Propagation of Ultrasonic Waves in Viscous Fluids

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1. Introduction

Viscosity is a property of fluids which can have a great importance in many fields of science and industry. It can be measured by conventional viscometers or by ultrasonic methods (Hertz & Al, 1991). Ultrasonic methods are based either on the determination of the characteristics of propagation (velocity or attenuation) or by the measurement of reflection coefficient (Hertz & Al, 1991) (Malcom J.W Povey, 1997). In the ultrasound based experimental devices, the measurements of the propagation velocity and the attenuation during the propagation allow to determine the physical properties of the medium (He & Zheng, 2001).

When an ultrasonic pulse is propagating into a viscous fluid, the pulse waveforms change because of the attenuation and the dispersion of the propagation medium. After its travelling through a medium, the transmitted is not a simply delayed and attenuated waveform copy of the pulse injected at the input. A modelling of the phenomenon of propagation is necessary for an adequate interpretation of the pulse waveforms.

During the 20 to 30 last years, a special attention has been devoted to the modelling of the ultrasonic wave propagation in viscous media. In almost these models, the phenomenon of propagation is represented by a slightly dispersive linear system for which the phase is a linear function of frequency (Hertz & Al, 1991). Unfortunately the response of such systems is not causal (D.T, 1995). Hilbert transform or Fractional Calculus are some of the main mathematical tools which have been used to develop theoretical models respecting causality (Hertz & Al, 1991) (D.T, 1969) (He, 1999). In the case of viscous media for which the attenuation of the waves is proportional to the frequency squared, the methods based on Laplace transform allow the derivation of impulse responses respecting the causality (Thomas.L, 1995) (D.T, 1969) (Norton & Purrington, 2009).

2. Theory

When an ultrasonic wave travels through a fluid, the phenomenon of attenuation results from several mechanisms of absorption. At the beginning of the propagation, the absorption is mainly due to the viscosity since the thermal agitation did not have enough time to exert its influence. Each layer of the fluid tends to slow down the displacement of the adjacent layers causing thus the damping of the wave as it penetrates into the fluid. Besides viscosity,

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there is appearance of the conduction thermal phenomenon which results from the heat transfer between the regions of dilatation and compression. The wave propagation causes a thermal agitation within the fluid characterized by the collisions of the atoms: it is the Brownian motion which causes the coupling or energy exchange between the wave motion and the internal motion (translation, vibration, rotation).
The Brownian motion is a random motion of microscopic particles with different velocities resulting from the increase in temperature and caused by the molecular shocks. The local periodic variation of the pressure due to the wave propagation involves a molecular displacement (rotation + translation). This displacement is caused by the increase in the internal energy of the fluid. The change of the potential energy causes a change of the structure of the fluid because of the modifications of the distances between the different atoms. To pass from one energy level to another, each atom acquires a certain energy, leaving a disorder explained by the presence of holes in the energy levels. These displacements are the same as those which leads to the relaxation mechanism, i.e. the return to equilibrium after the modification of the positions and the structure of the fluid molecules. The return to the equilibrium is achieved after a certain time named relaxation time. The entropy of the fluid increased and any increases in entropy means the establishment of an irreversible equilibrium state accompanied by energy dissipation and dispersion in the frequency domain.

3. Problem formulation

3.1 Theoretical model

The propagation of a plane wave in a viscous fluid can be interpreted by considering that the fluid can be regarded as a set of particles which undergo during the wave propagation a succession of dilatations and compressions. Each particle transmits the vibration which it receives from its neighbors and behaves as a secondary source. The model is represented by a shematic system made up of masses m, separated by a distance a at the equilibrium, and assembled in series with a spring and a piston coupled in parallel model of Voigt-Kelvin. Under the action of the wave, the system oscillates from its equilibrium position (Thomas.L, 2004).

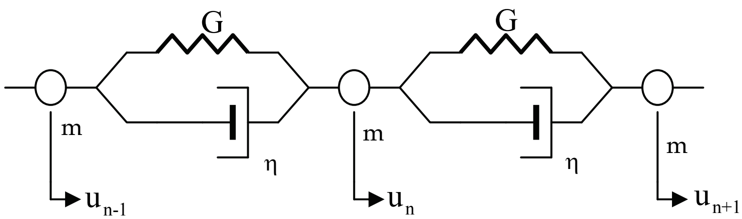

Fig. 1. Representation of the linear model of the discrete chain

3.2 Equation of propagation

The wave propagation equation of the acoustic pressure in a viscous medium is given by (Ludwig & Levin, 1995):

$$\frac{\partial^2 p}{\partial x^2} - \frac{1}{v_0^2}\frac{\partial^2 p}{\partial t^2} + \tau\frac{\partial^3 p}{\partial t \partial^2 x} = 0 \tag{1}$$

where $p(x,t)$, and $\tau = [\eta_V + 4\eta_S / 3] / \rho v_0^2$ are respectively the acoustic pressure and the relaxation time. η_S and η_V are the shear and bulk viscosity and v_0 is the propagation velocity.

Equation (1) can be solved by using Laplace transform. By taking account of the homogeneous initial conditions, the application of the transform of Laplace to the equation of propagation (1) gives:

$$\frac{\partial^2 P(x,s)}{\partial x^2} - \frac{s^2}{v_0^2(1+\tau s)}P(x,s) = 0 \tag{2}$$

$P(x,s)$ represents Laplace transform of $p(x,t)$. The solution that satisfies the boundary condition $\lim_{x\to\infty} P(x,s) = 0$ is:

$$P(x,s) = A \quad \exp^{\frac{-sx}{v_0\sqrt{1+s\tau}}} \tag{3}$$

A is a constant determined from the boundary conditions at $x = 0$. If the pressure at $x = 0$ is given by $p(0,t)$, equation (3) can be written as:

$$P(x,s) = P(0,s)\,H(x,s) \tag{4}$$

In this latter expression, $P(0,s)$ is the Laplace transform of $p(0,t)$ and $H(s)$ is a complex valued function defined by:

$$H(x,s) = e^{\frac{-xs}{v_0\sqrt{1+s\tau}}} \tag{5}$$

The inverse transform allows writing the pressure $p(x,t)$ as a convolution product:

$$p(x,t) = p(0,t) \otimes h(x,t) \tag{6}$$

where $h(x,t)$ is the inverse Laplace transform of $H(x,s)$, symbol $\otimes$ denotes the convolution in time. The impulse response $h(x,t)$ is defined by:

$$h(x,t) = \frac{1}{j\,2\pi}\int_{c-j\infty}^{c+j\infty} H(x,s)\; e^{st}\, ds \tag{7}$$

Integral (7) is evaluated in the complex plane along the Bromwich contour. This integral has to be carried out in the complex plan, along the straight line $\mathrm{Re}(s) = c$, with $s = c + jy$ where y varies from $-\infty$ à $+\infty$. The real number c is selected in such a way that c is on the right of all the singularities (poles, points of connections, or essential singularities).

The research of singularities shows that equation 7 presents a pole at: $s = -1/\tau$. To carry out the calculation of this integral, the following change for variable is made: $p = \frac{1}{\tau} + s$, $\varsigma = \frac{x}{\tau^{\frac{1}{2}} v_0}$ and $m = \frac{k}{\tau^{\frac{3}{2}} v_0}$.

Thus equation 7 becomes:

$$h(x,t) = e^{\frac{-t}{\tau}} \; TL^{-1}\left\{H_1(p)\, H_2(p)\right\} \tag{8}$$

Where $H_1(p)$ and $H_2(p)$ are defined by:

$$H_1(p) = e^{-k\sqrt{\varsigma}} \text{ et } H_2(p) = e^{\frac{m}{\sqrt{\varsigma}}} \tag{9}$$

Accounting of the convolution theorem, one obtains:

$$h(x,t) = e^{\frac{-t}{\tau}} \; h_1(x,t) \otimes h_2(x,t) \tag{10}$$

$h_1(x,t)$ and $h_2(x,t)$ being respectively the Laplace inverse transforms of $H_1(p)$ and $H_2(p)$.
The inverse transform $h_1(x,t)$ of $H_1(p)$ is:

$$h_1(x,t) = \frac{\varsigma}{2\sqrt{\pi t^3}} e^{\frac{-\varsigma^2}{4t}} \tag{11}$$

The inverse Laplace transform of the second term is defined by:

$$h_2(x,t) = \frac{1}{j2\pi} \int_{\gamma-i\infty}^{\gamma+i\infty} e^{\frac{m}{\sqrt{\varsigma}}} e^{\varsigma t} d\varsigma \tag{12}$$

This integral has a pole located at $\varsigma = 0$; γ must thus be a positive real number. The evaluation of this integral is made by considering the Bromwich contour.
The figure (2-a), represents this closed contour C of Bromwich made up of the segments at the right-hand side Γ and of the half rings C_1 of center $(\gamma, 0)$ and of radius R .This contour does not contain any pole, the integral on C is null according to Cauchy theorem:

$$\int_{\Gamma} = -\int_{C_1} \tag{13}$$

According to the Jordan Lemma, the integral on the half-circle C_1 cancels when the radius R tends towards the infinite. Then:

$$h_2(x,t) = \lim_{R\to\infty} \int_{\Gamma} = 0 \text{ pour } t \le 0 \tag{14}$$

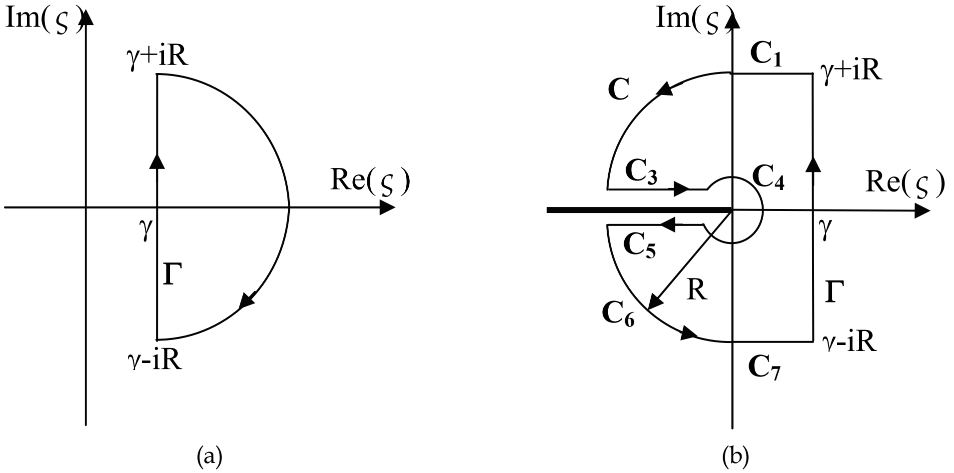

Fig. 2. (a) Contour of Bromwich for t ≤ 0 (b) Contour of Bromwich for t ≥ 0

Figure (2-b), represents a closed contour C made up of the segments on the right-hand side of Γ, C_1 and C_7 , arcs of a circle C_2 and C_6 of radius *R* centered at the origin, of an arc of a circle C_4 of radius unit and segments on the right-hand side C_3 and C_5. The half-line defined by Im(p)=0 and Re(p)≤0 constitutes a cut of this contour and does not contain any pole, so the integral on C is null according to Cauchy theorem , so :

$$\int_{\Gamma} = -\int_{C_1} - \int_{C_2} - \int_{C_3} - \int_{C_4} - \int_{C_5} - \int_{C_6} - \int_{C_7} \tag{15}$$

Integrals on the segments C_1 and C_7 cancel when *R* tends towards the infinite. And according to Jordan Lemma, integrals along circle arcs C_2 and C_6 cancel when the radius *R* tends towards the infinite, so ones obtain:

$$\begin{aligned} h_2(x,t) &= \lim_{R\to\infty} \int_{\Gamma} \\ h_2(x,t) &= -\lim_{R\to\infty}\left[\int_{C_3} + \int_{C_4} + \int_{C_5}\right] \end{aligned} \tag{16}$$

On the segment and contour C_3, the integral I3 is written under the form :

$$\begin{aligned} I_3(x,t) &= \int_{-\infty}^{-1} e^{\frac{m}{\sqrt{\varsigma}}} e^{\varsigma t} d\varsigma \\ \varsigma &= re^{j\pi} = r(\cos\pi + j\sin\pi) = -r \end{aligned} \tag{17}$$

r being a positive real number. So I3 is now given by:

$$I_3(x,t) = \int_{+1}^{+\infty} e^{-\frac{i\,m}{\sqrt{r}}}\, e^{-rt} dr \tag{18}$$

On the segment and C_5 contour, the integral I_5 and ς can be written as:

$$I_5(x,t) = \int_{-1}^{-\infty} e^{\frac{m}{\sqrt{\varsigma}}}\, e^{\varsigma t} d\varsigma \qquad \varsigma = re^{-j\pi} = r(\cos\pi + j\sin\pi) = -r \tag{19}$$

where r is a positive real number. Then, one obtains the expression of I_5 :

$$I_5(x,t) = -\int_{+1}^{+\infty} e^{\frac{i\,m}{\sqrt{r}}}\, e^{-rt} dr \tag{20}$$

The sum of the integrals on the segments C_3 and C_5 gives:

$$I_3 + I_5 = -2i \int_{+1}^{+\infty} \sin\left(\frac{m}{\sqrt{r}}\right) e^{-rt} dr \tag{21}$$

On C_4 contour $\varsigma = e^{-i\theta}$, so the expression of I_4 is written under the following form :

$$I_4(x,t) = -i \int_{+\pi}^{-\pi} e^{[m\cos(\frac{\theta}{2}) + t\cos(\theta)]} \times e^{i[t\sin(\theta) - m\sin(\frac{\theta}{2}) + \theta]} d\theta \tag{22}$$

The integrand is an odd function of θ

$$I_4(x,t) = -2i \int_0^{\pi} e^{[m\cos(\frac{\theta}{2}) + t\cos(\theta)]} \times \cos(t\sin(\theta) - m\sin(\frac{\theta}{2}) + \theta) d\theta \tag{23}$$

$h_2(x,t)$ can be written under the form:

$$h_2(x,t) = \frac{1}{\pi}\int_{+1}^{+\infty} \sin(\frac{m}{\sqrt{r}}) e^{-rt} dr + \frac{1}{\pi}\int_0^{\pi} e^{m\cos(\frac{\theta}{2}) + t\cos\theta} \times \cos(t\sin\theta - m\sin\frac{\theta}{2} + \theta) d\theta \tag{24}$$

To carry out the numerical calculation of the impulse response, one makes the following variable change for the integral going from 1 to $+\infty$:

$$r = \frac{1}{\sin\varphi}, \quad dr = \frac{-\cos\varphi}{\sin^2\varphi} d\varphi \tag{25}$$

$$\int_{+1}^{+\infty} \sin(\frac{m}{\sqrt{r}}) e^{-rt} dr = \int_0^{\pi/2} \sin(m\sqrt{\sin\varphi}) \times e^{-\frac{t}{\sin\varphi}} \frac{\cos^2\varphi}{\sin\varphi} d\varphi \tag{26}$$

$h_2(x,t)$ becomes then:

$$h_2(x,t) = \int_0^{\pi/2} \sin(m\sqrt{\sin\varphi}) \times e^{-\frac{t}{\sin\varphi}} \frac{\cos^2\varphi}{\sin\varphi} d\varphi \; + \tag{27}$$
$$\frac{1}{\pi}\int_0^{\pi} e^{[m\cos(\frac{\theta}{2}) + t\cos(\theta)]} \times \cos(t\sin(\theta) - m\sin(\frac{\theta}{2}) + \theta)d\theta$$

The integrals of the equation (27) are evaluated numerically by a Gauss quadrature method. The impulse response h(x,t) can then be calculated using the equation (12) and, knowing the pressure at the output of the source p(0,t), one can deduce the pressure at any point in the propagation medium by using equation (6).

4. Numerical simulation of the propagation

To represent the pressure injected at the input of the system, we use a damped sinusoid of the form:

$$p(0.t) = f(t) = e^{-\delta t}\sin(2\pi f t) \tag{28}$$

This function represents with a quite good precision the waveform of the pulses delivered by the ultrasonic generators. The damping ratio δ controls the frequency bandwidth of the input pulse around the nominal frequency f which is about several megahertz for current applications.

The output pressure at a given position is obtained by calculating the convolution of p(0,t) with the impulse response of filter propagation defined by equation 7.

The various functions used for the modeling of the propagation process (input pulse, impulse response and output pulse) are given on figure 3. By analyzing figure 3, we quote that the amplitude of the output signal is attenuated but one also notices a spreading in the time of the output pressure which arrives with a delay corresponding to the travel time. The comparison of the spectrum of the output pressure with that of the pressure at the input, shows a shift to lower frequencies. This results from the fact that the attenuation increases with frequency. One also notes the presence of a precursor such as it has been mentioned by other authors (Thomas.L, 1995) (He, 1999) (Ludwig & Levin, 1995). The presence of this precursor appears in the form of the spreading at the beginning of the waveform pulses which, in the absence of viscosity, would arrive at the point of x coordinate x at the moment $T = x/cL$.

5. Variation of the output pressure due to the propagation phenomena

5.1 Influence of the penetration distance

One notes the progressive reduction in the amplitude and a time-scale of the output signal as the distance from the beginning of the propagation increases figure 4. The pressure decrease of the output is due to the absorption caused by the viscous effects which attenuate the amplitude and cause a progressive filtering depending on the distance travelled by the wave. The frequency components are the beginning attenuated because of the dispersive nature of the medium.

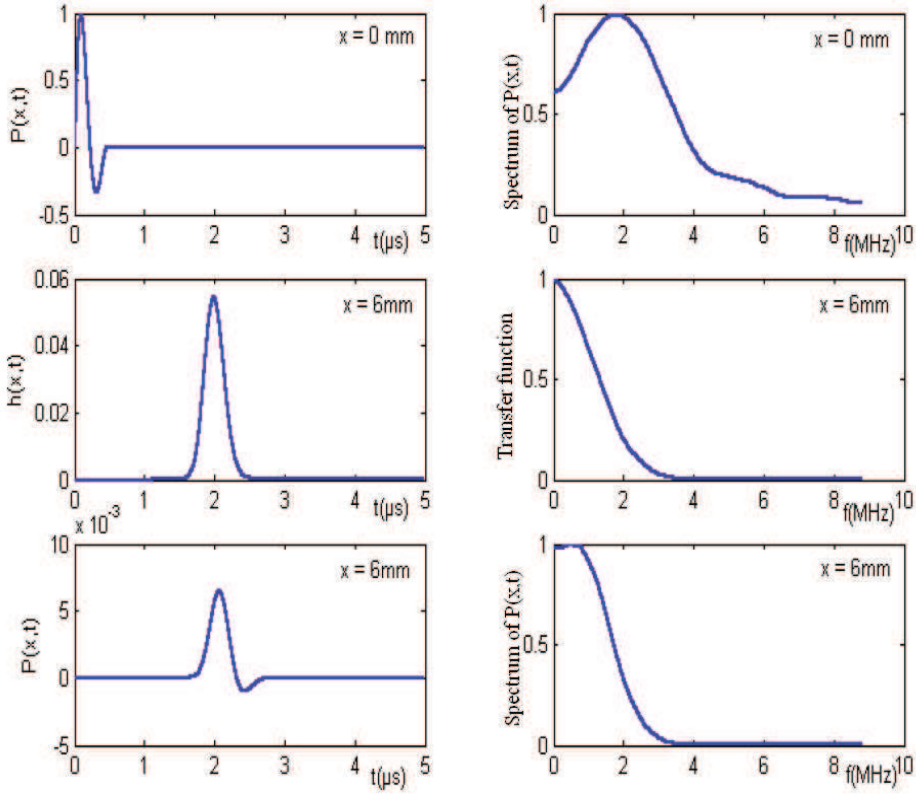

Fig. 3. Graphical representation of the input pulse, the impulse response of the output pulse at x = 6mm and the respective spectra for τ = 0.005s. The nominal frequency is f = 2.25 MHz

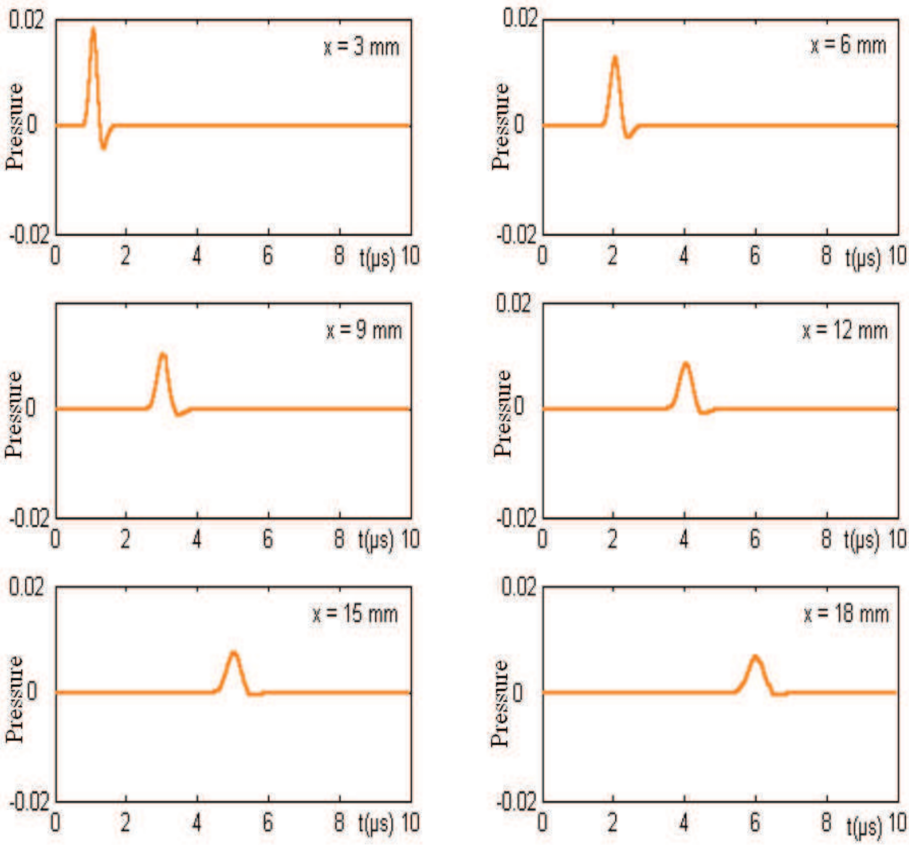

Fig. 4. Graphical representation of the output pulse for different distances for a wave having a center frequency f = 2.25MHz and for τ = 0.005s.

5.2 Influence of frequency variation

Figure 5 represents the variation of the pressure output versus frequency at a given position and for a fixed relaxation time. The increase in the frequency causes a more important reduction in the amplitude at the output. Besides this considerable reduction, we notice a contraction in time and a change of the waveform of the pulses.

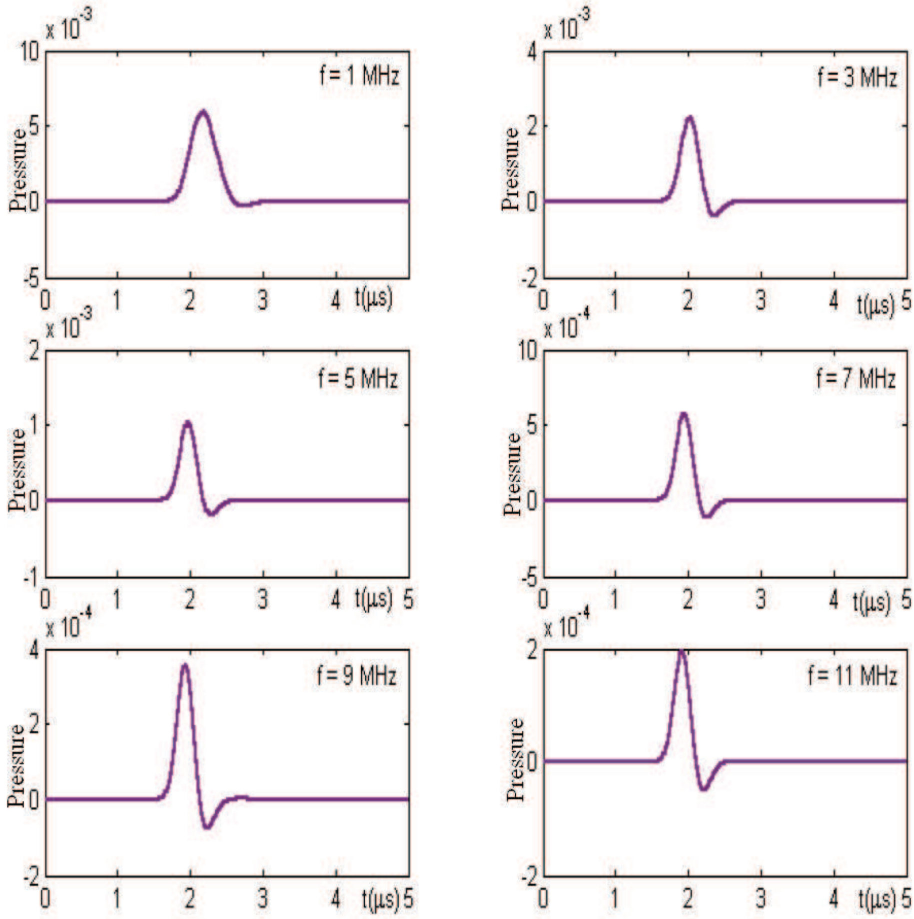

Fig. 5. Graphical representation of the output pulse at x = 6mm for different frequencies of the input pulse with τ = 0.005s

5.3 Influence of relaxation time

The effect of the relaxation time is represented on figure 6, we notice a rapid decrease of amplitude and distortion of the output pulses when the relaxation time increases and a spread in time of the output pressure.

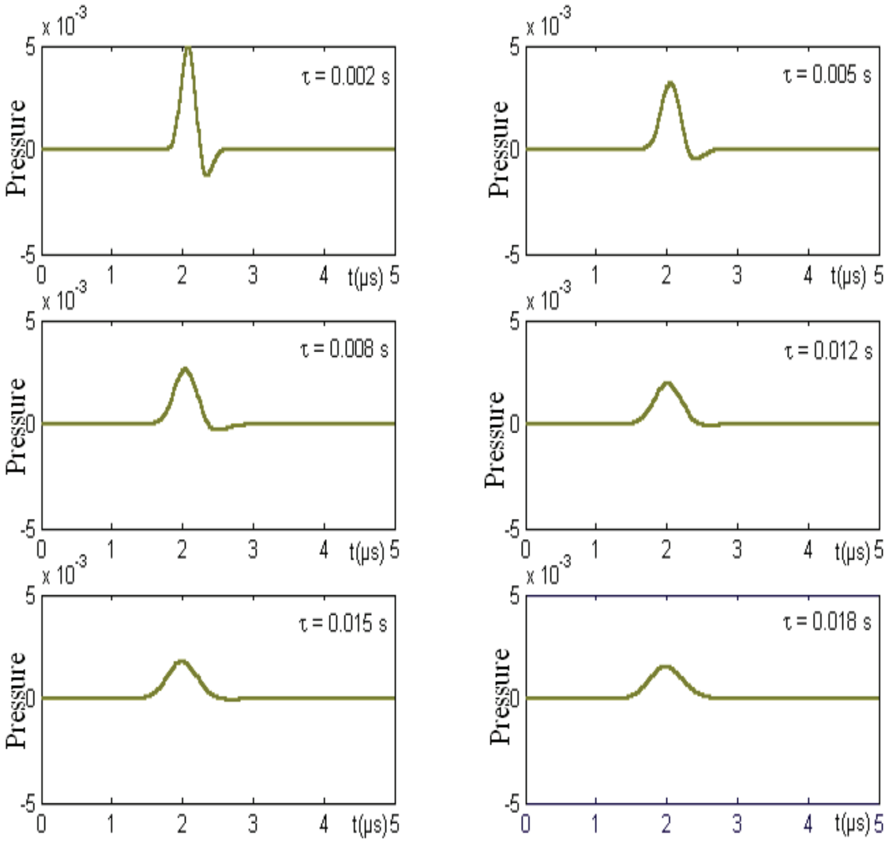

Fig. 6. Graphical representation of the output pulse at x = 6mm for different frequencies of the input pulse with τ = 0.005s

6. Validation of the theoretical model

The influence of the various mechanisms of absorption which contribute to the reduction in the amplitude during the wave propagation, have been checked in experiments by studying the variation of the propagation velocity ultrasonic and the parameter of attenuation versus the temperature in glycerine (Oudina & Djelouah, 2008). The variation of the propagation velocity versus the temperature is represented on figure 8 which shows that the propagation velocity decreases when the temperature increases; this decreasing result from the thermal agitation within the fluid because of the wave motion, and characterized by the collisions of the atoms. In the case of glycerol, the predominant molecular bindings are the OH bindings which are particularly sensitive to the temperature. In fact, a sudden change of temperature induced by the successive collision of atoms causes a break of the chemical bonds of OH type whose energy cohesion is the lowest, leading thus to a decreasing velocity when temperature increases. So it can be said that an increase in the entropy means the establishment of an irreversible steady state.

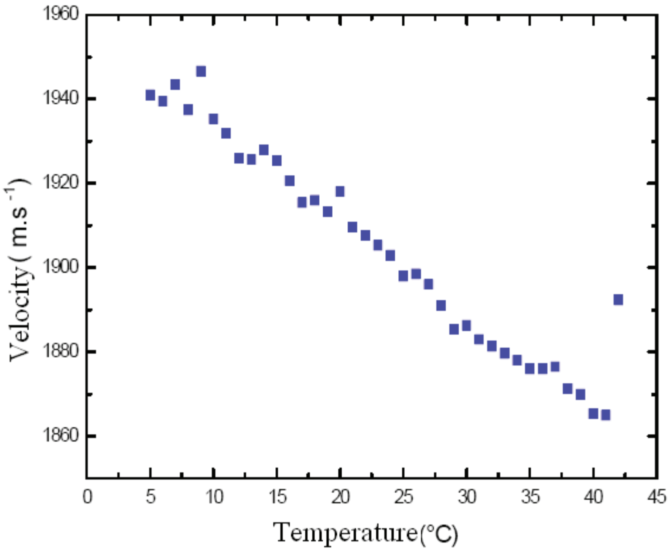

Fig. 8. Variation of the propagation velocity versus the temperature.

The study of the attenuation versus the temperature is represented on figure 9. It is noted that the attenuation decreases with increasing temperature, knowing that the attenuation parameter is closely related to the viscosity and the viscosity of a fluid strongly depends on the temperature; in particular it decreases when the temperature increases.

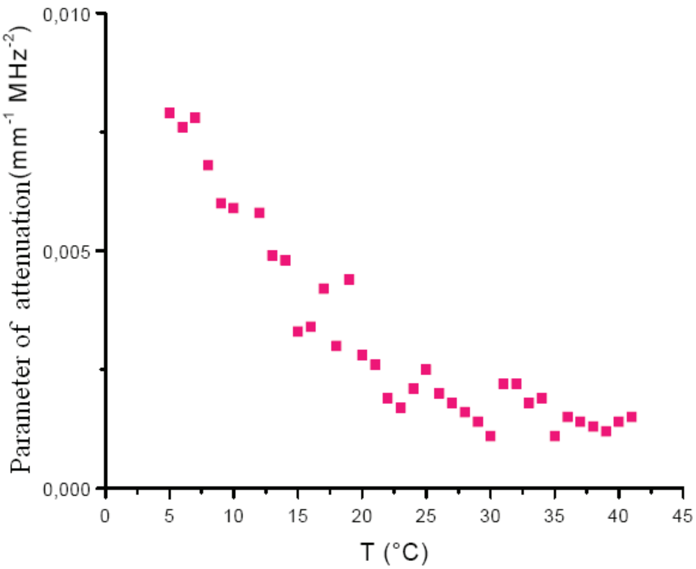

Fig. 9. Variation of the parameter of attenuation versus temperature.

7. Conclusion

The modeling of the equation of propagation in a viscous fluid by the transform of Laplace made it possible to highlight the phenomena of absorption by developing a theoretical model which allows to study the influence of these various parameters on the propagation in the viscous fluids.

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In the recent decades, there has been a growing interest in micro- and nanotechnology. The advances in nanotechnology give rise to new applications and new types of materials with unique electromagnetic and mechanical properties. This book is devoted to the modern methods in electrodynamics and acoustics, which have been developed to describe wave propagation in these modern materials and nanodevices. The book consists of original works of leading scientists in the field of wave propagation who produced new theoretical and experimental methods in the research field and obtained new and important results. The first part of the book consists of chapters with general mathematical methods and approaches to the problem of wave propagation. A special attention is attracted to the advanced numerical methods fruitfully applied in the field of wave propagation. The second part of the book is devoted to the problems of wave propagation in newly developed metamaterials, micro- and nanostructures and porous media. In this part the interested reader will find important and fundamental results on electromagnetic wave propagation in media with negative refraction index and electromagnetic imaging in devices based on the materials. The third part of the book is devoted to the problems of wave propagation in elastic and piezoelectric media. In the fourth part, the works on the problems of wave propagation in plasma are collected. The fifth, sixth and seventh parts are devoted to the problems of wave propagation in media with chemical reactions, in nonlinear and disperse media, respectively. And finally, in the eighth part of the book some experimental methods in wave propagations are considered. It is necessary to emphasize that this book is not a textbook. It is important that the results combined in it are taken “from the desks of researchers“. Therefore, I am sure that in this book the interested and actively working readers (scientists, engineers and students) will find many interesting results and new ideas.

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