An Adaptive Controller Design for Flexible-joint Electrically-driven Robots With Consideration of Time-Varying Uncertainties

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1. Abstract

Almost all present control strategies for electrically-driven robots are under the rigid robot assumption. Few results can be found for the control of electrically driven robots with joint flexibility. This is because the presence of the joint flexibility greatly increases the complexity of the system dynamics. What is worse is when some system dynamics are not available and a good performance controller is required. In this paper, an adaptive design is proposed to this challenging problem. A backstepping-like procedure incorporating the model reference adaptive control is employed to circumvent the difficulty introduced by its cascade structure and various uncertainties. A Lyapunov-like analysis is used to justify the closed-loop stability and boundedness of internal signals. Moreover, the upper bounds of tracking errors in the transient state are also derived. Computer simulation results are presented to demonstrate the usefulness of the proposed scheme.
Keywords: Adaptive control; Flexible-joint electrically-driven robot; FAT

2. Introduction

Control of rigid robots has been well understood in recent years, but most of the schemes ignore the dynamics coming from electric motors and harmonic drivers that are widely implemented in the industrial robots. However, actuator dynamics constitute an important part of the complete robot dynamics, especially in the cases of high-velocity movement and highly varying loads[1],[2]. The main reason for using a reduced model is to simplify complexity of controller design. For each joint, consideration of the flexibility from the

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harmonic driver results in an additional 2nd order dynamics. If the motor dynamics is also included, a totally 5th order dynamics should be considered. It is well-known that a multi-DOF rigid robot is a highly nonlinear and coupled system. If we consider the motor dynamics and joint flexibility for all joints, the controller design problem would become extremely difficult. In this paper, we would like to design a controller for a flexible-joint electrically-driven robot under more challenging conditions, that is, the robot system contains various uncertainties.

Better motion control performance was obtained by Tarn et. al.[2] under experimental verification for a rigid robot when considering the motor dynamics. For the robust control of rigid robots with consideration of actuator dynamics can be found in [3]-[9]. Important developments for the adaptive control of electrically-driven rigid robots can be seen in [10]-[18].

The above mentioned schemes are all for the control of rigid robots. For the control of flexible-joint electrically driven robots, few results can be found. Some robust designs were presented in [19]-[22]. However, to our best knowledge, no work has been reported on adaptive control of flexible-joint robot manipulators incorporating motor dynamics. The main contribution of the present paper is to propose an adaptive controller for this system. The controller does not need to calculate the regressor[23] which is required in conventional robot adaptive control. The design follows a backstepping-like procedure with the support of the model reference adaptive control. The function approximation technique (FAT)[24]-[35] is employed to deal with the system uncertainties. A Lyapunov-like analysis is used to justify the closed-loop stability and boundedness of internal signals.

This paper is organized as follows. Section 2 derives the proposed adaptive motion controller in detail. Section 3 presents simulation results of the motion control of a 2-D robot using the proposed controller. Section 4 concludes the paper.

3. Main Results

The dynamics of a rigid-link flexible-joint electrically-driven (RLFJED) robot can be described by

$$\mathbf{D}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \mathbf{K}(\boldsymbol{\theta} - \mathbf{q}) \tag{1}$$

$$\mathbf{J}\ddot{\boldsymbol{\theta}} + \mathbf{B}\dot{\boldsymbol{\theta}} + \mathbf{K}(\boldsymbol{\theta} - \mathbf{q}) = \mathbf{H}\mathbf{i} \tag{2}$$

$$\mathbf{L}\dot{\mathbf{i}} + \mathbf{R}(\mathbf{i},\dot{\boldsymbol{\theta}}) = \mathbf{u} \tag{3}$$

where $\mathbf{q} \in \Re^n$ is the vector of link angles, $\boldsymbol{\theta} \in \Re^n$ is the vector of actuator angles, $\mathbf{i} \in \Re^n$ is the motor armature currents, $\mathbf{u} \in \Re^n$ is the control input voltage, $\mathbf{D}(\mathbf{q})$ is the $n \times n$ inertia matrix, $\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}$ is an n-vector of centrifugal and Coriolis forces, and $\mathbf{g}(\mathbf{q})$ is the gravity vector. $\mathbf{J}$, $\mathbf{B}$ and $\mathbf{K}$ are $n \times n$ constant diagonal matrices of actuator inertia, damping and joint stiffness, respectively. $\mathbf{H} \in \Re^{n\times n}$ is an invertible constant diagonal matrix which characterizes the electro-mechanical conversion between current and torque, $\mathbf{L} \in \Re^{n\times n}$ is the constant diagonal matrix of electrical inductance, $\mathbf{R}(\mathbf{i},\dot{\boldsymbol{\theta}}) \in \Re^n$

represents the effect of the electrical resistance and the motor back-emf. Here, we would like to consider the case when the precise forms of $\mathbf{D}(\mathbf{q})$, $\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}$, $\mathbf{g}(\mathbf{q})$, $\mathbf{L}$ and $\mathbf{R}(\mathbf{i},\dot{\boldsymbol{\theta}})$ are not available and their variation bounds are not given. This implies that traditional adaptive control and robust control cannot be applicable. In the following, we would like to use the FAT to design an adaptive controller for the robot. Moreover, it is well-known that derivation of the regressor matrix for the adaptive control of high DOF rigid robot is generally tedious. For the RLFJED robot in (1), (2), and (3) its dynamics is much more complex than that of its rigid-joint counterpart. Therefore, the computation of the regressor matrix becomes extremely difficult. One of the contributions of the present paper is to propose an adaptive controller which does not need to calculate the regressor matrix needed in the conventional robot adaptive control.

Define $\boldsymbol{\tau}_t = \mathbf{K}(\boldsymbol{\theta} - \mathbf{q})$ [36,37] to be the vector of transmission torques, so (1) and (2) becomes

$$\mathbf{D}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau}_t \tag{4}$$

$$\mathbf{J}_t\ddot{\boldsymbol{\tau}}_t + \mathbf{B}_t\dot{\boldsymbol{\tau}}_t + \boldsymbol{\tau}_t = \mathbf{H}\mathbf{i} - \overline{\mathbf{q}}(\dot{\mathbf{q}},\ddot{\mathbf{q}}) \tag{5}$$

where $\mathbf{J}_t = \mathbf{J}\mathbf{K}^{-1}$, $\mathbf{B}_t = \mathbf{B}\mathbf{K}^{-1}$ and $\overline{\mathbf{q}}(\dot{\mathbf{q}},\ddot{\mathbf{q}}) = \mathbf{J}\ddot{\mathbf{q}} + \mathbf{B}\dot{\mathbf{q}}$. Define signal vector $\mathbf{s} = \dot{\mathbf{e}} + \boldsymbol{\Lambda}\mathbf{e}$ and $\mathbf{v} = \dot{\mathbf{q}}_d - \boldsymbol{\Lambda}\mathbf{e}$, where $\mathbf{q}_d \in \Re^n$ is the vector of desired states, $\mathbf{e} = \mathbf{q} - \mathbf{q}_d$ is the state error, and $\boldsymbol{\Lambda} = diag(\lambda_1, \lambda_2, \ldots, \lambda_n)$ with $\lambda_i > 0$ for all i=1, … n. Rewrite (4) in the form

$$\mathbf{D}\dot{\mathbf{s}} + \mathbf{C}\mathbf{s} + \mathbf{g} + \mathbf{D}\dot{\mathbf{v}} + \mathbf{C}\mathbf{v} = \boldsymbol{\tau}_t \tag{6}$$

A. Controller Design for Known Robot

Suppose $\mathbf{D}(\mathbf{q})$, $\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}$ and $\mathbf{g}(\mathbf{q})$ are known, and we may design a proper control law such that $\boldsymbol{\tau}$ follows the trajectory below

$$\boldsymbol{\tau}_t = \mathbf{g} + \mathbf{D}\dot{\mathbf{v}} + \mathbf{C}\mathbf{v} - \mathbf{K}_d\mathbf{s} \tag{7}$$

where $\mathbf{K}_d$ is a positive definite matrix. Substituting (7) into (6), the closed loop dynamics becomes $\mathbf{D}\dot{\mathbf{s}} + \mathbf{C}\mathbf{s} + \mathbf{K}_d\mathbf{s} = \mathbf{0}$. Define a Lyapunov function candidate as $V = \frac{1}{2}\mathbf{s}^T\mathbf{D}\mathbf{s}$. Its time derivative along the trajectory of the closed loop dynamics can be computed as $\dot{V} = -\mathbf{s}^T\mathbf{K}_d\mathbf{s} + \mathbf{s}^T(\dot{\mathbf{D}} - 2\mathbf{C})\mathbf{s}$. Since $\dot{\mathbf{D}} - 2\mathbf{C}$ can be proved to be skew-symmetric, the above equation becomes $\dot{V} = -\mathbf{s}^T\mathbf{K}_d\mathbf{s} \le 0$. It is easy to prove that $\mathbf{s}$ is uniformly bounded and square integrable, and $\dot{\mathbf{s}}$ is also uniformly bounded. Hence, $\mathbf{s} \to \mathbf{0}$ as $t \to \infty$, or we may say $\mathbf{e} \to \mathbf{0}$ as $t \to \infty$. To make the actual $\boldsymbol{\tau}$ converge to the perfect $\boldsymbol{\tau}$ in (7), let us consider the reference model

$$\mathbf{J}_r\ddot{\boldsymbol{\tau}}_r+\mathbf{B}_r\dot{\boldsymbol{\tau}}_r+\mathbf{K}_r\boldsymbol{\tau}_r=\mathbf{J}_r\ddot{\boldsymbol{\tau}}_{td}+\mathbf{B}_r\dot{\boldsymbol{\tau}}_{td}+\mathbf{K}_r\boldsymbol{\tau}_{td} \tag{8}$$

where $\boldsymbol{\tau}_r \in \Re^n$ is the state vector of the reference model and $\boldsymbol{\tau}_{td} \in \Re^n$ is the vector of desired states. Matrices $\mathbf{J}_r \in \Re^{n\times n}$, $\mathbf{B}_r \in \Re^{n\times n}$ and $\mathbf{K}_r \in \Re^{n\times n}$ are selected such that $\boldsymbol{\tau}_r \rightarrow \boldsymbol{\tau}_{td}$ exponentially. Define $\bar{\boldsymbol{\tau}}_{td}(\dot{\boldsymbol{\tau}}_{td},\ddot{\boldsymbol{\tau}}_{td})=\mathbf{K}_r^{-1}(\mathbf{B}_r\dot{\boldsymbol{\tau}}_{td}+\mathbf{J}_r\ddot{\boldsymbol{\tau}}_{td})$, we may rewrite (5) and (8) in the state space form as

$$\dot{\mathbf{x}}_p=\mathbf{A}_p\mathbf{x}_p+\mathbf{B}_p\mathbf{H}\mathbf{i}-\mathbf{B}_p\bar{\mathbf{q}} \tag{9}$$

$$\dot{\mathbf{x}}_m=\mathbf{A}_m\mathbf{x}_m+\mathbf{B}_m(\boldsymbol{\tau}_{td}+\bar{\boldsymbol{\tau}}_{td}) \tag{10}$$

where $\mathbf{x}_p=[\boldsymbol{\tau}_t \quad \dot{\boldsymbol{\tau}}_t]^T \in \Re^{2n}$ and $\mathbf{x}_m=[\boldsymbol{\tau}_r \quad \dot{\boldsymbol{\tau}}_r]^T \in \Re^{2n}$ are augmented state vectors. $\mathbf{A}_p=\begin{bmatrix}\mathbf{0} & \mathbf{I}_{n\times n}\\ -\mathbf{J}_t^{-1} & -\mathbf{J}_t^{-1}\mathbf{B}_t\end{bmatrix}\in\Re^{2n\times 2n}$ and $\mathbf{A}_m=\begin{bmatrix}\mathbf{0} & \mathbf{I}_{n\times n}\\ -\mathbf{J}_r^{-1}\mathbf{K}_r & -\mathbf{J}_r^{-1}\mathbf{B}_r\end{bmatrix}\in\Re^{2n\times 2n}$ are augmented system matrices. $\mathbf{B}_p=\begin{bmatrix}\mathbf{0}\\ \mathbf{J}_t^{-1}\end{bmatrix}\in\Re^{2n\times n}$ and $\mathbf{B}_m=\begin{bmatrix}\mathbf{0}\\ \mathbf{J}_r^{-1}\mathbf{K}_r\end{bmatrix}\in\Re^{2n\times n}$ are augmented input gain matrices, and the pair $(\mathbf{A}_m,\mathbf{B}_m)$ is controllable. Since all system parameters are assumed to be available at the present stage, we may select a perfect current trajectory in the form[38]

$$\mathbf{i}=\mathbf{H}^{-1}[\Theta\mathbf{x}_p+\Phi\boldsymbol{\tau}_{td}+\mathbf{h}(\bar{\boldsymbol{\tau}}_{td},\bar{\mathbf{q}})] \tag{11}$$

where $\Theta\in\Re^{n\times 2n}$ and $\Phi\in\Re^{n\times n}$ satisfy $\mathbf{A}_p+\mathbf{B}_p\Theta=\mathbf{A}_m$ and $\mathbf{B}_p\Phi=\mathbf{B}_m$, respectively, and $\mathbf{h}(\bar{\boldsymbol{\tau}}_{td},\bar{\mathbf{q}})=\Phi\bar{\boldsymbol{\tau}}_{td}+\bar{\mathbf{q}}$. Substituting (11) into (9) and after some rearrangements, we may have the system dynamics

$$\dot{\mathbf{x}}_p=\mathbf{A}_m\mathbf{x}_p+\mathbf{B}_m(\boldsymbol{\tau}_{td}+\bar{\boldsymbol{\tau}}_{td}) \tag{12}$$

Define $\mathbf{e}_m=\mathbf{x}_p-\mathbf{x}_m$ and we may have the error dynamics directly from (10) and (12)

$$\dot{\mathbf{e}}_m=\mathbf{A}_m\mathbf{e}_m \tag{13}$$

Let $\mathbf{e}_\tau=\boldsymbol{\tau}_t-\boldsymbol{\tau}_r$ be the output vector of the error dynamics (13) as

$$\mathbf{e}_\tau=\mathbf{C}_m\mathbf{e}_m \tag{14}$$

where $\mathbf{C}_m \in \Re^{n\times 2n}$ is the augmented output matrix such that the pair $(\mathbf{A}_m, \mathbf{C}_m)$ is observable and the transfer function $\mathbf{C}_m(s\mathbf{I}-\mathbf{A}_m)^{-1}\mathbf{B}_m$ is strictly positive real. Since $\mathbf{A}_m$ is stable, (13) implies $\mathbf{e}_m \to 0$ as $t\to\infty$. This further gives $\boldsymbol{\tau}_t \to \boldsymbol{\tau}_{td}$ as $t\to\infty$. To ensure the actual $\mathbf{i}$ to converge to the perfect $\mathbf{i}$ in (11), let us select the control input in (3) as

$$\mathbf{u} = \mathbf{L}\dot{\mathbf{i}}_d + \mathbf{R}(\mathbf{i},\dot{\boldsymbol{\theta}}) - \mathbf{K}_c\mathbf{e}_i \tag{15}$$

where $\mathbf{e}_i = \mathbf{i}-\mathbf{i}_d$ is the current error, $\mathbf{i}_d \in \Re^n$ is the desired current which is equivalent to the perfect current trajectory $\mathbf{i}$ in (11), and $\mathbf{K}_c \in \Re^{n\times n}$ is a positive definite matrix. Substituting (15) into (3), the closed loop dynamics becomes $\mathbf{L}\dot{\mathbf{e}}_i + \mathbf{K}_c\mathbf{e}_i = \mathbf{0}$. According to this, it is easy to prove that $\mathbf{i}\to\mathbf{i}_d$ as $t\to\infty$ with proper selection of $\mathbf{K}_c$.

In summary, if all parameters in the RLFJED robot (1), (2), and (3) are available, the desired transmission torque (7), the desired current (11), the control input (15) can give asymptotic convergence tracking performance.

B. Controller Design for Uncertain Robot

Suppose $\mathbf{D}(\mathbf{q})$, $\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}$, $\mathbf{g}(\mathbf{q})$, $\mathbf{L}$ and $\mathbf{R}(\mathbf{i},\dot{\boldsymbol{\theta}})$ are not available, and $\ddot{\mathbf{q}}$, $\ddot{\boldsymbol{\theta}}$ are not easy to measure, we would like to design a desired transmission torque $\boldsymbol{\tau}_d$ so that a proper controller $\mathbf{u}$ can be constructed to have $\boldsymbol{\tau}_t \to \boldsymbol{\tau}_{td}$. Instead of (7), let us design a desired transmission torque $\boldsymbol{\tau}_{td}$ as

$$\boldsymbol{\tau}_{td} = \hat{\mathbf{g}} + \hat{\mathbf{D}}\dot{\mathbf{v}} + \hat{\mathbf{C}}\mathbf{v} - \mathbf{K}_d\mathbf{s} \tag{16}$$

where $\hat{\mathbf{D}}$, $\hat{\mathbf{C}}$ and $\hat{\mathbf{g}}$ are estimates of $\mathbf{D}(\mathbf{q})$, $\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})$ and $\mathbf{g}(\mathbf{q})$, respectively. Using (16), we may have the closed loop dynamics

$$\mathbf{D}\dot{\mathbf{s}} + \mathbf{C}\mathbf{s} + \mathbf{K}_d\mathbf{s} = -\tilde{\mathbf{D}}\dot{\mathbf{v}} - \tilde{\mathbf{C}}\mathbf{v} - \tilde{\mathbf{g}} + (\boldsymbol{\tau}_t - \boldsymbol{\tau}_{td}) \tag{17}$$

where $\tilde{\mathbf{D}} = \mathbf{D}-\hat{\mathbf{D}}$, $\tilde{\mathbf{C}} = \mathbf{C}-\hat{\mathbf{C}}$ and $\tilde{\mathbf{g}} = \mathbf{g}-\hat{\mathbf{g}}$. If a proper controller and update laws for $\hat{\mathbf{D}}$, $\hat{\mathbf{C}}$ and $\hat{\mathbf{g}}$ can be designed, we may have $\boldsymbol{\tau}_t \to \boldsymbol{\tau}_{td}$, $\hat{\mathbf{D}}\to\mathbf{D}$, $\hat{\mathbf{C}}\to\mathbf{C}$ and $\hat{\mathbf{g}}\to\mathbf{g}$ so that (17) can give desired performance. Let us consider the desired current $\mathbf{i}_d$ instead of (11)

$$\mathbf{i}_d = \mathbf{H}^{-1}(\boldsymbol{\Theta}\mathbf{x}_p + \boldsymbol{\Phi}\boldsymbol{\tau}_d + \hat{\mathbf{h}}) \tag{18}$$

where $\hat{\mathbf{h}}$ is an estimate of $\mathbf{h}$. By (18), we may have the system dynamics

$$\dot{\mathbf{x}}_p = \mathbf{A}_m\mathbf{x}_p + \mathbf{B}_m(\boldsymbol{\tau}_d + \overline{\boldsymbol{\tau}}_d) + \mathbf{B}_p\mathbf{H}(\mathbf{i} - \mathbf{i}_d) + \mathbf{B}_p(\hat{\mathbf{h}} - \mathbf{h}) \tag{19}$$

Together with (10), we may have the error dynamics

$$\dot{\mathbf{e}}_m = \mathbf{A}_m\mathbf{e}_m + \mathbf{B}_p[\mathbf{H}\mathbf{e}_i + (\hat{\mathbf{h}} - \mathbf{h})] \tag{20}$$

$$\mathbf{e}_\tau = \mathbf{C}_m\mathbf{e}_m \tag{21}$$

If we may design a control input $\boldsymbol{u}$ and an appropriate update law such that $\mathbf{i} \to \mathbf{i}_d$ and $\hat{\mathbf{h}} \to \mathbf{h}$, then (20) implies $\mathbf{e}_m \to 0$ as $t \to \infty$. This further implies $\boldsymbol{\tau}_t \to \boldsymbol{\tau}_{td}$ as $t \to \infty$. Here, according to (15), let us select the control input in (3) as

$$\mathbf{u} = \hat{\mathbf{f}} - \mathbf{K}_c\mathbf{e}_i \tag{22}$$

where $\hat{\mathbf{f}}$ is an estimate of $\mathbf{f}(\dot{\mathbf{i}}_d, \mathbf{i}, \dot{\boldsymbol{\theta}}) = \mathbf{L}\dot{\mathbf{i}}_d + \mathbf{R}(\mathbf{i}, \dot{\boldsymbol{\theta}})$. Substituting (22) into (3), we may have the system dynamics

$$\mathbf{L}\dot{\mathbf{e}}_i + \mathbf{K}_c\mathbf{e}_i = \hat{\mathbf{f}} - \mathbf{f} \tag{23}$$

If an appropriate update law for $\hat{\mathbf{f}}$ can be selected, we may have $\mathbf{i} \to \mathbf{i}_d$. Since **D**, **C**, **g**, **h** and **f** are functions of time, traditional adaptive controllers are not directly applicable. To design the update laws, let us apply the function approximation representation

$$\begin{aligned} &\mathbf{D} = \mathbf{W}_\mathbf{D}^T\mathbf{Z}_\mathbf{D} + \boldsymbol{\varepsilon}_\mathbf{D}, \quad \mathbf{C} = \mathbf{W}_\mathbf{C}^T\mathbf{Z}_\mathbf{C} + \boldsymbol{\varepsilon}_\mathbf{C}, \quad \mathbf{g} = \mathbf{W}_\mathbf{g}^T\mathbf{Z}_\mathbf{g} + \boldsymbol{\varepsilon}_\mathbf{g}, \\ &\mathbf{h} = \mathbf{W}_\mathbf{h}^T\mathbf{Z}_\mathbf{h} + \boldsymbol{\varepsilon}_\mathbf{h}, \quad \mathbf{f} = \mathbf{W}_\mathbf{f}^T\mathbf{Z}_\mathbf{f} + \boldsymbol{\varepsilon}_\mathbf{f} \end{aligned} \tag{24a}$$

where $\mathbf{W}_\mathbf{D} \in \Re^{n^2\beta_D \times n}$, $\mathbf{W}_\mathbf{C} \in \Re^{n^2\beta_C \times n}$, $\mathbf{W}_\mathbf{g} \in \Re^{n\beta_g \times n}$, $\mathbf{W}_\mathbf{h} \in \Re^{n\beta_h \times n}$, and $\mathbf{W}_\mathbf{f} \in \Re^{n\beta_f \times n}$ are weighting matrices, $\mathbf{Z}_\mathbf{D} \in \Re^{n^2\beta_D \times n}$, $\mathbf{Z}_\mathbf{C} \in \Re^{n^2\beta_C \times n}$, $\mathbf{Z}_\mathbf{g} \in \Re^{n\beta_g \times 1}$, $\mathbf{Z}_\mathbf{h} \in \Re^{n\beta_h \times 1}$ and $\mathbf{Z}_\mathbf{f} \in \Re^{n\beta_f \times 1}$ are matrices of basis functions, and $\boldsymbol{\varepsilon}_{(\cdot)}$ are approximation error matrices. The number $\beta_{(\cdot)}$ represents the number of basis functions used. Using the same set of basis functions, the corresponding estimates can also be represented as

$$\begin{aligned} &\hat{\mathbf{D}} = \hat{\mathbf{W}}_\mathbf{D}^T\mathbf{Z}_\mathbf{D}, \quad \hat{\mathbf{C}} = \hat{\mathbf{W}}_\mathbf{C}^T\mathbf{Z}_\mathbf{C}, \quad \hat{\mathbf{g}} = \hat{\mathbf{W}}_\mathbf{g}^T\mathbf{Z}_\mathbf{g}, \\ &\hat{\mathbf{h}} = \hat{\mathbf{W}}_\mathbf{h}^T\mathbf{Z}_\mathbf{h}, \quad \hat{\mathbf{f}} = \hat{\mathbf{W}}_\mathbf{f}^T\mathbf{Z}_\mathbf{f} \end{aligned} \tag{24b}$$

Define $\tilde{\mathbf{W}}_{(\cdot)} = \mathbf{W}_{(\cdot)} - \hat{\mathbf{W}}_{(\cdot)}$, then equation (17), (20) and (23) becomes

$$\mathbf{D}\dot{\mathbf{s}} + \mathbf{C}\mathbf{s} + \mathbf{K}_d\mathbf{s} = (\boldsymbol{\tau}_t - \boldsymbol{\tau}_{td}) - \tilde{\mathbf{W}}_{\mathbf{D}}^T\mathbf{Z}_{\mathbf{D}}\dot{\mathbf{v}} - \tilde{\mathbf{W}}_{\mathbf{C}}^T\mathbf{Z}_{\mathbf{C}}\mathbf{v} - \tilde{\mathbf{W}}_{\mathbf{g}}^T\mathbf{Z}_{\mathbf{g}} + \boldsymbol{\varepsilon}_1 \tag{25}$$

$$\dot{\mathbf{e}}_m = \mathbf{A}_m\mathbf{e}_m - \mathbf{B}_p\tilde{\mathbf{W}}_{\mathbf{h}}^T\mathbf{Z}_{\mathbf{h}} + \mathbf{B}_p\mathbf{H}\mathbf{e}_i + \mathbf{B}_p\boldsymbol{\varepsilon}_2 \tag{26}$$

$$\mathbf{L}\dot{\mathbf{e}}_i + \mathbf{K}_c\mathbf{e}_i = -\tilde{\mathbf{W}}_{\mathbf{f}}^T\mathbf{Z}_{\mathbf{f}} + \boldsymbol{\varepsilon}_3 \tag{27}$$

where $\boldsymbol{\varepsilon}_1 = \boldsymbol{\varepsilon}_1(\boldsymbol{\varepsilon}_{\mathbf{D}}, \boldsymbol{\varepsilon}_{\mathbf{C}}, \boldsymbol{\varepsilon}_{\mathbf{g}}, \mathbf{s}, \ddot{\mathbf{q}}_d)$, $\boldsymbol{\varepsilon}_2 = \boldsymbol{\varepsilon}_2(\boldsymbol{\varepsilon}_{\mathbf{h}}, \mathbf{e}_m)$ and $\boldsymbol{\varepsilon}_3 = \boldsymbol{\varepsilon}_3(\boldsymbol{\varepsilon}_{\mathbf{f}}, \mathbf{e}_i)$ are lumped approximation errors. Since $\mathbf{W}_{(\cdot)}$ are constant matrices, their update laws can be easily found by proper selection of the Lyapunov-like function. Let us consider a candidate

$$\begin{aligned} V(\mathbf{s}, \mathbf{e}_m, \mathbf{e}_i, \tilde{\mathbf{W}}_{\mathbf{D}}, \tilde{\mathbf{W}}_{\mathbf{C}}, \tilde{\mathbf{W}}_{\mathbf{g}}, \tilde{\mathbf{W}}_{\mathbf{h}}, \tilde{\mathbf{W}}_{\mathbf{f}}) &= \frac{1}{2}\mathbf{s}^T\mathbf{D}\mathbf{s} + \mathbf{e}_m^T\mathbf{P}_t\mathbf{e}_m + \frac{1}{2}\mathbf{e}_i^T\mathbf{L}\mathbf{e}_i \\ &+ \frac{1}{2}Tr(\tilde{\mathbf{W}}_{\mathbf{D}}^T\mathbf{Q}_{\mathbf{D}}\tilde{\mathbf{W}}_{\mathbf{D}} + \tilde{\mathbf{W}}_{\mathbf{C}}^T\mathbf{Q}_{\mathbf{C}}\tilde{\mathbf{W}}_{\mathbf{C}} + \tilde{\mathbf{W}}_{\mathbf{g}}^T\mathbf{Q}_{\mathbf{g}}\tilde{\mathbf{W}}_{\mathbf{g}} + \tilde{\mathbf{W}}_{\mathbf{h}}^T\mathbf{Q}_{\mathbf{h}}\tilde{\mathbf{W}}_{\mathbf{h}} + \tilde{\mathbf{W}}_{\mathbf{f}}^T\mathbf{Q}_{\mathbf{f}}\tilde{\mathbf{W}}_{\mathbf{f}}) \end{aligned} \tag{28}$$

where $\mathbf{P}_t = \mathbf{P}_t^T \in \Re^{2n\times 2n}$ is a positive definite matrix satisfying the Lyapunov equation $\mathbf{A}_m^T\mathbf{P}_t + \mathbf{P}_t\mathbf{A}_m = -\mathbf{C}_m^T\mathbf{C}_m$. The matrices $\mathbf{Q}_{\mathbf{D}} \in \Re^{n^2\beta_{\mathbf{D}}\times n^2\beta_{\mathbf{D}}}$, $\mathbf{Q}_{\mathbf{C}} \in \Re^{n^2\beta_C\times n^2\beta_C}$, $\mathbf{Q}_{\mathbf{g}} \in \Re^{n\beta_g\times n\beta_g}$, $\mathbf{Q}_{\mathbf{h}} \in \Re^{n\beta_h\times n\beta_h}$ and $\mathbf{Q}_{\mathbf{f}} \in \Re^{n\beta_f\times n\beta_f}$ are positive definite. The notation *Tr*(.) denotes the trace operation of matrices. The time derivative of *V* along the trajectory of (25), (26), and (27) can be computed as

$$\begin{aligned} \dot{V} &= \mathbf{s}^T\mathbf{D}\dot{\mathbf{s}} + \frac{1}{2}\mathbf{s}^T\dot{\mathbf{D}}\mathbf{s} + \dot{\mathbf{e}}_m^T\mathbf{P}_t\mathbf{e}_m + \mathbf{e}_m^T\mathbf{P}_t\dot{\mathbf{e}}_m + \mathbf{e}_i^T\mathbf{L}\dot{\mathbf{e}}_i \\ &\quad - Tr(\tilde{\mathbf{W}}_{\mathbf{D}}^T\mathbf{Q}_{\mathbf{D}}\dot{\hat{\mathbf{W}}}_{\mathbf{D}} + \tilde{\mathbf{W}}_{\mathbf{C}}^T\mathbf{Q}_{\mathbf{C}}\dot{\hat{\mathbf{W}}}_{\mathbf{C}} + \tilde{\mathbf{W}}_{\mathbf{g}}^T\mathbf{Q}_{\mathbf{g}}\dot{\hat{\mathbf{W}}}_{\mathbf{g}} + \tilde{\mathbf{W}}_{\mathbf{h}}^T\mathbf{Q}_{\mathbf{h}}\dot{\hat{\mathbf{W}}}_{\mathbf{h}} + \tilde{\mathbf{W}}_{\mathbf{f}}^T\mathbf{Q}_{\mathbf{f}}\dot{\hat{\mathbf{W}}}_{\mathbf{f}}) \\ &= -\mathbf{s}^T\mathbf{K}_d\mathbf{s} + \mathbf{s}^T\mathbf{e}_\tau - \mathbf{e}_\tau^T\mathbf{e}_\tau + \mathbf{e}_m^T\mathbf{P}_t\mathbf{B}_p\mathbf{H}\mathbf{e}_i - \mathbf{e}_i^T\mathbf{K}_c\mathbf{e}_i + \mathbf{s}^T\boldsymbol{\varepsilon}_1 + \mathbf{e}_m^T\mathbf{P}_t\mathbf{B}_p\boldsymbol{\varepsilon}_2 + \mathbf{e}_i^T\boldsymbol{\varepsilon}_3 \\ &\quad - Tr[\tilde{\mathbf{W}}_{\mathbf{D}}^T(\mathbf{Z}_{\mathbf{D}}\dot{\mathbf{v}}\mathbf{s}^T + \mathbf{Q}_{\mathbf{D}}\dot{\hat{\mathbf{W}}}_{\mathbf{D}}) + \tilde{\mathbf{W}}_{\mathbf{C}}^T(\mathbf{Z}_{\mathbf{C}}\mathbf{v}\mathbf{s}^T + \mathbf{Q}_{\mathbf{C}}\dot{\hat{\mathbf{W}}}_{\mathbf{C}})] \\ &\quad - Tr[\tilde{\mathbf{W}}_{\mathbf{g}}^T(\mathbf{Z}_{\mathbf{g}}\mathbf{s}^T + \mathbf{Q}_{\mathbf{g}}\dot{\hat{\mathbf{W}}}_{\mathbf{g}}) + \tilde{\mathbf{W}}_{\mathbf{h}}^T(\mathbf{Z}_{\mathbf{h}}\mathbf{e}_m^T\mathbf{P}_t\mathbf{B}_p + \mathbf{Q}_{\mathbf{h}}\dot{\hat{\mathbf{W}}}_{\mathbf{h}})] \\ &\quad - Tr[\tilde{\mathbf{W}}_{\mathbf{f}}^T(\mathbf{Z}_{\mathbf{f}}\mathbf{e}_i^T + \mathbf{Q}_{\mathbf{f}}\dot{\hat{\mathbf{W}}}_{\mathbf{f}})] \end{aligned} \tag{29}$$

According to the Kalman-Yakubovic Lemma, we have $\mathbf{e}_m^T\mathbf{P}_t\mathbf{B}_p = \mathbf{e}_\tau^T$ by picking $\mathbf{B}_m = \mathbf{B}_p$ [39]. According to (29), the update laws can be selected as

$$\dot{\hat{\mathbf{W}}}_{\mathbf{D}} = -\mathbf{Q}_{\mathbf{D}}^{-1}\mathbf{Z}_{\mathbf{D}}\dot{\mathbf{v}}\mathbf{s}^T - \sigma_{\mathbf{D}}\hat{\mathbf{W}}_{\mathbf{D}}, \quad \dot{\hat{\mathbf{W}}}_{\mathbf{C}} = -\mathbf{Q}_{\mathbf{C}}^{-1}\mathbf{Z}_{\mathbf{C}}\mathbf{v}\mathbf{s}^T - \sigma_{\mathbf{C}}\hat{\mathbf{W}}_{\mathbf{C}},$$
$$\dot{\hat{\mathbf{W}}}_{\mathbf{g}} = -\mathbf{Q}_{\mathbf{g}}^{-1}\mathbf{Z}_{\mathbf{g}}\mathbf{s}^T - \sigma_{\mathbf{g}}\hat{\mathbf{W}}_{\mathbf{g}}, \quad \dot{\hat{\mathbf{W}}}_{\mathbf{h}} = -\mathbf{Q}_{\mathbf{h}}^{-1}\mathbf{Z}_{\mathbf{h}}\mathbf{e}_\tau^T - \sigma_{\mathbf{h}}\hat{\mathbf{W}}_{\mathbf{h}}, \tag{30}$$
$$\dot{\hat{\mathbf{W}}}_{\mathbf{f}} = -\mathbf{Q}_{\mathbf{f}}^{-1}\mathbf{Z}_{\mathbf{f}}\mathbf{e}_i^T - \sigma_{\mathbf{f}}\hat{\mathbf{W}}_{\mathbf{f}}$$

where $\sigma_{(\cdot)}$ are positive numbers. Then (29) becomes

$$\dot{V} = -\begin{bmatrix}\mathbf{s}^T & \mathbf{e}_\tau^T & \mathbf{e}_i^T\end{bmatrix}\mathbf{Q}\begin{bmatrix}\mathbf{s}\\ \mathbf{e}_\tau \\ \mathbf{e}_i\end{bmatrix} + \begin{bmatrix}\mathbf{s}^T & \mathbf{e}_\tau^T & \mathbf{e}_i^T\end{bmatrix}\begin{bmatrix}\boldsymbol{\varepsilon}_1\\ \boldsymbol{\varepsilon}_2 \\ \boldsymbol{\varepsilon}_3\end{bmatrix}$$
$$+\sigma_{\mathbf{D}}Tr(\tilde{\mathbf{W}}_{\mathbf{D}}^T\hat{\mathbf{W}}_{\mathbf{D}}) + \sigma_{\mathbf{C}}Tr(\tilde{\mathbf{W}}_{\mathbf{C}}^T\hat{\mathbf{W}}_{\mathbf{C}}) + \sigma_{\mathbf{g}}Tr(\tilde{\mathbf{W}}_{\mathbf{g}}^T\hat{\mathbf{W}}_{\mathbf{g}}) \tag{31}$$
$$+\sigma_{\mathbf{h}}Tr(\tilde{\mathbf{W}}_{\mathbf{h}}^T\hat{\mathbf{W}}_{\mathbf{h}}) + \sigma_{\mathbf{f}}Tr(\tilde{\mathbf{W}}_{\mathbf{f}}^T\hat{\mathbf{W}}_{\mathbf{f}})$$

where $\mathbf{Q} = \begin{bmatrix}\mathbf{K}_d & -\frac{1}{2}\mathbf{I}_{n\times n} & \mathbf{0}\\ -\frac{1}{2}\mathbf{I}_{n\times n} & \mathbf{I}_{n\times n} & -\frac{1}{2}\mathbf{H}\\ \mathbf{0} & -\frac{1}{2}\mathbf{H} & \mathbf{K}_c\end{bmatrix}$ is positive definite due to proper selections of $\mathbf{K}_d$ and $\mathbf{K}_c$. Owing to the existence of $\boldsymbol{\varepsilon}_1$, $\boldsymbol{\varepsilon}_2$, and $\boldsymbol{\varepsilon}_3$ the definiteness of $\dot{V}$ cannot be determined. According to **Appendix** *Lemma A.1*、*Lemma A.4* and *Lemma A.7,* the right hand side of (31) can be divided into two parts to derive following inequalities

$$-\begin{bmatrix}\mathbf{s}^T & \mathbf{e}_\tau^T & \mathbf{e}_i^T\end{bmatrix}\mathbf{Q}\begin{bmatrix}\mathbf{s}\\ \mathbf{e}_\tau \\ \mathbf{e}_i\end{bmatrix} + \begin{bmatrix}\mathbf{s}^T & \mathbf{e}_\tau^T & \mathbf{e}_i^T\end{bmatrix}\begin{bmatrix}\boldsymbol{\varepsilon}_1\\ \boldsymbol{\varepsilon}_2 \\ \boldsymbol{\varepsilon}_3\end{bmatrix}$$
$$\leq -\frac{1}{2}\left(\lambda_{\min}(\mathbf{Q})\left\|\begin{bmatrix}\mathbf{s}\\ \mathbf{e}_\tau \\ \mathbf{e}_i\end{bmatrix}\right\|^2 - \frac{1}{\lambda_{\min}(\mathbf{Q})}\left\|\begin{bmatrix}\boldsymbol{\varepsilon}_1\\ \boldsymbol{\varepsilon}_2 \\ \boldsymbol{\varepsilon}_3\end{bmatrix}\right\|^2\right) \tag{32a}$$

$$Tr(\tilde{\mathbf{W}}_{\mathbf{D}}^T\hat{\mathbf{W}}_{\mathbf{D}}) \leq \frac{1}{2}Tr(\mathbf{W}_{\mathbf{D}}^T\mathbf{W}_{\mathbf{D}}) - \frac{1}{2}Tr(\tilde{\mathbf{W}}_{\mathbf{D}}^T\tilde{\mathbf{W}}_{\mathbf{D}}) \tag{32b}$$

$$Tr(\tilde{\mathbf{W}}_{\mathbf{C}}^T\hat{\mathbf{W}}_{\mathbf{C}}) \le \frac{1}{2}Tr(\mathbf{W}_{\mathbf{C}}^T\mathbf{W}_{\mathbf{C}}) - \frac{1}{2}Tr(\tilde{\mathbf{W}}_{\mathbf{C}}^T\tilde{\mathbf{W}}_{\mathbf{C}}) \tag{32c}$$

$$Tr(\tilde{\mathbf{W}}_{\mathbf{g}}^T\hat{\mathbf{W}}_{\mathbf{g}}) \le \frac{1}{2}Tr(\mathbf{W}_{\mathbf{g}}^T\mathbf{W}_{\mathbf{g}}) - \frac{1}{2}Tr(\tilde{\mathbf{W}}_{\mathbf{g}}^T\tilde{\mathbf{W}}_{\mathbf{g}}) \tag{32d}$$

$$Tr(\tilde{\mathbf{W}}_{\mathbf{h}}^T\hat{\mathbf{W}}_{\mathbf{h}}) \le \frac{1}{2}Tr(\mathbf{W}_{\mathbf{h}}^T\mathbf{W}_{\mathbf{h}}) - \frac{1}{2}Tr(\tilde{\mathbf{W}}_{\mathbf{h}}^T\tilde{\mathbf{W}}_{\mathbf{h}}) \tag{32e}$$

$$Tr(\tilde{\mathbf{W}}_{\mathbf{f}}^T\hat{\mathbf{W}}_{\mathbf{f}}) \le \frac{1}{2}Tr(\mathbf{W}_{\mathbf{f}}^T\mathbf{W}_{\mathbf{f}}) - \frac{1}{2}Tr(\tilde{\mathbf{W}}_{\mathbf{f}}^T\tilde{\mathbf{W}}_{\mathbf{f}}) \tag{32f}$$

According to (28), we hav

$$\begin{aligned} V &= \frac{1}{2}[\mathbf{s}^T\mathbf{D}\mathbf{s} + \mathbf{e}_i^T\mathbf{L}\mathbf{e}_i + 2\mathbf{e}_m^T\mathbf{P}_t\mathbf{e}_m \\ &\quad + Tr(\tilde{\mathbf{W}}_{\mathbf{D}}^T\mathbf{Q}_{\mathbf{D}}\tilde{\mathbf{W}}_{\mathbf{D}} + \tilde{\mathbf{W}}_{\mathbf{C}}^T\mathbf{Q}_{\mathbf{C}}\tilde{\mathbf{W}}_{\mathbf{C}} + \tilde{\mathbf{W}}_{\mathbf{g}}^T\mathbf{Q}_{\mathbf{g}}\tilde{\mathbf{W}}_{\mathbf{g}} + \tilde{\mathbf{W}}_{\mathbf{h}}^T\mathbf{Q}_{\mathbf{h}}\tilde{\mathbf{W}}_{\mathbf{h}}) + \tilde{\mathbf{W}}_{\mathbf{f}}^T\mathbf{Q}_{\mathbf{f}}\tilde{\mathbf{W}}_{\mathbf{f}})] \\ &\le \frac{1}{2}\left[\lambda_{\max}(\mathbf{A})\left\|\begin{bmatrix}\mathbf{s}\\ \mathbf{e}_\tau\\ \mathbf{e}_i\end{bmatrix}\right\|^2 + \lambda_{\max}(\mathbf{Q}_{\mathbf{D}})Tr(\tilde{\mathbf{W}}_{\mathbf{D}}^T\tilde{\mathbf{W}}_{\mathbf{D}}) + \lambda_{\max}(\mathbf{Q}_{\mathbf{C}})Tr(\tilde{\mathbf{W}}_{\mathbf{C}}^T\tilde{\mathbf{W}}_{\mathbf{C}})\right. \\ &\quad + \lambda_{\max}(\mathbf{Q}_{\mathbf{g}})Tr(\tilde{\mathbf{W}}_{\mathbf{g}}^T\tilde{\mathbf{W}}_{\mathbf{g}}) + \lambda_{\max}(\mathbf{Q}_{\mathbf{h}})Tr(\tilde{\mathbf{W}}_{\mathbf{h}}^T\tilde{\mathbf{W}}_{\mathbf{h}}) + \lambda_{\max}(\mathbf{Q}_{\mathbf{f}})Tr(\tilde{\mathbf{W}}_{\mathbf{f}}^T\tilde{\mathbf{W}}_{\mathbf{f}})] \end{aligned} \tag{33}$$

where $\mathbf{A} = \begin{bmatrix}\mathbf{D} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & 2\mathbf{C}_m^T\mathbf{P}_t\mathbf{C}_m & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{L}\end{bmatrix}$. With (32) and (33), (31) can be further written as

$$\begin{aligned} \dot{V} &\le -\alpha V + \frac{1}{2}\left\{[\alpha\lambda_{\max}(\mathbf{A}) - \lambda_{\min}(\mathbf{Q})]\left\|\begin{bmatrix}\mathbf{s}\\ \mathbf{e}_\tau\\ \mathbf{e}_i\end{bmatrix}\right\|^2 + [\alpha\lambda_{\max}(\mathbf{Q}_{\mathbf{D}}) - \sigma_{\mathbf{D}}]Tr(\tilde{\mathbf{W}}_{\mathbf{D}}^T\tilde{\mathbf{W}}_{\mathbf{D}})\right. \\ &\quad + [\alpha\lambda_{\max}(\mathbf{Q}_{\mathbf{C}}) - \sigma_{\mathbf{C}}]Tr(\tilde{\mathbf{W}}_{\mathbf{C}}^T\tilde{\mathbf{W}}_{\mathbf{C}}) + [\alpha\lambda_{\max}(\mathbf{Q}_{\mathbf{g}}) - \sigma_{\mathbf{g}}]Tr(\tilde{\mathbf{W}}_{\mathbf{g}}^T\tilde{\mathbf{W}}_{\mathbf{g}}) \\ &\quad + [\alpha\lambda_{\max}(\mathbf{Q}_{\mathbf{h}}) - \sigma_{\mathbf{h}}]Tr(\tilde{\mathbf{W}}_{\mathbf{h}}^T\tilde{\mathbf{W}}_{\mathbf{h}}) + [\alpha\lambda_{\max}(\mathbf{Q}_{\mathbf{f}}) - \sigma_{\mathbf{f}}]Tr(\tilde{\mathbf{W}}_{\mathbf{f}}^T\tilde{\mathbf{W}}_{\mathbf{f}}) \\ &\quad + \frac{1}{\lambda_{\min}(\mathbf{Q})}\left\|\begin{bmatrix}\boldsymbol{\varepsilon}_1\\ \boldsymbol{\varepsilon}_2\\ \boldsymbol{\varepsilon}_3\end{bmatrix}\right\|^2 + \sigma_{\mathbf{D}}Tr(\mathbf{W}_{\mathbf{D}}^T\mathbf{W}_{\mathbf{D}}) + \sigma_{\mathbf{C}}Tr(\mathbf{W}_{\mathbf{C}}^T\mathbf{W}_{\mathbf{C}}) \\ &\quad \left. + \sigma_{\mathbf{g}}Tr(\mathbf{W}_{\mathbf{g}}^T\mathbf{W}_{\mathbf{g}}) + \sigma_{\mathbf{h}}Tr(\mathbf{W}_{\mathbf{h}}^T\mathbf{W}_{\mathbf{h}}) + \sigma_{\mathbf{f}}Tr(\mathbf{W}_{\mathbf{f}}^T\mathbf{W}_{\mathbf{f}})\right\} \end{aligned} \tag{34}$$

Although **D** and **L** are unknown, we know that $\exists \overline{D}$ and $\underline{D}$ s.t. $\underline{D} \le \|\mathbf{D}\| \le \overline{D}$、$\exists \overline{L}$ and $\underline{L}$ s.t. $\underline{L} \le \|\mathbf{L}\| \le \overline{L}$, $\exists \overline{\eta}_A, \underline{\eta}_A > 0$ s.t. $\lambda_{\max}(\mathbf{A}) \le \overline{\eta}_A$ and $\lambda_{\min}(\mathbf{A}) \ge \underline{\eta}_A$ [40]. Picking $\alpha \le \min\left\{\frac{\lambda_{\min}(\mathbf{Q})}{\overline{\eta}_A}, \frac{\sigma_{\mathbf{D}}}{\lambda_{\max}(\mathbf{Q_D})}, \frac{\sigma_{\mathbf{C}}}{\lambda_{\max}(\mathbf{Q_C})}, \frac{\sigma_{\mathbf{g}}}{\lambda_{\max}(\mathbf{Q_g})}, \frac{\sigma_{\mathbf{h}}}{\lambda_{\max}(\mathbf{Q_h})}, \frac{\sigma_{\mathbf{f}}}{\lambda_{\max}(\mathbf{Q_f})}\right\}$,
then we have

$$\dot{V} \le -\alpha V + \frac{1}{2\lambda_{\min}(\mathbf{Q})}\left\|\begin{bmatrix}\boldsymbol{\varepsilon}_1\\ \boldsymbol{\varepsilon}_2\\ \boldsymbol{\varepsilon}_3\end{bmatrix}\right\|^2 + \frac{1}{2}[\sigma_{\mathbf{D}}Tr(\mathbf{W}_{\mathbf{D}}^T\mathbf{W}_{\mathbf{D}}) + \sigma_{\mathbf{C}}Tr(\mathbf{W}_{\mathbf{C}}^T\mathbf{W}_{\mathbf{C}}) + \sigma_{\mathbf{g}}Tr(\mathbf{W}_{\mathbf{g}}^T\mathbf{W}_{\mathbf{g}}) + \sigma_{\mathbf{h}}Tr(\mathbf{W}_{\mathbf{h}}^T\mathbf{W}_{\mathbf{h}}) + \sigma_{\mathbf{f}}Tr(\mathbf{W}_{\mathbf{f}}^T\mathbf{W}_{\mathbf{f}})] \tag{35}$$

Hence, $\dot{V} < 0$ whenever

$$(\mathbf{s}, \mathbf{e}_\tau, \mathbf{e}_i, \tilde{\mathbf{W}}_{\mathbf{D}}, \tilde{\mathbf{W}}_{\mathbf{C}}, \tilde{\mathbf{W}}_{\mathbf{g}}, \tilde{\mathbf{W}}_{\mathbf{h}}, \tilde{\mathbf{W}}_{\mathbf{f}}) \in \{(\mathbf{s}, \mathbf{e}_\tau, \mathbf{e}_i, \tilde{\mathbf{W}}_{\mathbf{D}}, \tilde{\mathbf{W}}_{\mathbf{C}}, \tilde{\mathbf{W}}_{\mathbf{g}}, \tilde{\mathbf{W}}_{\mathbf{h}}, \tilde{\mathbf{W}}_{\mathbf{f}}) | V > \frac{1}{2\alpha}\left[\frac{1}{\lambda_{\min}(\mathbf{Q})}\sup_{\tau \ge t_0}\left\|\begin{bmatrix}\boldsymbol{\varepsilon}_1(\tau)\\ \boldsymbol{\varepsilon}_2(\tau)\\ \boldsymbol{\varepsilon}_3(\tau)\end{bmatrix}\right\|^2 + \sigma_{\mathbf{D}}Tr(\mathbf{W}_{\mathbf{D}}^T\mathbf{W}_{\mathbf{D}}) + \sigma_{\mathbf{C}}Tr(\mathbf{W}_{\mathbf{C}}^T\mathbf{W}_{\mathbf{C}}) + \sigma_{\mathbf{g}}Tr(\mathbf{W}_{\mathbf{g}}^T\mathbf{W}_{\mathbf{g}}) + \sigma_{\mathbf{h}}Tr(\mathbf{W}_{\mathbf{h}}^T\mathbf{W}_{\mathbf{h}}) + \sigma_{\mathbf{f}}Tr(\mathbf{W}_{\mathbf{f}}^T\mathbf{W}_{\mathbf{f}})\right]\}$$

This further concludes that **s**, $\mathbf{e}_\tau$, $\mathbf{e}_i$, $\tilde{\mathbf{W}}_{\mathbf{D}}$, $\tilde{\mathbf{W}}_{\mathbf{C}}$, $\tilde{\mathbf{W}}_{\mathbf{g}}$, $\tilde{\mathbf{W}}_{\mathbf{h}}$, and $\tilde{\mathbf{W}}_{\mathbf{f}}$ are uniformly ultimately bounded(*u.u.b.*). The implementation of the desired transmission torque (16), desired current (18), control input (22) and update law (30) does not need to calculate the regressor matrix which is required in most adaptive designs for robot manipulators. The convergence of the parameters, however, can be proved to depend on the persistent excitation condition of the input.

The above derivation only demonstrates the boundedness of the closed loop system, but in practical applications the transient performance is also of great importance. For further development, we may apply the comparison lemma[40] to (35) to have the upper bound for *V* as

$$V(t) \le e^{-\alpha(t-t_0)}V(t_0) + \frac{1}{2\alpha}\left[\frac{1}{\lambda_{\min}(\mathbf{Q})}\sup_{t_0<\tau<t}\left\|\begin{bmatrix}\boldsymbol{\varepsilon}_1(\tau)\\ \boldsymbol{\varepsilon}_2(\tau)\\ \boldsymbol{\varepsilon}_3(\tau)\end{bmatrix}\right\|^2 + \sigma_{\mathbf{D}}Tr(\mathbf{W}_{\mathbf{D}}^T\mathbf{W}_{\mathbf{D}}) + \sigma_{\mathbf{C}}Tr(\mathbf{W}_{\mathbf{C}}^T\mathbf{W}_{\mathbf{C}}) + \sigma_{\mathbf{g}}Tr(\mathbf{W}_{\mathbf{g}}^T\mathbf{W}_{\mathbf{g}}) + \sigma_{\mathbf{h}}Tr(\mathbf{W}_{\mathbf{h}}^T\mathbf{W}_{\mathbf{h}}) + \sigma_{\mathbf{f}}Tr(\mathbf{W}_{\mathbf{f}}^T\mathbf{W}_{\mathbf{f}})\right] \tag{36}$$

From (28), we obtain

$$V \geq \frac{1}{2}\left[\lambda_{\min}(\mathbf{A})\left\|\begin{bmatrix}\mathbf{s}\\ \mathbf{e}_\tau\\ \mathbf{e}_i\end{bmatrix}\right\|^2 + \lambda_{\min}(\mathbf{Q_D})Tr(\tilde{\mathbf{W}}_\mathbf{D}^T\tilde{\mathbf{W}}_\mathbf{D}) + \lambda_{\min}(\mathbf{Q_C})Tr(\tilde{\mathbf{W}}_\mathbf{C}^T\tilde{\mathbf{W}}_\mathbf{C})\right. \\ \left. + \lambda_{\min}(\mathbf{Q_g})Tr(\tilde{\mathbf{W}}_\mathbf{g}^T\tilde{\mathbf{W}}_\mathbf{g}) + \lambda_{\min}(\mathbf{Q_h})Tr(\tilde{\mathbf{W}}_\mathbf{h}^T\tilde{\mathbf{W}}_\mathbf{h}) + \lambda_{\min}(\mathbf{Q_f})Tr(\tilde{\mathbf{W}}_\mathbf{f}^T\tilde{\mathbf{W}}_\mathbf{f})\right] \tag{37}$$

Thus, the bound of $\left\|\begin{bmatrix}\mathbf{s}^T & \mathbf{e}_\tau^T & \mathbf{e}_i^T\end{bmatrix}^T\right\|^2$ for $t \geq t_0$ can be derived from (36) and (37) as

$$\left\|\begin{bmatrix}\mathbf{s}\\ \mathbf{e}_\tau\\ \mathbf{e}_i\end{bmatrix}\right\|^2 \leq \frac{1}{\underline{\eta}_A}[V - \lambda_{\min}(\mathbf{Q_D})Tr(\tilde{\mathbf{W}}_\mathbf{D}^T\tilde{\mathbf{W}}_\mathbf{D}) \\ - \lambda_{\min}(\mathbf{Q_C})Tr(\tilde{\mathbf{W}}_\mathbf{C}^T\tilde{\mathbf{W}}_\mathbf{C}) - \lambda_{\min}(\mathbf{Q_g})Tr(\tilde{\mathbf{W}}_\mathbf{g}^T\tilde{\mathbf{W}}_\mathbf{g}) \\ - \lambda_{\min}(\mathbf{Q_h})Tr(\tilde{\mathbf{W}}_\mathbf{h}^T\tilde{\mathbf{W}}_\mathbf{h}) - \lambda_{\min}(\mathbf{Q_f})Tr(\tilde{\mathbf{W}}_\mathbf{f}^T\tilde{\mathbf{W}}_\mathbf{f})] \\ \leq \frac{1}{\underline{\eta}_A}\left\{2e^{-\alpha(t-t_0)}V(t_0) + \frac{1}{\alpha}\left[\frac{1}{\lambda_{\min}(\mathbf{Q})}\sup_{t_0<\tau<t}\left\|\begin{bmatrix}\boldsymbol{\varepsilon}_1(\tau)\\ \boldsymbol{\varepsilon}_2(\tau)\\ \boldsymbol{\varepsilon}_3(\tau)\end{bmatrix}\right\|^2\right.\right. \\ + \sigma_\mathbf{D}Tr(\mathbf{W}_\mathbf{D}^T\mathbf{W}_\mathbf{D}) + \sigma_\mathbf{C}Tr(\mathbf{W}_\mathbf{C}^T\mathbf{W}_\mathbf{C}) + \sigma_\mathbf{g}Tr(\mathbf{W}_\mathbf{g}^T\mathbf{W}_\mathbf{g}) \\ \left. + \sigma_\mathbf{h}Tr(\mathbf{W}_\mathbf{h}^T\mathbf{W}_\mathbf{h}) + \sigma_\mathbf{f}Tr(\mathbf{W}_\mathbf{f}^T\mathbf{W}_\mathbf{f})\right] - \lambda_{\min}(\mathbf{Q_D})Tr(\tilde{\mathbf{W}}_\mathbf{D}^T\tilde{\mathbf{W}}_\mathbf{D}) \\ - \lambda_{\min}(\mathbf{Q_C})Tr(\tilde{\mathbf{W}}_\mathbf{C}^T\tilde{\mathbf{W}}_\mathbf{C}) - \lambda_{\min}(\mathbf{Q_g})Tr(\tilde{\mathbf{W}}_\mathbf{g}^T\tilde{\mathbf{W}}_\mathbf{g}) \\ \left. - \lambda_{\min}(\mathbf{Q_h})Tr(\tilde{\mathbf{W}}_\mathbf{h}^T\tilde{\mathbf{W}}_\mathbf{h}) - \lambda_{\min}(\mathbf{Q_f})Tr(\tilde{\mathbf{W}}_\mathbf{f}^T\tilde{\mathbf{W}}_\mathbf{f})\right\} \tag{38}$$

From the derivations above, we can conclude that the proposed design is able to give bounded tracking with guaranteed transient performance. The following theorem is a summary of the above results.

Theorem 1: Consider the RLFJED robot (1)-(3) with unknown parameters **D**, **C**, **g**, **L** and **R**, then desired transmission torque (16), desired current (18), control input (22) and update law (30) ensure that

(i) error signals **s**, $\mathbf{e}_\tau$, $\mathbf{e}_i$, $\tilde{\mathbf{W}}_\mathbf{D}$, $\tilde{\mathbf{W}}_\mathbf{C}$, $\tilde{\mathbf{W}}_\mathbf{g}$, $\tilde{\mathbf{W}}_\mathbf{h}$, and $\tilde{\mathbf{W}}_\mathbf{f}$ are *u.u.b.*

(ii) the bound of the tracking error vectors for $t \geq t_0$ can be derived as the form of (38), if the Lyapunov-like function candidates are chosen as (28).

Remark 1: The term with $\sigma_{(\cdot)}$ in (30) is to modify the update law to robust the closed-loop system for the effect of the approximation error[26]. Suppose a sufficient number of basis functions $\beta_{(\cdot)}$ is selected so that the approximation error can be neglected then we may have $\sigma_{(\cdot)} = 0$, and (31) becomes

$$\dot{V} = -\begin{bmatrix}\mathbf{s}^T & \mathbf{e}_\tau^T & \mathbf{e}_i^T\end{bmatrix}\mathbf{Q}\begin{bmatrix}\mathbf{s}\\ \mathbf{e}_\tau \\ \mathbf{e}_i\end{bmatrix} \le 0 \tag{39}$$

It is easy to prove that **s**, $\mathbf{e}_\tau$, and $\mathbf{e}_i$ are also square integrable. From (25), (26) and (27), $\dot{\mathbf{s}}$, $\dot{\mathbf{e}}_\tau$ and $\dot{\mathbf{e}}_i$ are bounded; as a result, asymptotic convergence of **s**, $\mathbf{e}_\tau$ and $\mathbf{e}_i$ can easily be shown by Barbalat's lemma. This further implies that $\mathbf{i} \to \mathbf{i}_d$, $\boldsymbol{\tau}_t \to \boldsymbol{\tau}_{td}$ and $\mathbf{q} \to \mathbf{q}_d$ even though **D**, **C**, **g**, **h**, **L**, and **f** are all unknown.
Remark 2: Suppose $\boldsymbol{\varepsilon}_1$, $\boldsymbol{\varepsilon}_2$, and $\boldsymbol{\varepsilon}_3$ cannot be ignored but their variation bounds are available[25,26] i.e. there exists positive constants δ_1, δ_2 and δ_3 such that $\|\boldsymbol{\varepsilon}_1\| \le \delta_1$, $\|\boldsymbol{\varepsilon}_2\| \le \delta_2$ and $\|\boldsymbol{\varepsilon}_3\| \le \delta_3$. To cover the effect of these bounded approximation errors, the desired transmission torque (16), the desired current (18), and the control input (22) are modified to be

$$\boldsymbol{\tau}_{td} = \hat{\mathbf{g}} + \hat{\mathbf{D}}\dot{\mathbf{v}} + \hat{\mathbf{C}}\mathbf{v} - \mathbf{K}_d\mathbf{s} + \boldsymbol{\tau}_{robust\,1} \tag{40}$$

$$\mathbf{i}_d = \mathbf{H}^{-1}[\boldsymbol{\Theta}\mathbf{x}_p + \boldsymbol{\Phi}\boldsymbol{\tau}_{td} + \hat{\mathbf{h}} + \boldsymbol{\tau}_{robust\,2}] \tag{41}$$

$$\mathbf{u} = \hat{\mathbf{f}} - \mathbf{K}_c\mathbf{e}_i + \boldsymbol{\tau}_{robust\,3},\ \mathbf{e}_i = \mathbf{i} - \mathbf{i}_d \tag{42}$$

where $\boldsymbol{\tau}_{robust1}$, $\boldsymbol{\tau}_{robust2}$ and $\boldsymbol{\tau}_{robust3}$ are robust terms to be designed. Let us consider the Lyapunov-like function candidate (28) and the update law (30) again. The time derivative of V can be computed as

$$\begin{aligned}\dot{V} = &-\begin{bmatrix}\mathbf{s}^T & \mathbf{e}_\tau^T & \mathbf{e}_i^T\end{bmatrix}\mathbf{Q}\begin{bmatrix}\mathbf{s}\\ \mathbf{e}_\tau \\ \mathbf{e}_i\end{bmatrix} + \delta_1\|\mathbf{s}\| + \delta_2\|\mathbf{e}_\tau\| + \delta_3\|\mathbf{e}_i\| \\ &+ \mathbf{s}^T\boldsymbol{\tau}_{robust1} + \mathbf{e}_\tau^T\boldsymbol{\tau}_{robust2} + \mathbf{e}_i^T\boldsymbol{\tau}_{robust3}\end{aligned} \tag{43}$$

By picking $\boldsymbol{\tau}_{robust1} = -\delta_1[\mathrm{sgn}(s_1) \ \cdots \ \mathrm{sgn}(s_n)]^T$, where s_k, k=1,…,n is the k-th element of **s**, $\boldsymbol{\tau}_{robust2} = -\delta_2[\mathrm{sgn}(e_{\tau_1}) \ \cdots \ \mathrm{sgn}(e_{\tau_n})]^T$ where e_{τ_k}, k=1,…,2n is the k-th element of $\mathbf{e}_{\tau}$, and $\boldsymbol{\tau}_{robust3} = -\delta_3[\operatorname{sgn}(e_{i_1}) \quad \cdots \quad \operatorname{sgn}(e_{i_n})]^T$, where e_{i_k}, k=1,…,n is the k-th element of $\mathbf{e}_i$, we may have $\dot{V} \le 0$, and asymptotic convergence of the state error can be concluded by Barbalat's lemma.

4. Simulation Study

Consider a 2-DOF planar robot (Fig.1) represented by the differential equation (1), (2) and (3). The quantities m_i, l_i, l_{ci} and I_i are mass, length, gravity center distance and inertia of link i, respectively. Actual values of link parameters in the simulation[34] are m_1=0.5kg, m_2=0.5kg, l_1=l_2=0.75m, l_{c1}=l_{c2}=0.375m, I_1=0.09375kg-m^2, and I_2=0.046975kg-m^2. The actuator inertias, damping, and joint stiffness are $\mathbf{J} = diag(0.02, 0.01)(kg \cdot m^2)$, $\mathbf{B} = diag(5,4)(Nm \cdot \sec/rad)$ and $\mathbf{K} = diag(100,100)(Nm/rad)$ respectively. The motor parameters are: $\mathbf{L} = diag(0.025, 0.025)(H)$, $\mathbf{H} = diag(10,10)(N-m/A)$. Considering the cases of high-velocity movement, we would like the end-point to track a 0.2m-radius circle centered at (0.8 m, 1.0 m) in 2 seconds without knowing its precise model. The initial conditions of the link angles and the motor angles are $\mathbf{q} = \boldsymbol{\theta} = [0.0022 \quad 1.5019 \quad 0 \quad 0]^T$. The initial value of the reference model state vector is $\boldsymbol{\tau}_r = [15.97 \quad -47.26 \quad 0 \quad 0]^T$ which is the same as the initial value of the desired reference input $\boldsymbol{\tau}_d$. The initial condition of the motor armature currents is $\mathbf{i} = [77.49 \quad -83.92]^T$ which is the same as the initial value of the desired reference current $\mathbf{i}_d$. The controller gains are selected as $\mathbf{K}_d = diag(20,20)$, $\boldsymbol{\Lambda} = diag(10,10)$, and $\mathbf{K}_c = diag(50,50)$. Each element of **D**, **C**, **g**, **h**, and **f** is approximated by the first 11 terms of the Fourier series. The simulation results are shown in Fig. 2 to 9. Fig. 2 shows the tracking performance of the end-point and the desired trajectory in the Cartesian space. It is observed that the end-point trajectory converges nicely to the desired trajectory, although the initial position error is quite large. Fig. 3 is the joint space tracking performance. It shows that the transient response vanishes very quickly. Fig. 4 is the control inputs in voltage. Fig. 5 to 9 are the performance of function approximation for **D**, **C**, **g**, **h**, and **f** respectively. Since the reference input does not satisfy the persistent excitation condition, some estimates do not converge to their actual values but remain bounded as desired. It is worth to note that in designing the controller we do not need much knowledge for the system. All we have to do is to pick some controller parameters and some initial weighting matrices.

5. Conclusions

An adaptive controller is proposed for RLFJED robots containing time-varying uncertainties. A backstepping-like procedure is developed to deal with the cascade structure in its dynamic equations. The function approximation technique is employed to cope with the time-varying uncertainties. The closed loop stability is proved by using the Lyapunov-like analysis. The realization of the proposed controller does not need to calculate the regressor which is required in most adaptive designs for robot manipulators. Simulation results justify the performance of the proposed controller in fast tracking operations although most of the robot parameters are not available.

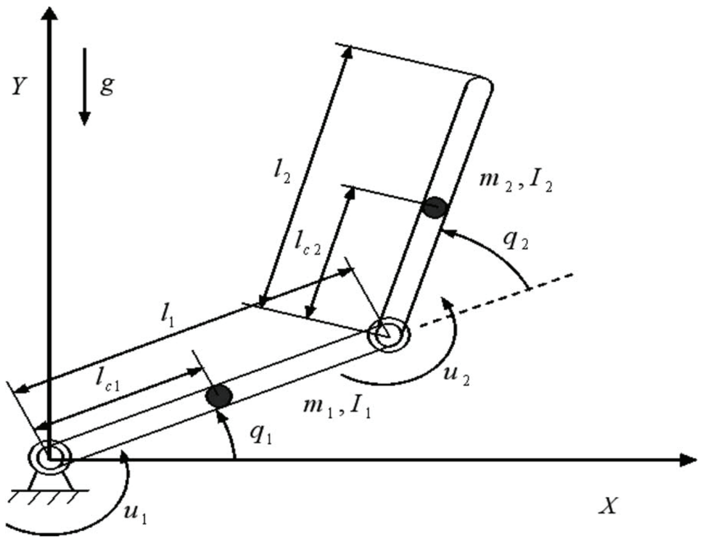

Figure 1. 2-DOF planar robot

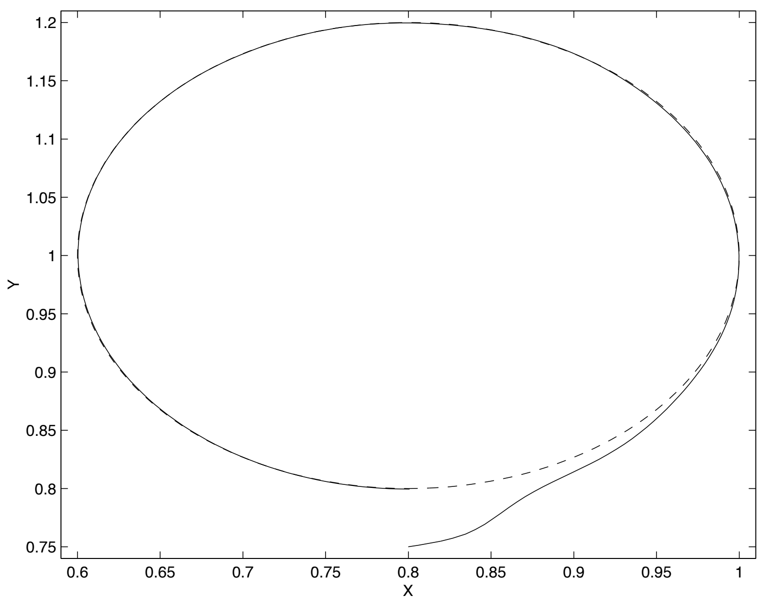

Figure 2. Tracking performance of end-point in the X-Y space (— actual; --- desired). The end-point is initialized at the point (0.8*m*, 0.75*m*) and is required to track a 0.2*m*-radius circle in 2 seconds. After some transient, the tracking error is very small, although we do not know precise dynamics of the robot

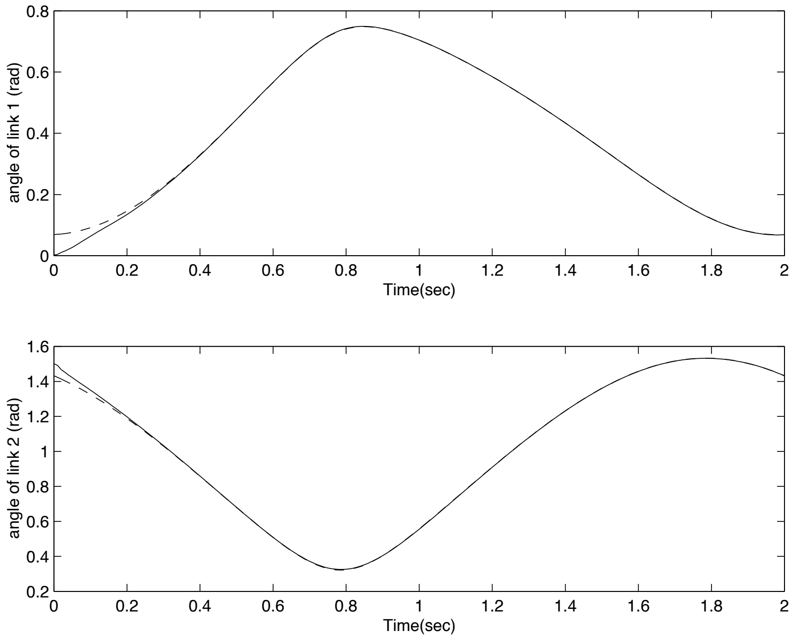

Figure 3. The joint space tracking performance(— actual; --- desired). The real trajectory converges very quickly

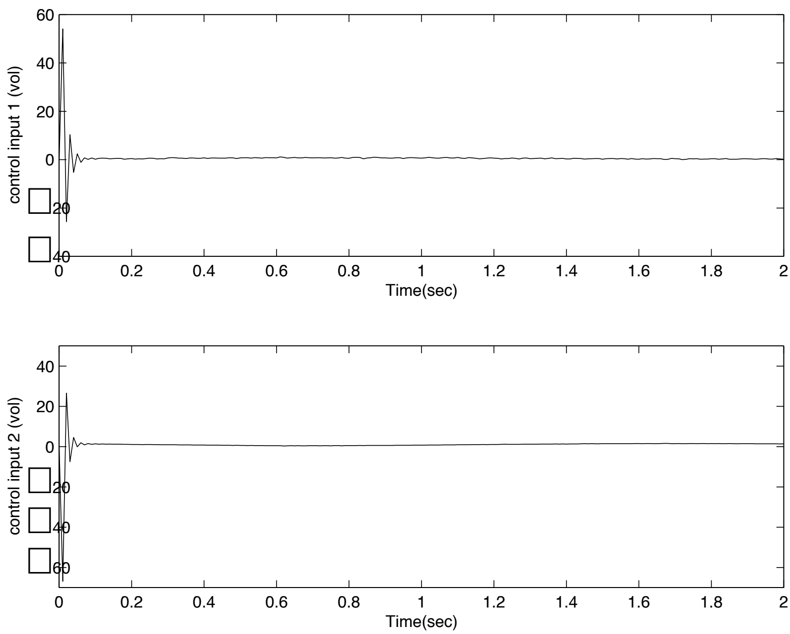

Figure 4. Control input voltage

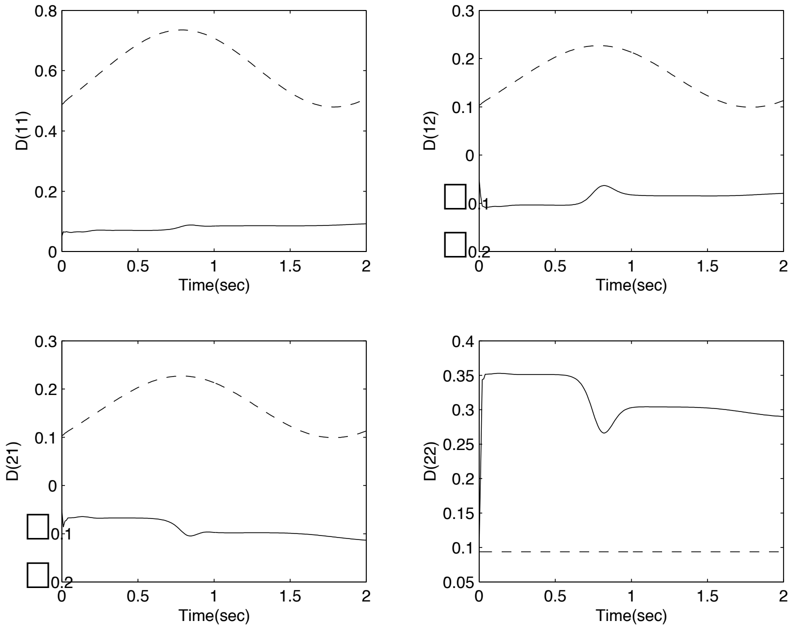

Figure 5. Approximation of **D** matrix(— estimate; --- real). Although the estimated values do not converge to the true values, they are bounded and small

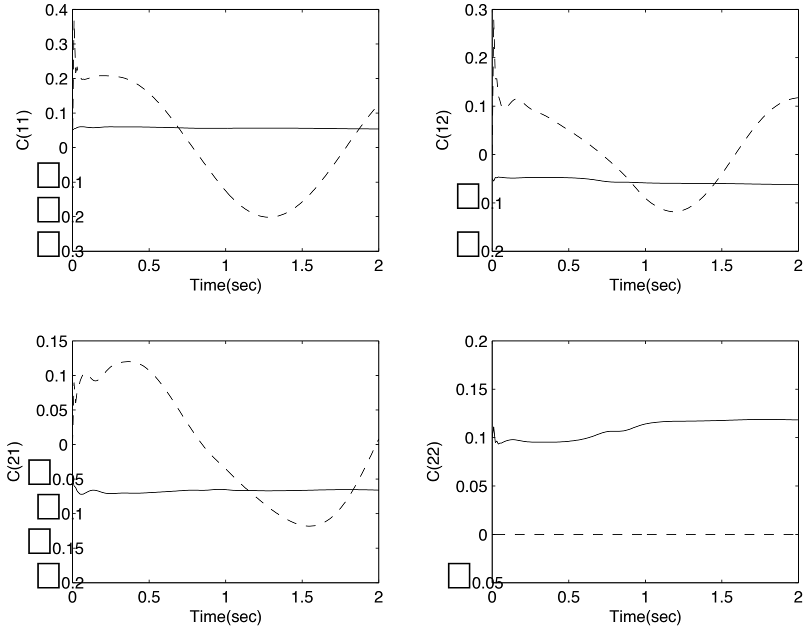

Figure 6. Approximation of **C** matrix(— estimate; --- real)

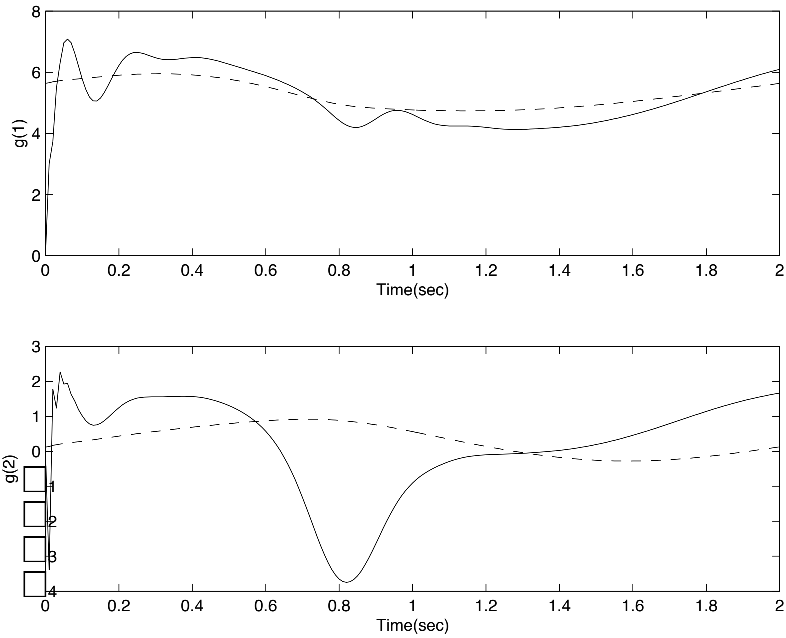

Figure 7. Approximation of vector **g**(— estimate; --- real)

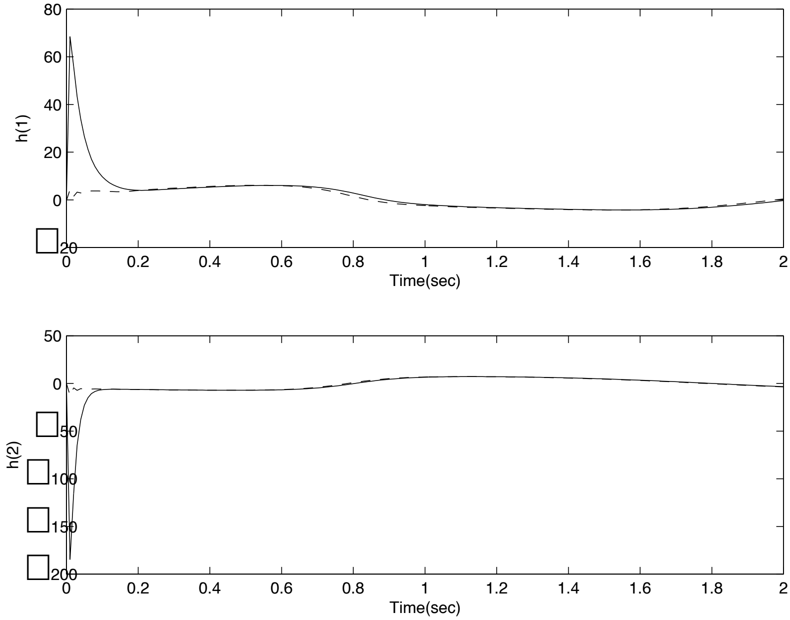

Figure 8. Approximation of vector **h**(— estimate; --- real)

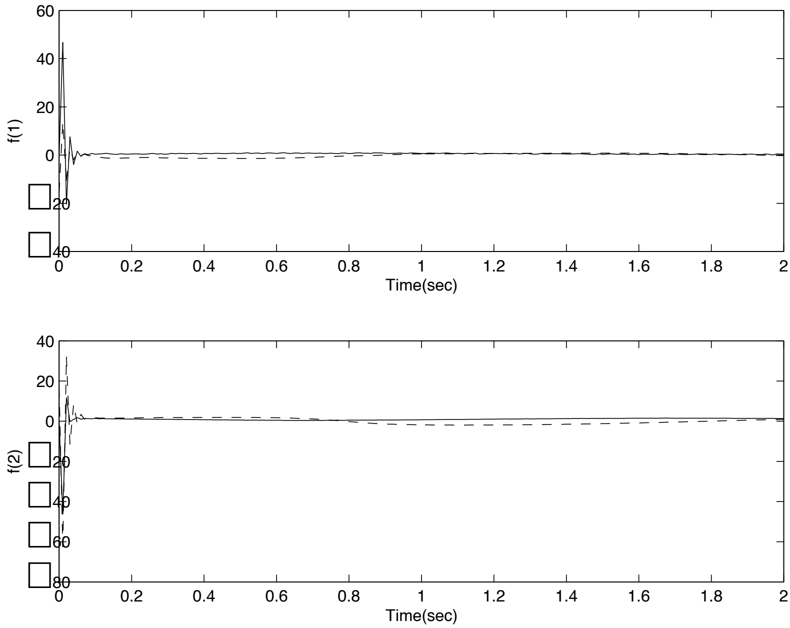

Figure 9. Approximation of vector **f**(— estimate; --- real)

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7. Appendix

Lemma A.1:

Let $\mathbf{s} \in \Re^n$, $\boldsymbol{\varepsilon} \in \Re^n$ and $\mathbf{K}$ is the $n \times n$ positive definite matrix. Then,

$$-\mathbf{s}^T \mathbf{K}\mathbf{s} + \mathbf{s}^T \boldsymbol{\varepsilon} \le \frac{1}{2}[\lambda_{\min}(\mathbf{K})\|\mathbf{s}\|^2 - \frac{\|\boldsymbol{\varepsilon}\|^2}{\lambda_{\min}(\mathbf{K})}]. \tag{A.1}$$

Proof:

$$\begin{aligned}-\mathbf{s}^T \mathbf{K}\mathbf{s} + \mathbf{s}^T \boldsymbol{\varepsilon} &\le [-\lambda_{\min}(\mathbf{K})\|\mathbf{s}\| + \|\boldsymbol{\varepsilon}\|]\|\mathbf{s}\| \\ &= -\frac{1}{2}[\sqrt{\lambda_{\min}(\mathbf{K})}\|\mathbf{s}\| - \frac{\|\boldsymbol{\varepsilon}\|}{\sqrt{\lambda_{\min}(\mathbf{K})}}]^2 \\ &\quad -\frac{1}{2}[\lambda_{\min}(\mathbf{K})\|\mathbf{s}\|^2 - \frac{\|\boldsymbol{\varepsilon}\|^2}{\lambda_{\min}(\mathbf{K})}] \\ &\le -\frac{1}{2}[\lambda_{\min}(\mathbf{K})\|\mathbf{s}\|^2 - \frac{\|\boldsymbol{\varepsilon}\|^2}{\lambda_{\min}(\mathbf{K})}]\end{aligned}$$

Q.E.D.

Lemma A.2:

Let $\mathbf{w}_i^T = [w_{i1} \quad w_{i2} \quad \cdots \quad w_{in}] \in \Re^{1\times n}$, i=1,…,m and $\mathbf{W}$ is a block diagonal matrix defined as $\mathbf{W} = diag\{\mathbf{w}_1, \mathbf{w}_2, \cdots, \mathbf{w}_m\} \in \Re^{mn\times m}$. Then,

$$Tr(\mathbf{W}^T \mathbf{W}) = \sum_{i=1}^{m} \|\mathbf{w}_i\|^2 . \tag{A.2}$$

The notation $Tr(.)$ denotes the trace operation.
Proof: The proof is straightforward as below:

$$\mathbf{W}^T\mathbf{W} = \begin{bmatrix} w_{11} & \cdots & w_{1n} & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ 0 & \cdots & 0 & w_{21} & \cdots & w_{2n} & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots & w_{m1} & \cdots & w_{mn} \end{bmatrix} \begin{bmatrix} w_{11} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ w_{1n} & 0 & \cdots & 0 \\ 0 & w_{21} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & w_{2n} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & w_{m1} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & w_{mn} \end{bmatrix}$$

$$= \begin{bmatrix} \mathbf{w}_1^T & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{w}_2^T & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{w}_m^T \end{bmatrix} \begin{bmatrix} \mathbf{w}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{w}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{w}_m \end{bmatrix}$$

$$= \begin{bmatrix} \mathbf{w}_1^T\mathbf{w}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{w}_2^T\mathbf{w}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{w}_m^T\mathbf{w}_m \end{bmatrix}$$

$$= \begin{bmatrix} \|\mathbf{w}_1\|^2 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \|\mathbf{w}_2\|^2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \|\mathbf{w}_m\|^2 \end{bmatrix}$$

The last equality holds because by definition $\mathbf{w}_i^T\mathbf{w}_i = w_{i1}^2 + w_{i2}^2 + \ldots + w_{im}^2 = \|\mathbf{w}_i\|^2$.

Therefore, we have $$Tr = (\mathbf{W}^T\mathbf{W}) = \sum_{i=1}^{m} \|\mathbf{w}_i\|^2 .$$ Q.E.D.

Lemma A.3:

Suppose $\mathbf{w}_i^T = [w_{i1} \quad w_{i2} \quad \cdots \quad w_{in}] \in \Re^{1\times n}$ and $\mathbf{v}_i^T = [v_{i1} \quad v_{i2} \quad \cdots \quad v_{in}] \in \Re^{1\times n}$, i=1,…,m. Let $\mathbf{W}$ and $\mathbf{V}$ be block diagonal matrices that are defined as $\mathbf{W} = diag\{\mathbf{w}_1, \mathbf{w}_2, \cdots, \mathbf{w}_m\} \in \Re^{mn\times m}$ and $\mathbf{V} = diag\{\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_m\} \in \Re^{mn\times m}$, respectively. Then,

$$Tr(\mathbf{V}^T\mathbf{W}) \le \sum_{i=1}^{m} \|\mathbf{v}_i\|\|\mathbf{w}_i\| \quad . \text{(A.3)}$$

Proof: The proof is also straightforward:

$$\mathbf{V}^T\mathbf{W} = \begin{bmatrix} \mathbf{v}_1^T & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{v}_2^T & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{v}_m^T \end{bmatrix} \begin{bmatrix} \mathbf{w}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{w}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{w}_m \end{bmatrix}$$

$$= \begin{bmatrix} \mathbf{v}_1^T\mathbf{w}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{v}_2^T\mathbf{w}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{v}_m^T\mathbf{w}_m \end{bmatrix}$$

Hence,

$$\begin{aligned} Tr(\mathbf{V}^T\mathbf{W}) &= \mathbf{v}_1^T\mathbf{w}_1 + \mathbf{v}_2^T\mathbf{w}_2 + \ldots + \mathbf{v}_m^T\mathbf{w}_m \\ &\le \|\mathbf{v}_1\|\|\mathbf{w}_1\| + \|\mathbf{v}_2\|\|\mathbf{w}_2\| + \ldots + \|\mathbf{v}_m\|\|\mathbf{w}_m\| \qquad \text{Q.E.D.} \\ &= \sum_{i=1}^{m} \|\mathbf{v}_i\|\|\mathbf{w}_i\| \end{aligned}$$

Lemma A.4:

Let $\mathbf{W}$ be defined as in *Lemma A.2*, and $\tilde{\mathbf{W}}$ is a matrix defined as $\tilde{\mathbf{W}} = \mathbf{W} - \hat{\mathbf{W}}$, where $\hat{\mathbf{W}}$ is a matrix with proper dimension. Then

$$Tr(\tilde{\mathbf{W}}^T\hat{\mathbf{W}}) \le \frac{1}{2}Tr(\mathbf{W}^T\mathbf{W}) - \frac{1}{2}Tr(\tilde{\mathbf{W}}^T\tilde{\mathbf{W}}) \,. \quad \text{(A.4)}$$

Proof:

$$
\begin{aligned}
Tr(\tilde{\mathbf{W}}^T\hat{\mathbf{W}}) &= Tr(\tilde{\mathbf{W}}^T\mathbf{W}) - Tr(\tilde{\mathbf{W}}^T\tilde{\mathbf{W}}) \\
&\le \sum_{i=1}^{m}(\|\tilde{\mathbf{w}}_i\|\|\mathbf{w}_i\| - \|\tilde{\mathbf{w}}_i\|^2) \quad \text{(by } \textit{Lemma A.2} \text{ and } \textit{A.3}) \\
&= \frac{1}{2}\sum_{i=1}^{m}[\|\mathbf{w}_i\|^2 - \|\tilde{\mathbf{w}}_i\|^2 - (\|\tilde{\mathbf{w}}_i\| - \|\mathbf{w}_i\|)^2] \\
&\le \frac{1}{2}\sum_{i=1}^{m}(\|\mathbf{w}_i\|^2 - \|\tilde{\mathbf{w}}_i\|^2) \\
&= \frac{1}{2}Tr(\mathbf{W}^T\mathbf{W}) - \frac{1}{2}Tr(\tilde{\mathbf{W}}^T\tilde{\mathbf{W}}) \quad \text{(by } \textit{Lemma A.2})
\end{aligned}
$$

Q.E.D.

In the above lemmas, we consider properties of a block diagonal matrix. In the following, we would like to extend the analysis to a class of more general matrices.

Lemma A.5:

Let $\mathbf{W}$ be a matrix in the form $\mathbf{W}^T = [\mathbf{W}_1^T \quad \mathbf{W}_2^T \quad \cdots \quad \mathbf{W}_p^T] \in \Re^{pmn\times m}$ where $\mathbf{W}_i = diag\{\mathbf{w}_{i1}, \mathbf{w}_{i2}, \cdots, \mathbf{w}_{im}\} \in \Re^{mn\times m}$, i=1,…,p, are block diagonal matrices with the entries of vectors $\mathbf{w}_{ij}^T = [w_{ij1} \quad w_{ij2} \quad \cdots \quad w_{ijn}] \in \Re^{1\times n}$, j=1,…,m. Then, we may have

$$
Tr(\mathbf{W}^T\mathbf{W}) = \sum_{i=1}^{p}\sum_{j=1}^{m}\|\mathbf{w}_{ij}\|^2 . \qquad \text{(A.5)}
$$

Proof:

$$
\begin{aligned}
\mathbf{W}^T\mathbf{W} &= [\mathbf{W}_1^T \quad \cdots \quad \mathbf{W}_p^T]\begin{bmatrix}\mathbf{W}_1 \\ \vdots \\ \mathbf{W}_p\end{bmatrix} \\
&= \mathbf{W}_1^T\mathbf{W}_1 + \cdots + \mathbf{W}_p^T\mathbf{W}_p
\end{aligned}
$$

Hence, we may calculate the trace as

$$Tr(\mathbf{W}^T\mathbf{W}) = Tr(\mathbf{W}_1^T\mathbf{W}_1) + \cdots + Tr(\mathbf{W}_p^T\mathbf{W}_p)$$
$$= \sum_{j=1}^{m}\left\|\mathbf{w}_{1j}\right\|^2 + \cdots + \sum_{j=1}^{m}\left\|\mathbf{w}_{pj}\right\|^2 \qquad \text{(by } Lemma\ A.1\text{)}$$
$$= \sum_{i=1}^{p}\sum_{j=1}^{m}\left\|\mathbf{w}_{ij}\right\|^2$$

Q.E.D.

Lemma A.6:

Let $\mathbf{V}$ and $\mathbf{W}$ be matrices defined in *Lemma A.5*, Then, $Tr(\mathbf{V}^T\mathbf{W}) \le \sum_{i=1}^{p}\sum_{j=1}^{m}\left\|\mathbf{v}_{ij}\right\|\left\|\mathbf{w}_{ij}\right\|$.

(A.6)

Proof:

$$Tr(\mathbf{V}^T\mathbf{W}) = Tr(\mathbf{V}_1^T\mathbf{W}_1) + \cdots + Tr(\mathbf{V}_p^T\mathbf{W}_p)$$
$$\le \sum_{j=1}^{m}\left\|\mathbf{v}_{1j}\right\|\left\|\mathbf{w}_{1j}\right\| + \cdots + \sum_{j=1}^{m}\left\|\mathbf{v}_{pj}\right\|\left\|\mathbf{w}_{pj}\right\| \qquad \text{(by } Lemma\ A.3\text{)}$$
$$= \sum_{i=1}^{p}\sum_{j=1}^{m}\left\|\mathbf{v}_{ij}\right\|\left\|\mathbf{w}_{ij}\right\|$$

Q.E.D.

Lemma A.7:

Let $\mathbf{W}$ be defined as in *Lemma A.5*, and $\tilde{\mathbf{W}}$ is a matrix defined as $\tilde{\mathbf{W}} = \mathbf{W} - \hat{\mathbf{W}}$, where $\hat{\mathbf{W}}$ is a matrix with proper dimension. Then

$$Tr(\tilde{\mathbf{W}}^T\hat{\mathbf{W}}) \le \frac{1}{2}Tr(\mathbf{W}^T\mathbf{W}) - \frac{1}{2}Tr(\tilde{\mathbf{W}}^T\tilde{\mathbf{W}}). \qquad \text{(A.7)}$$

Proof:

$$
\begin{aligned}
Tr(\tilde{\mathbf{W}}^T\hat{\mathbf{W}}) &= Tr(\tilde{\mathbf{W}}^T\mathbf{W}) - Tr(\tilde{\mathbf{W}}^T\tilde{\mathbf{W}}) \\
&\le \sum_{i=1}^{p}\sum_{j=1}^{m}(\|\tilde{\mathbf{w}}_{ij}\|\|\mathbf{w}_{ij}\| - \|\tilde{\mathbf{w}}_{ij}\|^2) \qquad \text{(by } \textit{Lemma A.5} \text{ and } \textit{A.6}) \\
&= \frac{1}{2}\sum_{i=1}^{p}\sum_{j=1}^{m}[\|\mathbf{w}_{ij}\|^2 - \|\tilde{\mathbf{w}}_{ij}\|^2 - (\|\tilde{\mathbf{w}}_{ij}\| - \|\mathbf{w}_{ij}\|)^2] \\
&\le \frac{1}{2}\sum_{i=1}^{p}\sum_{j=1}^{m}(\|\mathbf{w}_{ij}\|^2 - \|\tilde{\mathbf{w}}_{ij}\|^2) \\
&= \frac{1}{2}Tr(\mathbf{W}^T\mathbf{W}) - \frac{1}{2}Tr(\tilde{\mathbf{W}}^T\tilde{\mathbf{W}}) \qquad \text{(by } \textit{Lemma A.5})
\end{aligned}
$$

Q.E.D

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