4

Closed-Loop Feedback Systems in Automation and Robotics, Adaptive and Partial Stabilization

G. R. Rokni Lamooki
Center of Excellence in Biomathematics, Faculty of mathematics statistics and Computer Science, College of Science, University of Tehran
Iran

1. Introduction

Feedback controls have applications in various fields including engineering, mechanics, biomathematics, and mathematical economics; see (Ogata, 1970), (de Queiroz, et al. 2000), (Murray, 2002), and (Seierstad & Sydsaeter, 1987) for more details. Lyapunov based control of mechanical system is a well-known technique. This includes Lyapunov direct/indirect methods. Such techniques can be employed to control the whole state variables or a part of the state variables. Sometimes there are some uncertainties or some reference trajectories which requires adaptive control. Back-stepping is a yet powerful approach to design the required controller. However, this approach leads to a complicated controller, especially when the chain of integrators is long. Back-stepping can also be used when the aim of control is the stability with respect to a part of the variables. These three concepts emerge in a mechanical system like a robot. Adaptive control can be carried out through two different approaches: indirect and direct adaptive control. Nevertheless there are some drawbacks in such control systems which are a matter of concern. For example, when there is the possibility of fault or it is considered to turn off the adaptation for saving energy, when the system seems to be relaxed at its equilibrium situation, the outcome can be dramatically destructive. Adaptively controlled systems with unknown parameters exhibit partial stability phenomenon when the persistence of excitation is not assumed to be satisfied by the designed controllers. Partial stability technique is most useful when a fully stabilized system losses some control engine or some phase variables are not actively controlled. Such situation is most applicable for automatic systems which need to work remotely without a proper access to maintenance; e.g., satellite, robots to work on other planets or under hard conditions which are required to continue their mission even if some fault happens, or when a minimum of controller is required. It is also applicable to biped robots when one of the engines is turned off, or weakened, for lack of energy or fault or when the robot is passively designed. It is worth noting that another useful aspect of partial stability and control is the possibility of controlling the required part of the phase variables without spending energy to control the part of the variables which is not relevant to the mission of the designed system. These concepts will be explained through some examples. The results will be illustrated by numerical computations. This chapter is organized as follows. In section 2 the notion of stability and partial stability will be briefly discussed. In section 3 the adaptive back stepping design will be introduced with two examples of fully stabilized and partially stabilized systems. The notion of single-wedge bifurcation will be discussed. In section 4, the question is: whether in mechanical system single-wedge bifurcation is likely to appear or not? If so, what sort of instability may occur when such bifurcation takes place? In this section an example of a simple mechanical system with unknown parameter will be studied. This mechanical system is a pendulum with one unknown parameter. The reason of considering such simple system is to emphasize that such undesirable situation is more likely to take place in more complicated mechanical systems when that is possible in a simple case. In section 5 a robot will be studied where only one of the phase variables is actively controlled while there are a reference trajectory and some unknown parameters. This falls into the category of adaptive stabilization with respect to a part of the variables. Such technique does not always leads to the objective of the control. We would like to see that how the geometric boundedness of the system can lead to a successful design.

2. Stability and partial stability

Consider the differential equation

$$\dot{x} = f(x). \tag{1}$$

For any initial value x_0 the solution $\phi_t(x_0) = x(t, x_0)$ is called the flow of the system (1). The point x^* is called an equilibrium for (1) if $\phi_t(x^*) = x^*$ for all $t \geq 0$. Such points satisfy $f(x^*) = 0$. Suppose that the vector field f is complete so that the solutions exist for all time. We call x^* an asymptotic stable equilibrium if for any neighborhood U around x^* there is another neighborhood V such that all solutions starting in V are bounded by U and converge to x^* asymptotically. In order to check the stability, one needs to resort different techniques. Lyapunov has developed important techniques for the problem of stability, so-called *direct* and *indirect* methods. Lyapunov indirect method basically guarantees local stability of the nonlinear system. Here, the eigenvalues of the linearization of the system, about the equilibrium x^* are examined. If all of them have negative real parts then the linearized system is globally stable. However, the original nonlinear system is typically stable only for small perturbations of initial conditions around the equilibrium. The set of admissible initial perturbations is usually a difficult task to determine. On the other hand, Lyapunov direct method examines the vector field directly. It is based on the existence of a so-called Lyapunov function, a positive-definite function defined in a neighborhood of the equilibrium x^*, with a negative-definite time derivative. This guarantees the stability of the system in a neighborhood of x^*.

The case where the Lyapunov function is not negative-definite, but just negative can only guarantees the stability, but not asymptotic stability. However, through some invariant properties we can have asymptotic stability too. This is formulated in La' Salle invariant principle (Khalil, 1996).

Now, we consider the system

$$\dot{x} = f(x, w), \qquad x = (y, z) \in R^{p+q}, \qquad w \in R^s, p + q = n. \tag{2}$$

Here, $f(0,0) = 0$, x is the state and $w = w(x)$ is the feedback controller such that $w(0) = 0$. The vector field f is considered smooth. In the standard Lyapunov based stabilization with respect to all variables $x = (y, z)$ around the equilibrium, lets say $x = 0$, we choose a control $w(x)$ such that there exists a positive-definite Lyapunov function with a negative-definite time derivative in a domain around the equilibrium, which then guarantees the asymptotic stability of $x = 0$. In the problem of stabilization with respect to a part of the variables the notion of $y-$positive-definite Rumyantsev function (Rumyantsev, 1957) plays a key role. The domain of a Rumyantsev function is a cylinder

$$D = \{(y, z) \mid \qquad \| y \| \leq H, \qquad \| z \| \leq \infty \ \}, \tag{3}$$

for some $H > 0$.

Definition: The function $V : D \to R$ is called a $y-$positive definite Rumyantsev function if there exists a continuous function $W(y)$ with $W(0) = 0$ which is positive in cylinder (2) so that $V(y, z) \geq W(y)$ for all $(y, z) \in D$.

Definition: The system $\dot{x} = f(x, w(x))$ is called $y-$stable or stable with respect to y if for any $\varepsilon > 0$ there exists $\delta > 0$ such that for all initial conditions x_0 with $\| x_0 \| < \delta$ the solution $y(t)$ satisfies $\| y(t) \| < \varepsilon$. The system $\dot{x} = f(x, w(x))$ is called asymptotically $y-$stable or asymptotically stable with respect to y if, in addition, there exists a number $\Delta > 0$ such that for all initial condition x_0 with $\| x_0 \| < \Delta$ the solution $y(t)$ satisfies $\lim_{t\to\infty} y(t) = 0$.

There are several approaches towards analyzing the partial stability. These approaches are given by (Rumyantsev, 1957); (Rumyantsev, 1970); and (Rumyantsev & Oziraner, 1987); see also (Vorotnikov, 1998).

There are two major directions to prove asymptotic $y-$stability: the method of sign-definite time derivative Rumyantsev function and the method of sign-constant time derivative Rumyantsev function. The former requires a Rumyantsev function with a $y-$negative-definite time-derivative, whereas the later considers a Rumyantsev function with a $y-$negative time-derivative. For simplicity, we refer to these methods by terms sign-definite and sign-constant method respectively. See (Rumyantsev, 1957), (Rumyantsev, 1970) and (Vorotnikov, 1998) for more details. The method of the sign-constant is based on two concepts of the boundedness and precompactness; see (Andreev, 1991), (Andreev, 1987) and (Oziraner, 1973).

3. Adaptive back-stepping design

Consider the following system with one fixed unknown parameter

$$\begin{cases} \dot{x} = f_1(x, y, \theta^*), \\ \dot{y} = f_2(x, y, u). \end{cases} \tag{4}$$

Assume $f_1(0,0,\theta)=0$ for all θ. Adaptive back-stepping has two steps. First a feedback $y=\kappa(x,\hat{\theta})$ is designed with $\kappa(0,\hat{\theta})=0$ for all $\hat{\theta}$, using an estimation $\hat{\theta}$ for the unknown parameter θ^*. The estimation $\hat{\theta}$ is updated according to the adaptation $\dot{\hat{\theta}}=G(x,\theta)$ such that the $x-$equation is stabilized. In the next step we need to specify the actual controller u and parameter adaptation so that $\varsigma(t)=y(t)-\kappa(x(t),\hat{\theta}(t))$ and $x(t)$ converge to zero as time goes to infinity. As an example, consider the system

$$\begin{cases} \dot{x}=y+\theta^*\varphi(x), \\ \dot{y}=u. \end{cases} \tag{5}$$

Here, $x,y\in R$ are state variables, u is the controller and $\theta^*\in R$ is the unknown parameter. Suppose ϕ is smooth and $\phi(0)=0$. Using the back-stepping technique, one can construct the following controller and parameter adaptation.

$$\begin{cases} u=-\nu(\varsigma)-x-\left(\mu'(x)+\phi'(x)\hat{\phi}\right)\left(-\mu(x)+\varsigma\right)-\phi(x)\hat{\theta}, \\ \dot{\hat{\theta}}=\phi(x)\left[x+\varsigma\left(\mu'(x)+\phi'(x)\hat{\theta}\right)\right], \end{cases} \tag{6}$$

to achieve the following closed-loop system.

$$\begin{cases} \dot{x}=-\mu(x)+\varsigma+\tilde{\theta}\phi(x), \\ \dot{\varsigma}=-x-\nu(\varsigma)+\left(\mu'(x)+\phi'(x)(\theta^*-\tilde{\theta}\right)\tilde{\theta}\phi(x), \\ \dot{\tilde{\theta}}=-\phi(x)\left(x+\varsigma\left(\mu'(x)+\phi'(x)(\theta^*-\tilde{\theta}\right)\right). \end{cases} \tag{7}$$

Here, $\tilde{\theta}=\theta-\hat{\theta}$ is the error of estimation. One can observe that in such system $\tilde{\theta}$ is bounded and indeed converges to some fixed value depends on initial cinditions. This fixed value defines a non-adaptive controlle so called limit controller which is accordingly corresponding to a non-adaptive closed system so called limit system. Surprisingly, such limit system is not guaranteed to be stabilized. Sometimes such limit system attracts a large subset of all initial conditions. The occurrence of this situation is called single-wedge bifurcation. The term single-wedge reffers to the fact that the shape of all initial conditions absorbed to such destabilized non-adaptive limit systems looks like a wedge. The system (7), dramatically undergoes a singl-wedge bifurcation; that is a transcritical bifurcation corresponding to a destabilized limit system, possibly with finite escape time, and with a large basin of attraction; see (Townley, 1999) and (Rokni, et al. 2003) for more details on this issue and derivation of (6)-(7). The problem is not merely about the destabilizing limit system, that is also about the finite escape time.
Now, we focus on the system

$$\begin{cases} \dot{x}=f\ (x,w,\theta^*), \\ \dot{w}=h\ (x,w,u), \end{cases} \quad x=(y,z)\in R^{p+q}, \qquad w\in R^s, p+q=n. \tag{8}$$

Here x, w are the phase variables, θ^* is a vector of unknown parameters, and $u \in R^m$ is the controller. Suppose $f(0,0,\theta) = 0, h(0,0,0) = 0$ for all θ. The aim is to design a controller u such that the closed-loop system is stabilized with respect to y while other variables including parameter adaptation stay bounded. We use the back-stepping design, but at each step we only aim to stabilize y. We use the partial stability approach described in section 2 to design a controller u together with a $y-$positive definite function V with $y-$negative-definite $\dot{V}$. In case of sign constant $\dot{V}$, we also need the boundedness property of non-stabilized variables. Consider the following example.

$$\begin{cases} x = \begin{bmatrix} y & z \end{bmatrix}^T \in R^2, \\ \dot{y} = bw + \theta^* \phi^1(y,z), \\ \dot{z} = cw + \theta^* \phi^2(y,z) \\ \dot{w} = u. \end{cases} . \tag{9}$$

Suppose ϕ is smooth and $\phi(0,0) = 0$. The adaptive partial stabilization of this system has two stages. First we stabilize the $x-$equation with respect to y by assuming that w is the controller. At this stage we can define $w = \kappa(x, \hat{\theta}) = -b^{-1}(\hat{\theta}\phi^1 + h(y))$ where $\hat{\theta}$ is the estimation for θ. Here h satisfies $yh(y) > 0$. Next, we stabilize two variables $\varsigma = w - \kappa(x, \hat{\theta})$ and y using a suitable controller u. This leads to

$$\begin{cases} u = -\left[by + b^{-1}\,\phi^1\hat{\theta} + b^{-1}\hat{\theta}\left(\frac{\partial \phi^1}{\partial y} + h' \right)(b\varsigma - h(y)) \right] \\ \qquad - \left[b^{-1}\hat{\theta} \frac{\partial \phi^1}{\partial z}\left(c\varsigma - cb^{-1}\hat{\theta}\phi^1 - cb^{-1}h(y) + \hat{\theta}\phi^2 \right) \right] - \mu(\varsigma), \\ \dot{\hat{\theta}} = \varsigma\left(b^{-1}\hat{\theta}\left(\frac{\partial \phi^1}{\partial y} + h' \right)\phi^1 + b^{-1}\hat{\theta}\frac{\partial \phi^1}{\partial z}\phi^2 \right) + y\phi^1. \end{cases} . \tag{10}$$

Here, μ is another function satisfying $\varsigma\mu(\varsigma) > 0$. It can be shown that under some mild conditions on ϕ, in this closed-loop system, the error of parameter estimation $\tilde{\theta} = \theta - \hat{\theta}$ converges to some value depending on initial conditions. The variable w converges to zero and z stay bounded. This system exhibits destabilized limit systems, but no single-wedge type behavior.

Partial stability phenomena frequently appear in mechanical systems, for example, in rotating bodies. One classical example is Euler's equations for tumbling box when one or more controller is omitted. Another well-known case of partially stabilized systems is adaptively controlled systems without persistence of excitation. Sometimes the system capability requires partial stabilization and sometimes the control strategy implies that. In mathematical model of certain biological systems of $n-$ spices a chain of integrators appears with the controller located at the last integrator; see (Murray, 2002). Such systems are referred to as strict feedback form and are locally asymptotically stabilizable about the nominal equilibrium via a recursive design. Such controller is usually very complicated and contains many unnecessary cancellations; see (Krstić, et al. 1995) for some techniques for avoiding unnecessary cancellations. However, it might not be necessary to stabilize all the spices. If that is required, or enough, to fully control a part of these spices while the other stay bounded, then the designed controller will be simpler and more economic. In these types of systems, unknown parameters are likely to appear. Therefore, that is vital to study the possibility of single-wedge bifurcation to avoid destabilizing when the adaptation turns off. In this chapter we focus on mechanical cases, but the method can be applied to other fields too.

4. Simple pendulum

A simple pendulum with fixed given length and mass can be represented by

$$\ddot{\phi} + \alpha\dot{\phi} - k\sin\phi = u, \tag{11}$$

Here, ϕ is the angle between the rod and the vertical axis, and $\alpha > 0$ represents the friction. The pendulum is inverted when $k > 0$ and is not inverted when $k < 0$. We assume $k \in R$ to cover both situations. The absolute value of k is proportional to the gravitation constant which is assumed to be fixed but unknown. The aim is to design an adaptive controller which works for any value of k. Note that the case $k = 0$, no gravity, is not generic. The purpose of the control is $(\phi, \dot{\phi}) \to 0$ asymptotically. The focus is the possibility of single-wedge bifurcation. Suppose that there is no friction; that is $\alpha = 0$. Suppose $\hat{k}$ is the estimation of k and $\tilde{k} = k - \hat{k}$ is the error of the estimation. Through a recursive back-stepping design we can find an adaptive controller with a tuning function for parameter adaptation. We denote $x = \phi$ and $y = \dot{\phi}$. Then, the equation (11) becomes

$$\begin{cases} \dot{x} = y, \\ \dot{y} = k\sin x - \alpha y + u. \end{cases} \tag{12}$$

It needs to remind that we assumed $\alpha = 0$. We use the adaptive back-stepping approach to design an adaptive controller. At first step, we consider y as the controller for $x-$equation. Using $2V_1 = x^2$ as the Lyapunov function the time derivative of V_1 is negative definite by choosing $y = -h(x)$, where h satisfies $xh(x) > 0$. Then, we apply the change of variable $\varsigma = y + h(x)$. In the new system of coordinate, the equation (12) becomes

$$\begin{cases} \dot{x} = \varsigma - h(x), \\ \dot{\varsigma} = (\hat{k} + \tilde{k})\sin x + (\varsigma - h(x))h'(x)) + u. \end{cases} \tag{13}$$

Now, we propose the Lyapunov function $2V = x^2 + \varsigma^2 + \tilde{k}^2$. The time derivative of V is

$$\dot{V} = -xh(x) + \varsigma\left[x + \hat{k}\sin x + \varsigma h'(x) - h(x)h'(x) + u\right] + \tilde{k}\left[\varsigma\sin x - \dot{\hat{k}}\right]. \tag{14}$$

We choose

$$\begin{cases} u = -\mu(\varsigma) - x - \hat{k}\sin x - \varsigma h'(x) + h(x)h'(x), \\ \dot{\hat{k}} = \varsigma \sin x. \end{cases} \tag{15}$$

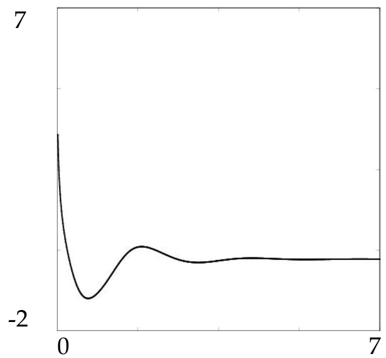
Here, μ is a function satisfying $\varsigma\mu(\varsigma) > 0$, then

$$\dot{V} = -xh(x) - \varsigma\mu(\varsigma). \tag{16}$$

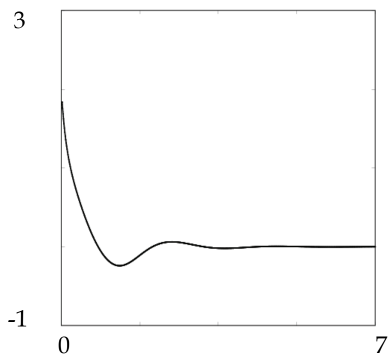
The three-dimensional auxiliary closed-loop system is

$$\begin{cases} \dot{x} = \varsigma - h(x), \\ \dot{\varsigma} = \tilde{k}\sin x - x - \mu(\varsigma), \\ \dot{\tilde{k}} = -\varsigma \sin x. \end{cases} \tag{17}$$

The closed-loop system (17) is partially asymptotically stabilized with respect to (x,ς) . To see this, one can observe that the auxiliary closed loop system (17) is $\tilde{k}-$bounded. This boundedness property together with the fact that V is $(x,\varsigma,\tilde{k})-$positive definite while $\dot{V}$ is sign constant results the required partial stability. Therefore, the origin of the actual closed-loop system (11) and (15) is partially asymptotically stabilized with respect to $(\phi,\dot{\phi})$ regardless the actual value of k and its initial condition. This stabilization is global. In Fig. 1 $x(t)$ and $\varsigma(t)$ are drawn for $h(x) = x + x^2 + x^3$ and $\mu(\varsigma) = \varsigma + \varsigma^2 + \varsigma^3$ for initial condition $(x,\varsigma,\tilde{k}) = (2,6,-6)$.



(a)



(b)

Fig. 1. $x(t)$ and $\varsigma(t)$ are drawn for $h(x) = x + x^2 + x^3$ and $\mu(\varsigma) = \varsigma + \varsigma^2 + \varsigma^3$ for initial condition $(x,\varsigma,\tilde{k}) = (2,6,-6)$. The horizontal axis is time. The vertical axis in (a) is $\varsigma(t)$ and in (b) is $x(t)$.

The closed-loop system (17) has a one dimensional manifold of equilibria defined by $(x,\varsigma)=0$. Every equilibrium on this manifold has one zero eigenvalue due to the degeneracy appeared in $\tilde{k}-$equation as a result of adaptive back-stepping design. The stability type of equilibria on this manifold is characterized by two more eigenvalues given by the linearization of the vector field around those equilibria. One can observe that the arbitrary equilibrium $(0,0,\tilde{k}_\infty)$ has two eigenvalues given by the polynomial

$$\lambda^2+(h_1+\mu_1)\lambda+h_1\mu_1+1-\tilde{k}_\infty=0, \tag{18}$$

where $h_1=h'(0)$ and $\mu_1=\mu'(0)$. The single-wedge bifurcation may take place when $h_1\mu_1+1-\tilde{k}_\infty=0$, and $h_1+\mu_1>0$. This later condition is always the case as long as both linear parts of h and μ are not simultaneously zero. We denote this critical equilibrium with $(0,0,\tilde{k}_c)$. We use the change of variables

$$\begin{bmatrix} x \\ \varsigma \end{bmatrix}=M^{-1}\begin{bmatrix} p \\ q \end{bmatrix}, \qquad M=\begin{bmatrix} 1 & 1 \\ -\mu_1 & -h_1 \end{bmatrix}. \tag{19}$$

Here, the column of M are the eigenvectors of the linearization matrix of the $(x,\varsigma)-$part of the vector field (17) around the critical equilibrium corresponding to the eigenvalues $\lambda_1=-h_1-\mu_1<0$ and $\lambda_2=0$ respectively. Such transformation keeps $\tilde{k}$ invariant. In order to analyze the closed-loop system (17) around its critical equilibrium, we first represent the system (17) in terms of $(p,q,\tilde{k})$ and then reduce the resultant system to the center manifold. Here, the center manifold is given by $p=H(q,\tilde{k})$. The center manifold is invariant and tangent to the linear eigenspace corresponding to the eigenvalue $\lambda_2=0$. Therefore,

$$\dot{p}=\frac{\partial H}{\partial q}\dot{q}+\frac{\partial H}{\partial \tilde{k}}\dot{\tilde{k}}. \tag{20}$$

The lengthy, but straightforward procedure of center manifold calculation leads to the following truncation of the reduced system

$$\begin{cases} \dot{q}=\beta_1 q^2+\beta_2 q\tilde{k}+qO(|q,\tilde{k}|^2), \\ \dot{\tilde{k}}=\gamma_1 q^2+qO(|q,\tilde{k}|^2). \end{cases} \tag{21}$$

Here,

$$\beta_1=\frac{\mu_1 h_2+h_1^2\mu_2}{h_1+\mu_1}, \quad \beta_2=\frac{1}{h_1+\mu_1}, \quad \gamma_1=-h_1, \tag{22}$$

where, $2h_2 = h''(0)$ and $2\mu_2 = \mu''(0)$. It can be observed that the reduced system (21) is degenerate. We utilize the singular time reparametrization (Dumortier & Roussarie, 2000); that is $t = \frac{1}{q}\tau$ to achieve the divided out system

$$\begin{cases} \dfrac{dq}{d\tau} = q' = \beta_1 q + \beta_2 \tilde{k} + O(|q,\tilde{k}|^2), \\ \dfrac{d\tilde{k}}{d\tau} = \tilde{k}' = \gamma_1 q + O(|q,\tilde{k}|^2), \end{cases} \tag{23}$$

which is generically hyperbolic around the origin. The singular time reparametrization keeps the orbits but the direction of which are reversed when $q < 0$. In order to have single-wedge bifurcation for the closed-loop system (17), it is sufficient that the origin of the system (23) become a node, either stable or unstable. The characteristic equation of the origin of the system (23) is

$$\lambda^2 - \beta_1\lambda - \beta_2\gamma_1 = 0. \tag{24}$$

The sign of the discriminant of this algebraic equation is equivalent to the sign of

$$\delta = A\mu_2^2 + Bh_2^2 + C\mu_2 h_2 - \omega, \tag{25}$$

where,

$$A = h_1^4, \quad B = \mu_1^2, \quad C = 2\mu_1 h_1^2, \quad \omega = 4h_1^2 + 4h_1\mu_1. \tag{26}$$

The origin of the divided out system (23) is a center when $\delta < 0$. This implies that the center manifold (21) has a semi-center at the origin. This causes that the critical equilibrium of the closed-loop system (17) to have a semi-center. However, when $\delta > 0$, the origin of the divided out system (23) is a node; therefore, the critical equilibrium of the closed-loop system (17) undergoes a single-wedge bifurcation. It can be observed that $\delta < 0$ is corresponding to the stripe

$$-\frac{2}{h_1}\sqrt{1+\frac{\mu_1}{h_1}} < \mu_2 + \frac{C}{2A}h_2 < \frac{2}{h_1}\sqrt{1+\frac{\mu_1}{h_1}}, \tag{27}$$

in $(\mu_2, h_2)-$ parameter space. It is worth noting that the second order derivatives of h and μ control the occurrence of the single-wedge bifurcation. When $\mu_2 = h_2 = 0$, we have $\delta < 0$ and there will be no single-wedge bifurcation. When $\|(\mu_2, h_2)\|$ is large enough, that is $|\mu_2 + C/(2A)h_2| > 2/h_1(1+\mu_1/h_1)^{0.5}$, the parameter δ become positive and single-wedge bifurcation takes place. With $\mu_2 = 0$, the critical values of h_2 are $h_2^c = \pm 2h_1/\mu_1(1+h_1/\mu_1)^{0.5}$, and for $h_2 = 0$, the critical values of μ_2 are $\mu_2^c = \pm 2/h_1(1+\mu_1/h_1)^{0.5}$.

In Fig. 2, the single-wedge bifurcation appeared in the reduced system (23) is shown for the case $h(x) = x + 2x^2 + x^3$ and $\mu(\varsigma) = \varsigma + 2\varsigma^2 + \varsigma^3$. Here $\beta_1 = 4$, $\beta_2 = 0.5$, $\gamma_1 = -1$. The wedge region is the set of all initial conditions attracting to the origin where the limit system is unstable. The black curves are orbits converging the bifurcation point. The wedge is the lower right area limited by the border, tick horizontal line and one of the orbits.

Remark 1: The single-wedge bifurcation generically appears due to the nonlinear terms in feedbacks. One might argue that by applying linear feedbacks such situation can be avoided. However, linear feedbacks are applying through devices which may introduce some amount of nonlinearities. The width of the stripe defined by (27) depends on h_1 and μ_1 and is bounded by $\eta = 4 / h_1(1 + \mu_1 / h_1)^{0.5}$. The linear coefficients h_1 and μ_1 determine the local convergence. When they are small the convergence will slow down. One can observe that if we expect similar rate of local convergence for both x and ς, then $\eta = 4\sqrt{2} / h_1$ approximately. For large enough h_1 the area where the single-wedge bifurcation does not take place will narrow down. To extend this region one needs to slow down the convergence process which may not be desirable. Another reason to consider this situation is to illustrate that how such behavior happens in a simple system. In a more complicated case, such dramatic behavior may occur generically.

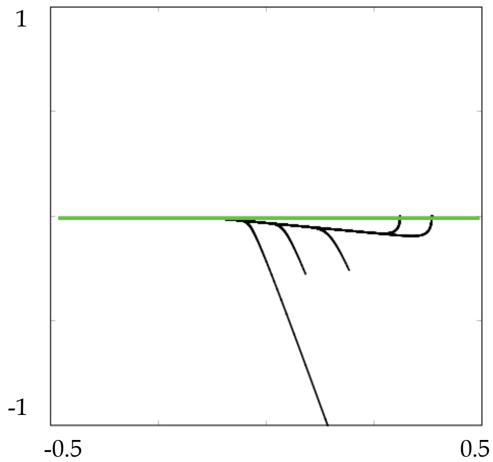


Fig. 2. The single-wedge bifurcation is shown for $h(x) = x + 2x^2 + x^3$ and $\mu(\varsigma) = \varsigma + 2\varsigma^2 + \varsigma^3$. The horizontal axis is $\tilde{k}$ and the vertical axis is q. The tick horizontal line represents the manifold of equilibria.

Remark 2: In our analysis, we assumed that $\alpha = 0$. By some algebraic calculation, it can be observed that including the term αy with $\alpha \neq 0$ will only shift the value of μ_1 by the amount of α. It can be understood from equation (21) that the limit system corresponding to $\tilde{k}_\infty > 0$ is unstable, but due to the linear part $\beta_2 \tilde{k}_\infty q$, the limit system will be only unstable and finite escape time will not arise. It suggests that the closed-loop inverted pendulum with limit controller and without parameter adaptation can stay stabilized if it will not fall into the basin of attraction of the equilibrium $(0, 0, \tilde{k}_c)$.

5. Biped robots

A passive bipedal robot with elastic elements has been studied in (Asano & Wei Luo, 2007), where a feedback control has been designed. Here we consider the same model when there is an unknown parameter. The governing equation is

$$M(q)\ddot{q} + h(q,\dot{q}) + \frac{\partial Q}{\partial q} = Su. \tag{28}$$

Here $q = \begin{bmatrix} \theta_1 & \theta_2 & b_2 \end{bmatrix}$ are the geometrical variables of the robot, $M(q)$ is the inertia matrix, $h(q,\dot{q})$ is the vector of Coriolis centrifugal and gravity forces. The elastic energy is defined as

$$Q = \frac{1}{2}k(b_2 - b_0)^2, \tag{29}$$

where, b_0 is the normal length of the leg and k is an unknown parameter; see (Asano & Wei Luo, 2007) for more details.

The vector $S = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$ requires that the walk is passive and only the elastic element is under control. We introduce the following variables

$$X = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} = \begin{bmatrix} \theta_1 & \theta_2 & b_2 \end{bmatrix}, \quad Y = \dot{X}, \quad X_0 = b_0 S. \tag{30}$$

This leads to

$$\begin{cases} \dot{X} = Y, \\ \dot{Y} = -M^{-1}h - kM^{-1}(X - X_0) + M^{-1}Su. \end{cases} \tag{31}$$

We omitted the arguments of the functions for simplicity, but all changes in variables need to be applied in functions arguments. Suppose $X_d = Sx_d$ is the reference signal. We define the error by $e = X_d - X$. The equation (31) becomes

$$\begin{cases} \dot{e} = \dot{X}_d - Y, \\ \dot{Y} = -M^{-1}h - kM^{-1}(X_d - e - X_0) + M^{-1}Su. \end{cases} \tag{32}$$

We proceed with adaptive back-stepping technique to partially stabilize the system with respect to $(e_3, \dot{e}_3)$. Suppose $|e_3|^2 \le V_1$ is an e_3- positive definite Rumyantsev function with time derivative

$$\dot{V}_1 = \frac{\partial V_1}{\partial e}\dot{e} = \frac{\partial V_1}{\partial e}(\dot{X}_d - Y). \tag{33}$$

The first step of back-stepping approach can be proceeded by considering Y as the controller of $X-$ equation. We can choose

$$Y = \dot{X}_d + \mu(e). \tag{34}$$

The time derivative of V_1 will become

$$\dot{V}_1 = -\frac{\partial V_1}{\partial e}\mu(e) = -w(e). \tag{35}$$

By choosing a suitable function μ we achieve an e_3- positive definite w. Now, we introduce an auxiliary variable

$$\varsigma = Y - (\dot{X}_d + \mu(e)). \tag{36}$$

For simplicity we take

$$V_1 = \frac{1}{2}e_3^2 \tag{37}$$

In this new coordinates we get the following auxiliary system

$$\begin{cases} \dot{e}_3 = -\varsigma_3 - \mu(e_3), \\ \dot{\varsigma}_3 = S^T\left[-M^{-1}h - kM^{-1}(X_d - e - X_0) - \ddot{X}_d - \mu'(e)\dot{e}\right] + \eta u. \end{cases} \tag{38}$$

Here, $\eta = S^T M^{-1} S$. Suppose $k = \hat{k} + \tilde{k}$, where $\hat{k}$ is the estimation of k and $\tilde{k}$ is the error of estimation. We introduce the following Rumyantsev function

$$V = V_1(e) + V_2(\varsigma) + V_3(\tilde{k}). \tag{39}$$

Without loss of generality we can take

$$V_2 = \frac{1}{2}\varsigma_3^2, \quad V_3 = \frac{1}{2}\tilde{k}^2. \tag{40}$$

The time derivative of V becomes

$$\begin{aligned} \dot{V} &= -e_3\mu(e_3) \\ &+ \varsigma_3\left[S^T[-M^{-1}h - \hat{k}M^{-1}(X_d - e - X_0) - \ddot{X}_d - \mu'\mu + \mu'\varsigma - e_3] + \eta u\right] \\ &+ \tilde{k}\left[-\dot{\hat{k}} - \varsigma_3 S^T M^{-1}(X_d - e - X_0)\right]. \end{aligned} \tag{41}$$

We choose the controller and the parameter adaptation as

$$\eta u = -\nu(\varsigma_3) \\ - S^T\left(-M^{-1}h - \hat{k}M^{-1}(X_d - e - X_0) - \ddot{X}_d - \mu'\mu + \mu'\varsigma - e_3\right), \tag{42}$$
$$\dot{\hat{k}} = -\varsigma S^T M^{-1}(X_d - e - X_0).$$

We choose a suitable function ν such that $\eta\varsigma_3\nu > 0$. These leads to

$$\dot{V} = -\frac{\partial V}{\partial e}\mu(e) - \eta\varsigma_3\nu(.\varsigma_3). \tag{43}$$

The function V is positive definite with respect to $(e_3, \varsigma_3, \tilde{k})$, but (43) states that its time derivative is negative semidefinite, because $\tilde{k}$ is not included in (43). One can observe that two angels θ_1, θ_2 are always bounded; see (Asano & Wei Luo, 2007). It is also clear that the vector field (31) is smooth. We can also assume that feedbacks are smooth. Therefore, the non-stabilized variables stay bounded. So we can construct the cylinder (3) and employ the boundedness property stated in section 2 to achieve the required partial stability.

6. Conclusion

We have seen that in relatively simple mechanical systems like a pendulum, having an unknown parameter may leads to an adaptive controller which undergoes an undesirable behaviour, dramatically. According to the questions addressed in introduction of this chapter, we have found that the destabilising limit system with a large basin of attraction does not perform a finite escape time. Instead, that will be only unstable. It is clear that when the pendulum is not inverted, we do not expect to see such situation. That is apparent from the centre manifold analysis too. It is worth noting that, lack of adaptation, does not mean that there is no control. It only means that the controller is converged to a limit controller, but the system is still closed-loop. For inverted pendulum, such non-adaptive limit controller works perfectly, as long as the system does not fall into the region of attraction of the critical limit system. This shows a drawback of back-stepping approach. There is still a question: how such situation can be overcome without further knowledge of the system?

When we design a partially stabilized system, the method of sign definite and sign constant work in two different ways. When the time derivative of Rumyantsev function is not negative definite, one would employ boundedness or precompactness. None of them can be directly applied to the system, without any further knowledge of the system's dynamics or geometry. In case of section 5, assuming that two angels are both bounded during procedure and that the vector field is smooth, we can conclude that the closed-loop system is indeed stabilized with respect to leg's length. Otherwise, such conclusion would not be straightforward. The difficulty relates to differences between the appearance of non-stabilized variables and the unknown parameters. One can assume that non-stabilized variables satisfy the precompactness property. In another assumption, one can observe that the parameter estimation stay bounded if the controller is designed properly. However, in many systems, these two sets of non-stabilized variables and parameter estimation may belong to different categories satisfying precompactness or boundedness. In the example of section 5, both stayed bounded and we achieved the aim of stabilization. However, this method has a drawback. Stabilization with respect to one variable and the boundedness of others does not guarantee that the system works properly, since they are just bounded. One would not worry about the parameter estimation as long as that is bounded and converges to some value depending on initial conditions, but the phase variables may exceed the mechanical capacity of the system. Therefore, after designing a partially adaptive controller for a system, one needs to work out on mechanical advantage and disadvantage of the closed-loop system. Such procedure is not accomplished in section 5. Another issue in controller designed by (42) is the asymptotic convergence. This is always the case when we have some unknown parameter.

7. References

Andreev, A. S. (1987) Investigation of partial asymptotic stability and instability based on the limiting equations, *J. Appl. Maths. Mechs*., 2, 51, 196--201.

Andreev, A. S. (1991). An investigation of partial asymptotic stability, *J. Appl. Maths. Mechs*., 4, 5, 429-435.

Asano, F. & Wei Luo, Zh. (2007) Parametrically excited dynamic bipedal walking, in Habib, M. K. (2007) *Bioinspiration and Robotics: Walking and Climbing Robots*, I-Tech Education and Publishing, Vienna, Austria, 1-14.

Dumortier, F. & Roussarie, R. (2001) Geometric singular perturbation theory beyond normal hyperbolicity, in Jones, C. K. R. T. and Khibnik, A. I. (editors) Multiple-time-scale dynamical systems, Springer-Verlag, New York, 29-63.

Khalil, H. K. (1996) *Nonlinear systems*, Prentice-Hall, London.

Krstić, M. & Kanellakopoulos, I. & Kokotović (1995) *Nonlinear and Adaptive Control Design*, John-Wiley & Sons, New York.

Murray, J. D. (2002) *Mathematical biology*, 3rd ed., Springer, Berlin.

Ogata, K. (1970) *Modern control engineering*, Prentice-Hall, London.

Oziraner, A. S. (1973) On asymptotic stability and instability relative to a part of variables, *J. Appl. Maths. Mechs,* 37, 659-665.

de Queiroz, M. S. & Dawson, D. M. & Nagarkatti, S. P. & Zhang, F. (2000) *Lyapunov-based control of mechanical systems*, Birkhauser, Boston.

Rokni Lamooki, G. R. & Townley, S. & Osinga, H. M. (2003) Bifurcation and limit dynamics in adaptive control systems, *Int. J. Bif. Chaos,* 15, 5, 1641-1664.

Rumyantsev, V. V. (1957) On the stability of motion with respect to the part of the variables, *Vestnik Moscow. Univ. Ser. Mat. Mech Fiz. Astron. Khim*., 4, 9-16.

Rumyantsev, V. V. (1970) On the optimal stabilization of controlled systems, *J. Appl. Maths. Mechs*, 3, 34, 440-456.

Rumyantsev, V. V. and Oziraner, A. S. (1987) *The Stability and Stabilization of Motion with Respect to some of the Variables*, Nauka, Moscow.

Seierstad, A. & Sydsaeter, K. (1987) *Optimal control theory with economic applications*, North-Holland, Netherland.

Townley, S. (1999) An example of a globally stabilizing adaptive controller with a generically destabilizing parameter estimate, *IEEE Trans. Automat. Control*, 44, 11, 2238-2241.

Vorotnikov, V. I. (1998) *Partial Stability and Control*, Birkhauser , Boston, MA.

Automation and Robotics
Edited by Juan Manuel Ramos Arreguin

ISBN 978-3-902613-41-7
Hard cover, 388 pages
Publisher I-Tech Education and Publishing
Published online 01, May, 2008
Published in print edition May, 2008