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# Matlab-Simulink-Based Compound Model Reference Adaptive Control for DC Motor

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Additional information is available at the end of the chapter

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## Abstract

The objective of this chapter is to develop a compound Model Reference Adaptive Control (MRAC) of the dc motor by using the Matlab/Simulink software. The purpose of the chapter is to serve as a tutorial for the students or researchers in the field correlating step by step the presented theory with the Matlab/Simulink programming environment. The supraunitary relative degree model reference adaptive control is proposed as a solution to the parameters variation of the electric drives. The numerical simulation results confirm the robustness of the proposed solution at unmodelled dynamics or parameter variation of the process. The conventional control of the dc drive based on the cascaded loops is also treated in this chapter.

**Keywords:** dc motor, PI control, adaptive control, Matlab-Simulink, supraunitary relative degree

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## 1. Introduction

The conventional control of the dc motor supposes the use of the modulus and symmetric criteria, adapted to the fast processes (Kessler variant), by the optimum choosing of the tuning parameters of the corresponding controllers. Therefore, the cascaded control loops are the most appropriate choice for the conventional dc drive system. The advantages of using the conventional drive are: the complex process is divided into simple subprocesses having only one significant time constant compensated by an independent controller; it assures the minimum time response; thanks to the feedback path of the control loop, the stability of the dc drive system is maintained; the null steady state error; the fast compensation of the dc drive perturbations.

The main disadvantage of the conventional control is its inadaptability to the parameters variations of the processes. Taking into account that the normal operation of the processes is

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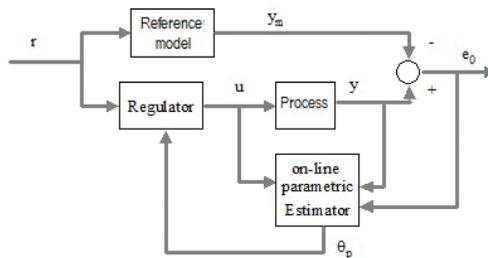
assured through the electric motors, the values of the electric parameters are not constant. It is well-known that the parameters of the controllers depend on the parameters of the process. Therefore, a change of the process parameters conducts to the deterioration of the drive system performances. Moreover, the uncertainty of the parameters and the structural mismatch of the processes are the main barriers of avoiding the use of the conventional control. In these particular cases the conventional control, even the robust control is inefficient due to the invariance of the controller parameters. An adaptive control is desired in order to maintain the imposed performances on the electric drive. Consequently, an additional adaptive loop is inserted to the conventional feedback system. The purpose of inserting the adaptive loop is to update the controller parameters related to the parameter variations of the process or to the modified structure of the physical process. According to the exogenous variation (reference and perturbation), the adaptive mechanism of the regulation part is performed by the minimization of the specific performance criterion. The result of the designed regulation algorithm consists of a new set of the controller parameters. The closed loop adaptive systems (widely spread) assure the appropriate control correlation with the process uncertainty (modified parameters or structure). The adaptive loop delivers the real time control based on the designed on-line controller.

The signals in **Figure 1** are defined as follows:  $r$ - the reference signal;  $y_m$ - the output signal of the reference model;  $u$ - the adaptive control;  $y$ -the output of the process (the measurable signals);  $e_0$  – the tracking error vector;  $\theta_p$  – the parameters of the process.

In this manner, the structural mismatch and the parametric uncertainty are compensated. In the mechatronic domain or the rolling mill the adaptive control is mandatory.

The name of the adaptive control structure comes from the use of the on-line parameters estimator. Based on it, the process parameters are provided by measuring the control, the process output and the tracking error. Taking into account the reference signal and the process parameters, the adaptive algorithm delivers the appropriate estimation of the control.

There are two types of the adaptive control structures [2]. The adaptive control can be in explicit (indirect) form or in implicit (direct) form. The indirect adaptive control delivers the estimation of the process parameters. The direct adaptive control delivers the estimation of the controller parameters.



**Figure 1.** The principle of the adaptive control [1].

In this chapter the adaptive control with supraunitary relative degree is introduced through the dc drive system.

The design procedure of the adaptive control is based on the strictly positive real (SPR) concept.

In order to attain the SPR condition, the equivalent transfer function should satisfy the following tasks:

1. strictly stable (the poles are situated in the left half plane of the complex space);
2. the relative degree must be 0 or 1;
3.  $\text{Re}[H_m(j\omega)] \geq 0$ , for any  $\omega > 0$ , i.e., the transfer function of the reference model,  $H_m(s)$ , should be minimum of phase (without dead time and zero situated in the right half plane).

The model reference adaptive control (MRAC) contains three aspects:

1. MRAC in simple form;
2. variable structure form; and
3. compound MRAC.

In order to obtain a perfect tracking,  $\lim_{t \rightarrow \infty} e_0(t) \rightarrow 0$ , the asymptotic limit of the tracking error vanishes, and the relative degree of the reference model should be greater or at least of the same order with the relative degree of the process:

$$n_m^* \geq n_p^* \quad (1)$$

## 2. Conventional dc drive system

In this Section the control methodology of the conventional dc drive system is presented.

The drive system consists of the full controlled dc motor connected to the load. At the high power, the controlled six pulses full bridge ac-dc power converter is involved; at the low – medium power the full bridge dc-dc power converter connected in series with the uncontrolled ac-dc power rectifier is used. From the point of view of commutation, there is a substantial difference between the above presented solutions: while at low and medium power, the force commutation is used (based on the high frequency switching power transistors), at high power, the natural commutation is used (based on thyristors).

In order to design the conventional control for fast processes, the entire system should be modeled under certain assumptions:

### 2.1. Assumptions for mathematical modeling of the dc motor

The magnetizing flux is maintained at the rated value; the permeability of the ferromagnetic core is infinite; the unsaturated magnetic circuit is considered, a compensated dc motor is taken into consideration; there are auxiliaries poles; the brushes are situated on the neutral axis; and the brush droop voltage is neglected.



$$H_{fn}(s) = \frac{1}{1 + sT_{Fn}}, \tag{2}$$

where  $T_{Fn}$  is the time constant of the speed transducer.

The corresponding Simulink block diagram is shown below in **Figure 3**.

The control design procedure starts with the armature current loop design (inner loop). The imposed performances for the inner closed loop are:

**I. the steady state regime:**

- a. null steady state error  $\varepsilon_{st} = 0$ ;
- b. the rejection of the perturbation influence (load torque is the main perturbation).

**II. the dynamic regime:**

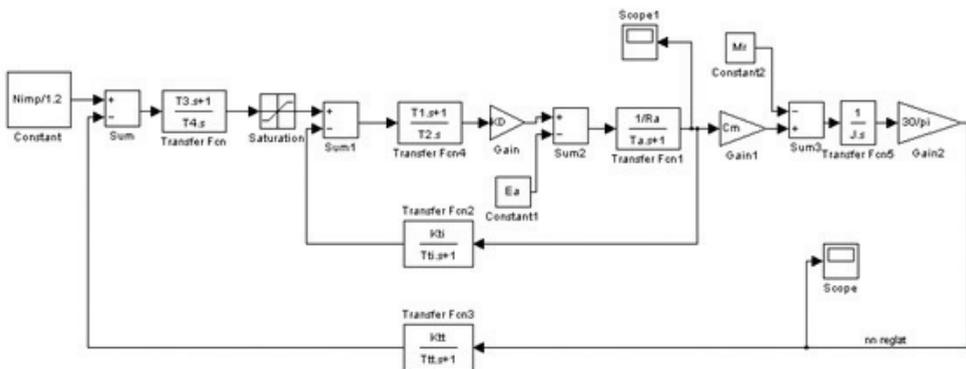
- a. current overshoot,  $\sigma = 4.3\%$ ;
- b. response time depends on the cutting frequency of the loop:  $t_r = 2.35/\omega_{cl}$ ;
- c. phase margin of the current loop:  $\gamma = 63^\circ 26'$ .

In order to satisfy the above mentioned performances the modulus criterion is used. The Kessler criterion guarantees of the above mentioned performances on condition that the open loop transfer function of the inner loop has the form:

$$H_{cutl}(s) = \frac{1}{2sT_{\Sigma I}(1 + sT_{\Sigma I})} \tag{3}$$

where,  $T_{\Sigma I}$  – the parasitic time constant of the current loop is considerably lower than the armature time constant:

$$T_{\Sigma I} = T_{FI} + T_{TI} \ll T_A \tag{4}$$



**Figure 3.** Simulink block diagram of the dc drive system.

in which  $T_{FI}$  is the current filter time constant, and  $T_{TI}$  is the current transducer time constant. According to modulus criterion, the Proportional Integral (PI) controller is suitable to control the armature current:

$$H_{RI}(s) = \frac{1 + s\tau_1}{s\tau_2}, \tag{5}$$

with the appropriately controller parameters:

$$\tau_2 = T_A \tag{6}$$

$$\tau_1 = 2T_{\Sigma I}K_dK_{TI} \frac{1}{R_A}. \tag{7}$$

Eq. (8) contains the mathematical model of the dc-dc power converter:

$$K_d = \frac{U_{Ar}}{10}, \tag{8}$$

and the attenuation factor of the current transducer (obtained by imposing the time response of the current loop) has the form:

$$K_{TI} = \frac{10}{I_{Amax}} \left[ \frac{V}{A} \right]. \tag{9}$$

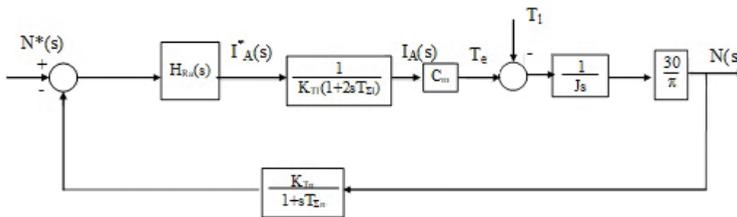
The closed loop armature current transfer function is as follows:

$$H_{ol}(s) = \frac{1}{k_{TI}(1 + 2sT_{\Sigma I})} \tag{10}$$

The equivalent block diagram of the dc drive system is shown in **Figure 4**.

Taking into account the reduced block diagram of the dc drive system (**Figure 4**), the symmetrical optimum criterion could be applied.

According to symmetrical optimum criterion, by using the following open loop transfer function (Kessler) the required performances of the closed loop system are attained:



**Figure 4.** The operational model of the dc drive system.

$$H_{cut,n}(s) = \frac{1 + 4sT_{\Sigma n}}{8s^2T_{\Sigma n}^2(1 + sT_{\Sigma n})}. \quad (11)$$

By comparing the equivalent open loop transfer function (**Figure 4**) with the imposed open loop transfer function (11), the PI speed controller is obtained:

$$H_{Rn}(s) = \frac{1 + s\tau_3}{s\tau_4}. \quad (12)$$

Moreover, the following speed controller parameters are obtained:

$$\tau_3 = 4T_{\Sigma n}, \quad (13)$$

$$\tau_4 = \frac{8T_{\Sigma n}^2 C_m \cdot 30}{K_{\pi} J \pi K_{Tn}}. \quad (14)$$

In order to well understand the tuning procedure of the speed controller, the additional parameters are provided:

$$T_{\Sigma n} = 2T_A + T_{Tn}, \quad (15)$$

$$C_m = \frac{T_{lr}}{I_{Ar}}, \quad (16)$$

$$K_{Tn} = \frac{10}{1.2 \cdot n_r} \left[ \frac{\text{rot}/\text{min}}{\text{V}} \right], \quad (17)$$

in which  $T_{\Sigma n}$  is the parasitic time constant of the speed loop,  $T_{lr}$  is the rated load torque,  $C_m$ - the mechanical constant of the dc motor, and  $T_{Tn}$  is the time constant of the speed transducer (it is obtained by imposing the time response of the speed loop).

The numerical implementation of the dc drive system is based on the Simulink block diagram shown in **Figure 5**.

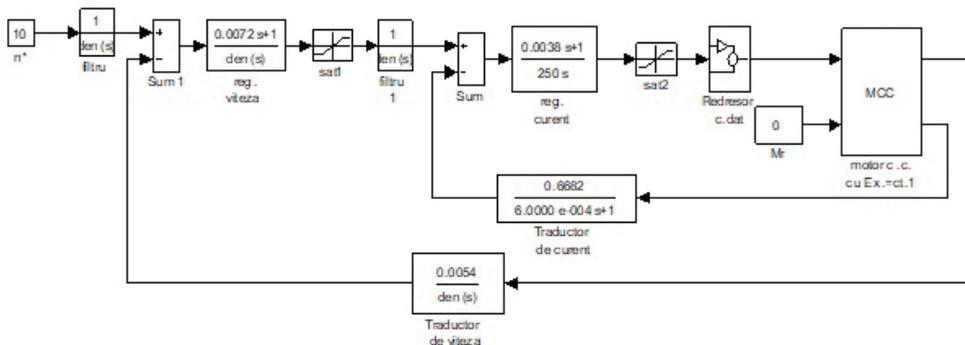


Figure 5. The numerical implementation of the dc drive system.

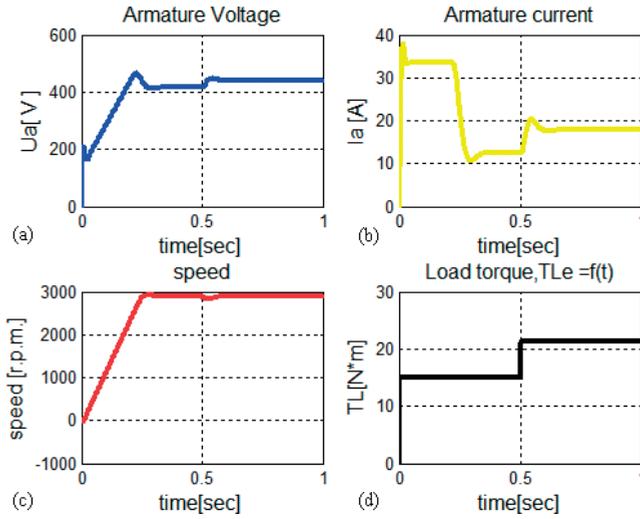


Figure 6. The dc drive numerical simulations results under load step variation.

By considering a dc motor with the following nameplate data: rated power  $P_r = 5.1$  kW, the maximum armature voltage  $U_r = 440$  V, the nominal armature current  $I_{Ar} = 17.8$  A, the reduced moment of inertia  $J = 0.02$  kgm<sup>2</sup>, the viscous force  $F_v = 0.0006$  Nms/rad, rated speed  $n_r = 2700$  rpm, the simulation results are obtained (Figure 6a–d). The motor data can be obtained based on the nameplate values by using the detailed Matlab software provided in [3].

Figure 6a–d shows the  $0.7T_r$  load starting simulation results of the dc conventional control based on the dc-dc full bridge power converter. Figure 6a contains the obtained armature voltage of the dc motor. The armature current varies according to Figure 6b, the speed varies as in Figure 6c under rated value of the load torque  $T_1 = 22.8$  Nm (Figure 6d). The load torque is applied at  $t = 0.5$  s.

### 3. Adaptive control

There are three assumptions available [4, 5]: the mathematical model of the process is linear, strictly proper and of minimum phase, having the supraunitary relative degree  $n_p^* = 2n_p^* = 2$ ; the reference model has the relative degree greater than one ( $n_m^* = 2$ ), is stable and of minimum phase; the reference signal should be bounded limit, being a continuous function. The second order mathematical model of the dc motor is used in this chapter. This supposes the transfer function of the process has the form:

$$H_p(s) = k_p \frac{N_p(s)}{D_p(s)}. \tag{18}$$

At the same time, the pole excess is known:

$$\partial[D_p(s)] - \partial[N_p(s)] = n_p^* > 1. \tag{19}$$

Taking into account that the relative degree of the process is supraunitary, the second order of the reference model is chosen,  $n_m^* = n_p^*$ .

$$H_m(s) = k_m \frac{N_m(s)}{D_m(s)}. \tag{20}$$

Due to the supraunitary relative degree, the strictly real positive condition for the reference model cannot be accomplished. This condition that the tracking error differs from the identification error [2–6] implies, in this case, the use of the augmented error.

The augmented error depends on the gain factor knowing.

### 3.1. The case of knowing only the sign of $k_p$ factor

In order to obtain a stable system the following signals vector is inserted:

$$\mathbf{v}_1 = \begin{bmatrix} \mathbf{v}_u \\ \mathbf{v}_y \\ \mathbf{y}_p \end{bmatrix}, \tag{21}$$

The dynamic filters ( $\Lambda$ ,  $\mathbf{h}$ ) are placed on the command  $\mathbf{v}_u$  and on the output of the process  $\mathbf{v}_{yp}$ :

$$\begin{cases} \dot{\mathbf{v}}_u(t) = \Lambda \mathbf{v}_u(t) + \mathbf{h} \mathbf{u}(t) \\ \dot{\mathbf{v}}_y(t) = \Lambda \mathbf{v}_y(t) + \mathbf{h} \mathbf{y}_p(t) \end{cases} \tag{22}$$

The solution of the dynamical filter is implemented in Matlab as in **Figure 7** (applied only for the first equation).

The ( $\Lambda$ ,  $\mathbf{h}$ ) pair is chosen in controllable canonical form,  $\Lambda \in \mathfrak{R}^{(n_p-1) \times (n_p-1)}$ ,  $\mathbf{h} \in \mathfrak{R}^{n_p}$ , such that:

$$\det(s\mathbf{I} - \Lambda) = N_m(s) \cdot \lambda_1(s), \tag{23}$$

in which:  $\lambda_1(s)$  is an arbitrary Hurwitz polynomial having the degree [7]:

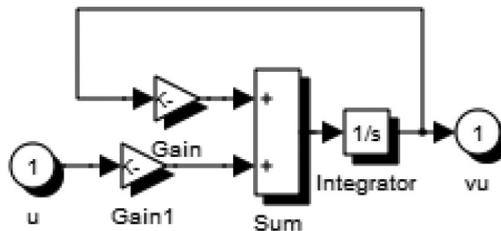


Figure 7. L-h ( $\Lambda$ ,  $\mathbf{h}$ )  $v_u$  filter block.

$$\partial[\mathbf{N}_m(s)\lambda_1(s)] = n_p - 1. \quad (24)$$

Therefore, the  $\lambda_1(s)$  polynomial is a design component.

The Hurwitz polynomial  $L(s)$  is chosen such that the transfer function  $H_m(s)L(s)$  becomes SPR. The degree of the  $L(s)$  is  $\partial[L(s)] = n_m^* - 1$ . If the  $L(s)$  is Hurwitz polynomial, then  $L^{-1}$  is stable.

The parameters vectors  $\mathbf{v}, \xi \in \mathfrak{R}^{2n_p}$  consist of

$$\mathbf{v} = \begin{bmatrix} v_u \\ v_{yp} \\ y_p \\ r \end{bmatrix}, \quad \xi = L^{-1}(s)[\mathbf{v}(t)], \quad (25)$$

The  $\mathbf{v}_1, \xi_1 \in \mathfrak{R}^{2n_p-1}$  are defined as follows:

$$\mathbf{v}_1 = \begin{bmatrix} v_u \\ v_{yp} \\ y_p \end{bmatrix}, \quad \xi_1 = \begin{bmatrix} \xi_n \\ \xi_y \\ \xi_p \end{bmatrix}. \quad (26)$$

The auxiliary error is computed on-line:

$$e_a = \theta^T \xi - L^{-1}(s)[\theta^T(t)\mathbf{v}(t)], \quad (27)$$

where:

$$\xi(t) = L^{-1}(s)[\mathbf{v}(t)]. \quad (28)$$

The augmented error is defined as:

$$e_c = e_0 + H_m(s)L(s)[K_1 e_a - \xi_1^T \xi_1 e_c], \quad (29)$$

and the on-line gradient adjustable parameter  $K_1$  depends only by the augmented error:

$$\overset{\circ}{K}_1 = -e_c e_a \quad (30)$$

### 3.2. The parametric adjustment laws for the compound adaptive control

#### 3.2.1. Gradient

The gradient law [2–4] is expressed as:

$$\overset{\circ}{\theta}_g = -\gamma_g \text{sign}(K_p) e_c \xi(t) \frac{1}{1 + \xi^T \xi} \quad (31)$$

$$e_a = (\theta_g + \theta_v)^T \xi - L^{-1}(s)[(\theta_g + \theta_v)^T \mathbf{v}(t)] \quad (32)$$

$$\xi(t) = L^{-1}(s)[\mathbf{v}(t)]; \text{ where } \xi = \begin{bmatrix} \xi_u \\ \xi_{yp} \\ \xi_p \end{bmatrix}. \quad (33)$$

### 3.2.2. Deduction of the variable structure parameter $\theta_v$

The parameter  $\theta_v$  can be deduced by using the following [2]:

$$\theta_v = \bar{\theta}_v [(e^{K_e \xi} - 1) / (e^{K_e \xi} + 1)] \text{sign}(K_p) \tag{34}$$

$$\dot{\bar{\theta}}_v = -\lambda \bar{\theta}_v - \gamma_v |\xi e_c|. \tag{35}$$

The block diagram of obtaining the augmented error is depicted in **Figure 8**, in which  $\Phi$  is the vector of the parameter estimation errors

$$\Phi = \theta - \theta^0 \tag{36}$$

The vector of the parameter  $\theta$  is obtained by using the compound structure:  $\theta = \theta_g + \theta_v$ .

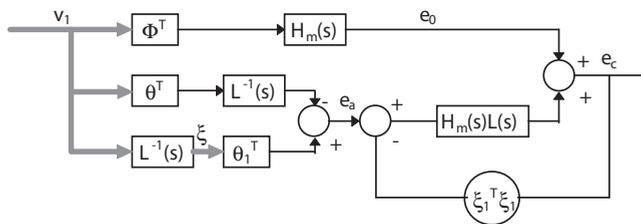
In the adaptive control, there is a commutation function; usually the *signum* function conducts toward a sliding mode regime such that the evolution to the equilibrium point is very fast. Therefore, the compound adaptive law is used:

$$u(t) = (\theta_g + \theta_v)^T \mathbf{v} + \left[ -\gamma_p \text{sign}(K_p) \xi e_0 + \frac{\dot{\theta}_v}{\theta_v} \frac{e^{K \xi e_0} - 1}{e^{K \xi e_0} + 1} \right]^T \xi. \tag{37}$$

The adaptive control provides robust characteristics to external disturbances and to unmodelled dynamics.

### 3.3. The stability of the solution

The perturbation of the dc drive system can leads to the instability of the system. The signals in the variable structure law are bounded. Therefore, the adaptive control assures a global stability [4].



**Figure 8.** The block diagram of the augmented error determination  $e_c$ .

## 4. Numerical simulation results

Taken into account the dc machine from the conventional control, operating at the constant flux, a speed cycle is applied in order to test the compound adaptive control. The speed cycle contains the dynamic regimes (starting, braking, reversing) and the steady state regime (**Figures 9–22**).

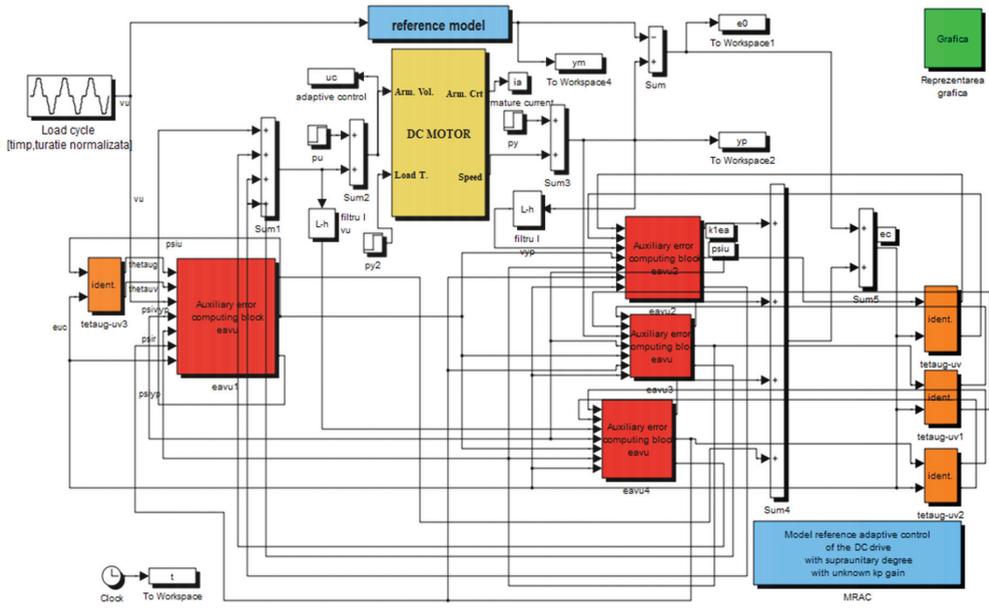


Figure 9. The Simulink implementation of the compound adaptive dc drive with supraunitary degree and with unknown gain.

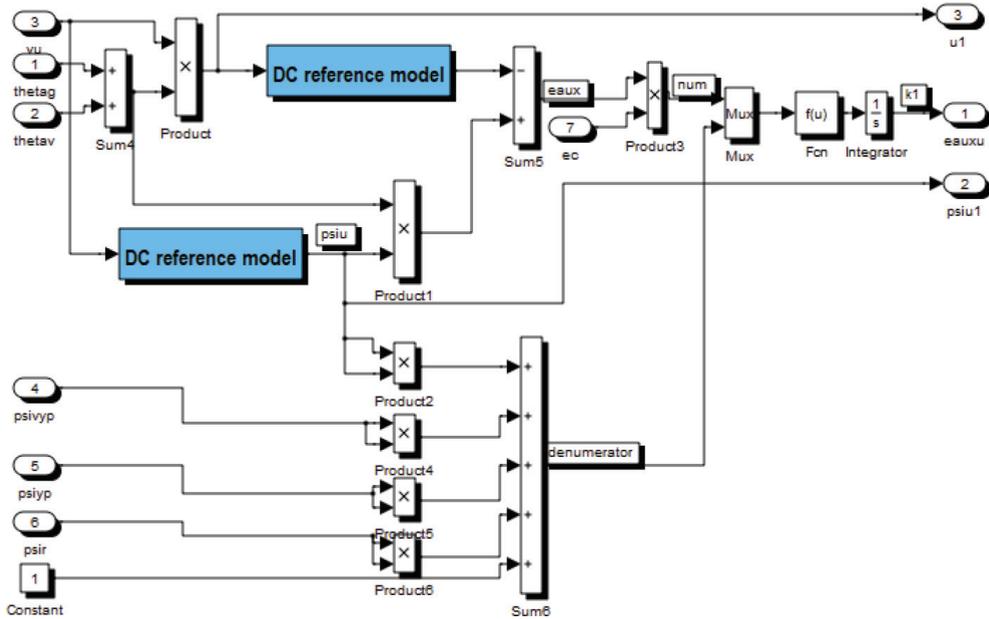


Figure 10. The Simulink implementation of the augmented error deduction  $ea_{vu}$ .

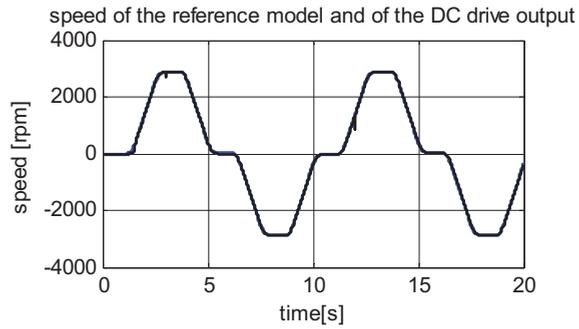


Figure 11. The speed cycle. At  $t = 12$  s, the rated load torque is applied.

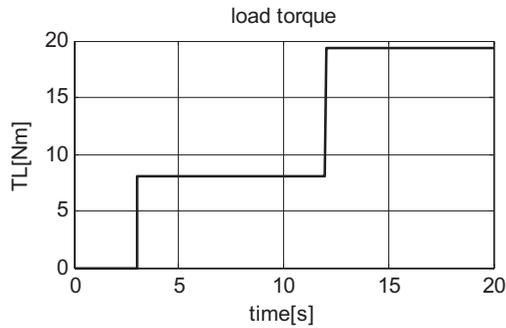


Figure 12. The load torque. At  $t = 12$  s, the rated load torque is applied.

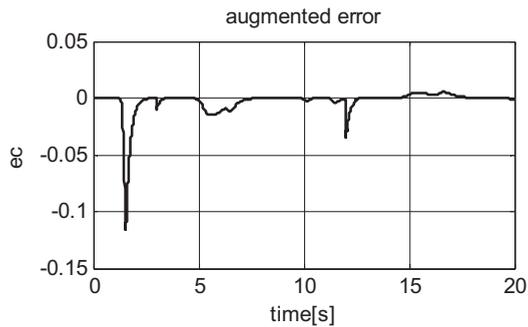


Figure 13. The augmented error. At  $t = 12$  s, the rated load torque is applied.

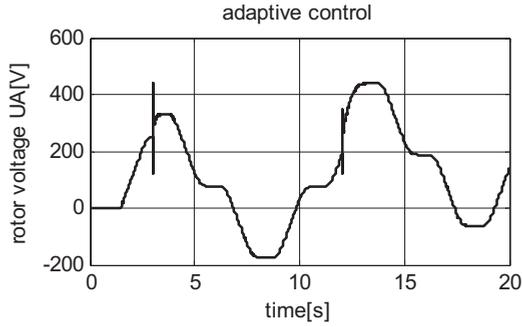


Figure 14. The adaptive control. At  $t = 12$  s, the rated load torque is applied.

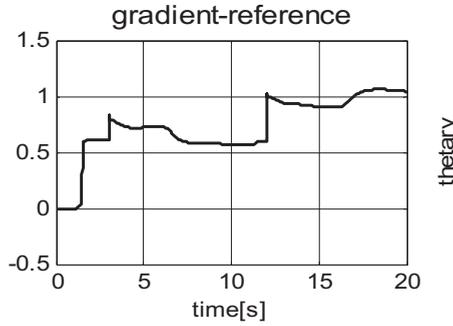


Figure 15. The adaptive mechanism of the parameter vector  $\theta$  obtained by using the compound structure:  $\theta_{rv}$  gradient reference.

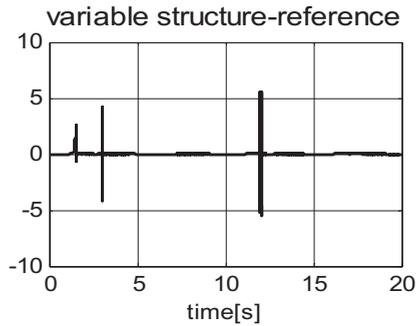


Figure 16. The adaptive mechanism of the parameter vector  $\theta$  obtained by using the compound structure:  $\theta_{rv}$  variable structure.

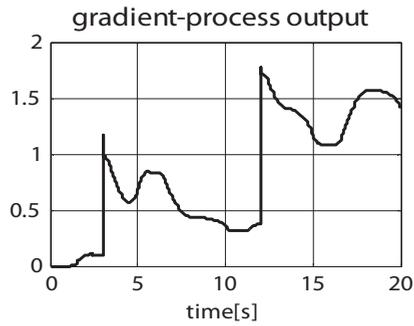


Figure 17. The adaptive mechanism of the parameter vector  $\theta$  obtained by using the compound structure:  $\theta_{yg}$  gradient-process output.

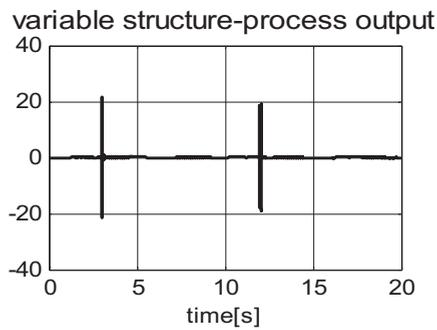


Figure 18. The adaptive mechanism of the parameter vector  $\theta$  obtained by using the compound structure:  $\theta_{vs}$  variable structure process output.

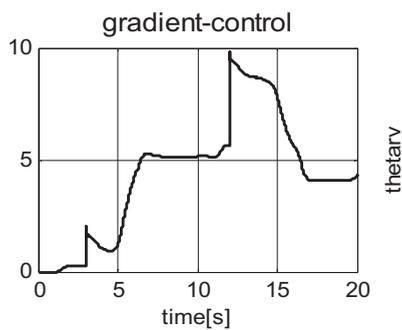


Figure 19. The adaptive mechanism of the parameter vector  $\theta$  obtained by using the compound structure:  $\theta_{ug}$  gradient control.

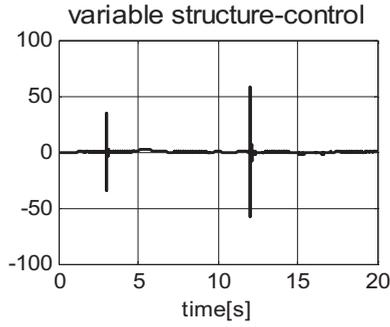


Figure 20. The adaptive mechanism of the parameter vector  $\theta$  obtained by using the compound structure:  $\theta_{us}$  variable structure control.

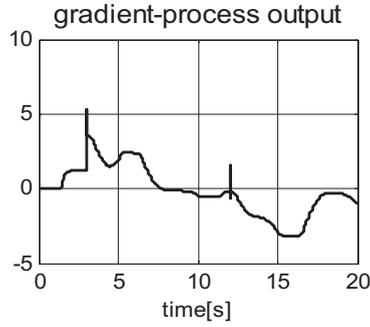


Figure 21. The adaptive mechanism of the parameter vector  $\theta$  obtained by using the compound structure:  $\theta_{pg}$  gradient-process output.

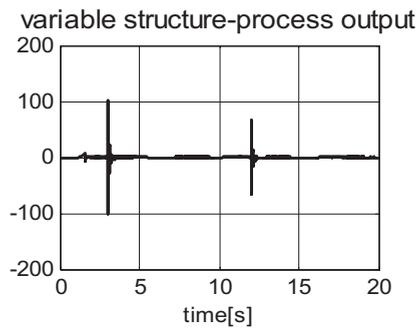


Figure 22. The adaptive mechanism of the parameter vector  $\theta$  obtained by using the compound structure:  $\theta_{ps}$  variable structure process output.

## 5. Conclusions

The conventional control and the compound adaptive control have been investigated by using a dc drive system. The complete methodology of tuning the controller parameters for the conventional control is provided. Under the assumptions mentioned in this chapter, the dc drive system has been implemented in Matlab-Simulink software. The adequate numerical simulation results have been obtained (**Figure 6**) for. Therefore, the regulation capability of the PI controller is tested under a variation of the load torque for a starting. Both, the dynamic and steady state regimes are investigated. In **Figure 6b**, the maximum load torque of the dc motor is used for a starting under the 70% load conditions. The maximum limit is maintained during the dynamic regime, the torque decreases in steady state due to the armature current decreasing. The constant parameter values have been considered. In case of the gradient and variable structure laws, the adaptive system is more robust to parameter uncertain or to unmodelled dynamics of the dc drive, and the increased regulation performances are obtained. The supraunitary relative degree model reference adaptive control has been considered.

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