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# Swarm Intelligence in Portfolio Selection

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## 1. Introduction

Portfolio selection problems in investments are among the most studied in modern finance, because of their computational intractability. The basic perception in modern portfolio theory is the way that upon it investors construct diversified portfolio of financial securities so as to achieve improved tradeoffs between risk and return.
Portfolio optimization is a procedure for generating the composition that best achieves the portfolio manager's objectives. One of the first to apply mathematical programming models to portfolio management was the quadratic programming model of Markowitz (1952), who proposed that risk be represented as the variance of the return (a quadratic function), which is to be minimized subject to achieving a minimum expected return on investment (a linear constraint). This single-period model is explained in detail by Luenberger (1998). The inputs of this analysis are security expected returns, variances, and covariance for each pair of securities, and these are all estimated from past performances of the securities. However, it is not realistic for real ever-changing asset markets. In addition, it would be so difficult to find the efficient portfolio when short sales are not allowed.
Mathematical programming (e.g., linear programming, integer linear programming, nonlinear programming, and dynamic programming) models have been applied to portfolio management for at least half a century. For a review on the application of mathematical programming models to financial markets refer to Board and Sutcliffe (1999).
Several portfolio optimization strategies have been proposed to respond to the investment objectives of individuals, corporations and financial firms, where the optimization strategy is selected according to one's investment objective. Jones (2000) gives a framework for classifying these alternative investment objectives.
Although the most obvious applications of portfolio optimization models are to equity portfolios, several mathematical programming methods (including linear, mixed integer, quadratic, dynamic, and goal programming) have also been applied to the solution of fixed income portfolio management problems since the early 1970s.
Recently, many Evolutionary Computation (EC) techniques (Beyer, 1996) such as Genetic Algorithm (GA), Particle Swarm Optimization (PSO) (Xu et al., 2006), (Delvalle et al., 2007) have been applied to solve combinatorial optimization problems (Angeline, 1995). These techniques use a set of potential solutions as a population, and find the optimal solution through cooperation and contest among the particles of the population. In comparison, in

optimization problems with computation complexity, EC techniques find often optimal solution faster than traditional algorithms (Pursehouse and Fleming, 2007).
In this study, the portfolio selection problem is concerned, in case that expected return rates are stochastic variables and the breeding swarm algorithm is applied to solve this problem. The First, the stochastic portfolio model and reliable decision are presented. The Second, the global evolutionary computation algorithm–breeding swarm is proposed in order to overcome the computational complexity and local and global searching limitation of traditional optimization methods. Finally, a numerical example of portfolio selection problem is given. Findings endorse the effectiveness of the newly proposed algorithm in comparison to particle swarm optimization method. The results show that the breeding swarm approach to portfolio selection has high potential to achieve better solution and higher convergence.

## 2. Stochastic Portfolio Model

The mean-variance model of Markowitz, to find an efficient portfolio is led to solve the following optimization problem (Markowitz, 1952):

$$\begin{aligned} &\textit{minimize} && \sigma^2(R'X) \\ &\textit{subject to} && \begin{cases} \rho(X)=\rho \\ e'(X)=1 \end{cases} \end{aligned} \tag{1}$$

where $\rho(X)$ is the reward on the portfolio $X$, $\rho$ is a constant target reward for a specific investor, and $e'$ is the transpose of the vector $e \in \Re^n$ of all 1s. The risk, $\sigma^2(R'X)$, of portfolio $X \in \Re^n$ is defined as the variance of its return $R'X$ . $R$ is the random vector of return rates. The expectation of $R$ will be denoted by $\overline{R}$ , that is, $E(R)=\overline{R}$ . Conveniently, we set: $X=(x_1,x_2,\ldots,x_n)'$, $\overline{R}=(\bar{r}_1,\bar{r}_2,\ldots,\bar{r}_n)'$, $e=(1,1,\ldots,1)'$.
This model can be rewritten by the following quadratic programming:

$$\begin{aligned} &\min && X'\Sigma X \\ &s.t. && R'X=\rho \\ & && e'(X)=1 \end{aligned} \tag{2}$$

where $\Sigma$ is the covariance matrix of the random variables $R$ .
In real ever-changing asset markets, returns of risky assets are not constant over the time. So we need to estimate $E(\overline{R})$ and $\Sigma=(\sigma_{ij})_{n\times n}$ in practical. The notion of efficient portfolio is a relative concept. The dominance between two portfolios can be defined in many different ways, and each one is expected to produce a different set of efficient portfolios. Only efficient portfolios are presented to the investor to make his/her final choice according to his/her taste toward risk. All investors in the same class, say risk averters, when the return of portfolio satisfy the expected value select the security with the lowest risk. According to above perceptions, the stochastic portfolio model can be described as:
*A: Stochastic portfolio model without risk-free asset*

$$\begin{aligned} \min \quad & X'\Sigma X \\ s.t. \quad & X'R \ge R_0 \\ & e'X = 1 \\ & X \ge 0 \end{aligned} \tag{3}$$

On condition that short sales are allowed, the stochastic portfolio model without risk-free asset can be described by eliminating the constraint $X \ge 0$.

*B: Stochastic portfolio model with risk-free asset*

$$\begin{aligned} \min \quad & X'\Sigma X \\ s.t. \quad & X'\overline{R} + (1 - X'e)R_f \ge R_0 \\ & e'X = 1 \\ & X \ge 0 \end{aligned} \tag{4}$$

where $R_f$ is the return rate of risk-free asset.

## 3. Reliable Decision of Portfolio Selection

Because of randomness of the condition $X'\overline{R} \ge R_0$, the feasible solution to the model (3) and (4) maybe achievable or not. Due to the degree of probability available in model (3) and (4), we define reliability and construct a model with limited probability. The new model can be described as follows:

$$\begin{aligned} \min \quad & X'\Sigma X \\ s.t. \quad & P(X'\overline{R} \ge R_0) \ge \alpha \\ & e'X = 1 \\ & X \ge 0 \end{aligned} \tag{5}$$

in the case that a risk-free asset exists:

$$\begin{aligned} \min \quad & X'\Sigma X \\ s.t. \quad & P(X'\overline{R} + (1 - e'X)R_f \ge R_0) \ge \alpha \\ & e'X = 1 \\ & X \ge 0 \end{aligned} \tag{6}$$

The model (5) and (6) is defined as the reliable model (3) and (4), and the possible solution of (5) and (6) is named $\alpha$ feasible solution of model (3) and (4) and is defined as $\alpha$ reliable decision for the portfolio. Since $\alpha$ reliable decision demonstrates that the portfolio decision is stochastic decision, is more important and practical and reflect the inconsistence of asset markets.

The model (5) and (6) can be converted into the determinate decision model. Defining constant $M$ by the following formula:

$$P\left(\frac{X'\overline{R} - X'R}{\sqrt{X'\Sigma X}} \ge M\right) = \alpha$$

The condition $P(X'\overline{R} \geq R_0) \geq \alpha$ would be equivalent to the determinate condition $X'R + M\sqrt{X'MX} \geq R_0$. The proof can be so followed:

*if* $P(X'\overline{R} \geq R_0) \geq \alpha$ *then* $P(X'\overline{R} \leq R_0) \leq 1-\alpha.$ Since $P(X'\overline{R} \leq X'R + M\sqrt{X'\Sigma X}) = 1-\alpha$, then according to unchanging nondecreasing manner of the distribution function of random variable, we obtain $X'R + M\sqrt{X'\Sigma X} \geq R_0$. Contradictorily, if $X'R + M\sqrt{X'\Sigma X} \geq R_0$ then $P(X'\overline{R} \geq R_0) \geq P(X'\overline{R} \geq X'R + M\sqrt{X'\Sigma X}) = \alpha$

Hence, the model (5) and (6) can be described with determinate constraint model (7) and (8):

$$\begin{aligned} \min \quad & J = X'\Sigma X \\ s.t. \quad & X'R + M\sqrt{X'\Sigma X} \geq R_0 \\ & e'X = 1 \\ & X \geq 0 \end{aligned} \tag{7}$$

in the case that we have a risk-free asset:

$$\begin{aligned} \min \quad & J = X'\Sigma X \\ s.t. \quad & X'R + M\sqrt{X'\Sigma X} - X'eR_f \geq R_0 - R_f \\ & e'X = 1 \\ & X \geq 0 \end{aligned} \tag{8}$$

If risky assets follow normal distribution $N(R_i, \sigma_i^2)$, constant $M$ can be obtained by following formula:

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{M} e^{\frac{-x^2}{2}} dx = 1-\alpha.$$

where $\alpha$ is reliable decision for the portfolio. Since $\alpha$ reliable decision demonstrates that the portfolio decision is stochastic decision, is more important and practical and reflect the inconsistence of asset markets (Xu et al., 2006).

## 4. Breeding Swarm Optimization Method for Stochastic Portfolio Selection

### 4.1 structure of Breeding Swarm Model

Angeline (1998) and Eberhart and Shi (1998) proposed that a hybrid model of GA and PSO can produce a very effective search strategy. In this context, our goal is to introduce a hybrid GA/PSO model. It has been shown that the performance of the PSO is not sensitive to the population size (Shi and Eberhart, 1999). Therefore, the PSO will work well (with a low number of particles) compared to the number of individuals needed for the GA. Since, each particle has one fitness function to be evaluated per iteration, the number of fitness function evaluations can be reduced or more iteration can be performed. The hybrid PSOs combine the traditional velocity and position update rules with the idea of breeding and subpopulations.

In this study, the hybrid model is tested and compared with the standard PSO model. This is done to illustrate that PSO with breeding strategies has the potential to achieve faster convergence and better solution. Our results show that with the correct combination of GA and PSO, the hybrid can outperform, or perform as well as, both the standard PSO and GA models. The hybrid algorithm combines the standard velocity and position update rules of PSOs (Xiao et al., 2004) with the ideas of selection, crossover and mutation from GAs. An additional parameter, the breeding ratio ($\Psi$), determines the proportion of the population which undergoes breeding (selection, crossover and mutation) in the current generation. Values for the breeding ratio parameter range from 0.0 to 1.0. In each generation, after the fitness values of all the individuals in the same population are calculated, the bottom ($N \cdot \Psi$), where N is the population size, is discarded and removed from the population. The remaining individual's velocity vectors are updated, acquiring new information from the population. The next generation is then created by updating the position vectors of these individuals to fill $N \cdot (1-\Psi)$ individuals in the next generation. The $N \cdot \Psi$ individuals needed to fill the population are selected from the individuals whose velocities are updated to undergo crossover and mutation and the process is repeated.

### 4.2 The Breeding Swarm Optimization Approach for Portfolio Selection

As mentioned in the previous section, domain of variables $x_i$ is [0, 1] and the number of particles required for simultaneous computation is 7. These particles represent the investment rate to asset i**.** We considered the population size equal to 20 and then generated a random initial population. Cost function J in (7) is defined as fitness function and used for evaluation of initial chromosomes. In this stage some particles are strong and others are weak (some of them produce lower value for fitness function and vice versa). These particles are floated in a 7-dimensional (7-D) space. After ranking the particles based on their fitness functions the best particles are selected. First each particle changes its position according to its own experience and its neighbors. So, first we have to define a neighborhood in the corresponding population and then describe the relations between particles that fall in that neighborhood. In this context, we have many topologies such as: Star, Ring, and Wheel. In this study we use the ring topology. In ring topology, each particle is related with two neighbors and intends to move toward the best neighbor. Each particle attempts to imitate its best neighbor by moving closer to the best solution found within the neighborhoods. It is important to note that neighborhoods overlap, which facilitates the exchange of information between neighborhoods and convergence to a single solution. In addition, we are using mutation and crossover operators for offspring from the selected particles to generate new populations. Therefore new populations are generated using two approaches: PSO and GA. The local best of BS algorithm is associated with the following topology (Settles et al., 2005):

1. Initialize a swarm of P particles in D-dimensional space, where D is the number of weights and biases.
2. Evaluate the fitness $f_p$ of each particle $p$ as the $J$.
3. If $f_p < p_{best}$ then $p_{best} = f_p$ and $x_{pbest} = x_p$, where $p_{best}$ is the current best fitness achieved by particle $p$, $x_p$ is the current coordinates of particle $p$ in $D$-dimensional weights space, and $x_{pbest}$ is the coordinate corresponding to particle $p$'s best fitness so far.
4. If $f_p < l_{best}$ then $l_{best} = p$, where $l_{best}$ is the particle having the overall best fitness over all particles in the swarm.

5. Select the first K best of P particles
6. Generate new population
   A: Change K particles velocity with equation:

$$\bar{v}_i = \bar{v}_i(t-1) + \rho_1(\bar{x}_{pbest_i} - \bar{x}_i(t)) + \rho_2(\bar{x}_{lbest_i} - \bar{x}_i(t))$$

   where $\rho_1$ , $\rho_2$ are accelerate constants and rand return uniform random number between 0 and 1. Then fly each particle *K* to $x_K$ + $V_{K.}$
   B: Then each *K* particles are used to offspring with mutation and crossover operators.
7. Loop to step 2 until convergence.

After completion of above processes, a new population is produced and the current iteration is completed. We iterate the above procedures until a certain criterion is met. At this point, the best fitted particle represents the optimum values of $x_i$ .

## 5. Experimental Results

In this part we present experimental results to illustrate the effectiveness of breeding swarm optimization method for the stochastic portfolio selection. The problem of portfolio selection is considered here with seven risky assets. In addition, we only examine model (7) by the breeding swarm optimization, but the optimal solution can be obtained for model (8) by the same algorithm. The return rate and covariance chart of returns are shown in Table 1 (Xu et al., 2006). Denote $F^{\bullet}$ as the obtained result of the risk of portfolio, $R^{\bullet}$ as the obtained result of the return of the portfolio.

### 5.1 Simulation Results

We used the following values for parameters in our experiments: the size of the population is 20, and for each experimental setting, 20 trials were performed. For the stochastic model, the expected portfolio return rate is $R_0 = 0.175$ , $M = 0.42$. Finally, the optimal portfolio of assets is obtained as follows. By 2000 iterations we found:
$X^{\bullet} = \{\ 0.8076, 0.3918, 0.0019, 0.0213, 0.0066, 0.2985, 0.0245\ \}$, the risk of portfolio is:
$\sigma^2(X^{\bullet}) = 0.0018$, the return of the portfolio is: $R(X^{\bullet}) = 0.1716$.
By 5000 iteration we obtained the following result for the optimal portfolio:
$X^{\bullet} = \{0.9137, 0.385, 0.0115, 0.0018, 0.2997, 0.0089\}$, the risk of portfolio is: $\sigma^2(X^{\bullet}) = 0.0011$, the return of the portfolio is: $R(X^{\bullet}) = 0.1812$.

### 5.2 Illustration and Analysis

The efficiency of the breeding swarm algorithm for portfolio selection, can be appraised from $F^{\bullet}$ (the risk of portfolio), $R^{\bullet}$ (the return of portfolio), number of iterations and the convergence rate. The results of simulation for two different iteration numbers are listed in Table 2., Fig. 1., Fig. 2., and Fig. 3.. In Table 2. the precision of the solutions for different iteration numbers is showed. From Fig. 1., Fig. 2., and Fig. 3. it can be found that the breeding swarm algorithm has so fast convergence rate for different iteration numbers.

These figures show the average fitness function of risk in 20 trials in three different iterations.

| Return | Covariance | | | | | | |
|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 0.120 | 0.141 | -0.189 | 0.167 | 0.188 | -0.186 | -0.194 | 0.161 |
| 0.090 | -0.189 | 0.260 | -0.220 | -0.248 | 0.253 | 0.255 | -0.216 |
| 0.100 | 0.167 | -0.220 | 0.224 | 0.238 | -0.217 | -0.238 | 0.209 |
| 0.100 | 0.188 | -0.248 | 0.238 | 0.270 | -0.247 | -0.260 | 0.220 |
| 0.009 | -0.186 | 0.253 | -0.238 | -0.260 | 0.256 | 0.279 | -0.230 |
| 0.115 | -0.194 | 0.255 | -0.238 | -0.260 | 0.256 | 0.279 | -0.230 |
| 0.110 | 0.161 | -0.216 | 0.209 | 0.220 | -0.217 | -0.230 | 0.209 |

Table 1**.** Return rate and covariance chart (Xu et al., 2006)

| Iterations | Best $F^{\bullet}$ | Best $R^{\bullet}$ | Average $F^{\bullet}$ | Average $R^{\bullet}$ |
|---|---|---|---|---|
| 2000 | 0.0018 | 0.1716 | 0.0038 | 0.1588 |
| 5000 | 0.0011 | 0.1812 | 0.0026 | 0.1638 |

Table 2. BS Algorithm Evaluation Results

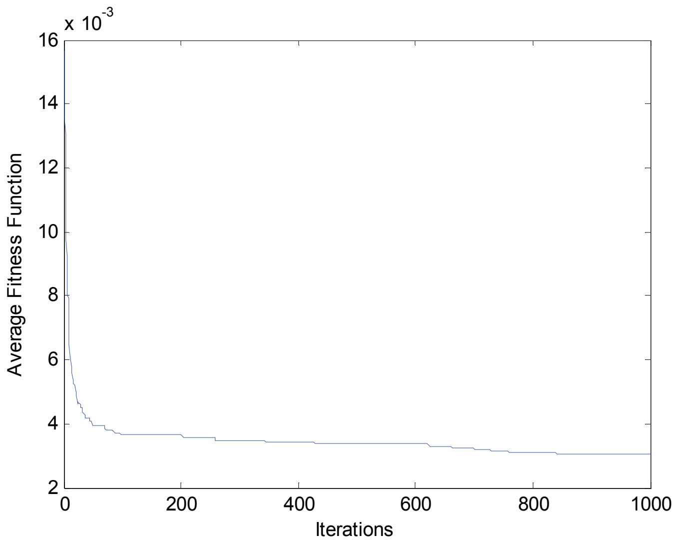


Figure 1. The performance and convergence rate with 1000 iterations

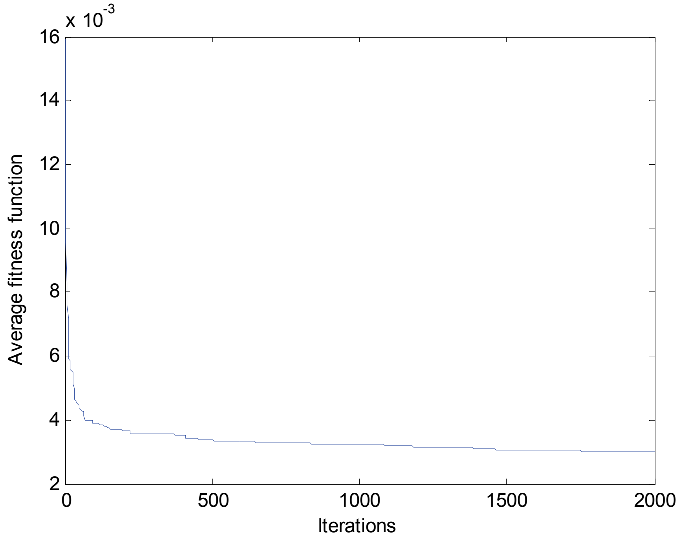


Figure 2. The performance and convergence rate with 2000 iterations

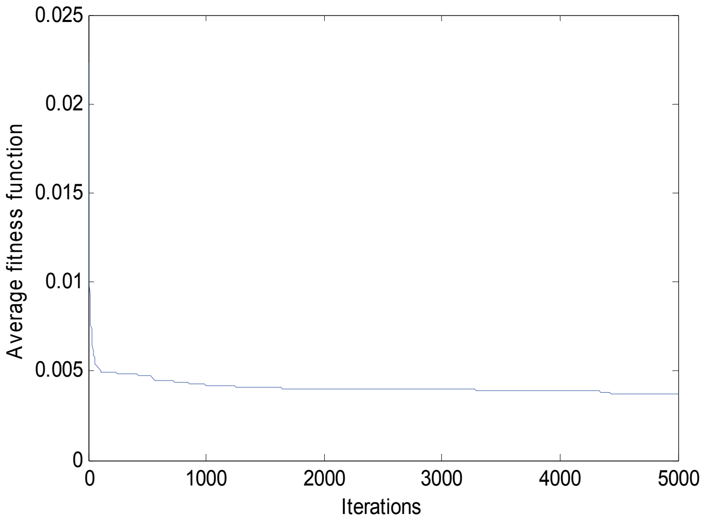


Figure 3. The performance and convergence rate with 5000 iterations

It is obvious from the figures that the BS algorithm has achieved to its efficient solution by nearly 1000 iterations. These results approve that the BS algorithm can find the solution of portfolio selection problem with high accuracy and convergence rate. The best results of Limited Velocity Particle Swarm Optimization (LVPSO) approach (Xu et al., 2006) are summarized in Table 3 to compare with our results.

| Method | Iterations | Average Iterations | Best $F^*$ | Best $R^*$ | Average $F^*$ | Average $R^*$ |
|---|---|---|---|---|---|---|
| LVPSO (Xu et al., 2006) | 7544 | 5006 | 0.009311 | 0.112622 | 0.009926 | 0.111531 |
| | 5415 | 3444 | 0.010612 | 0.110619 | 0.011098 | 0.107835 |
| BS | 5000 | 4850 | 0.001100 | 0.181200 | 0.002600 | 0.163800 |
| | 2000 | 1920 | 0.001800 | 0.171600 | 0.003800 | 0.158800 |

Table 3. Compare best results of two approaches LVPSO and BS

## 6. Conclusion

In this study, a new optimization method is used for portfolio selection problem which is powerful to select the best portfolio proportion with minimum risk and high return. One of the advantages of this hybrid approach is the high speed of convergence to the best solution, because it uses both advantages of GA and PSO approaches. Simulation results demonstrate that the BS approach can achieve better solutions to stochastic portfolio selection compared to PSO method.

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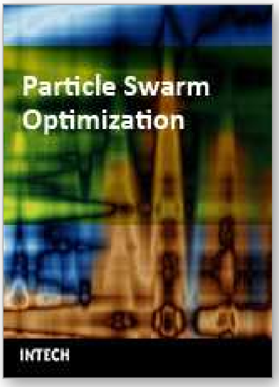


**Particle Swarm Optimization**

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Particle swarm optimization (PSO) is a population based stochastic optimization technique influenced by the social behavior of bird flocking or fish schooling.PSO shares many similarities with evolutionary computation techniques such as Genetic Algorithms (GA). The system is initialized with a population of random solutions and searches for optima by updating generations. However, unlike GA, PSO has no evolution operators such as crossover and mutation. In PSO, the potential solutions, called particles, fly through the problem space by following the current optimum particles. This book represents the contributions of the top researchers in this field and will serve as a valuable tool for professionals in this interdisciplinary field.