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# Failure Concepts in Fiber Reinforced Plastics

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## Abstract

The anisotropic nature of composite materials, specifically fiber reinforced plastics (FRPs), constitutes them a material category with adaptable mechanical properties, appropriate for the application they are being designed for. The stacking sequence choice of FRP laminates allows for the optimization of their strength, stiffness, and weight to the desired design requirements. The anisotropic nature of composites is also responsible for the different failure modes that they experience, which are based on the accumulation of damage, rather than crack initiation and propagation as the majority of homogeneous isotropic materials. This chapter discusses the background theory for determining the stress distribution in a laminated FRP, the possible failure modes occurring in composites, the failure criteria predicting the onset of failure, as well as cumulative damage models predicting the fatigue life of laminates.

**Keywords:** anisotropy, transverse isotropy, stacking sequence, fiber reinforced plastics (FRPs), laminates, classical lamination theory, failure criteria, cumulative damage models, fatigue

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## 1. Introduction

Lighter and more durable structures have become the focus of the majority of industries. From aviation and automotive to the battery industry, research and development sectors turn to polymer based reinforced composite materials as an alternative to metals. Fiber reinforced plastics (FRPs) are synthetic composites of epoxy resin and fibrous high strength materials. They have high strength and stiffness, are much lighter than their metal counterparts, and in the majority of cases show high resistance to corrosion [1].

FRPs offer the option of building components tailored to the desired properties for the destined applications. It is specifically FRP laminates, having unidirectional, long fiber reinforcement that can be optimized to provide the desired strength and stiffness at a low desirable

weight. As it will be shown later in this chapter, optimizing the fiber orientation in FRPs helps design and build optimum laminated structures.

Composite materials, by definition, are made of two or more constituent materials insoluble in each other. As a result, they are heterogeneous in nature. Specifically FRPs, composed of an epoxy resin matrix and a fibrous reinforcing phase, are heterogeneous and also highly anisotropic. It is imperative therefore, to account for this anisotropic nature and how it can affect the failure mechanisms of FRPs.

However, before the failure concepts in FRPs are addressed, it is imperative to define failure. What is failure, and how is it perceived? Failure is best defined as the point where a component ceases to perform adequately for the application it is designed for. Whether failure is expressed as catastrophic, similar to fracture of a metal component, or as degradation of mechanical properties of the material, for example due to creep, it is important to understand the mechanisms that can lead to any type of undesired failure and design the component against it.

This chapter will address the anisotropic and heterogeneous nature of FRPs, when it is most important to be accounted for, and how it is linked to estimating the strength and mechanical properties of FRPs. It will then discuss the different possible mechanical failure modes in composites and introduce some common criteria to predict the onset of failure. Cumulative damage is a failure concept for FRPs, and it is closely related to fatigue. The effect of some cumulative damage models will be shown.

## 2. Anisotropic heterogeneous materials

As mentioned above, composite materials are the result of the combination of two or more material phases insoluble in each other. FRPs in particular, have two phases: an epoxy resin matrix and a fiber-reinforcing phase. Each phase offers different properties and qualities to the composite, and depending on how the two phases are combined together, i.e. what volume percent of the total material is occupied by the matrix and the fibers, the properties of the composite will be different.

The matrix phase in FRPs is assumed to be a homogeneous isotropic material. However, it is the reinforcing phase that is responsible for the anisotropic nature of the composite. Fiber reinforcement may take different forms. FRPs may be reinforced by long unidirectional fibers, by long fibers woven in different ways, or short, chopped fibers scattered in the matrix. Although the failure modes discussed later in this chapter refer to all three types of FRPs presented above, the failure criteria and some examples of glass fiber reinforced plastics (GFRPs) discussed below, only concern laminated unidirectional FRPs reinforced with long fibers.

A convenient manufacturing process for FRPs is lamination. Laminated FRPs are structures composed of more than one layer of FRPs, often referred to as plies or laminae. Each lamina has a specified fiber orientation, as well as a given volumetric fraction of each phase. The way in which the laminae of different orientations are stacked on top of each other constitutes the

stacking sequence of the laminate, which determines to a large degree the mechanical properties of the laminate.

Many times in the analysis of composite components, an FRP laminate is viewed as a homogeneous, isotropic material having bulk mechanical properties. This is a practice often followed when the laminate is viewed macroscopically and attention is paid more on its geometry and loading conditions, rather than the specific effect of its properties. For example, a laminated beam may be considered as a homogenous component if its deflection is to be estimated under prescribed loading conditions, as is the case of an airplane wing. On the other hand, when investigating the strength, stiffness, and designing against failure, the anisotropic and heterogeneous nature of the FRP should be accounted for. This is when analysis moves from the macroscopic level to the lamina or even to the microscopic level, investigating each of the constituent materials, matrix and reinforcement, and their interface individually. The failure criteria presented below will focus on the lamina level.

As mentioned above, the composite material mechanical properties are different from those of the constituent materials, and depend on the volume fraction of each phase present. Rules of mixtures, is a simple model that allows calculation of the elastic properties of the FRP, from those of its constituent materials. Rules of mixtures and the Halpin-Tsai equations are shown below as a means of determining Young’s modulus ( $E_i$ ) and Poisson’s ratios ( $\nu_{ij}$ ), as bulk material properties. The Young’s modulus of the composite will vary in the three directions of the material, due to its anisotropic nature, as shown in Eqs. (1) and (2). The Shear Modulus ( $G_{ij}$ ) (Eq. (3)) is estimated by Halpin-Tsai equations, while the shear moduli  $G_{23}=G_{32}$  are estimated using Rules of Mixtures. The bulk moduli ( $K$ ) are expressed in in Eq. (5 a-c), and are used in calculating Poisson’s Ratios using Rules of Mixtures (Eqs. (6) and (7)) [2].

$$E_1 = (1 - f)E_m + fE_f \tag{1}$$

$$E_2 = E_3 = E_m \frac{(1 + \xi\eta f)}{(1 - \eta f)} \tag{2}$$

$$G_{12} = G_{21} = G_{13} = G_{31} = \frac{G_m(1 + \xi\eta f)}{(1 - \eta f)} \tag{3}$$

where the subscripts m and f, refer to matrix and fiber, respectively, and 1,2,3, to the directionality of the material. The constant f is the volume fraction of fibers in the composite such that  $0 \leq f \leq 1$ ,  $\xi \approx 1$ , and

$$\eta = \frac{\left(\frac{E_f}{E_m} - 1\right)}{\left(\frac{E_f}{E_m} + \xi\right)} \text{ or } \eta = \frac{\left(\frac{G_f}{G_m} - 1\right)}{\left(\frac{G_f}{G_m} + \xi\right)} \tag{4}$$

$$K = \left[ \frac{f}{K_f} + \frac{(1-f)}{K_m} \right]^{-1} \tag{5a}$$

$$K_f = \frac{E_f}{3(1 - 2\nu_f)} \quad (5b)$$

$$K_m = \frac{E_m}{3(1 - 2\nu_m)} \quad (5c)$$

$$\nu_{12} = (1 - f)\nu_m + f\nu_f \quad (6)$$

$$\nu_{23} = 1 - \nu_{21} - \frac{E_2}{3K} \quad (7)$$

The above equations show the effect of the constituent materials volume fractions to the stiffness of the composite.

The strength of the FRP becomes not only an important aspect in designing against failure, but also a parameter that can be optimized in order to build a lamina and consequently a laminate of high strength, desirable stiffness, and low weight. As will be shown below, the strength of the FRP can be optimized using an optimum stacking sequence, which involves laminae of different fiber orientations stacked in a specific order. Determining the strength involves therefore, not only accounting for the heterogeneity of the material as in rules of mixtures, but most importantly for the anisotropy of the material, and using this anisotropic nature to affect the properties of the FRP.

Setting up constitutive relationships for FRPs, using the generalized Hooke's Law (Eq. (8)), would require a total of 81 elastic constants in order to fully characterize the material behavior, which could be brought down to 36 constants in the case that both stresses and strains are assumed to be symmetric.

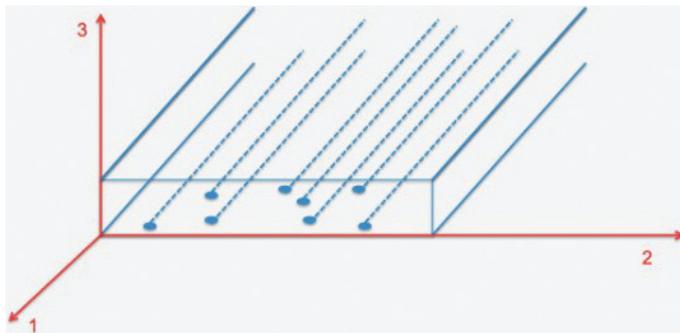
$$\sigma_{ij} = E_{ijkl} \varepsilon_{kl} \quad (8)$$

The nature of FRPs allows for manipulating their anisotropy in the lamina level to further decrease the number of elastic constants required to build constitutive relationships. At the lamina level there are two sets of axis that can be used to express the directionality of the material. One set, the global axis, refers to a reference frame of the laminate, where typically the horizontal, transverse, and vertical directions coincide with the dimensional directions of the laminate. However, each lamina, will have fibers oriented a certain way, therefore a second set of axis, referred to as local or principal axis are used, where the longitudinal direction is always parallel to the longitudinal fibers, thus making an angle with the global horizontal direction equal to that of the orientation angle of the fibers. In this manner each lamina has three mutually orthogonal axis of rotational symmetry, which reduced the number of required elastic constants to 12, where 9 of these are independent. Taking a closer look to the case of a unidirectional FRP lamina it can be observed that the lamina has two axis of symmetry (2,3) making a plane of isotropy in the material (**Figure 1**). This is because the properties of unidirectional FRPs are the same along the 2 and 3 directions, thus plane 23 becomes the isotropy plane. This becomes a special case of orthotropic materials, called transversely isotropic materials, which allows for further decrease of the independent elastic constants from 9 to 5.

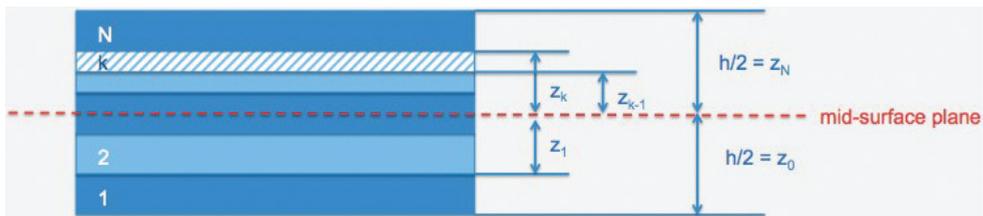
The Classical Lamination Theory (CLT), applicable only to orthotropic continuous laminated composite materials, is the set of equations that allows for the development of a constitutive relationship that determines the state of stress in each layer of a laminate [3–5].

CLT investigates lamina, first as a separately structure and then by taking into account its position within the laminate, in order to determine the stress and strain distribution in the laminate. It can therefore, be understood how stacking sequence selection is significant to the strength and performance of a laminate. Consequently, the position of each laminae in the laminate should be clearly defined. There is a specific way to number the lamina in a laminate, and following this configuration the position of the laminae is used in the CLT. Laminae numbering begins from the bottom lamina in a laminate as shown in **Figure 2**. The fictitious separation plane that goes through the mid-section of the laminate, called the mid-surface plane, serves as a datum from where the position of each lamina is determined. As a result, the position of the lamina may be positive, if the lamina is above the mid-surface plane, and negative, when it is below this plane. Such laminae numbering configuration is important and useful in understanding stacking sequence nomenclature, as well as in communicating stress concentration regions in the laminate.

Apart from elastic properties, which can be determined either experimentally or though the Rules of Mixtures and Halpin-Tsai equations, CLT requires knowledge of thermal expansion properties, estimated at each of the three directions of the FRP composite, and in many cases hygroscopic coefficients. In the case of the transversely isotropic materials, only two sets of material properties are required: one set for direction 1 and one for direction 2, which has the



**Figure 1.** Orientation of the fibers of a unidirectional composite along direction 1.



**Figure 2.** Nomenclature of laminae stacking.

same properties as direction 3. The first step of CLT is to determine stiffness matrices for each lamina. The stiffness matrices ( $Q$  and  $\bar{Q}$ ) are evaluated for each lamina and as a result, the effect of the fiber orientation is taken into account (Eq. (9)). The bar over  $Q$  shows that the fiber orientation in the matrix is other than  $0^\circ$ , constituting the lamina an off-axis lamina, meaning that it is not along the  $x$ -direction of the laminate.<sup>1</sup> The equations for the stiffness matrix element involve elastic property information, as well as transformation matrix components to apply the effects of fiber orientation. The subscript  $k$  in Eq. (9) denotes the  $k^{\text{th}}$  lamina in the laminate.

$$\bar{Q}_k = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{21} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{61} & \bar{Q}_{62} & \bar{Q}_{66} \end{bmatrix} \quad (9)$$

For the case of on-axis laminae, where the fibers are parallel to the global  $x$ -axis direction, the equations for the stiffness matrix becomes:

$$[Q]_k = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \quad (10)$$

where

$$Q_{11} = \frac{E_1}{1 - \nu_{12}\nu_{21}} \quad (11)$$

$$Q_{12} = Q_{21} = \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}} \quad (12)$$

$$Q_{22} = \frac{E_2}{1 - \nu_{12}\nu_{21}} \quad (13)$$

$$Q_{66} = G_{12} \quad (14)$$

When the fibers of the lamina make an angle with the global  $x$ -axis direction, the lamina is called off-axis, and the stiffness matrix in Eq. (9) is populated based on the following equations

$$\bar{Q}_{11} = Q_{11}\cos^4\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta\cos^2\theta + Q_{22}\sin^4\theta \quad (15)$$

$$\bar{Q}_{12} = \bar{Q}_{21} = (Q_{11} + Q_{22} - 4Q_{66})\sin^2\theta\cos^2\theta + Q_{22}(\sin^4\theta + \cos^4\theta) \quad (16)$$

$$\bar{Q}_{22} = Q_{11}\sin^4\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta\cos^2\theta + Q_{22}\cos^4\theta \quad (17)$$

$$\bar{Q}_{16} = \bar{Q}_{61} = (Q_{11} - Q_{12} - 2Q_{66})\sin\theta\cos^3\theta + (Q_{12} - Q_{22} + 2Q_{66})\cos\theta\sin^3\theta \quad (18)$$

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<sup>1</sup>It is often practice to account the global orientation of the laminate  $x,y,z$ , as coinciding with the local orientation 1,2,3 of the  $0^\circ$  fiber orientation. As a result, the  $0^\circ$  fibers, along local direction 1, are also along the  $x$ -direction of the laminate. Such laminae are called on-axis and any lamina of different orientation is termed off-axis.

$$\bar{Q}_{26} = \bar{Q}_{62} = (Q_{11} - Q_{12} - 2Q_{66}) \cos \theta \sin^3 \theta + (Q_{12} - Q_{22} + 2Q_{66}) \sin \theta \cos^3 \theta \quad (19)$$

$$\bar{Q}_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{66} (\sin^4 \theta + \cos^4 \theta) \quad (20)$$

where  $m = \cos \theta$  and  $n = \sin \theta$ .

To develop a constitutive equation for the  $k^{\text{th}}$  lamina (Eq. (21)) stress distribution in each layer is related to the strain in each layer through the stiffness matrix. The strain distribution depends on the mid-surface strains ( $\varepsilon_{ij}^o$ ) and curvatures ( $\kappa_{ij}$ ) developed in the laminate. Mid-surface strains and curvatures are laminate parameters, and therefore are the same for all laminae. They depend on the loading conditions, including the effects of thermal ( $\alpha_{ij}$ ) and hygral conditions ( $\beta_{ij}$ ) in the laminate. Therefore, thermal strains and hygral effects, which are responsible for residual strains in the laminate, induced during manufacturing and curing, should be subtracted from the mid-surface strains and curvatures.

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k = [\bar{Q}_k] \left( \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} + z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} - \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_k \Delta T - \begin{Bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{Bmatrix}_k \bar{M} \right) \quad (21)$$

In order to build the stress and strain distributions in the laminate, there are three more important matrices in CLT that need to be considered. These are: the Extension Stiffness Matrix,  $A_{ij}$ , the Extension-Bending Coupling Matrix,  $B_{ij}$ , and the Bending Stiffness Matrix,  $D_{ij}$ . These three matrices bring together the stiffness effects from each lamina, and consequently fiber orientation, always accounting for the position of each lamina in the laminate. The thickness,  $t$ , of each lamina is also an important factor in CLT and is accounted for through these three matrices. Each of these matrices determines the stress distribution of the laminate due to the different loading conditions that may be applied. Matrix A considers the tension-compression effects of longitudinal and transverse loading, matrix D considers the effects of bending moments, while matrix B couples the effects of both types of loading. Eqs. (22)–(24) refer to these three matrices, while Eq. (25), builds the relationship by calculating the normal forces and moments [6].

$$[A_{ij}] = \sum_{k=1}^n [\bar{Q}_{ij}]_k t_k \quad (22)$$

$$[B_{ij}] = \sum_{k=1}^n [\bar{Q}_{ij}]_k t_k \bar{z}_k \quad (23)$$

$$[D_{ij}] = \sum_{k=1}^n [\bar{Q}_{ij}]_k \left( t_k \bar{z}_k^2 + \frac{t_k^3}{12} \right) \quad (24)$$

$$\begin{Bmatrix} \hat{N} \\ \dots \\ \hat{M} \end{Bmatrix} = \begin{pmatrix} A & \vdots & B \\ \dots & \dots & \dots \\ B & \vdots & D \end{pmatrix} \begin{Bmatrix} \varepsilon^o \\ \dots \\ \kappa \end{Bmatrix} \quad (25)$$

<sup>2</sup> All loading conditions, including thermal and hygral effects, are accounted for in  $\hat{N}$  and  $\hat{M}$ .

### 3. How do composite materials fail?

The aforementioned concepts showed how the mechanical properties and the strength of the anisotropic material are evaluated. However, an important question to ask when designing FRP components against failure is how do composites fail.

Depending on how we evaluate the strength of a laminated composite; macroscopically or microscopically, different modes of failure will become important to examine. While homogeneous materials, as is the case of metals, fail through crack initiation and propagation, often leading to fracture, composites accumulate damage and their strength degrades slowly. Degradation of the strength is often due to different failure modes and fatigue [2, 7, 8].

It has been briefly mentioned above that it is the reinforcing phase of FRPs that possesses the higher stiffness and strength. Therefore, it is the fibers that hold the load carrying capacity of the FRP composite. The load carrying capacity of composites is hidden in their fibers. As a result, the anisotropic nature of composites not only allows them to have different strengths in different directions, but also to have their constituents failing under different loads. In addition to this, due to the different fiber orientations and laminae positions in composite materials, these composites have a great advantage in failure. All lamina will not fail under the same loading conditions. Although one or more laminae may fail, the load may still be carried by the remaining strong laminae and the composite component may still be operational [9]. The matrix serves mostly as a mean of holding these fibers together and shaping the composite component, while its contribution to the load carrying capacity of the material is minimal. As a result, failure is usually aggravated when it involves degradation of the fiber load carrying capacity.

Looking at the constituent material level and their interface, i.e. microscopically, there are three failure modes that should be examined. These are:

1. Failure of the matrix phase, which is usually realized as in most homogeneous materials through crack initiation and propagation.
2. Failure at the reinforcing phase, which is the fracture of one or more fibers of the reinforcing phase.
3. Failure at the interface of the two constituents, referred to as debonding, where the fibers detach from the matrix material.

Each of the above failure modes, although they are responsible for the degradation of the composite mechanical properties, they affect the strength and performance of the material differently. Since fibers hold the most significant part of the composite's load carrying capacity, it is fracture of the fibers that can significantly impact and impair this capacity. It is the longitudinal axis of the fiber that holds their strength, and as a result, fracture of a fiber means a discontinuity in this strength along the longitudinal direction of the fiber, and consequently a degradation of this strength in the composite. Fractured fibers cannot be replaced, and therefore, this is a failure mode that will permanently degrade the strength of the material.

Although the majority of matrix materials in FRPs are thermosetting polymers, which means that a fractured matrix or debonding may not be repairable failure, the fact that the matrix

holds an insignificant load carrying capacity, constitutes these two failure modes as lower intensity modes compared to fiber fracture.

At the laminate level, i.e. looking at the composite macroscopically, there exists another failure mode called delamination. Delamination is the separation, often also referred to as debonding, of consecutive plies. Delamination is one of the most common failure modes observed macroscopically and can significantly affect the performance of the laminated component, depending on the application the laminate is designed for. Delamination may be the result of poor manufacturing and it is for this reason that extra attention is being paid during the lamination of components to avoid contaminants entering between the layers, and also avoid the formation of voids due to air entrapment during the laminating process. Depending on the application, and polymer matrix of the FRP, delamination is a serious failure mode, but also one that may be repaired by a second curing process.

In both homogeneous and heterogeneous materials, there exist failure modes that are associated with the environment of the application the components are designed for. Different materials have different chemical properties and therefore are not always suited for all environments. Polymeric composites see many applications in highly corrosive environments [1], however, depending on the matrix or reinforcing phase they may not be suitable for applications at high temperatures, UV exposure, or high moisture content. As can be seen from the constitutive equation for FRP laminae, thermal stresses and moisture absorption will affect significantly the stresses developed in the lamina. Thermal gradients will not only affect the matrix material, but will also induce undesirable residual stresses in-between the lamina that can affect the strength of the composite, and lead to delamination or matrix fracture [6]. Similarly the moisture absorbing capacity of fibers is something that should be considered when selecting fiber-reinforcing phases. As an example, natural fibers, such as Hemp fibers, are high strength fibers investigated as a replacement to glass fibers in many applications in the automotive industry [10–12]. However, they are highly hygroscopic and their mechanical properties deteriorate faster due to moisture absorption [7, 12–16].

#### **4. Predicting failure and damage in FRPs**

The above failure modes may differ in the way that failure occurs, however, they all result in deteriorating the strength of the composite material. Examining the failure mechanism in each mode means working at the materials level, which is beyond the scope of this section. Delamination on the other hand, may be a failure mode that is often examined as a fracture mechanism, and for this reason will not be considered in the discussion that follows. However the failure result of the modes, to the extent that they affect the stress distribution in the composite, can be predicted by the following criteria.

Failure theories, or preferably failure criteria, such as the Tresca and von Mises are commonly used in ductile isotropic materials. These criteria, apart from the fact that are specific to ductile isotropic materials, they deal with parameters (stress and strain, respectively) in each direction separately [2, 6, 9]. The anisotropy of composite materials plays, as it has already been shown

above, a significant role in the way that strength is built in the composite, and how failure can affect the composite's performance. Therefore, failure criteria should be selected based on their capacity to account for the interaction of stresses in different directions in the composite, where the material properties vary. The former type of failure criteria, as are the Tresca and von Mises, are referred to as non-interactive, because they account for each direction separately, while the latter, most suitable for composite materials, are referred to as interactive failure criteria. There exist various failure criteria that have been developed to address anisotropic materials, and specifically the case of FRPs. These criteria, although they account for the interaction of stresses, and consequently properties in different directions, cannot give an exact estimate or prediction for the stress conditions to cause failure. These criteria account for just the interaction of stresses irrespective of the failure mode or any other conditions (environmental, thermal, etc.), which may be responsible for composite failure. As a result, they are used as an estimate for the conditions at the onset of failure.

The majority of failure criteria are polynomial expansions, treating the stress tensor ( $\sigma_{ij}$ ) as a means of characterizing the onset of failure in a composite material. As mentioned above, failure criteria can only give approximate estimates of the onset of failure, and have been developed based on comparisons to experimental data. The polynomial expansion is tailored to the case of transversely isotropic materials, which reduces significantly the number of material parameters required [2].

The criteria that will be discussed below compare the stress state in each lamina to failure stress under plane stress conditions and determine whether the lamina has failed or not. They are therefore, observing the composite material at the lamina level, and not down to the constituent material and interface level.

*Tsai-Hill Failure Criterion:*

$$\frac{\sigma_1^2}{X^2} - \frac{\sigma_1\sigma_2}{X^2} + \frac{\sigma_2^2}{Y^2} + \frac{\sigma_{12}^2}{S^2} \leq 1 \quad (26)$$

The Tsai-Hill criterion in Eq. (26) compares longitudinal ( $\sigma_1$ ), transverse ( $\sigma_2$ ), and shear stresses ( $\sigma_{12}$ ) in each lamina to the ultimate longitudinal tensile and compressive ( $X$  and  $X'$ ), transverse tensile and compressive ( $Y$  and  $Y'$ ), and shear ( $S$ ) ultimate strengths. A total of 5 parameters are required, while only 3 are involved in the equation. The criterion states that when the above is equal or greater than 1 failure has occurred.

*Tsai-Wu Failure Criterion:*

$$\begin{aligned} & \left(\frac{1}{X} - \frac{1}{X'}\right)\sigma_{11} + \left(\frac{1}{Y} - \frac{1}{Y'}\right)(\sigma_{22} + \sigma_{33}) + \frac{\sigma_{11}^2}{XX'} + \frac{1}{YY'}(\sigma_{22} + \sigma_{33})^2 + 2F_{12}\sigma_{11}(\sigma_{22} + \sigma_{33}) \\ & + \frac{1}{S^2}(\sigma_{23}^2 - \sigma_{22}\sigma_{33}) + \frac{1}{S^2}(\sigma_{12}^2 + \sigma_{31}^2) \leq 1 \end{aligned} \quad (27)$$

Similar to the Tsai-Hill, the Tsai-Wu criterion investigates failure at the lamina level stating that failure occurs when the above equation is equal to 1. As can be seen in Eq. (27), there are 6

constants involving the materials parameters of tensile and compressive strengths in the longitudinal and transverse directions, as well as shear strengths. Therefore, this criterion requires a total of 7 material parameters. The Tsai-Wu criterion terms can be evaluated by the assumption of uniaxial tension and compression results, which is based on experimental data [2, 6]. The interaction parameter ( $F_{12}$ ) due to its interactive nature is often estimated from multiaxial stress data [2].

Both theories can give an estimated result as to when a lamina will fail. Tsai-Hill tends to overestimate failure, while Tsai-Wu tends to underestimate failure [17]. **Table 1** shows data using the above two failure criteria predicting the onset of failure in a 6-layer GFRP laminate under 3-point bending [17]. As can be observed from the data, the two theories for the case of the anti-symmetric laminate do not agree as to the exact magnitude of the criterion estimate, nor as to the possible first ply to fail. For this reason, more than one failure theories may be used in the design against failure of an FRP laminated component.

Lamina	Fiber Orientation	Tsai-Hill	Tsai-Wu
1	-45°	0.0539	-0.3632
2	0°	0.0989	-0.0027
3	90°	0.1289	-0.2924
4	90°	0.4688	-0.3083
5	90°	0.4688	-0.3083
6	90°	0.4688	-0.3083

**Table 1.** Example of failure prediction using Tsai-Hill and Tsai-Wu criteria in GFRP laminate.

As it was previously mentioned failure modes are often exhibited in different constituents of the composite, more specifically either the matrix or the fibers. The above example of how the Tsai-Hill and Tsai-Wu criteria over- and underestimate, respectively, the onset of failure, shows the approximate nature of the failure criteria, and the possible need to focus on the failing constituent. Of course, in the case where the fibers are significantly stiffer than the matrix, and failure is observed in the matrix, the fibers should be able to maintain the strength of the composite, while in the case of fiber fracture, the same will not hold true. Therefore, it is prudent that in situations of high anisotropy, an observation of failure in the matrix and fibers be made separately. The Christensen Criterion, also of the polynomial-expansion type, makes use of this differentiation between the two constituents, and requires 5 material parameters [2].

Christensen Failure Criterion:

Failure in the matrix

$$\left(\frac{1}{X} - \frac{1}{X'}\right)(\sigma_{22} - \sigma_{33}) + \frac{1}{YY'} \left[ (\sigma_{22} - \sigma_{33})^2 + 4\sigma_{23}^2 \right] + \frac{(\sigma_{12}^2 + \sigma_{31}^2)}{S^2} \leq 1 \quad (28)$$

Failure in the fibers

$$\left(\frac{1}{\bar{X}} - \frac{1}{\bar{X}'}\right)(\sigma_{11}) + \frac{\sigma_{11}^2}{\bar{X}\bar{X}'} - \frac{1}{4}\left(\frac{1}{\bar{X}} + \frac{1}{\bar{X}'}\right)^2 \sigma_{11}(\sigma_{22} + \sigma_{33}) \leq 1 \quad (29)$$

The above criteria, similar to all failure criteria developed for FRP composites, deal with the onset of failure in the lamina level. However, when designing against failure in a laminated FRP component, it is always useful to know under what conditions the component may fail and not which lamina will fail first. The advantage of composite materials is that they fail due to damage accumulation, which may be more delayed than catastrophic failure in an isotropic material, such as metal, due to cracking or yielding. The anisotropic nature of composites, with the fact that fibers hold the load carrying capacity of the composite material, allow the structure to continue performing under given conditions, although the properties in one or more laminae are being or have been degraded. Predicting when a composite structure will fail, meaning when all laminae have failed under one or more of the above criteria, is useful information and progressive damage is the process to follow. Progressive damage works by applying the above criteria of failure to determine which lamina will fail first, what we refer to as first-ply failure, and then continue to determine the next lamina, up to the last one. Every time a lamina fails, the properties of the composite are degraded and the failure of the remaining lamina is evaluated under new conditions of stress distribution and loading [2, 9].

Composite materials have been recognized as an optimum alternative to metals and have been replacing metals in many industries. For example, the automotive industry in an effort to decrease carriage weight, and consequently fuel emissions, has introduced composite suspensions. There are therefore, applications for composites that involve cyclic loading, which leads to fatigue failure. The above criteria may be helpful in estimating and approximating predictions of the stresses to initiate failure in a composite lamina, and together with progressive damage predict when the whole laminate will fail. However, the fatigue behavior of an FRP laminate should also be considered.

Cumulative Damage Theory calculates the damage caused during cyclic loading, as well as its accumulation during cyclic loading under various stress amplitudes [18]. There are two options when considering the concept of cumulative damage: a) calculating the residual strength of the component, being the instantaneous static strength maintained by the material after loading to stress levels that can cause damage, and b) estimating the damage accumulated in the material, using damage models [2].

As it has been shown above, for a composite to fail damage should be accumulated in each lamina and start degrading the lamina, and consequently laminate strength. This is also what happens during cyclic loading, where the strength of the material starts to decrease at a low rate in the early fatigue life, and at a faster rate close to the end, leading to possible failure [19].

Cumulative damage models do not focus on material data, rather on the loading conditions of the component in question. As a result, they predict damage accumulation in a general sense. Therefore, to decide upon one or more appropriate models to predict the fatigue life of a composite material, attention should be paid not to the material properties but to the type of stresses that cause failure in composites. Because of the stiff reinforcing phase holding the load

carrying capacity of the lamina, it is higher stresses that will more likely degrade the strength of the material, than lower ones. As a result, cumulative damage models should focus on low cycle fatigue (LCF) where stresses are higher [20].

Three cumulative damage models are compared below for the case of GFRP laminates, reinforced with E-Glass fibers. This is a common FRP material considered in automotive applications as a substitute for steel. The three models require information on the number of cycles ( $n_i$ ) under an applied stress ( $\sigma_i$  and  $\sigma_k$ ), the number of cycles to failure ( $N_i$ ) under this same applied stress, the ultimate strength of the material ( $\sigma_{ultimate}$ ), the ratio of the applied stress to the ultimate material strength ( $S_k$ ), and the number of repetitions of the loading cycle ( $C$ ). Cumulative damage models denote failure, when the model equations equal to 1 [8, 18, 19, 21]. As a result, under the assumption that the material accumulates 100% damage to the full extent of its fatigue life, these models may be used to estimate the fatigue life, in cycles to failure ( $N_i$ ), by setting the models to equal 1.

*Palmgren-Miner:*

$$\left( \sum_{i=1}^n \frac{n_i}{N_i} \right) C = 1 \tag{30}$$

The Palmgren-Miner cumulative damage model is maybe one of the simplest and most commonly used models in metal fatigue. The model defines the damage accumulated in the material in the form of life fractions. Each fraction is a percentage of composite life consumed during the cyclic loading application [18, 22]. When the sum of all fractions equals 1, there is no remaining residual life to be expended, and the material is assumed to have failed. The Palmgren-Miner model does not account for the loading sequence in the case of different applied stress amplitudes.

*Broutman-Sahu:*

$$\left( \sum_{i=1}^n \frac{(\sigma_{ultimate} - \sigma_i)}{(\sigma_{ultimate} - \sigma_{i+1})} \frac{n_i}{N_i} \right) C = 1 \tag{31}$$

*Hashin-Rotem:*

$$\left( \sum_{k=1}^{i-1} \left( \frac{n(k-1)}{N(k-1)} \right)^{\frac{1-S_k}{1-S_{k-1}}} + \frac{n_i}{N_i} \right) C = 1 \tag{32}$$

$$S_k = \frac{\sigma_k}{\sigma_{Ultimate}} \tag{32a}$$

$$S_{k-1} = \frac{\sigma_{k-1}}{\sigma_{Ultimate}} \tag{32b}$$

Although the Palmgren-Miner and Hashin-Rotem have been initially designed as cumulative damage models for metals, they have been both used for FRP, and more specifically GFRP fatigue life predictions. Hashin-Rotem is designed as a two-stress level loading damage model,

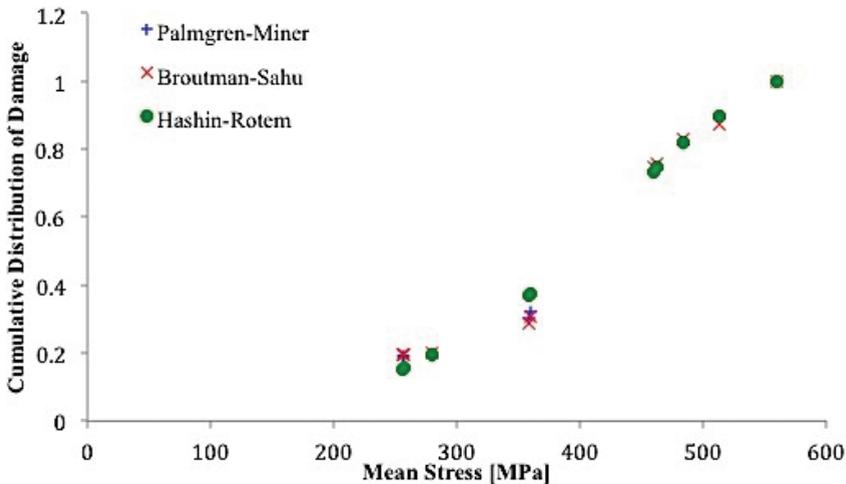
and can be expanded to multi-stress level loading using damage curve families that represent residual lifetimes and considering that equivalent residual lives are expended by components undergoing different loading schemes [18]. The Broutman-Sahu model was developed and tested on GFRP laminates.

The linearity and non-linearity of the above models is important and is determined based on the required parameters for their calculation [8]. As a result, Palmgren-Miner is a linear stress-independent model, Broutman-Sahu a linear stress-dependent model, and Hashin-Rotem a non-linear stress-dependent model.

Similar to the failure criteria, these models give an approximation of the accumulated damage during cyclic loading, and consequently when used to estimate cycles to failure, the fatigue life of the laminate. To evaluate the applicability of these models to composite materials, and choose those that predict the fatigue life of a GFRP laminate more accurately, calculations of the above models were compared to fatigue life experiments [23–24]. In order to estimate the probability to failure for the GFRP laminate using each model a standard two-parameter Weibull analysis was followed.

**Figure 3**, shows comparison of the three models for an E-glass GFRP beam cycled between 256 and 560 MPa. All three models give similar results for the cumulative distribution of damage in the GFRP laminate. Specifically, the damage predicted by the Palmgren-Miner and Broutman-Sahu models is almost identical, and the two linear models show a constant probability of failure of 19%, at low mean stresses up to 280 MPa. All three models predict complete failure at a mean stress of 560 MPa. This mean stress level corresponds to a maximum stress of 1.1 GPa, which exceeds the ultimate strength of the material (**Figure 4**) [23–24].

Comparing the results of the three models to experimental fatigue data, it can be concluded that similar to the failure criteria, cumulative damage models can be used to approximately



**Figure 3.** Cumulative distribution of damage vs. mean stress in E-glass fiber/epoxy laminate.

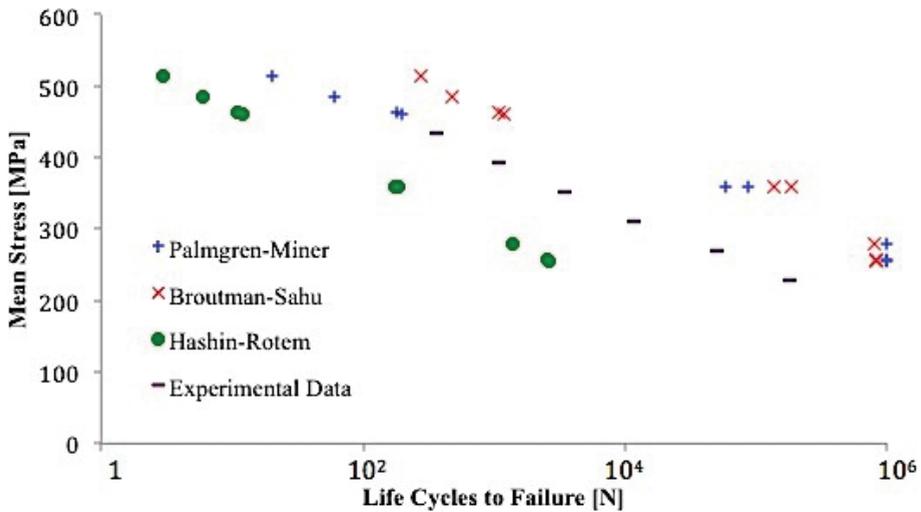


Figure 4. Mean stress vs. life to failure in E glass fiber/epoxy.

predict the fatigue life of a composite component, without getting accurate life predictions. In the case of the GFRP of Figure 4, the two linear models give overestimated predictions of fatigue life, and the non-linear model a significantly underestimated prediction, when model data is compared to experimental data. The linear stress-dependent model, is the one to estimate a predicted life having closer agreement with experimental results.

## 5. Conclusion

The versatility of composite materials, and specifically the case of FRPs, have made laminates ideal alternative structures to metallic components. The ability to design against failure by optimizing the stacking sequence of laminates while at the same time designing not just a strong component but also one with desirable stiffness, have opened new horizons to the use of FRPs.

This chapter discussed the theory behind building constitutive relationships for laminae and consequently laminate structures. Using the stress information from the CLT, it was shown how to estimate the onset of failure under different loadings. Of course, when designing against failure it is prudent to consider and consult more than one failure criteria, as they do not give exact predictions for the onset of failure, rather approximations. By considering one or more models, bounds can be drawn to limit the onset of failure conditions. The same holds true for the case of fatigue life, where cumulative damage models also tend to either overestimate or underestimate the fatigue life of laminates.

The above criteria and models, however, besides their predictive character, which to a large degree is due to the anisotropic nature of the laminates, as well as to the fact that they consider

only mechanical failure in the composite, can be applied to laminates of more than one constituents, as well as hybrid composite laminates. Therefore, if the optimization of a component requires more than one matrix and/or reinforcing phase, the above analysis could be followed to get estimates for the onset of failure and fatigue life of the component. The same holds true for hybrid laminates, where a third material, usually a reinforcing phase, other than fibers, for example a metallic reinforcement, is introduced. As a result, the above criteria and models can see applications in current designs of new composite materials, as the components built in n-d printing where the structures have not only varying material orientations, but also material properties.

The failure criteria and cumulative damage models, become therefore, an invaluable set of tools to the design against failure of laminate components. This design with the use of CLT can provide optimum laminates that have the desired stacking sequence to optimize mechanical properties and weight, as well as cost requirements. However, the discussion of this chapter did not account for failure that is not mechanical. The failure modes discussed concentrate on matrix or fiber cracking, debonding, and delamination, which was not examined at the laminate level. However, thermal and hygral effects are only accounted for in CLT and are not part of the failure criteria presented in this chapter. As a result, when failure is due to chemical degradation of the matrix under UV rays, or creep which should involve the time parameter, failure should be considered using different models which were not the scope of this chapter.

## Nomenclature

$E_{ijkl}$	Young's Modulus
$G_{ij}$	Shear Modulus
$K$	Bulk Modulus
$f$	Volume Fraction
$\nu_{ij}$	Poisson's Ratio
$\sigma_{ij}$	Stress Tensor
$\varepsilon_{kl}$	Strain Tensor
$\kappa_{ij}$	Curvature
$\varepsilon_{ij}^o$	Mid-Surface Strains
$\alpha_{ij}$	Coefficient of Thermal Expansion
$\beta_{ij}$	Hygroscopic Coefficient
$\bar{Q}$	Stiffness Matrix
$A_{ij}$	Extension Stiffness Matrix

$B_{ij}$	Extension-Bending Coupling Matrix
$D_{ij}$	Bending Stiffness Matrix
$z$	Position of layer in laminate
$X$ and $X'$	Longitudinal Tensile and Compressive Strength.
$Y$ and $Y'$	Transverse Tensile and Compressive Strength.
$S$	Shear Strength
$n_i$	Number of cycles under applied stress
$N_i$	Cycles to failure
$C$	Repetitions of cyclic loading

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