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Adaptive Inverse Optimal Control of a Magnetic Levitation System

Yasuyuki Satoh[1], Hisakazu Nakamura[1],
Hitoshi Katayama[2] and Hirokazu Nishitani[1]
[1]*Nara Institute of Science and Technology,* [2]*Shizuoka University*
Japan

1. Introduction

In recent years, control Lyapunov functions (CLFs) and CLF-based control designs have attracted much attention in nonlinear control theory. Particularly, CLF-based inverse optimal controllers are some of the most effective controllers for nonlinear systems [Sontag (1989); Freeman & Kokotović (1996); Sepulchre et al. (1997); Li & Krstić (1997); Krstić & Li (1998)]. These controllers minimize a meaningful cost function and guarantee the optimality and a stability margin. Moreover, we can obtain the optimal controller without solving the Hamilton-Jacobi equation. An inverse optimal controller with input constraints has also been proposed [Nakamura et al. (2007)]. On the other hand, these controllers assume that the desired state of the controlled system is an equilibrium state. Then, if the controlled system does not satisfy the assumption, we have to use a pre-feedback control design method to the assumption is virtually satisfied. However, a pre-feedback control design causes the luck of robustness. This implies that a stability margin of inverse optimal controllers is lost. Hence the designed controller does not asymptotically stabilize the system if there exists a parameter uncertainty in the system.

In this article, we study how to guarantee a stability margin when the pre-feedback controller design is used. We consider a magnetic levitation system as an actual control example and propose an adaptive inverse optimal controller which guarantees a gain margin for the system. The proposed controller consists of a conventional inverse optimal controller and a pre-feedback compensator with an adaptive control mechanism. By introducing adaptive control law based on adaptive control Lyapunov functions (ACLFs), we can successfully guarantee the gain margin for the closed loop system. Furthermore, we apply the proposed method to the actual magnetic levitation system and confirm its effectiveness by experiments.

This article is organized as follows. Section 2 introduces some mathematical notation and definitions, and outlines the previous results of CLF-based inverse optimal control design. Section 3 describes the experimental setup of the magnetic levitation system and its mathematical model. In section 4, we design an inverse optimal controller with a pre-feedback compensator for the magnetic levitation system. The problem with the designed controller is demonstrated by the experiment in section 5. To deal with the problem, we propose an adaptive inverse optimal controller in section 6. The effectiveness of the proposed controller is confirmed by the experiment in section 7. Section 8 is devoted to concluding remarks.

2. Preliminaries

In this section, we introduce some mathematical definitions and preliminary results of CLF-based inverse optimal control. We also refer to ACLF-based adaptive control techniques.

2.1 Mathematical notations and definitions

We use the notation $R_{\geq 0} := [0, \infty)$.

Definition 1 A function $\operatorname{sgn}(y)$ is defined for $y \in R$ by the following equation:

$$\operatorname{sgn}(y) = \begin{cases} -1 & (y < 0) \\ 0 & (y = 0) \\ 1 & (y > 0). \end{cases} \tag{1}$$

In this section, we consider the following input affine nonlinear system:

$$\dot{x} = f(x) + g(x)u, \tag{2}$$

where $x \in R^n$ is a state vector, $u \in U \subseteq R^m$ is an input vector and U is a convex subspace containing the origin $u = 0$. We assume that $f : R^n \to R^n$ and $g : R^n \to R^{n \times m}$ are continuous vector fields, and $f(0) = 0$. Let $L_f V$ and $L_g V$ be the Lie derivative of $f(x)$ and $g(x)$ respectively, which are defined by

$$L_f V(x) = \frac{\partial V}{\partial x} f(x), \tag{3}$$

$$L_g V(x) = \frac{\partial V}{\partial x} g(x). \tag{4}$$

For simplicity of notations, we shall drop (x) in the remaining of this article. We suppose that a local control Lyapunov function is given for system (2).

Definition 2 A smooth proper positive-definite function $V : X \to R_{\geq 0}$ defined on a neighborhood of the origin $X \subset R^n$ is said to be a local control Lyapunov function (local CLF) for system (2) if the condition

$$\inf_{u \subset U} \{L_f V + L_g V \cdot u\} < 0 \tag{5}$$

is satisfied for all $x \in X \setminus \{0\}$. Moreover, $V(x)$ is said to be a control Lyapunov function (CLF) for system (2) if $V(x)$ is a function defined on entire R^n and condition (3) is satisfied for all $x \in R^n \setminus \{0\}$.

If there exists no input constraint ($U = R^m$), a smooth proper positive-definite function $V : R^n \to R_{\geq 0}$ is a CLF if and only if

$$L_g V = 0 \Rightarrow L_f V < 0, \forall x \neq 0. \tag{6}$$

In this article, we guarantee the robustness of controllers by sector margins and gain margins.

Definition 3 A locally Lipschitz continuous mapping $\phi(u) \in R^m$ is said to be a sector nonlinearity in (α, β) with respect to $u \in R^m$ if the following conditions are satisfied:

$$\begin{aligned} \alpha u^T u < u^T \phi(u) < \beta u^T u, \forall u \neq 0, \\ \phi(0) = 0. \end{aligned} \tag{7}$$

Definition 4 System (2) is said to have a sector margin (α, β) with respect to $u \in R^m$ if the closed system

$$\dot{x} = f(x) + g(x)\phi(u) \tag{8}$$

is asymptotically stable, where $\phi(u)$ is any sector nonlinearity in (α, β) with respect to $u \in R^m$.

Definition 5 System (2) is said to have a gain margin (α, β) with respect to $u \in R^m$ if the closed system (8) is asymptotically stable, when $\phi(u)$ is given as follows:

$$\phi(u) = \kappa u, \kappa \in (\alpha, \beta). \tag{9}$$

By the definition, gain margins are the special case of sector margins. If system (2) has a sector margin (α, β), it also has a gain margin (α, β).

2.2 Inverse optimal controller

We introduce the inverse optimal controller proposed by Nakamura et al [Nakamura et al. (2007)]. The following results are obtained for system (2) with input constraint

$$U_k^C := \left\{ u \in R^m \,\middle|\, \|u\|_k = \left(\sum_{i=1}^{m} |u_i|^k \right)^{\frac{1}{k}} < C(x) \right\}, \tag{10}$$

where $1 < k < \infty$ is a constant and $C(x) > 0$ is continuous on R^n.

Theorem 1 We consider system (2) with input constraint (10). Let $V(x)$ be a local CLF for system (2) and $a_1 > 0$ be the maximum number satisfying

$$\inf_{u \in U_k^C} \{L_f V + L_g V \cdot u\} < 0, \quad \forall x \in W_1 := \{x \mid V(x) < a_1\}. \tag{11}$$

Then, W_1 is a domain in which the origin is asymptotically stabilizable. If $V(x)$ is a CLF, then $a_1 = \infty$ and $W_1 = R^n$.

Theorem 2 We consider system (2) with input constraint (10) . Let $V(x)$ be a local CLF for system (2) , $P(x)$ be a function defined by

$$P(x) = \frac{L_f V}{C(x) \|L_g V\|_{\frac{k}{k-1}}}, \tag{12}$$

and $a_r \in (0, a_1)$ be the maximum number such that the condition

$$\inf_{u \in U_k^C} \left\{ L_f V + \frac{k-1}{k} L_g V \cdot u \right\} < 0, \quad \forall x \in W_r := \{x \mid V(x) < a_r\} \tag{13}$$

is satisfied, and d be a positive constant. Then, input

$$u_i = -\frac{1}{R(x)} \left| L_{g_i} V^{\frac{1}{k-1}} \right| \mathrm{sgn}(L_{g_i} V) \qquad (i = 1, \ldots, m), \tag{14}$$

$$R(x) = \begin{cases} \dfrac{(2 + q(x) \|L_g V\|_{\frac{k}{k-1}}^{\frac{1}{k-1}}) \|L_g V\|_{\frac{k}{k-1}}^{\frac{1}{k-1}}}{\frac{k}{k-1}(P + |P|) + q(x) \|L_g V\|_{\frac{k}{k-1}}^{\frac{1}{k-1}}} & (L_g V \neq 0) \\ \dfrac{2}{q(x)} & (L_g V = 0), \end{cases} \tag{15}$$

$$q(x) = dC^{\frac{1}{k-1}}(x) \tag{16}$$

asymptotically stabilizes the origin in W_r, and minimizes the cost function:

$$J = \int_0^\infty \left\{ l(x) + \frac{R^{k-1}(x)}{k} \|u\|_k^k \right\} dt, \\ l(x) = \frac{k-1}{k} \cdot \frac{1}{R(x)} \|L_g V\|_{\frac{k}{k-1}}^{\frac{k}{k-1}} - L_f V. \tag{17}$$

Moreover, it achieves at least a sector margin (α, β) in W_r.

2.3 Adaptive control problem

We consider an adaptive control problem for nonlinear systems. In this section, we introduce some definitions and properties. We consider the following input affine nonlinear system:

$$\dot{x} = f_0(x) + f_1(x)\theta + g(x)u, \tag{18}$$

where $x \in R^n$ is a state vector, $u \in R^m$ is an input vector, and $\theta \in R^p$ is a constant unknown parameter vector. We assume that $f_0 : R^n \to R^n$, $g : R^n \to R^{n\times m}$ and $f_1 : R^n \to R^{n\times p}$ are continuous vector fields, and $f_0(0) = 0$. Note that there exists no input constraint.
The stabilizability of the system with unknown parameters is defined as the following.

Definition 6 Let $\hat{\theta}$ be an estimate of θ. We say that (18) is globally adaptively stabilizable if there exist a function $\alpha(x, \hat{\theta})$ continuous on $R^n \setminus \{0\} \times R^p$ with $\alpha(0, \hat{\theta}) \equiv 0$, a continuous function $\tau(x, \hat{\theta})$, and a positive definite symmetric $p \times p$ matrix Γ, such that the dynamic controller

$$u = \alpha(x, \hat{\theta}), \tag{19}$$

$$\dot{\hat{\theta}} = \Gamma \tau(x, \hat{\theta}) \tag{20}$$

guarantees that the solution $(x, \hat{\theta})$ is globally bounded, and $x \to 0$ as $t \to \infty$ for any value of the unknown parameter $\theta \in R^p$.
For the stabilization problem, we introduce an adaptive control Lyapunov function (ACLF) as the following.

Definition 7 We consider system (18) and assume that $V_a(x, \theta)$ is a CLF for system (18). Then, $V_a(x, \theta)$ is called an adaptive control Lyapunov function (ACLF) for system (18) if there exists a positive-definite symmetric matrix Γ such that for each $\theta \in R^p$, V_a is a CLF for the modified system

$$\dot{x} = f_0(x) + f_1(x)\left(\theta + \Gamma\left(\frac{\partial V_a}{\partial \theta}\right)^T\right) + g(x)u. \tag{21}$$

Krstić et al. (1995) proved the following theorem.
Theorem 3 The following two statements are equivalent:

(1) There exists a triple (α, V_a, Γ) such that $\alpha(x, \hat{\theta})$ globally asymptotically stabilizes (21) at $x = 0$ for each $\theta \in R^p$ with respect to the Lyapunov function $V_a(x, \theta)$.

(2) There exists an ACLF $V_a(x, \theta)$ for system (18).

3. Magnetic Levitation System

3.1 System configuration

We consider a stabilization problem of a magnetic levitation system shown in Fig. 1[Mizutani et al. (2004)]. The system consists of a magnet with a disk, a glass guide rod, upper and lower magnetic drive coils that generate a magnetic field in response to a DC current and two laser-based sensors that measure the magnetic position using the reflection of the disk surface.

3.2 Mathematical model of the system

In this article, we control the position of the magnet using attractive force generated by the upper drive magnetic coil. The force diagram is illustrated in Fig. 2. ξ is the position of the magnet from the upper coil, and F_u is an attractive force for the magnet generated by the upper drive magnetic coil.
The dynamical equation for the magnet is described by

$$m\ddot{\xi} = F_u - m\mu\dot{\xi} - mg_0, \tag{22}$$

where m is the mass of the magnet, μ is a friction constant. g_0 is the gravitational acceleration.

Fig. 1. Magnetic levitation system

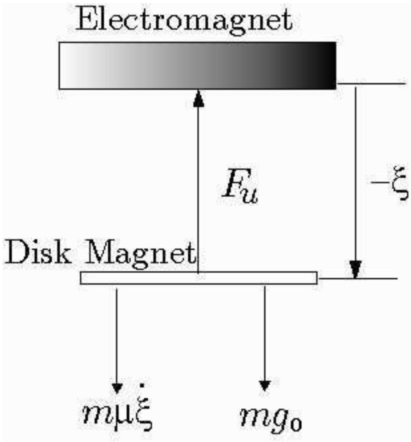

Fig. 2. Force diagram of the magnetic levitation system

Here, F_u is modeled by

$$F_u = \frac{u}{a(-\xi + b)^4}, \tag{23}$$

where a and b are constants determined by numerical modeling of the magnetic configuration, and u is a control input voltage for the upper coil. The position ξ is measured by the upper laser sensor.

Let ξ^* be the desired position of the magnet, $x_1 = \xi - \xi^*$, and $x_2 = \dot{x}_1$. We set $x = [x_1, x_2]^T$. Then we obtain the following state equation:

$$\dot{x} = f(x) + g(x)u, \tag{24}$$

where $f(x)$ and $g(x)$ are defined as

$$f(x) = \begin{bmatrix} x_2 \\ -\mu x_2 - g_0 \end{bmatrix}, g(x) = \begin{bmatrix} 0 \\ \dfrac{1}{ma(-x_1 - \xi^* + b)^4} \end{bmatrix}. \tag{25}$$

The system parameters are shown in Table 1.

m [kg]	μ [-]	g_0 [m/s²]	a [V/N·m⁴]	b [m]
0.12	4.5	9.80665	40118.9	0.056464

Table 1. Parameter values of the magnetic levitation system

There exists the following input constraint in system (24):

$$\|u\|_2 = \sqrt{u^2} < 5\,[\mathrm{V}]. \tag{26}$$

By the above discussion, the control problem is reduced to the stabilization problem of system (24) with the input constraint (26).

4. Pre-feedback Gravity Compensation

In system (2), we assume that $f(0) = 0$. However, $f(0) \neq 0$ in system (24). Therefore, we cannot directly apply the inverse optimal controller (14) to system (24). To achieve $f(0) = 0$, we design a controller to compensate for gravity by a pre-feedback input.
We consider the following gravity compensation input $u_c(x)$ as

$$u_c(x) = mg_0 a(-x_1 - \xi^* + b)^4. \tag{27}$$

Substituting (27) into (24), the gravitational acceleration g_0 is successfully canceled. Then, we split the input $u(x)$ using $u_c(x)$ as

$$u(x) = u_c(x) + u_s(x), \tag{28}$$

where u_s is an asymptotic stabilizing input for system (24) when $g_0 = 0$.
By using (26) and (28), the input constraint is rewritten to

$$\|u(x)\|_2 = \|u_c(x) + u_s(x)\|_2 < 5. \tag{29}$$

To handle input constraint (29) as a norm constraint, we rewrite (29) as

$$\|u_s(x)\| < \big\| 5 - |u_c(x)| \,\big\|_2 := C(x). \tag{30}$$

(30) represents a constraint depending on the state. Note that constraint (30) is more severe than the original constraint (29). The problem of designing controller (28) is reduced to the problem of designing controller $u_s(x)$ with input constraint $\|u_s(x)\|_2 < C(x)$.
To apply inverse optimal controller (14), we construct a CLF for system (24). In general, the controller performance often depends on a CLF. However, it is unclear which CLF achieves the best control performance. Hence, we construct a CLF with a design parameter. Using the integrator backstepping method, a CLF $V(x)$ can be carried out as

$$V(x) = \frac{1}{2}(r^2 + 1)x_1^{\,2} + r x_1 x_2 + \frac{1}{2}x_2^{\,2}, \tag{31}$$

where r is a positive constant and also a design parameter.
Now, we construct input u_s. Let $f_0(x)$ be the function defined by

$$f_0(x) = f(x)|_{g_0=0} = \begin{bmatrix} x_2 \\ -\mu x_2 \end{bmatrix}. \tag{32}$$

By using (31), we can calculate $L_{f_0}V$ and L_gV as

$$L_{f_0}V = \{(r^2+1) - \mu r\}x_1x_2 + (r-\mu){x_2}^2, \tag{33}$$

$$L_gV = \frac{rx_1 + x_2}{ma(-x_1 - \xi^* + b)^4}. \tag{34}$$

Substituting (33) and (34) into (14) and (15), we get the following input $u_s(x)$.

$$u_s(x) = -\frac{1}{R_1(x)} L_gV,\text{'} \tag{35}$$

$$R_1(x) = \begin{cases} \dfrac{(2+q(x)\|L_gV\|_2)\|L_gV\|_2}{2(P_1+|P_1|)+q(x)\|L_gV\|_2} & (L_gV \neq 0) \\ \dfrac{2}{q(x)} & (L_gV = 0), \end{cases} \tag{36}$$

$$P_1(x) = \frac{L_{f_0}V}{C(x)\|L_gV\|_2}, \tag{37}$$

$$q(x) = dC(x) = d\left\| 5 - |u_c(x)| \right\|_2. \tag{38}$$

According to Theorem 2, $u_s(x)$ has a sector margin $(1/2, \infty)$.
Finally, the following controller $u(x)$ is obtained:

$$u(x) = mg_0a(-x_1 - \xi^* + b)^4 - \frac{1}{R_1(x)} L_gV. \tag{39}$$

5. Experiment 1

We apply controller (39) to the magnetic levitation system. We set $x(0) = [-1.4, 0.0]^T$ and $\xi^* = -2.0$ [cm]. The controller is implemented by MATLAB/SIMULINK. The sampling

interval is 1×10^{-3} [sec] and control parameters are $r = 8$ and $d = 1.25 \times 10^{-4}$, respectively.
The time response of the controlled system is shown in Fig. 3. Although the velocity x_2 vibrates due to sensor noise, the input constraint (26) is satisfied. However, the position x_1 does not converge to zero (an offset error remains). Then, the actual magnetic levitation system is not asymptotically stabilized by the proposed controller (39).
The biggest reason for the offset error is the lack of robustness with respect to u_c. If there exists a parameter uncertainty in $g(x)$, the gravitational acceleration g_0 is not completely canceled by the pre-feedback $u_g(x)$. Therefore, the proposed controller $u(x)$ does not guarantee the robustness for the system (24) even if the stabilizing input $u_s(x)$ guarantees the sector margin $(1/2, \infty)$ for the system

$$\dot{x} = f_0(x) + g(x)u_s. \tag{40}$$

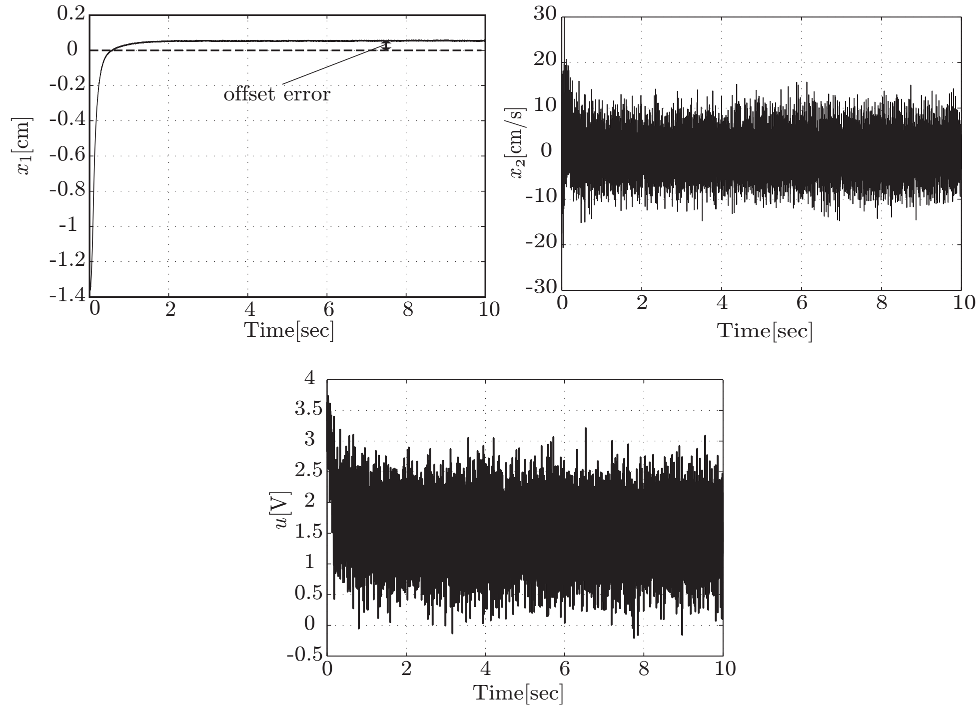

Fig. 3. Experimental result of controller (39)

6. Adaptive Inverse Optimal Controller Design

6.1 Robustness recovery via adaptive control

To solve the problem stated in section 5, we propose a controller that guarantees a gain margin for u. We apply an adaptive control technique to achieve a gain margin for input u. Before applying the adaptive controller, we rewrite the system (24) to

$$\dot{x} = f_0(x) + f_1(x)g_0 + g(x)u \ , \tag{41}$$

where $f_0(x)$ is defined by (31) and $f_1(x) = [0,-1]^T$. Additionally, to consider a gain margin for (41), we rewrite the system to

$$\dot{x} = f_0(x) + \kappa f_1(x)\theta + \kappa g(x)u \ , \tag{42}$$

where κ is an unknown constant and $\theta := g_0 / \kappa$ is a constant unknown parameter. Note that the range of κ, in which the origin of the system (42) is asymptotically stable, is a gain margin for input u. Furthermore, we consider the following input:

$$u(x,\hat{\theta}) = u_c(x,\hat{\theta}) + u_s'(x,\hat{\theta}), \tag{43}$$

where $\hat{\theta} := g_0 / \hat{\kappa}$ and $\hat{\kappa}$ is an estimate of κ. We suppose that input $u_s'(x,\hat{\theta})$ asymptotically stabilizes the system (40) and guarantees the gain margin $(1/2,\infty)$. Let $u_g(x,\hat{\theta})$ be a gravity compensation input defined as follows:

$$u_c(x,\hat{\theta}) = ma\hat{\theta}(-x_1 - \xi^* + b)^4. \tag{44}$$

Remark 1 In this section, we do not mention whether the input constraints exist or not.

Then, we construct an adaptive law $\dot{\hat{\theta}}$ such that the input (43) stabilizes the system (42) and show the input (43) has a gain margin $(1/2,\infty)$.
In this section, we use an ACLF to construct an adaptive law. The following lemma is available for constructing an ACLF.

Lemma 1 We consider system (42). Let $V(x)$ be a CLF for system (41). Then, $V(x)$ is an ACLF for system (42).

Proof: If $V(x)$ is an ACLF for system (42), $V(x)$ is a CLF for the following system:

$$\dot{x} = f_0(x) + \kappa f_1(x)\left(\theta + \gamma \frac{\partial V}{\partial \theta}\right) + \kappa g(x)u, \tag{45}$$

where γ is a positive constant. Note that $\partial V / \partial \theta = 0$, the above system is rewritten to

$$\dot{x} = f_0(x) + \kappa f_1(x)\theta + \kappa g(x)u. \tag{46}$$

Sytem (46) is asymptotically stabilized by the input

$$u(x) = \theta u_c(x) + u_s(x), \tag{47}$$

where $u_c(x)$ and $u_s(x)$ are defined by (27) and (35) respectively. This implies all CLFs for system (41) are ACLFs for system (42).

By Lemma 1, CLF (31) is applicable to an ACLF for system (42).

Lemma 2 We consider system (42) and assume that an ACLF $V(x)$ for (42) is obtained. Let $V'(x,\hat{\theta})$ be a function defined by

$$V'(x,\hat{\theta}) = V(x) + \frac{\kappa}{2\gamma}(\theta - \hat{\theta})^2 = V(x) + \frac{\kappa}{2\gamma}\tilde{\theta}^2, \tag{48}$$

where $1/2 < \kappa < \infty$ and $\tilde{\theta} := \theta - \hat{\theta}$. Let the adaptive law $\dot{\hat{\theta}}$ be

$$\dot{\hat{\theta}} = \gamma\tau(x) = \gamma \frac{\partial V}{\partial x} f_1(x). \tag{49}$$

Then, $V'(x,\hat{\theta})$ is a Lyapunov function for the closed loop system of (42).

Proof: Let the origin of system (42) be $(x,\hat{\theta}) = (0,\theta)$. Then, V' is a positive definite function. Assume u is input (43) and note that $\dot{\tilde{\theta}} = -\dot{\hat{\theta}}$. Then,

$$\begin{aligned} \dot{V}'(x,\hat{\theta}) &= \frac{\partial V}{\partial x}\left[f_0(x) + \kappa\left\{f_1(x)\hat{\theta} + g(x)\left(u_c(x,\hat{\theta}) + u_s'(x,\hat{\theta})\right)\right\}\right] \\ &= \frac{\partial V}{\partial x}\left[f_0(x) + \kappa g(x) u_s'(x,\hat{\theta})\right] \le 0. \end{aligned} \tag{50}$$

Since the input $u_s'(x,\hat{\theta})$ has a gain margin $(1/2,\infty)$, $\dot{V}'(x,\hat{\theta})$ is less than or equal to zero. Then $V'(x,\hat{\theta})$ is a Lyapunov function for the closed loop system of (42) and the origin $(x,\hat{\theta}) = (0,\theta)$ is stable.

Remark 2 Lyapunov function (48) contains an unknown constant κ. However, it does not become a problem because both input (43) and adaptive law (49) do not contain κ.

Lemma 3 We consider system (42) and assume that an ACLF $V(x)$ for (42) is obtained. Then, if $1/2 < \kappa < \infty$, $x \to 0 (t \to \infty)$ and $\hat{\theta} \to \theta (t \to \infty)$ are achieved by input (43) and adaptive law (49).

Proof: By Lemma 2, we can construct a Lyapunov function $V'(x,\hat{\theta})$ (47) for system (41). The input and the adaptive law are given by (42) and (48), respectively. Then, we obtain $\dot{V}'(x,\hat{\theta}) \le 0 (x \ne 0)$ because the input $u_s'(x,\hat{\theta})$ has a gain margin $(1/2,\infty)$. Let S be a set defined by

$$\begin{aligned} S &:= \{(x,\hat{\theta}) | \dot{V}'(x,\hat{\theta}) = 0, x \in R^n, \hat{\theta} \in R\}, \\ &= \{(x,\hat{\theta}) | x = 0, \hat{\theta} \in R\}. \end{aligned} \tag{51}$$

We show that the largest invariant set contained in S consists of only a point $(x,\hat{\theta}) = (0,\theta)$. Consider the following solution of (42) belonging to S:

$$x(t) \equiv 0, t \ge 0. \tag{52}$$

Note that $u_s'(0,\hat{\theta}) = 0$, we obtain the following equation for (42):

$$\begin{aligned} \dot{x} &= f_0(0) + \kappa\{f_1(0)\theta + g(0)u(0,\hat{\theta})\}, \\ &= \kappa\{f_1(0)\theta + g(0)u_c(x,\hat{\theta})\}, \\ &= \kappa f_1(\theta - \hat{\theta}) \equiv 0, \end{aligned} \tag{53}$$

where $\kappa \ne 0$ and $f_1(0) \ne 0$, we obtain $\hat{\theta} \equiv \theta$. On the other hand, if $x = 0$ and $\hat{\theta} \ne \theta$, we obtain $\dot{x} \ne 0$ by (50). Therefore, the largest invariant set contained in S is a set $\{(0,\theta)\}$. Finally, we obtain $x \to 0$ and $\hat{\theta} \to \theta$ when $t \to \infty$ by LaSalle's invariance principle [Khalil (2002)].

The following theorem is obtained by Lemmas 2 and 3.

Theorem 4 We consider system (42), controller (43) and adaptive law (49). Then, the controller has a gain margin $(1/2,\infty)$.

6.2 Adaptive inverse optimal controller

We calculate $\dot{\hat{\theta}}$ of (49) by using CLF (31) as:

$$\begin{aligned} \dot{\hat{\theta}} &= \gamma\left[(r^2+1)x_1 + rx_2 \quad rx_1 + x_2\right] \begin{bmatrix} 0 \\ -1 \end{bmatrix} \\ &= -\gamma(rx_1 + x_2). \end{aligned} \tag{54}$$

Furthermore, taking into consideration the input constraint, we obtain the following controller:

$$u(x,\hat{\theta}) = u_c(x,\hat{\theta}) + u_s'(x,\hat{\theta}) = ma\hat{\theta}(-x_1 - \xi^* + b)^4 + u_s'(x,\hat{\theta}), \tag{55}$$

$$u_s'(x,\hat{\theta}) = -\frac{1}{R_2(x,\hat{\theta})} L_g V, ' \tag{56}$$

$$R_2(x,\hat{\theta}) = \begin{cases} \dfrac{(2 + q(x,\hat{\theta})\|L_g V\|_2)\|L_g V\|_2}{2(P_2 + |P_2|) + q(x,\hat{\theta})\|L_g V\|_2} & (L_g V \neq 0) \\ \dfrac{2}{q(x,\hat{\theta})} & (L_g V = 0), \end{cases} \tag{57}$$

$$P_2(x,\hat{\theta}) = \frac{L_{f_0} V}{C(x,\hat{\theta})\|L_g V\|_2}, \tag{58}$$

$$q(x,\hat{\theta}) = dC(x,\hat{\theta}), \tag{59}$$

$$C(x,\hat{\theta}) = \left\| 5 - |u_c(x,\hat{\theta})| \right\|_2, \tag{60}$$

where we use $u_s(x)$ given by (35) as $u_s'(x,\hat{\theta})$. Then, note that the input constraint $C(x)$ is rewritten to $C(x,\hat{\theta})$ given by (60). According to Lemma 2 and the result of [Nakamura et al. (2007)], we can show the input $u_s'(x,\hat{\theta})$ minimizes the following cost function:

$$J = \int_0^\infty l(x,\hat{\theta}) + \frac{R_2(x,\hat{\theta})}{2} u_s'^2 dt, \tag{61}$$

where

$$l(x,\hat{\theta}) = \frac{1}{2R_2(x,\hat{\theta})}\|L_g V\|_2^2 - L_{f_0} V. \tag{62}$$

It is obvious that a gain margin $(1/2, \infty)$ is guaranteed for controller (55) at least in the neighborhood of the origin.

7. Experiment 2

In this section, we apply controller (55) to the magnetic levitation system and confirm its effectiveness by the experiment. To consider the input constraint, we employ the following adaptive law with projection instead of (54):

$$\dot{\hat{\theta}} = \begin{cases} 0 & \hat{\theta} = 2g_0, rx_1 + x_2 > 0 \\ 0 & \hat{\theta} = 0, rx_1 + x_2 < 0 \\ -\gamma(rx_1 + x_2) & otherwise \end{cases} . \quad (63)$$

We set the adaptation gain $\gamma = 160$ and the initial value of the estimate $\hat{\theta}(0) = 820$. The other experimental conditions and control parameters are the same as in section 5. The experimental result is shown in Fig. 5. Position x_1 converges to zero without any tuning of control parameters. The gain margin guaranteed by the adaptive law seems quite effective. We can observe that the input is larger than the non-adaptive controller (39), however, the input constraint is satisfied. The parameter estimate $\hat{\theta}$ also tends to converge to the true value θ. As a result, the effectiveness of the proposed controller (55) is confirmed.

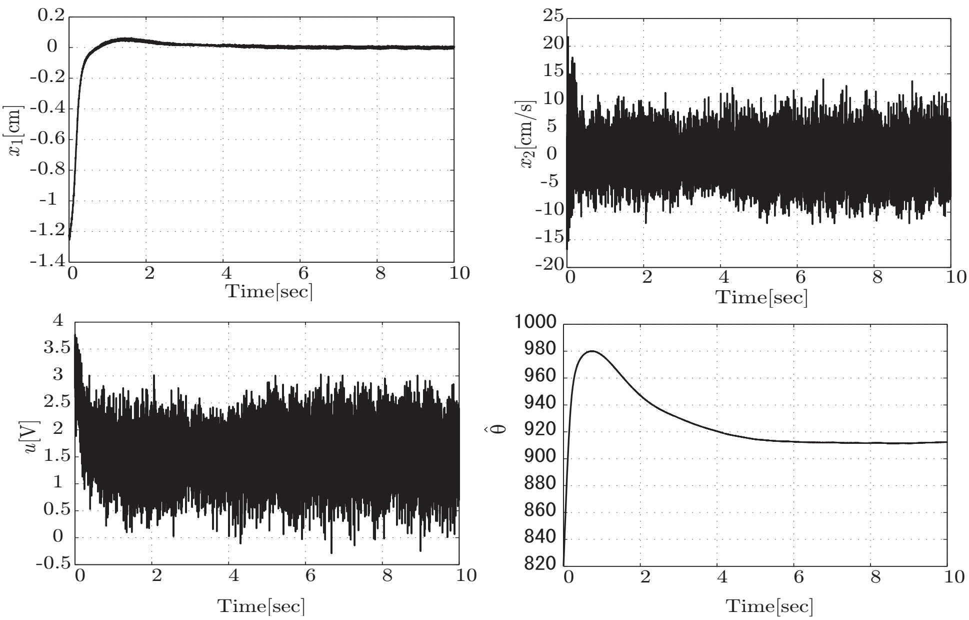

Fig. 4. Experimental result of controller (55)

8. Conclusion

In this article, we proposed an adaptive inverse optimal controller for the magnetic levitation system. First, we designed an inverse optimal controller with a pre-feedback gravity compensator and applied it to the magnetic levitation system. However, this controller cannot guarantee any stability margin. We demonstrated that the controller did not work well (offset error remained) in the experiment. Hence, we proposed an improved controller via an adaptive control technique to guarantee the stability margin. Finally, we confirmed the effectiveness of the proposed adaptive inverse optimal controller by the experiment. As a result, we achieved offset-free control performance.

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Adaptive control has been a remarkable field for industrial and academic research since 1950s. Since more and more adaptive algorithms are applied in various control applications, it is becoming very important for practical implementation. As it can be confirmed from the increasing number of conferences and journals on adaptive control topics, it is certain that the adaptive control is a significant guidance for technology development.The authors the chapters in this book are professionals in their areas and their recent research results are presented in this book which will also provide new ideas for improved performance of various control application problems.

InTech Europe
University Campus STeP Ri
Slavka Krautzeka 83/A
51000 Rijeka, Croatia
Phone: +385 (51) 770 447
Fax: +385 (51) 686 166
www.intechopen.com

InTech China
Unit 405, Office Block, Hotel Equatorial Shanghai
No.65, Yan An Road (West), Shanghai, 200040, China
中国上海市延安西路65号上海国际贵都大饭店办公楼405单元
Phone: +86-21-62489820
Fax: +86-21-62489821