1.02 \angle 0^\circ + (15 - j50)(1.0115 - j0.0270)] \right) \right\} \\ &= 1.0000 \text{ pu} \end{aligned}$$

The voltage $V_3^{[2]}$ is calculated as follows:

$$\begin{aligned} V_3^{[2]} &= \frac{1}{y_{31} + y_{32}} \left(\frac{P_3^{sch} - jQ_3^{[2]}}{V_3^{[1]*}} + y_{31}V_1 + y_{32}V_2^{[2]} \right) \\ &= \frac{1}{10 - j40 + 15 - j50} \left(\frac{1.5 - j1}{1.03 \angle 0^\circ} + (10 - j40)1.02 \angle 0^\circ + (15 - j50)(1.0115 - j0.0270) \right) \\ &= 1.0298 - j0.0030 \text{ pu} \end{aligned}$$

Only imaginary value of the calculated $V_3^{[2]}$ is retained and the real part is calculated based on the retained imaginary values and the scheduled $|V_3|$.

$$Re_i^{new} = \sqrt{1.03^2 - 0.0030^2} = 1.03 \text{ pu}$$

Therefore,

$$V_3^{[2]} = 1.03 - j0.003 = 1.03 \angle -0.1644^\circ \text{ pu}$$

The iterative solution is presented in **Table 2**.

Iteration	$V_2^{[k]}$	$V_3^{[k]}$
1	1.0123 \angle -1.4717°	1.03 \angle -0.1226°
2	1.0119 \angle -1.5273°	1.03 \angle -0.1644°
3	1.0119 \angle -1.5598°	1.03 \angle -0.1846°
4	1.0119 \angle -1.5750°	1.03 \angle -0.1941°
5	1.0118 \angle -1.5823°	1.03 \angle -0.1986°
6	1.0118 \angle -1.5857°	1.03 \angle -0.2008°

Table 2.
Gauss-Seidel iterative solution.

4. Newton-Raphson technique

The Newton-Raphson (N-R) technique, also known as the method of successive approximation, is based on Taylor's expansion approximation. The unknown x in the function $f(x) = c$ can be determined using Taylor's expansion approximation. Starting with an initial estimate $x^{[0]}$, the deviation from the correct solution is $\Delta x^{[0]}$. Applying Taylor's expansion, the function can be written as follows:

$$f(x^{[0]} + \Delta x^{[0]}) = c \quad (15)$$

$$f(x^{[0]} + \Delta x^{[0]}) = f(x^{[0]}) + f'(x^{[0]})\Delta x^{[0]} + \frac{1}{2!}f''(x^{[0]})\Delta x^{[0]^2} + \frac{1}{3!}f'''(x^{[0]})\Delta x^{[0]^3} + \dots = c \quad (16)$$

Assuming $\Delta x^{[0]}$ is small, the higher order terms ($\frac{1}{2!}f''(x^{[0]})\Delta x^{[0]^2} + \frac{1}{3!}f'''(x^{[0]})\Delta x^{[0]^3} + \dots$) are neglected and the function can be approximated by the first two terms.

$$f(x^{[0]} + \Delta x^{[0]}) \approx f(x^{[0]}) + f'(x^{[0]})\Delta x^{[0]} = c \quad (17)$$

Based on $x^{[0]}$, the deviation from the correct solution can be iteratively calculated.

$$\Delta x^{[0]} = \frac{c - f(x^{[0]})}{f'(x^{[0]})} = \frac{\Delta f(x^{[0]})}{f'(x^{[0]})} \quad (18)$$

The improved solution can be calculated iteratively.

$$x^{[1]} = x^{[0]} + \Delta x^{[0]} \quad (19)$$

The iterative process is stopped when the mismatch between scheduled and calculated value ($\Delta f^{[k]} = c - f(x^{[k]})$) is within acceptable limits $|\Delta f^{[k]}| \leq \epsilon$.

In the context of power flow problems, unknown variables x are both voltage magnitude and angles ($|V_i| \angle \delta_i$) at load buses, as well as voltage angles (δ_i) at regulated buses. The scheduled (specified) quantities (c) are both net real (P_i^{sch}) and reactive power (Q_i^{sch}) values at load buses and real power at generation buses as shown in **Table 1**. The iterative values of reactive power are calculated using Eqs. (12) or (13). Similarly, the iterative values of real power are calculated using Eqs. (20) or (21):

$$P_i^{[k+1]} = Re \left\{ V_i^{*[k]} \left(V_i^{[k]} \sum_{j=0}^n y_{ij} - \sum_{\substack{j=1 \\ j \neq i}}^n y_{ij} V_j^{[k \text{ or } k+1]} \right) \right\} \quad (20)$$

$$P_i^{[k+1]} = \sum_{j=1}^n |V_i|^{[k]} |V_j|^{[k \text{ or } k+1]} |Y_{ij}| \cos(\theta_{ij} - \delta_i^{[k]} + \delta_j^{[k]}) \quad (21)$$

The Newton-Raphson power flow formulation can be written as follows:

$$x^{[k]} = \begin{bmatrix} \delta_i^{[k]} \\ |V_i|^{[k]} \end{bmatrix}, c = \begin{bmatrix} P_i^{sch} \\ Q_i^{sch} \end{bmatrix}, f = \begin{bmatrix} P_i^{[k]} \\ Q_i^{[k]} \end{bmatrix}, \text{ and } \begin{bmatrix} P_i^{sch} \\ Q_i^{sch} \end{bmatrix} - \begin{bmatrix} P_i^{[k]} \\ Q_i^{[k]} \end{bmatrix} = \begin{bmatrix} \Delta P_i^{[k]} \\ \Delta Q_i^{[k]} \end{bmatrix}.$$

When more variables are used, the derivative f' is replaced by partial derivatives with respect to different variables. The partial derivative matrix is called the Jacobian matrix.

$$f' = \begin{bmatrix} \frac{\partial P_i}{\partial \delta_i} & \frac{\partial P_i}{\partial |V_i|} \\ \frac{\partial Q_i}{\partial \delta_i} & \frac{\partial Q_i}{\partial |V_i|} \end{bmatrix} = \begin{bmatrix} J_{P\delta} & J_{P|V|} \\ J_{Q\delta} & J_{Q|V|} \end{bmatrix} \quad (22)$$

Therefore, the Newton-Raphson power flow formulation can be solved using the below equation:

$$\begin{bmatrix} P_i^{sch} \\ Q_i^{sch} \end{bmatrix} - \begin{bmatrix} P_i^{[k]} \\ Q_i^{[k]} \end{bmatrix} = \begin{bmatrix} \Delta P_i^{[k]} \\ \Delta Q_i^{[k]} \end{bmatrix} = \begin{bmatrix} \frac{\partial P_i^{[k]}}{\partial \delta_i} & \frac{\partial P_i^{[k]}}{\partial |V_i|} \\ \frac{\partial Q_i^{[k]}}{\partial \delta_i} & \frac{\partial Q_i^{[k]}}{\partial |V_i|} \end{bmatrix} \begin{bmatrix} \Delta \delta_i^{[k]} \\ \Delta |V_i|^{[k]} \end{bmatrix} \quad (23)$$

To solve for the deviation, the inverse of the Jacobian matrix is required for every iteration.

$$\begin{bmatrix} \Delta \delta^{[k]} \\ \Delta |V_i|^{[k]} \end{bmatrix} = \begin{bmatrix} J_{P\delta}^{[k]} & J_{P|V|}^{[k]} \\ J_{Q\delta}^{[k]} & J_{Q|V|}^{[k]} \end{bmatrix}^{-1} \begin{bmatrix} \Delta P_i^{[k]} \\ \Delta Q_i^{[k]} \end{bmatrix} \quad (24)$$

Then, new values are calculated:

$$\begin{bmatrix} \delta_i^{[k]} \\ |V_i|^{[k]} \end{bmatrix} = \begin{bmatrix} \delta_i^{[k-1]} \\ |V_i|^{[k-1]} \end{bmatrix} + \begin{bmatrix} \Delta \delta_i^{[k]} \\ \Delta |V_i|^{[k]} \end{bmatrix} \quad (25)$$

The iterative process stops when the mismatch between calculated and scheduled quantities is within $\begin{bmatrix} \Delta P_i^{[k]} \\ \Delta Q_i^{[k]} \end{bmatrix} \leq \epsilon$.

Example 3: Newton-Raphson power flow solution.

Solve the power flow problem shown in **Figure 3** using the Newton-Raphson technique. Perform two iterations.

Solution:

The first step in Newton-Raphson Power Flow technique is building Y-bus using Eqs. (2) and (3).

$$Y = \begin{bmatrix} 15 - j55 & -5 + j15 & -10 + j40 \\ -5 + j15 & 20 - j65 & -15 + j50 \\ -10 + j40 & -15 + j50 & 25 - j90 \end{bmatrix} \\ = \begin{bmatrix} 57.01 \angle -74.74^\circ & 15.81 \angle 108.43^\circ & 41.23 \angle 104.04^\circ \\ 15.81 \angle 108.43^\circ & 68.01 \angle -72.90^\circ & 52.20 \angle 106.70^\circ \\ 41.23 \angle 104.04^\circ & 52.20 \angle 106.70^\circ & 93.41 \angle -74.48^\circ \end{bmatrix}$$

Since the unknown variables are δ_2 , δ_3 , and $|V_2|$; the scheduled quantities are P_2^{sch} , P_3^{sch} , and Q_2^{sch} , the following problem formulation can be written.

$$\begin{bmatrix} P_2^{sch} \\ P_3^{sch} \\ Q_2^{sch} \end{bmatrix} - \begin{bmatrix} P_2^{[k]} \\ P_3^{[k]} \\ Q_2^{[k]} \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2^{[k]}}{\partial \delta_2} & \frac{\partial P_2^{[k]}}{\partial \delta_3} & \frac{\partial P_2^{[k]}}{\partial |V_2|} \\ \frac{\partial P_3^{[k]}}{\partial \delta_2} & \frac{\partial P_3^{[k]}}{\partial \delta_3} & \frac{\partial P_3^{[k]}}{\partial |V_2|} \\ \frac{\partial Q_2^{[k]}}{\partial \delta_2} & \frac{\partial Q_2^{[k]}}{\partial \delta_3} & \frac{\partial Q_2^{[k]}}{\partial |V_2|} \end{bmatrix} \begin{bmatrix} \Delta \delta_2^{[k]} \\ \Delta \delta_3^{[k]} \\ \Delta |V_2|^{[k]} \end{bmatrix}$$

To calculate the Jacobian matrix elements, P_2 , P_3 , and Q_2 equations are obtained using (4) and (5).

$$\begin{aligned} P_2 &= \sum_{j=1}^n |V_2||V_j||Y_{2j}| \cos(\theta_{2j} - \delta_2 + \delta_j) \\ &= |V_2||V_1||Y_{21}| \cos(\theta_{21} - \delta_2 + \delta_1) + |V_2|^2|Y_{22}| \cos(\theta_{22}) + |V_2||V_3||Y_{23}| \cos(\theta_{23} - \delta_2 + \delta_3) \\ \frac{\partial P_2}{\partial \delta_2} &= |V_2||V_1||Y_{21}| \sin(\theta_{21} - \delta_2 + \delta_1) + |V_2||V_3||Y_{23}| \sin(\theta_{23} - \delta_2 + \delta_3) \\ \frac{\partial P_2}{\partial \delta_3} &= -|V_2||V_3||Y_{23}| \sin(\theta_{23} - \delta_2 + \delta_3) \end{aligned}$$

$$\frac{\partial P_2}{\partial |V_2|} = |V_1||Y_{21}| \cos(\theta_{21} - \delta_2 + \delta_1) + 2|V_2||Y_{22}| \cos(\theta_{22}) + |V_3||Y_{23}| \cos(\theta_{23} - \delta_2 + \delta_3)$$

$$\begin{aligned} P_3 &= \sum_{j=1}^n |V_3||V_j||Y_{3j}| \cos(\theta_{3j} - \delta_3 + \delta_j) \\ &= |V_3||V_1||Y_{31}| \cos(\theta_{31} - \delta_3 + \delta_1) + |V_3||V_2||Y_{32}| \cos(\theta_{32} - \delta_3 + \delta_2) + |V_3|^2|Y_{33}| \cos(\theta_{33}) \\ \frac{\partial P_3}{\partial \delta_2} &= -|V_3||V_2||Y_{32}| \sin(\theta_{32} - \delta_3 + \delta_2) \end{aligned}$$

$$\frac{\partial P_3}{\partial \delta_3} = |V_3||V_1||Y_{31}| \sin(\theta_{31} - \delta_3 + \delta_1) + |V_3||V_2||Y_{32}| \sin(\theta_{32} - \delta_3 + \delta_2)$$

$$\frac{\partial P_3}{\partial |V_2|} = |V_3||Y_{32}| \cos(\theta_{32} - \delta_3 + \delta_2)$$

$$\begin{aligned} Q_2 &= -\sum_{j=1}^n |V_2||V_j||Y_{2j}| \sin(\theta_{2j} - \delta_2 + \delta_j) \\ &= -|V_2||V_1||Y_{21}| \sin(\theta_{21} - \delta_2 + \delta_1) - |V_2|^2|Y_{22}| \sin(\theta_{22}) - |V_2||V_3||Y_{23}| \sin(\theta_{23} - \delta_2 + \delta_3) \\ \frac{\partial Q_2}{\partial \delta_2} &= |V_2||V_1||Y_{21}| \cos(\theta_{21} - \delta_2 + \delta_1) + |V_2||V_3||Y_{23}| \cos(\theta_{23} - \delta_2 + \delta_3) \end{aligned}$$

$$\frac{\partial Q_2}{\partial \delta_3} = -|V_2||V_3||Y_{23}| \cos(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial Q_2}{\partial |V_2|} = -|V_1||Y_{21}| \sin(\theta_{21} - \delta_2 + \delta_1) - 2|V_2||Y_{22}| \sin(\theta_{22}) - |V_3||Y_{23}| \sin(\theta_{23} - \delta_2 + \delta_3)$$

Iteration #1: assume $V_2^{[0]} = 1.00 \angle 0^\circ$ and $V_3^{[0]} = 1.03 \angle 0^\circ$.

$$\text{The calculated quantities: } \begin{bmatrix} P_2^{[1]} \\ P_3^{[1]} \\ Q_2^{[1]} \end{bmatrix} = \begin{bmatrix} -0.5500 \\ 0.5665 \\ -1.8000 \end{bmatrix}$$

$$\text{The scheduled quantities: } \begin{bmatrix} P_2^{sch} \\ P_3^{sch} \\ Q_2^{sch} \end{bmatrix} = \begin{bmatrix} -2 \\ 1.5 \\ -0.5 \end{bmatrix}$$

The mismatch power matrix:
$$\begin{bmatrix} \Delta P_2^{[1]} \\ \Delta P_3^{[1]} \\ \Delta Q_2^{[1]} \end{bmatrix} = \begin{bmatrix} -2 \\ 1.5 \\ -0.5 \end{bmatrix} - \begin{bmatrix} -0.5500 \\ 0.5665 \\ -1.8000 \end{bmatrix} = \begin{bmatrix} -1.4500 \\ 0.9335 \\ 1.3000 \end{bmatrix}$$

The Jacobian matrix (J) elements for iteration # 1 are as follows:

$$J = \begin{bmatrix} 66.8 & -51.5 & 19.45 \\ -51.5 & 93.52 & -15.45 \\ -20.55 & 15.45 & 63.2 \end{bmatrix}$$

Newton-Raphson formulation is as follows:

$$\begin{bmatrix} -1.4500 \\ 0.9335 \\ 1.3000 \end{bmatrix} = \begin{bmatrix} 66.8 & -51.5 & 19.45 \\ -51.5 & 93.52 & -15.45 \\ -20.55 & 15.45 & 63.2 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{[1]} \\ \Delta\delta_3^{[1]} \\ \Delta|V_2|^{[1]} \end{bmatrix}$$

$$\begin{bmatrix} \Delta\delta_2^{[1]} \\ \Delta\delta_3^{[1]} \\ \Delta|V_2|^{[1]} \end{bmatrix} = \begin{bmatrix} 66.8 & -51.5 & 19.45 \\ -51.5 & 93.52 & -15.45 \\ -20.55 & 15.45 & 63.2 \end{bmatrix}^{-1} \begin{bmatrix} -1.4500 \\ 0.9335 \\ 1.3000 \end{bmatrix} = \begin{bmatrix} -0.0279 \text{ rad} \\ -0.0033 \text{ rad} \\ 0.0123 \text{ pu} \end{bmatrix}$$

$$\begin{bmatrix} \delta_2^{[1]} \\ \delta_3^{[1]} \\ |V_2|^{[1]} \end{bmatrix} = \begin{bmatrix} \delta_2^{[0]} \\ \delta_3^{[0]} \\ |V_2|^{[0]} \end{bmatrix} + \begin{bmatrix} \Delta\delta_2^{[1]} \\ \Delta\delta_3^{[1]} \\ \Delta|V_2|^{[1]} \end{bmatrix}$$

$$\begin{bmatrix} \delta_2^{[1]} \\ \delta_3^{[1]} \\ |V_2|^{[1]} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1.00 \end{bmatrix} + \begin{bmatrix} -0.0279 \text{ rad} \\ -0.0033 \text{ rad} \\ 0.0123 \text{ pu} \end{bmatrix} = \begin{bmatrix} -0.0279 \text{ rad} \\ -0.0033 \text{ rad} \\ 1.0123 \text{ pu} \end{bmatrix} = \begin{bmatrix} -1.5986^\circ \\ -0.1891^\circ \\ 1.0123 \text{ pu} \end{bmatrix}$$

Iteration #2: Consider $V_2^{[1]} = 1.0123 \angle -1.5986^\circ$ and $V_3^{[1]} = 1.03 \angle -0.1891^\circ$.

The calculated quantities:
$$\begin{bmatrix} P_2^{[2]} \\ P_3^{[2]} \\ Q_2^{[2]} \end{bmatrix} = \begin{bmatrix} -2.0109 \\ 1.5202 \\ -0.4621 \end{bmatrix}$$

The mismatch power matrix:
$$\begin{bmatrix} \Delta P_2^{[2]} \\ \Delta P_3^{[2]} \\ \Delta Q_2^{[2]} \end{bmatrix} = \begin{bmatrix} -2 \\ 1.5 \\ -0.5 \end{bmatrix} - \begin{bmatrix} -2.0109 \\ 1.5202 \\ -0.4621 \end{bmatrix} = \begin{bmatrix} 0.0109 \\ -0.0202 \\ -0.0379 \end{bmatrix}$$

The Jacobian matrix elements are calculated as follows:

$$J = \begin{bmatrix} 67.07 & -51.74 & 18.26 \\ -52.50 & 94.49 & -14.18 \\ -22.51 & 16.91 & 65.34 \end{bmatrix}$$

$$\begin{bmatrix} \Delta\delta_2^{[2]} \\ \Delta\delta_3^{[2]} \\ \Delta|V_2|^{[2]} \end{bmatrix} = \begin{bmatrix} 67.07 & -51.74 & 18.26 \\ -52.50 & 94.49 & -14.18 \\ -22.51 & 16.91 & 65.34 \end{bmatrix}^{-1} \begin{bmatrix} 0.0109 \\ -0.0202 \\ -0.0379 \end{bmatrix}$$

$$\begin{bmatrix} \Delta\delta_2^{[2]} \\ \Delta\delta_3^{[2]} \\ \Delta|V_2|^{[2]} \end{bmatrix} = \begin{bmatrix} 0.1272 \times 10^{-3} \text{ rad} \\ -0.2154 \times 10^{-3} \text{ rad} \\ -0.4810 \times 10^{-3} \text{ pu} \end{bmatrix}$$

$$\begin{bmatrix} \delta_2^{[2]} \\ \delta_3^{[2]} \\ |V_2|^{[2]} \end{bmatrix} = \begin{bmatrix} \delta_2^{[1]} \\ \delta_3^{[1]} \\ |V_2|^{[1]} \end{bmatrix} + \begin{bmatrix} \Delta\delta_2^{[2]} \\ \Delta\delta_3^{[2]} \\ \Delta|V_2|^{[2]} \end{bmatrix}$$

$$\begin{bmatrix} \delta_2^{[2]} \\ \delta_3^{[2]} \\ |V_2|^{[2]} \end{bmatrix} = \begin{bmatrix} -0.0279 \text{ rad} \\ -0.0033 \text{ rad} \\ 1.0123 \text{ pu} \end{bmatrix} + \begin{bmatrix} 0.1272 \times 10^{-3} \text{ rad} \\ -0.2154 \times 10^{-3} \text{ rad} \\ -0.4810 \times 10^{-3} \text{ pu} \end{bmatrix}$$

$$\begin{bmatrix} \delta_2^{[2]} \\ \delta_3^{[2]} \\ |V_2|^{[2]} \end{bmatrix} = \begin{bmatrix} -0.0277 \text{ rad} \\ -0.0035 \text{ rad} \\ 1.0118 \text{ pu} \end{bmatrix} = \begin{bmatrix} -1.5871^\circ \\ -0.2005^\circ \\ 1.0118 \text{ pu} \end{bmatrix}$$

It is worth noting that the mismatch between calculated and scheduled quantities diminishes very quickly.

$$\begin{bmatrix} \Delta P_2^{[1]} \\ \Delta P_3^{[1]} \\ \Delta Q_2^{[1]} \end{bmatrix} = \begin{bmatrix} -1.4500 \\ 0.9335 \\ 1.3000 \end{bmatrix} \text{ and } \begin{bmatrix} \Delta P_2^{[2]} \\ \Delta P_3^{[2]} \\ \Delta Q_2^{[2]} \end{bmatrix} = \begin{bmatrix} 0.0109 \\ -0.0202 \\ -0.0379 \end{bmatrix}$$

The iterative solution is presented in **Table 3**.

Iteration	$V_2^{[k]}$	$V_3^{[k]}$
1	1.0123 \angle -1.5986°	1.03 \angle -0.1891°
2	1.0118 \angle -1.5871°	1.03 \angle -0.2005°
3	1.0118 \angle -1.5871°	1.03 \angle -0.2005°

Table 3.
 Newton-Raphson iterative solution.

5. Fast-decoupled technique

In high voltage transmission systems, the voltage angles between adjacent buses are relatively small. In addition, X/R ratio is high. These two properties result in a strong coupling between real power and voltage angle and between reactive power and voltage magnitude. In contrary, the coupling between real power and voltage magnitude, as well as reactive power and voltage angle, is weak. Considering adjacent buses, real power flows from the bus with a higher voltage angle to the bus with a lower voltage angle. Similarly, reactive power flows from the bus with a higher voltage magnitude to the bus with a lower voltage magnitude.

Fast-decoupled power flow technique includes two steps: (1) decoupling real and reactive power calculations; (2) obtaining of the Jacobian matrix elements directly from the Y-bus.

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_{P\delta} & 0 \\ 0 & J_{Q|V|} \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta|V| \end{bmatrix} \quad (26)$$

or

$$[\Delta P] = [J_{P\delta}][\Delta\delta] \quad (27)$$

$$[\Delta Q] = [J_{Q|V}] [\Delta |V|] \quad (28)$$

Next step is to obtain $J_{P\delta}$ and $J_{Q|V}$ from the Y-bus as flowing:

$$[\Delta \delta] = -[B']^{-1} \left[\frac{\Delta P}{|V|} \right] \quad (29)$$

$$[\Delta |V|] = -[B'']^{-1} \left[\frac{\Delta Q}{|V|} \right] \quad (30)$$

Where the $[B']$ and $[B'']$ are relevant imaginary part of the Y-bus matrix elements. $[B']$ is related to the buses at which real power is scheduled (δ is unknown) and $[B'']$ is related to the buses at which reactive power is scheduled ($|V|$ is unknown).

$$B_{ij} = |Y_{ij}| \sin(\theta_{ij}) \quad (31)$$

The fast-decoupled technique requires more iterations to converge compared to the Newton-Raphson power flow formulation, especially if X/R ratio is not high. Another advantage of this method is that the Jacobian matrix has constant term elements which are obtained and inverted once at the beginning of the iterative process.

Example 4: Fast-decoupled power flow solution.

Solve the power flow problem shown in **Figure 3** using fast-decoupled power flow technique. Perform two iterations.

Solution:

The first step in fast-decoupled power flow technique is obtaining $[B']$ and $[B'']$ from the Y-bus.

$$Y = \begin{bmatrix} 15 - j55 & -5 + j15 & -10 + j40 \\ -5 + j15 & 20 - j65 & -15 + j50 \\ -10 + j40 & -15 + j50 & 25 - j90 \end{bmatrix}$$

Since the unknown voltage angles are δ_2 and δ_3 , elements for $[B']$ are obtained from the Y-bus (intersection of columns numbers 2 & 3 and rows numbers 2 & 3) as follows:

$$[B'] = \begin{bmatrix} -65 & 50 \\ 50 & -90 \end{bmatrix}$$

Since the unknown voltage magnitude is $|V_2|$ at bus 2, $[B'']$ contains one element only (intersection of the column and the row number 2):

$$[B''] = [-65]$$

The fast-decoupled power flow formulation becomes as follows:

$$\begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \end{bmatrix} = -[B']^{-1} \begin{bmatrix} \frac{\Delta P_2}{|V_2|} \\ \frac{\Delta P_3}{|V_3|} \end{bmatrix} \quad \text{and} \quad [\Delta |V_2|] = -[B'']^{-1} \left[\frac{\Delta Q_2}{|V_2|} \right]$$

Or

$$\begin{bmatrix} \Delta\delta_2^{[k]} \\ \Delta\delta_3^{[k]} \end{bmatrix} = -[B']^{-1} \begin{bmatrix} \frac{\Delta P_2^{[k]}}{|V_2|^{[k-1]}} \\ \frac{\Delta P_3^{[k]}}{|V_3|^{[k-1]}} \end{bmatrix} \quad \text{and} \quad \Delta|V_2|^{[k]} = -[B'']^{-1} \left[\frac{\Delta Q_2^{[k]}}{|V_2|^{[k-1]}} \right]$$

Iteration #1: assume $V_2^{[0]} = 1.00 \angle 0^\circ$ and $V_3^{[0]} = 1.03 \angle 0^\circ$.

The calculated quantities:

$$\begin{bmatrix} P_2^{[1]} \\ P_3^{[1]} \\ Q_2^{[1]} \end{bmatrix} = \begin{bmatrix} -0.5500 \\ 0.5665 \\ -1.8000 \end{bmatrix}$$

The mismatch power matrix:

$$\begin{bmatrix} \Delta P_2^{[1]} \\ \Delta P_3^{[1]} \\ \Delta Q_2^{[1]} \end{bmatrix} = \begin{bmatrix} -2 \\ 1.5 \\ -0.5 \end{bmatrix} - \begin{bmatrix} -0.5500 \\ 0.5665 \\ -1.8000 \end{bmatrix} = \begin{bmatrix} -1.4500 \\ 0.9335 \\ 1.3000 \end{bmatrix}$$

Calculation of δ_2 and δ_3 :

$$\begin{aligned} \begin{bmatrix} \Delta\delta_2^{[1]} \\ \Delta\delta_3^{[1]} \end{bmatrix} &= -[B']^{-1} \begin{bmatrix} \frac{\Delta P_2^{[1]}}{|V_2|^{[0]}} \\ \frac{\Delta P_3^{[1]}}{|V_3|^{[0]}} \end{bmatrix} = - \begin{bmatrix} -65 & 50 \\ 50 & -90 \end{bmatrix}^{-1} \begin{bmatrix} -1.4500 \\ 0.9335 \\ 1.03 \end{bmatrix} \\ &= \begin{bmatrix} -0.0254 \text{ rad} \\ -0.0041 \text{ rad} \end{bmatrix} = \begin{bmatrix} -1.4569^\circ \\ -0.2324^\circ \end{bmatrix} \\ \begin{bmatrix} \delta_2^{[1]} \\ \delta_3^{[1]} \end{bmatrix} &= \begin{bmatrix} \delta_2^{[0]} \\ \delta_3^{[0]} \end{bmatrix} + \begin{bmatrix} \Delta\delta_2^{[1]} \\ \Delta\delta_3^{[1]} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -1.4569^\circ \\ -0.2324^\circ \end{bmatrix} = \begin{bmatrix} -1.4569^\circ \\ -0.2324^\circ \end{bmatrix} \end{aligned}$$

Calculation of $|V_2|$:

$$\begin{aligned} \Delta|V_2|^{[1]} &= -[-65]^{-1} \left[\frac{1.3}{1.00} \right] = 0.0200 \\ \Delta|V_2|^{[1]} &= |V_2|^{[0]} + \Delta|V_2|^{[1]} = 1.00 + 0.0200 = 1.020 \text{ pu} \end{aligned}$$

Iteration #2: considering $V_2^{[1]} = 1.020 \angle -1.4569^\circ$ and $V_3^{[0]} = 1.03 \angle -0.2324^\circ$.

The calculated quantities:

$$\begin{bmatrix} P_2^{[2]} \\ P_3^{[2]} \\ Q_2^{[2]} \end{bmatrix} = \begin{bmatrix} -1.6671 \\ 1.2133 \\ -0.0239 \end{bmatrix}$$

The mismatch power matrix:

$$\begin{bmatrix} \Delta P_2^{[2]} \\ \Delta P_3^{[2]} \\ \Delta Q_2^{[2]} \end{bmatrix} = \begin{bmatrix} -2 \\ 1.5 \\ -0.5 \end{bmatrix} - \begin{bmatrix} -1.6671 \\ 1.2133 \\ -0.0239 \end{bmatrix} = \begin{bmatrix} -0.3329 \\ 0.2867 \\ -0.4761 \end{bmatrix}$$

Calculation of δ_2 and δ_3 :

$$\begin{bmatrix} \Delta\delta_2^{[2]} \\ \Delta\delta_3^{[2]} \end{bmatrix} = -[B']^{-1} \begin{bmatrix} \frac{\Delta P_2^{[2]}}{|V_2|^{[1]}} \\ \frac{\Delta P_3^{[2]}}{|V_3|^{[1]}} \end{bmatrix} = - \begin{bmatrix} -65 & 50 \\ 50 & -90 \end{bmatrix}^{-1} \begin{bmatrix} \frac{-0.3329}{1.02} \\ \frac{0.2867}{1.03} \end{bmatrix}$$

$$\begin{bmatrix} \Delta\delta_2^{[2]} \\ \Delta\delta_3^{[2]} \end{bmatrix} = \begin{bmatrix} -0.0046 \text{ rad} \\ 0.0005 \text{ rad} \end{bmatrix} = \begin{bmatrix} -0.0300^\circ \\ 0.0286^\circ \end{bmatrix}$$

$$\begin{bmatrix} \delta_2^{[2]} \\ \delta_3^{[2]} \end{bmatrix} = \begin{bmatrix} \delta_2^{[1]} \\ \delta_3^{[1]} \end{bmatrix} + \begin{bmatrix} \Delta\delta_2^{[2]} \\ \Delta\delta_3^{[2]} \end{bmatrix} = \begin{bmatrix} -1.4569^\circ \\ -0.2324^\circ \end{bmatrix} + \begin{bmatrix} -0.0300^\circ \\ 0.0286^\circ \end{bmatrix} = \begin{bmatrix} -1.7213^\circ \\ -0.2021^\circ \end{bmatrix}$$

Calculation of $|V_2|$:

$$\begin{bmatrix} \Delta|V_2|^{[2]} \end{bmatrix} = -[-65]^{-1} \begin{bmatrix} \frac{-0.4761}{1.02} \end{bmatrix} = -0.0072$$

$$\begin{bmatrix} |V_2|^{[2]} \end{bmatrix} = |V_2|^{[1]} + \Delta|V_2|^{[2]} = 1.02 - 0.0072 = 1.0128 \text{ pu}$$

The remaining five iterations are shown in **Table 4**. It shows that this method converges slower than Newton-Raphson method.

Iteration	$V_2^{[k]}$	$V_3^{[k]}$
1	1.0200 \angle -1.4569°	1.03 \angle -0.2324°
2	1.0128 \angle -1.7213°	1.03 \angle -0.2021°
3	1.0111 \angle -1.6019°	1.03 \angle -0.2029°
4	1.0118 \angle -1.5759°	1.03 \angle -0.2026°
5	1.0119 \angle -1.5877°	1.03 \angle -0.2027°

Table 4.
Fast-decoupled iterative solution.

6. DC power flow technique

DC power is an extension to the Fast-decoupled power flow formulation. In DC power flow method, the voltage is assumed constant at all buses; therefore, the $(\Delta|V|, \Delta Q)$ equation is neglected. The $(\Delta\delta, \Delta P)$ equation can be further simplified to a linear problem that does not require iterative solution:

$$-[B][\delta] = [P] \quad (32)$$

Or

$$[\delta] = -[B]^{-1}[P] \quad (33)$$

Example 5: DC power flow.

Solve the power flow problem shown in **Figure 3** using the DC power flow technique.

Solution:

The first step in DC power flow technique is obtaining $[B]$ from the Y-bus.

$$Y = \begin{bmatrix} 15 - j55 & -5 + j15 & -10 + j40 \\ -5 + j15 & 20 - j65 & -15 + j50 \\ -10 + j40 & -15 + j50 & 25 - j90 \end{bmatrix}$$

For this problem, Since the unknown voltage angles are δ_2 and δ_3 , elements for $[B]$ are obtained from the Y-bus as follows:

$$[B] = \begin{bmatrix} -65 & 50 \\ 50 & -90 \end{bmatrix}$$

$$\begin{bmatrix} \delta_2 \\ \delta_3 \end{bmatrix} = - \begin{bmatrix} -65 & 50 \\ 50 & -90 \end{bmatrix}^{-1} \begin{bmatrix} P_2 \\ P_3 \end{bmatrix} = - \begin{bmatrix} -65 & 50 \\ 50 & -90 \end{bmatrix}^{-1} \begin{bmatrix} -2 \\ 1.5 \end{bmatrix}$$

$$= \begin{bmatrix} -0.0313 \text{ rad} \\ -0.0007 \text{ rad} \end{bmatrix} = \begin{bmatrix} -1.7958^\circ \\ -0.0428^\circ \end{bmatrix}$$

7. Slack bus power and losses calculations

The main objective of power flow calculations is to determine the voltages (magnitude and angle) for a given load and generation conditions. Once voltages are known for all buses, slack bus power, as well as line flows and losses, can be calculated. The slack bus real and reactive power are calculated using Eqs. (4) and (5), respectively. Overall system losses are the difference between generation and load.

$$S_L = S_{Gen} - S_{Load} \quad (34)$$

Specific branch losses are calculated using branch power flow. For example, the losses in the line $i - j$ are the algebraic sum of the power flow.

$$S_{L \text{ } ij} = S_{ij} + S_{ji} \quad (35)$$

S_{ij} and S_{ji} are defined as follows:

$$S_{ij} = V_i I_{ij}^* \quad (36)$$

$$S_{ji} = V_j I_{ji}^* \quad (37)$$

The current between buses I_{ij} is a function of the voltages and the admittance between V_i and V_j . It is worth noting that $I_{ji} = -I_{ij}$

$$I_{ij} = (V_i - V_j)y_{ij} \quad (38)$$

Example 6: Slack bus power and losses.

For the 3-bus system shown in **Figure 3**, the voltages at buses 2 and 3 were iteratively calculated: $V_1 = 1.02 \angle 0^\circ$, $V_2 = 1.0118 \angle -1.5871^\circ$ and $V_3 = 1.03 \angle -0.2005^\circ$

- a. Calculate the slack bus power.
- b. Calculate the total system losses.
- c. Calculate individual branch losses.

Solution:

The polar form of the Y-bus is used.

$$Y = \begin{bmatrix} 57.01 \angle -74.74^\circ & 15.81 \angle 108.43^\circ & 41.23 \angle 104.04^\circ \\ 15.81 \angle 108.43^\circ & 68.01 \angle -72.90^\circ & 52.20 \angle 106.70^\circ \\ 41.23 \angle 104.04^\circ & 52.20 \angle 106.70^\circ & 93.41 \angle -74.48^\circ \end{bmatrix}$$

- a. The slack bus power

The slack bus power can be calculated using real and reactive power equations:

$$\begin{aligned} P_1 &= \sum_{j=1}^n |V_1| |V_j| |Y_{1j}| \cos(\theta_{1j} - \delta_1 + \delta_j) \\ &= |V_1|^2 |Y_{11}| \cos(\theta_{11}) + |V_1| |V_2| |Y_{12}| \cos(\theta_{12} - \delta_1 + \delta_2) + |V_1| |V_3| |Y_{13}| \cos(\theta_{13} - \delta_1 + \delta_3) \\ &= 0.5195 \text{ pu} \end{aligned}$$

$$\begin{aligned} Q_1 &= -\sum_{j=1}^n |V_1| |V_j| |Y_{1j}| \sin(\theta_{1j} - \delta_1 + \delta_j) \\ &= -|V_1|^2 |Y_{11}| \sin(\theta_{11}) - |V_1| |V_2| |Y_{12}| \cos(\theta_{12} - \delta_1 + \delta_2) - |V_1| |V_3| |Y_{13}| \sin(\theta_{13} - \delta_1 + \delta_3) \\ &= -0.4572 \text{ pu} \end{aligned}$$

- b. The total system losses

The total real power losses can be calculated using the total net injected real a power at all buses:

$$\begin{aligned} P_L &= P_1 + P_2 + P_3 \\ P_L &= P_1 + P_2 + P_3 = 0.5195 - 2 + 1.5 = 0.0195 \text{ pu} \end{aligned}$$

Similarly, the reactive power losses can be calculated.

$$Q_L = Q_1 + Q_2 + Q_3$$

However, Q_3 is unknown and can be calculated based on the given bus voltages:

$$\begin{aligned} Q_3 &= -\sum_{j=1}^n |V_3| |V_j| |Y_{3j}| \sin(\theta_{3j} - \delta_3 + \delta_j) \\ &= -|V_3| |V_1| |Y_{31}| \sin(\theta_{31} - \delta_3 + \delta_1) - |V_3| |V_2| |Y_{32}| \sin(\theta_{32} - \delta_3 + \delta_2) - |V_3|^2 |Y_{33}| \sin(\theta_{33}) \\ &= 1.0220 \text{ pu} \end{aligned}$$

Once Q_2 is calculated, the total reactive power losses are calculated:

$$Q_L = Q_1 + Q_2 + Q_3 = -0.4572 - 0.5 + 1.0220 = 0.0648 \text{ pu}$$

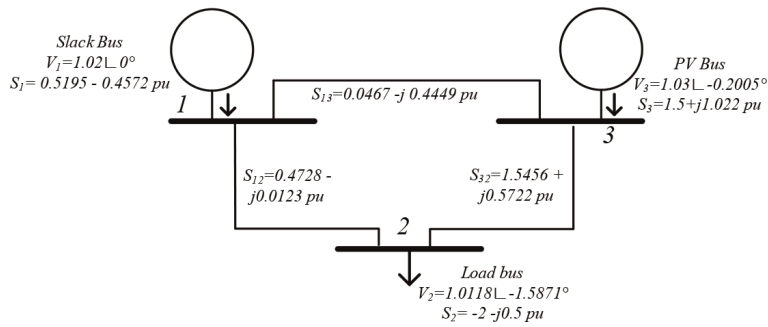


Figure 6.
 Power flow results.

c. Branch losses

To calculate the losses in line 1–2, $S_{L\ 12}$ is calculated by summing S_{12} and S_{21} . These flows are calculated as follows: $S_{12} = V_1 I_{12}^*$ and $S_{21} = V_2 I_{21}^*$

$$I_{12} = (V_1 - V_2)y_{12} = (1.02 \angle 0^\circ - 1.0118 \angle -1.5871^\circ)(-5 + j15) = 0.4635 + j0.0121 \text{ pu}$$

$$I_{21} = (V_2 - V_1)y_{21} = -I_{12} = -0.4635 - j0.0121 \text{ pu}$$

$$S_{12} = V_1 I_{12}^* = 1.02 \angle 0^\circ \times (0.4635 - j0.0121) = 0.4728 - j0.0123 \text{ pu}$$

$$S_{21} = V_2 I_{21}^* = 1.0118 \angle -1.5871^\circ \times (-0.4635 + j0.0121) = -0.4685 + j0.0252 \text{ pu}$$

$$S_{L\ 12} = S_{12} + S_{21} = (0.4728 - j0.0123) + (-0.4685 + j0.0252) = 0.0043 + j0.0129 \text{ pu}$$

Similarly, power flow and losses in other branches are calculated.

Line 1–3:

$$S_{31} = -0.0456 + j0.4494$$

$$S_{13} = 0.0467 - j0.4449$$

$$S_{L\ 13} = 0.0011 + j0.0045$$

Line 2–3:

$$S_{23} = -1.5315 - j0.5252$$

$$S_{32} = 1.5456 + j0.5722$$

$$S_{L\ 23} = 0.0141 + j0.0470$$

Total losses:

$$S_L = S_{L\ 13} + S_{L\ 23} + S_{L\ 12} = 0.0195 + j0.0648 \text{ pu}$$

The generation and load at different buses is shown in **Figure 6**.

8. Conclusions

Power flow analysis, or load flow analysis, has a wide range of applications in power systems operation and planning. This chapter presents an overview of the power flow problem, its formulation as well as different solution methods. The

power flow model of a power system can be built using the relevant network, load, and generation data. Outputs of the power flow model include voltages (magnitude and angles) at different buses. Once nodal voltages are calculated, real and reactive power flows in different network branches can be calculated. The calculation of branch power flows enables technical loss calculation in different network branches, as well as the total system technical losses.

Power flow analysis is performed by solving nodal power balance equations. Since these equations are nonlinear, iterative techniques such as the Gauss-Seidel, the Newton-Raphson, and the fast-decoupled power flow methods are commonly used to solve this problem. In general, the Gauss-Seidel method is simple but converges slower than the Newton-Raphson method. However, the latter method required the Jacobian matrix formation of at every iteration. The fast-decoupled power flow method is a simplified version of the Newton-Raphson method. This simplification is achieved in two steps: 1) decoupling real and reactive power calculations; 2) obtaining of the Jacobian matrix elements directly from the Y-bus matrix. The DC power method is an extension to the fast-decoupled power flow formulation. In DC power flow method, the voltage is assumed constant at all buses and the problem becomes linear.

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