

Modeling of a Thirteen-link 3D Biped and Planning of a Walking Optimal Cyclic Gait using Newton-Euler Formulation

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1. Introduction

Preliminaries. The design of walking cyclic gaits for legged robots and particularly the bipeds has attracted the interest of many researchers for several decades. Due to the unilateral constraints of the biped with the ground and the great number of degrees of freedom, this problem is not trivial. Intuitive methods can be used to obtain walking gaits as in (Grishin et al. 1994). Using physical considerations, the authors defined polynomial functions in time for an experimental planar biped. This method is efficient. However to build a biped robot and to choose the appropriate actuators or to improve the autonomy of a biped, an optimization algorithm can lead to very interesting results. In (Rostami & Besonnet 1998) the Pontryagin's principle is used to design impactless nominal trajectories for a planar biped with feet. However the calculations are complex and difficult to extend to the 3D case. Furthermore the adjoint equations are not stable and highly sensitive to the initial conditions (Bryson & Ho 1995). As a consequence a parametric optimization is a useful tool to find optimal motion. For example in robotics, basis functions as polynomial functions, splines, truncated fourier series are used to approximate the motion of the joints, (Chen 1991; Luca et al. 1991; Ostrowski et al. 2000; Dürrbaum et al. 2002; Lee et al 2005; Miossec & Aoustin 2006; Bobrow et al 2006). The choice of optimization parameters is not unique. The torques, the Cartesian coordinates or joint coordinates can be used. Discrete values for the torques defined at sampling time are used as optimization parameters in (Roussel et al. 2003). However it is necessary, when the torque is an optimized variable, to use the direct dynamic model to find the joint accelerations and integrations are used to obtain the evolution of the reference trajectory in velocity and in position. Thus this approach requires much calculations: the direct dynamic model is complex and many evaluations of this model is used in the integration process. In (Beletskii & Chudinov 1977; Besonnet et al. 2002; Channon et al. 1992; Zonfrilli & Nardi 2002; Chevallereau & Aoustin 2001; Miossec & Aoustin 2006) to overcome this difficulty, directly the parametric optimization defines the reference trajectories of Cartesian coordinates or joint coordinates

for 2D bipeds with feet or without feet. An extension of this strategy is given in this paper to obtain a cyclic walking gait for a 3D biped with twelve motorized joints.

Methodology. A half step of the cyclic walking gait is uniquely composed of a single support and an instantaneous double support which is modeled by passive impulsive equations. This walking gait is simpler than the human gait. But with this simple model the coupling effect between the motion in frontal plan and sagittal plane can be studied. A finite time double support phase is not considered in this work currently because for rigid modeling of robot, a double support phase can usually be obtained only when the velocity of the swing leg tip before impact is null. This constraint has two effects. In the control process it will be difficult to touch the ground with a null velocity, as a consequence the real motion of the robot will be far from the ideal cycle. Furthermore, large torques are required to slow down the swing leg before the impact and to accelerate the swing leg at the beginning of the single support. The energy cost of such a motion is higher than a motion with impact in the case of a planar robot without feet (Chevallereau & Aoustin 2001; Miossec & Aoustin 2006). The evolution of joint variables are chosen as spline functions of time instead of usual polynomial functions to prevent oscillatory phenomenon during the optimization process (see Chevallereau & Aoustin 2001; Saidouni & Bessonnet 2003 or Hu & Sun 2006). The coefficients of the spline functions are calculated as functions of initial, intermediate and final configurations, initial and final velocities of the robot. These configuration and velocity variables can be considered as optimization variables. Taking into account the impact and the fact that the desired walking gait is periodic, the number of optimization variables is reduced. In other study the periodicity conditions are treated as equality constraints (Marot 2007). The cost functional considered is the integral of the torque norm, which is a common criterion for the actuators of robotic manipulators, (Chen 1991; Chevallereau & Aoustin 2001; Bobrow et al. 2001; Garg & Kumar 2002). During the optimization process, the constraints on the dynamic balance, on the ground reactions, on the validity of impact, on the limits of the torques, on the joints velocities and on the motion velocity of the biped robot are taken into account. Therefore an inverse dynamic model is calculated during the single phase to obtain the torques for a suitable number of sampling times. An impulsive model for the impact on the ground with complete surface of the foot sole of the swing leg is deduced from the dynamic model for the biped in double support phase. Then it is possible to evaluate cost functional calculation, the constraints during the single support and at the impact.

Contribution. The dynamic model of a 3D biped with twelve degrees of freedom is more complex than for a 2D biped with less degrees of freedom. So its computation cost is important in the optimization process and the use of Newton-Euler method to calculate the torque is more appropriate than the Lagrange method usually used. Then for the 3D biped, in single support, our model is founded on the Newton Euler algorithm, considering that the reference frame is connected to a stance foot. The walking study includes impact phase. The problem solved in (Lee et al. 2005; Huang & Metaxas 2002) is to obtain an optimal motion beginning at a given state and ending at another given state. Furthermore authors used Lie theoretic formulation of the equations of motion. In our case the objective is to define cyclic walking for the 3D Biped. Lie theoretic formulation is avoided because for rigid bodies in serial or closed chains, recursive ordinary differential equations founded on the Newton-Euler algorithm is appropriate see (Angeles 1997).

Structure of the paper. The paper is organized as follows. The 3D biped and its dynamic model are presented in Section 2. The cyclic walking gait and the constraints are defined in Section 3. The optimization parameters, optimization process and the cost functional are discussed in Section 4. A summarize of the global optimization process is given in Section 5. Simulation results are presented in Section 6. Section 7 contains our conclusion and perspectives.

2. Model of the biped robot

2.1 Biped model

We considered an anthropomorphic biped robot with thirteen rigid links connected by twelve motorized joints to form a serial structure. It is composed of a torso, which is not directly actuated, and two identical open chains called legs which are connected at the hips. Each leg is composed of two massive links connected by a joint called knee. The link at the extremity of each leg is called foot which is connected at the leg by a joint called ankle. Each revolute joint is assumed to be independently actuated and ideal (frictionless). The ankles of the biped robot consist of the pitch and the roll axes, the knees consist of the pitch axis and the hips consist of the roll, pitch and yaw axes to constitute a biped walking system of two 2-DoF (two degrees of freedom) ankles, two 1-DoF knees and two 3-DoF hips as shown in figure 1. The action to walk associates single support phases separated by impacts with full contact between the sole of the feet and the ground, so that a model in single support, and an impact model are derived. The dynamic model in single support is used to evaluate the required torque thus only the inverse dynamic model is necessary. The impact model is used to determine the velocity of the robot after the impact, the torques are zero during the impact, thus a direct impact model is required. Since we use the Newton Euler equations to derive the dynamic model, the direct model is much more complicated to obtain than the inverse model.

The periodic walk studied includes a symmetrical behavior when the support is on right leg and left leg, thus only the behavior during an half step is computed, the behavior during the following half step is deduced by symmetry rules. As a consequence only the modeling on leg 1 is considered in the following.

2.2 Geometric description of the biped

For a planar robot any parameterization of the robot can be used, for a 3D model of robot with many degrees of freedom a systematic parameterization of the robot must be developed. Many studies have been conducted for the manipulator robot, thus the parameterization proposed for the manipulator robot is re-used for the walking robot. The first difficulty is to choose a base link for a walking robot. Since the leg one is in support during all the studied half step. The supporting foot is considered as base link.

To define the geometric structure of the biped walking system we assume that the link 0 (stance foot) is the base of the biped robot while the link 12 (swing foot) is the terminal link. Therefore we have a simple open loop robot which the geometric structure can be described using the notation of (Khalil & Kleinfinger 1985). The definition of the link frames is presented in figure 1 and the corresponding geometric parameters are given in table 1. Frame R_0 , which is fixed to the tip of the right foot (determined by the width l_p and the

length L_p), is defined such that the axis z_0 is along the axis of frontal joint ankle. The frame R_{13} is fixed to the tip of the left foot in the same way as R_0 .

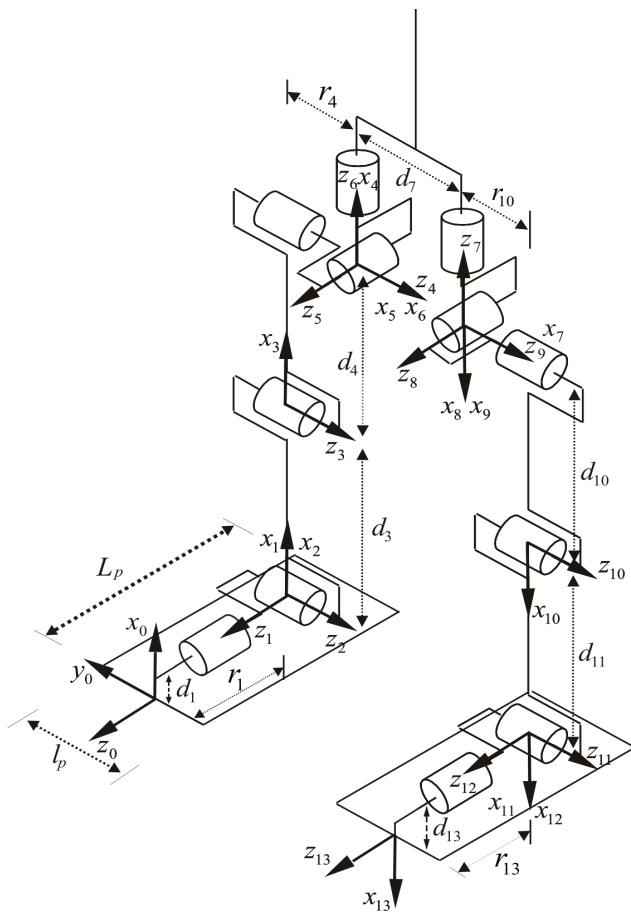


Fig. 1. Coordinate frame assignment for the biped robot.

| j | a_j | α_j | q_j | r_j | d_j |
|---|-------|------------------|----------------------------|-------|-------|
| 1 | 0 | 0 | θ_1 | r_1 | d_1 |
| 2 | 1 | $\frac{\pi}{2}$ | θ_2 | 0 | 0 |
| 3 | 2 | 0 | θ_3 | 0 | d_3 |
| 4 | 3 | 0 | θ_4 | r_4 | d_4 |
| 5 | 4 | $-\frac{\pi}{2}$ | $\theta_5 - \frac{\pi}{2}$ | 0 | 0 |
| 6 | 5 | $-\frac{\pi}{2}$ | θ_6 | 0 | 0 |

| j | a _j | α _j | q _j | r _j | d _j |
|----|----------------|------------------|----------------------------|-----------------------------------|----------------------------------|
| 7 | 6 | 0 | θ ₇ | 0 | d ₇ |
| 8 | 7 | $\frac{\pi}{2}$ | $\theta_8 - \frac{\pi}{2}$ | 0 | 0 |
| 9 | 8 | $-\frac{\pi}{2}$ | θ ₉ | 0 | 0 |
| 10 | 9 | 0 | θ ₁₀ | r ₁₀ = r ₄ | d ₁₀ = d ₄ |
| 11 | 10 | 0 | θ ₁₁ | 0 | d ₁₁ = d ₃ |
| 12 | 11 | $\frac{\pi}{2}$ | θ ₁₂ | 0 | 0 |
| 13 | 12 | 0 | θ ₁₃ | r ₁₃ = -r ₁ | d ₁₃ = d ₁ |

Table 1. Geometric parameters of the biped.

2.3 Dynamic model in single support phase

During the single support phase, our objective is only to determine the inverse dynamic model. The joint position, velocity and acceleration are known. The actuator torques must be calculated. Since the contact between the stance foot and the ground is unilateral, the ground reaction (forces and torques) must also be deduced. The Newton-Euler algorithm (see Khalil & Dombre 2002) must be adapted to determine the ground wrench. During the single support phase the stance foot is assumed to remain in flat contact on the ground, i.e., no sliding motion, no take-off, no rotation. Therefore the biped is equivalent to a 12-DoF manipulator robot. Let $q \in \mathbb{R}^{12}$ be the generalized coordinates, where q_1, \dots, q_{12} denote the relative angles of the joints, $\dot{q} \in \mathbb{R}^{12}$ and $\ddot{q} \in \mathbb{R}^{12}$ are the velocity vector and the acceleration vector respectively. The dynamic model is represented by the following relation:

$$\begin{bmatrix} R_{F_r} \\ \Gamma \end{bmatrix} = f(q, \dot{q}, \ddot{q}, F_t) \tag{1}$$

where $\Gamma \in \mathbb{R}^{12}$ is the joint torques vector, R_{F_r} is the ground wrench on the stance foot and F_t represents the external wrenches (forces and torques), exerted on links 1 to 12. In single support phase we assume that $F_t = 0$.

The Newton-Euler method is used to calculate the dynamic model as defined in equation (1). This method proposed by Luh, Walker et Paul (Luh et al. 1980) is based on two recursive calculations. Associated with our choice of parameterization the following algorithm is obtained (Khalil & Dombre 2002). The forward calculation, from the base (stance foot) to the terminal link (swing foot) determines the velocity, the accelerations and the total forces and moments on each link. Then the backward calculations, from swing foot to stance foot, gives the joint torques and reaction forces using equation of equilibrium of each link successively.

Forward recursive equations

Taking into account that the biped robot remains flat on the ground, the initial conditions are:

$$\omega_0 = \mathbf{0}, \dot{\omega}_0 = \mathbf{0} \text{ and } \ddot{\omega}_0 = -\mathbf{g} \tag{2}$$

the real acceleration is $\ddot{\omega}_0 = 0$ but the choice to write $\ddot{\omega}_0 = -g$ allows to take into account the gravity effect.

For the link j with its associated frame R_j , and considering the link $j-1$ as its antecedent, its angular velocity ${}^j\omega_j$, and the linear velocity jV_j of the origin O_j of R_j are :

$${}^j\omega_j = {}^j\omega_{j-1} + \bar{\sigma}_j \dot{\phi}_j^j a_j \tag{2}$$

$${}^jV_j = {}^jA_{j-1} \left({}^{j-1}V_{j-1} + {}^{j-1}\omega_{j-1} \times {}^{j-1}P_j \right) + \sigma_j \dot{\phi}_j^j a_j \tag{3}$$

with ${}^jA_{j-1}$, the orientation matrix of the frame R_{j-1} in the frame R_j , $\sigma_j = 0$ when the j joint is a revolute joint, $\sigma_j = 1$ when the j joint is prismatic joint and $\bar{\sigma}_j = 1 - \sigma_j$, ${}^j a_j$ is a unit vector along the z_j axis, ${}^{j-1}P_j$ is the vector expressing the origin of frame R_j in frame R_{j-1} . The angular acceleration of link j and the linear acceleration of the origin O_j of R_j are:

$${}^j\dot{\omega}_j = {}^jA_{j-1} {}^{j-1}\dot{\omega}_{j-1} + \bar{\sigma}_j \left(\dot{\phi}_j^j a_j + {}^j\omega_{j-1} \times \dot{\phi}_j^j a_j \right) \tag{4}$$

$${}^j\dot{V}_j = {}^jA_{j-1} \left({}^{j-1}\dot{V}_{j-1} + {}^{j-1}\dot{\omega}_{j-1} \times {}^{j-1}P_j \right) + \sigma_j \left(\dot{\phi}_j^j a_j + 2 {}^j\omega_{j-1} \times \dot{\phi}_j^j a_j \right) \tag{5}$$

where ${}^jU_j = {}^j\dot{\omega}_j + {}^j\dot{\omega}_j$. Matrices ${}^j\dot{\omega}_j \in \mathbb{R}^{3 \times 3}$ and ${}^j\dot{\omega}_j \in \mathbb{R}^{3 \times 3}$ designate the skew matrices associated with the vectors ${}^j\dot{\omega}_j \in \mathbb{R}^{3 \times 3}$ and ${}^j\omega_j \in \mathbb{R}^{3 \times 3}$ respectively.

$${}^j\dot{\omega}_j = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} \tag{7}$$

The total inertial forces and moments for link j are:

$${}^jF_j = M_j {}^j\dot{V}_j + {}^jU_j {}^jMS_j \tag{6}$$

$${}^jM_j = {}^jJ_j {}^j\dot{\omega}_j + {}^j\omega_j \times ({}^jJ_j {}^j\omega_j) + {}^jMS_j \times {}^j\dot{V}_j \tag{8}$$

with jJ_j inertia tensor of link j with respect to R_j frame, jMS_j is the first moments vector

of link j around the origin of R_j frame and M_j the mass of the link j . The antecedent link to the link 0 (stance foot) is not defined. For the iteration of the stance foot, only the equations (6) and (Pogreška! Izvor reference nije pronađen.) are used.

Backward recursive equations

The backward recursive equations are given as, for $j = 12, \dots, 0$:

$${}^j f_j = {}^j F_j + {}^j f_{j+1} \tag{7}$$

$${}^{j-1} f_j = {}^{j-1} A_j {}^j f_j \tag{8}$$

$${}^j m_j = {}^j M_j + {}^j A_{j+1} {}^{j+1} m_{j+1} + {}^j P_{j+1} \times {}^j f_{j+1} \tag{9}$$

These recursive equations will be initialized by the forces and moments exerted on the terminal link by the environment ${}^j f_{j+1}$ and ${}^j m_{j+1}$. In single support ${}^j f_{j+1} = 0$, ${}^j m_{j+1} = 0$. When $j=0$, ${}^0 f_0$ and ${}^0 m_0$ are the forces exerted on the link 0 or the ground reaction force and moment rewritten as ${}^0 F_R$ and ${}^0 M_R$ expressed in the frame R_0 .

If we neglect the friction and the motor inertia effects, the torque (or the force) Γ_j , is obtained by projecting m_j (or f_j) along the joint axis (z_j):

$$\Gamma_j = {}^j M_j + (\sigma_j {}^j f_j + \bar{\sigma}_j {}^j m_j)^T {}^j a_j \tag{10}$$

Γ_0 is not defined, since there is no actuator.

The ground reaction wrench is known in the frame R_0 . This frame is associated with the stance foot, and the axis y_0, z_0 defined the sole of the stance foot. The position of the zero moment point (ZMP) position which is the point of the sole such that the moment exerted by the ground is zero along the axis y_0 and z_0 is such that:

$$y_{ZMP} = \frac{{}^0 m_{0z}}{{}^0 f_{0x}} \tag{11}$$

$$z_{ZMP} = \frac{{}^0 m_{0y}}{{}^0 f_{0x}} \tag{12}$$

If the position of ZMP is within the support polygon, the biped robot is in dynamic equilibrium, the stance foot remains flat on the ground.

2.4. Impact model for the instantaneous double support

At the impact, the previous supporting foot becomes the swing foot, and its velocity after the impact can be different from zero. As a consequence the modeling of the robot must be able to describe a non fixed stance foot. Since the dynamic model is calculated with the Newton-Euler algorithm, it is very convenient to define the velocity of the link 0 with the Newton variables: V_0 the linear velocity of the origin of frame R_0 and ω_0 the angular velocity.

For the impact model, or the double support model the robot position is expressed by $X = [X_0, \alpha_0, q]^T \in R^{18}$, X_0 and α_0 are the position and the orientation variables of frame R_0 ; the robot velocity is $V = [{}^0V_0, {}^0\omega_0, \dot{q}]^T \in R^{18}$ and the robot acceleration is $\dot{V} = [{}^0\dot{V}_0, {}^0\dot{\omega}_0, \ddot{q}]^T \in R^{18}$.

The impact model is deduced from the dynamic model in double support, when we assume that the acceleration of the robot and the reaction force are Dirac delta-functions.

The dynamical model in double support can be written:

$$D(X)\dot{V} + C(V, q) + G(X) + D_f R_f = D_\Gamma + D_R R_{FR} \tag{13}$$

where $D \in R^{18 \times 18}$ is the symmetric definite positive inertia matrix, $C \in R^{18}$ represents the Coriolis and centrifugal forces, $G \in R^{18}$ is the vector of gravity. $R_{FR} = [F_R, M_R]^T \in R^6$ is the vector of the ground reaction forces on the stance foot, $R_f \in R^6$ represents the vector of forces F_{12} and moments M_{12} exerted by the swing foot on the ground, D_f , D_Γ and D_R are matrices that allow to take into account the forces and torques in the dynamic model. The model of impact can be deduced from (13) and is:

$$D(X)\Delta V + D_f I_{R_f} = D_R I_{R_{FR}} \tag{14}$$

where I_{R_f} and $I_{R_{FR}}$ are the intensity of Dirac delta-function for the forces R_f and R_{FR} . ΔV is the variation of velocity at the impact, $\Delta V = V^+ - V^-$, where V^- is the velocity of the robot before impact and V^+ its velocity after impact.

The impact is assumed to be inelastic with complete surface of the foot sole touching the ground. This means that the velocity of the swing foot impacting the ground is zero after impact. Two cases are possible after an impact: the rear foot takes off the ground or both feet remain on the ground. In the first case, the vertical component of the velocity of the taking-off foot just after an impact must be directed upwards and the impulsive ground reaction in this foot equals zero $I_{R_{FR}} = 0$. In the second case, the rear foot velocity has to be zero just after an impact. The ground produces impulsive forces in both feet. This implies that the vertical component of the impulsive ground reaction in the rear foot (as in the fore foot) is directed upwards. An impacting foot with zero velocity at impact, is a solution of the two cases, there is no impact, the reaction forces on the two leg are zero and the velocity of the two feet after impact is zero.

For our numerical tests, for the robot studied, only the first case gives a valid solution. The swing foot is zero velocity before the impact (and there is no impact) or the previous stance foot does not remain on the ground after the impact. Thus, the impact dynamic model is (see (Formal'skii 1982) and (Sakaguchi 1995)):

$$D(X)\Delta V = -D_f I_{R_f} \tag{15}$$

$$D_f^T V^+ = 0 \tag{16}$$

$$\begin{bmatrix} {}^0V_0^- \\ {}^0\omega_0^- \end{bmatrix} = \begin{bmatrix} 0_{3 \times 1} \\ 0_{3 \times 1} \end{bmatrix} \tag{17}$$

These equations form a system of linear equations which determines the impulse forces I_{R_f} and the velocity vector of the biped after impact V^+ .

$$I_{R_f} = (D_f^T D^{-1} D_f)^{-1} D_f^T V^- \tag{18}$$

$$V^+ = -D^{-1} D_f (D_f^T D^{-1} D_f)^{-1} D_f^T V^- + V^- \tag{19}$$

As the wrench R_f is naturally expressed in the frame R_{12} : ${}^{12}F_{12}$, ${}^{12}M_{12}$. The matrix D_f is the transpose of the Jacobian of velocity of the link R_{12} with respect to the robot velocity V . The velocities of link 12 can be expressed as :

$$\begin{bmatrix} V_{12} \\ \omega_{12} \end{bmatrix} = \begin{bmatrix} V_0 + \omega_0 \times {}^0P_{12} \\ \omega_{12} \end{bmatrix} + J_{12} \mathcal{Q} \tag{20}$$

where ${}^0R_{12}$ is the vector linking the origin of frame R_0 and the origin of frame R_{12} expressed in frame R_0 , $J_{12} \in \mathbb{R}^{6 \times 12}$ is the Jacobian matrix of the robot, $J_{12} \mathcal{Q}$ represents the effect of the joint velocities on the Cartesian velocity of link 12. The velocities V_{12} and ω_{12} must be expressed in frame R_{12} , thus we write:

$$\begin{bmatrix} {}^{12}V_{12} \\ {}^{12}\omega_{12} \end{bmatrix} = \begin{bmatrix} {}^{12}A_0 & -{}^{12}A_0 {}^0P_{12} \\ 0_{3 \times 3} & {}^{12}A_0 \end{bmatrix} \begin{bmatrix} {}^0V_0 \\ {}^0\omega_0 \end{bmatrix} + {}^{12}J_{12} \mathcal{Q} \tag{21}$$

where ${}^{12}A_0 \in \mathbb{R}^{3 \times 3}$ is the rotation matrix, which defines the orientation of frame R_0 with respect to frame R_{12} . Term ${}^0P_{12}$ is the skew-symmetric matrix of the vector product associated with vector ${}^0P_{12}$.

For the calculation of the inertia matrix D , following the same way, as the force R_{F_R} is applied on the stance leg, in equation (13), $D_R = [I_{6 \times 6} \mid 0_{12 \times 6}]^T \in \mathbb{R}^{18 \times 6}$. The matrix D_Γ defines the actuated joint thus we have : $D_\Gamma = [0_{6 \times 12} \mid I_{12 \times 12}]^T \in \mathbb{R}^{18 \times 12}$.

When no force is applied on the swing leg, the dynamic model (13) becomes:

$$D(X)\ddot{X} + C(V, q) + G(X) = \begin{bmatrix} R_{F_R} \\ \Gamma \end{bmatrix} \quad (22)$$

Since the stance foot is assumed to remain in flat contact, the resultant ground reaction force/moment R_{F_R} and the torques Γ can be computed using the Newton-Euler algorithm (see section 2.3). According to the method of (Walker & Orin 1982), the matrix D is calculated by the algorithm of Newton-Euler, by noting from (13), that the i^{th} column of D is equal to $\begin{bmatrix} R_{F_R} \\ \Gamma \end{bmatrix}$ if

$$V = 0, g = 0, \ddot{X} = e_i, R_i = 0 \quad (25)$$

$e_i \in \mathbb{R}^{18 \times 1}$ is the unit vector, whose elements are zero except the i^{th} element which is equal to 1. The vectors $C(V, q)$ and $G(X)$ can be obtained in the same way that D , however for the impact model the knowledge of these vectors are not necessary.

3. Definition of the walking cycle

Because a walking biped gait is a periodical phenomenon our objective is to design a cyclic biped gait. A complete walking cycle is composed of two phases: a single support phase and a double support phase which is modeled through passive impact equations. The single support phase begins with one foot which stays on the ground while the other foot swings from the rear to the front. We shall assume that the double support phase is instantaneous. This means that when the swing leg touches the ground the stance leg takes off. There are two facets to be considered for this problem. The definition of reference trajectories and the method to determine a particular solution of it. This section is devoted to the definition of reference trajectories. The optimal process to choose the best solution of parameters, allowing a symmetric half step, from the point of view of a given cost functional will be described in the next section.

3.1. Cyclic walking trajectory

Since the initial configuration is a double support configuration, both feet are on the ground, the twelve joint coordinates are not independent. Because the absolute frame is attached to the right foot we define the situation of the left foot by $(y_{lf}, z_{lf}, \phi_{lf})$ and the situation of the middle of the hips $(x_h, y_h, z_h, \theta_h)$, both expressed in R_0 frame. (y_{lf}, z_{lf}) are the Cartesian coordinates, in the horizontal plane, of the left foot position, ϕ_{lf} denotes the left foot yawing

motion, (x_h, y_h, z_h) is the hip position and θ_h defines the hip pitching motion. The two others parameters, orientation for the middle of the hips in frontal and transverse planes, are considered to be equal to zero. The values of the joint variables are solution of the inverse kinematics problem for a leg, which may also be considered as a 6-link manipulator. The problem is solved with a symbolic software, (SYMORO+, see (Khalil & Kleinfinger 1985)). Let us consider, for the cyclic walking gait, the current half step in the time interval $[0, T_s]$. In order to deduce the final configuration of the biped robot at time $t = T_s$, we impose a symmetric role of the two legs, therefore from the initial configuration $q_0 = q(t=0)$ in double support, the final configuration $q_{T_s} = q(t = T_s)$ in double support is deduced as:

$$q_{T_s} = E q_0 \tag{23}$$

where $E \in \mathbb{R}^{12 \times 12}$ is an inverted diagonal matrix which describes the exchange of legs. Taking into account the impulsive impact (15)-(17), we can compute the velocity vector of the biped after the impact. Therefore, the joint velocities after impact, $\dot{\mathcal{Q}}$ can be calculated when the joint velocities before the impact, $\dot{\mathcal{Q}}$ is known. The use of the defined matrix E allows us to calculate the initial joint velocities $\dot{\mathcal{Q}}_0 = \dot{\mathcal{Q}}(t=0)$ for the current half step as:

$$\dot{\mathcal{Q}}_0 = E \dot{\mathcal{Q}} \tag{24}$$

By this way the conditions of cyclic motion are satisfied.

3.2. Constraints

In order to insure that the trajectory is possible, many constraints have to be considered.

Magnitude constraints on position, velocities and torque:

- Each actuator has physical limits such that:

$$|\Gamma_i| - \Gamma_{i,max} \leq 0, \quad \text{for } i = 1, \dots, 12 \tag{25}$$

where $\Gamma_{i,max}$ denotes the maximum value for each actuator.

$$|\dot{\mathcal{Q}}_i| - \dot{\mathcal{Q}}_{i,max} \leq 0, \quad \text{for } i = 1, \dots, 12 \tag{26}$$

where $\dot{\mathcal{Q}}_{i,max}$ denotes the maximum velocity for each actuator.

- The upper and lower bounds of joints for the configurations during the motion are:

$$q_{i,\min} \leq q_i \leq q_{i,\max}, \quad \text{for } i = 1, \dots, 12 \tag{27}$$

$q_{i,\min}$ and $q_{i,\max}$ respectively stands for the minimum and maximum joint limits.

Geometric constraints in double support phase:

- The distance $d(\text{hip}, \text{foot})$ between the foot in contact with the ground and the hip must remain within a maximal value, i.e., :

$$d(\text{hip}, \text{foot}) \leq l_{\text{hip}} \tag{28}$$

This condition must hold for initial and final configurations of the double support phase.

- In order to avoid the internal collision of both feet through the lateral axis the heel and the toe of the left foot must satisfy:

$$y_{\text{heel}} \leq -a \quad \text{and} \quad y_{\text{toe}} \leq -a \tag{31}$$

with $a > \frac{l_p}{2}$ and l_p is the width of right foot.

Walking constraints:

- During the single support phase to avoid collisions of the swing leg with the stance leg or with the ground, constraints on the positions of the four corners of the swing foot are defined.
- We must take into account the constraints on the ground reaction $R_{F_R} = [R_{F_{R_x}}, R_{F_{R_y}}, R_{F_{R_z}}]^T$ for the stance foot in single support phase as well as impulsive forces $I_{R_i} = [I_{R_{i_x}}, I_{R_{i_y}}, I_{R_{i_z}}]^T$ on the foot touching the ground in instantaneous double support phase. The ground reaction in single support and the impulsive forces at the impact must be inside a friction cone defined by the friction coefficient μ . This is equivalent to write:

$$\sqrt{R_{F_{R_y}}^2 + R_{F_{R_z}}^2} \leq \mu R_{F_{R_x}} \tag{32}$$

$$\sqrt{I_{R_{i_y}}^2 + I_{R_{i_z}}^2} \leq \mu I_{R_{i_x}} \tag{33}$$

- The ground reaction forces in single support and the impulsive forces at the impact only can push from the ground but cannot pull from ground, then the conditions of no take off are deduced:

$$R_{f_x} \geq 0 \tag{34}$$

$$I_{R_{f_x}} \geq 0 \tag{35}$$

- In order to maintain the balance in dynamic walking, the Zero Moment Point which is equivalent to the Center of Pressure (CoP), (Vukobratovic & Borovac 2004; Vukobratovic & Stepanenko 1972; Vukobratovic & Borovac 1990), of the biped’s stance foot must be within the interior of the support polygon. Then for a rectangular foot the (CoP) must satisfy:

$$\frac{-l_p}{2} \leq \text{CoP}_y \leq \frac{l_p}{2} \tag{36}$$

$$-L_p \leq \text{CoP}_z \leq 0 \tag{37}$$

where l_p is the width and L_p is the length of the feet.

4. Parametric optimization

4.1. The cubic spline

To describe the joint motion by a finite set of parameters we choose to use for joint i , ($i = 1, \dots, 12$) a piecewise function of the form:

$$q_i = \varphi_i(t) = \begin{cases} \varphi_{i1}(t) & \text{if } t_0 \leq t \leq t_1 \\ \varphi_{i2}(t) & \text{if } t_1 \leq t \leq t_2 \\ \cdot & \cdot \\ \cdot & \cdot \\ \varphi_{in}(t) & \text{if } t_{n-1} \leq t \leq t_n \end{cases} \tag{38}$$

where $\varphi_k(t)$ are polynomials of third-order such that:

$$\varphi_{ik}(a_{ik}, t) = \sum_{j=0}^3 a_{ikj}(t - t_{k-1})^j, \quad k = 1, \dots, n \quad \forall t \in [t_0, t_n] \tag{39}$$

where a_{ikj} are calculated such that the position, velocity and acceleration are always continuous in t_1, \dots, t_{n-1} . The motion is defined by specifying an initial configuration q_0 , an initial velocity \dot{q}_0 , a final configuration q_T and a final velocity \dot{q}_T in double support, with

$n - 1$ intermediate configurations in single support and T_s the duration of this single support.

4.2. Optimization parameters

A parametric optimization problem has to be solved to design a cyclic bipedal gait with successive single supports and passive impacts (no impulsive torques are applied at impact). For a half step defined on the time interval $[0, T_s]$ this problem depends on parameters to prescribe the $n - 1$ intermediate configurations, the final velocity $\dot{\mathbf{q}}_s$ in the single support phase and, using the geometric model, the limit configuration of the biped at impact. Taking into account the conditions (23) and (24) the minimal number of parameters necessary to define the joint are:

1. $(n - 1) \times 12$ parameters are needed to define the $n - 1$ intermediate configurations in single support phase.
2. The joint velocities of the biped before the impact are also prescribed by twelve parameters, $\dot{\mathbf{q}}_i$ ($i = 1, \dots, 12$).
3. The left foot yawing motion denoted by ϕ_{lf} and its position (y_{lf}, z_{lf}) in the horizontal plane as well as the situation of the middle of the hips defined by $(x_h, y_h, z_h, \theta_h)$ in double support phase are chosen as parameters.

Then the total number of parameters is: $19 + (n - 1) \times 12$. Let us remark that to define the initial and final configurations for the half step, when both feet touch the ground, nine parameters are required. However we define these configurations with six parameters only.

These six parameters are defined by the vector $\mathbf{p} = [p_1 \ p_2 \ p_3 \ p_4 \ p_5 \ p_6]^T$ with the following geometric configuration data:

- p_1 : height of pelvis.
- p_2 : distance between the feet in the frontal plane each foot.
- p_3 : distance of the trunk with respect to the hip of the stance leg in the frontal plane.
- p_4 : orientation of the trunk in the sagittal plane.
- p_5 : position of the stance foot following y in frame R_0 .
- p_6 : position of the stance foot following z in frame R_0 .

The two others parameters, orientation of the middle of the hips in frontal and transverse planes, are fixed to zero. The duration of a half step, T_s , is arbitrarily fixed.

For our numerical tests $n = 3$ and then two intermediate configurations q_{int1} and q_{int2} of the 3D biped in single support are considered. To summarize, considering \mathbf{q} , $\dot{\mathbf{q}}$ and $\ddot{\mathbf{q}}$ of which the components equal the basis functions q_i (**Pogreška! Izvor reference nije pronađen.**) and their associated time derivatives \dot{q}_i and \ddot{q}_i , $i = 1, \dots, 12$, we can write:

$$\mathbf{q} = \mathbf{q}(q_0, \dot{\mathbf{q}}_0, q_{int1}, q_{int2}, q_{T_s}, \dot{\mathbf{q}}_s) \quad (40)$$

$$\mathcal{Q} = \mathcal{Q}(q_0, \mathcal{Q}_0, q_{int1}, q_{int2}, q_{T_s}, \mathcal{Q}_{T_s}) \tag{41}$$

$$\mathcal{P} = \mathcal{P}(q_0, \mathcal{Q}_0, q_{int1}, q_{int2}, q_{T_s}, \mathcal{Q}_{T_s}) \tag{42}$$

where φ is the vector of components $\varphi_i(t)$ (**Pogreška! Izvor reference nije pronađen.**) defining the cubic splines for joint i , $i = 1, \dots, 12$. The chosen vector of optimization parameters P_o can be written:

$$P_o = \begin{bmatrix} P_o(1) \\ P_o(2) \\ P_o(3) \\ P_o(4) \end{bmatrix} = \begin{bmatrix} q_{int1} \\ q_{int2} \\ \mathcal{Q}_{T_s} \\ P \end{bmatrix} \tag{43}$$

4.3. Cost functional

In the optimization process we consider, as cost functional J_r , the integral of the norm of the torque divided by the half step length. In other words we are minimizing a quantity proportional to the lost energy in the actuators for a motion on a half step of duration T_s . This general form of minimal energy performance represents the losses by Joule effects for the electrical motors to cover distance d .

$$J_r = \frac{1}{d} \int_0^{T_s} \Gamma^T \Gamma dt \tag{44}$$

4.4. Statement of the optimization problem to design a cyclic walking gait for the 3D biped

Generally, many values of parameters can give a periodic bipedal gait satisfying constraints (25)-(**Pogreška! Izvor reference nije pronađen.**). A parametric optimization process, that objective is to minimize J_r under nonlinear constraints, is used to find a particular nominal motion with the splines (**Pogreška! Izvor reference nije pronađen.**) as basis functions. This optimization problem can be formally stated as:

$$\left. \begin{array}{l} \text{Minimize } J_r(P_o) \\ \text{subject to } g_j(P_o) \leq 0 \quad j = 1, 2, \dots, l \end{array} \right\} \tag{45}$$

where $J_r(P_o)$ is the cost functional to minimize with l constraints $g_j(P_o) \leq 0$ to satisfy. These constraints are given in section 3.2. The optimization problem (**Pogreška! Izvor reference nije pronađen.**) is numerically solved by using the Matlab function *fmincon*. This optimization function provides an optimization algorithm based on the Sequential Quadratic Programming (SQP). There are forty-three parameters for this nonlinear optimization problem: twenty-four for the two intermediate configurations in single

support, twelve for the joint velocities before the impact and seven to solve the inverse kinematics problem, subject to the constraints given by (25)-(Pogreška! Izvor reference nije pronađen.).

5. Algorithm for generating an optimal cyclic walking gait

In this section the algorithm to obtain an optimal cyclic walking gait for the biped is given.

- **Step 1:** Given initial values for each components of parameter vector P_O (Pogreška! Izvor reference nije pronađen.).
- **Step 2:** With the parameters $P_O(4) = p$ compute the initial configuration and from the equation (23) the final configuration.
- **Step 3:** With the initial and final configurations, the parameters $P_O(3) = \mathbf{p}$ and the equations (15), (16) and (24) compute the initial velocity \mathbf{q}_0 .
- **Step 4:** For time $t = 0$ to $t = T_s$, compute the spline functions (Pogreška! Izvor reference nije pronađen.) for the initial and final configurations and the parameters $P_O(1) = q_{int1}$ and $P_O(2) = q_{int2}$. Compute their first and second derives with respect to time.
- **Step 5:** For sampling time $\{0, \dots, t_k, \dots, T_s\}$, solve recursively the inverse dynamics (Pogreška! Izvor reference nije pronađen.)-(10) to compute the torques, the position of the Center of Pressure CoP , the constraints.
- **Step 6:** For sampling time $\{0, \dots, t_k, \dots, T_s\}$, approximate the integral of the square vector of torques to compute the cost functional.
- **Step 7:** Check convergence. If yes, terminate. If no, go to step1 for a new parameter vector P_O and begin a new optimization process.

6. Simulation results

To validate our proposed method, we present the results of an optimal motion for the biped, SPEJBL, shown in figure 2. SPEJBL has been designed in the Department of Control Engineering of the Technical University in Praha. Its physical parameters are given in table 2. The inertia of each link are also taken into account in the dynamic model. The results shown have been obtained with $T_s = 0.58s$. The optimal motion is such that the step length is 0.18m and the optimal velocity is 0.315m / s. These values are results of the optimization process presented in Section IV, with the minimization of the cost functional (Pogreška! Izvor reference nije pronađen.) satisfying the constraints given by (25)-(Pogreška! Izvor reference nije pronađen.). The simulation of the optimal motion for one half step is illustrated in figure 3 and for 3 walking steps in figure 4. The normal components of the ground reactions, in function of time, of the stance foot during one half step in single support are presented in figure 5. The average vertical reaction force is 20 N, which is coherent with the weight of the robot which the mass equals 2.14 Kg. The chosen friction coefficient is 0.7. The figure 6 shows the CoP trajectory which is always inside the support polygon determined by $l_p = 0.11$ m and $L_p = 0.18$ m, that is, the robot maintains the balance during the motion. Because the minimal distance between of CoP and the boundary of the

foot is large, smaller foot is acceptable for this cyclic motion. The figure 7 shows the evolutions of joint variables $q_i(t)$ $i=1,\dots,12$, versus time, defined by the third-order spline function presented in Section III, in the single support phase during one half step. Let us remark that the evolution of each joint variable depends on the boundary conditions ($q_i(0), q_i(T_s)$ for $i=1,\dots,12$) and also on the intermediate configurations $q_{i,int1}, q_{i,int2}$ for $i=1,\dots,12$ whose values are computed in the optimal process. For a set of motion velocities, the evolution of criterion J_r is presented in figure 8. With respect to the evolution of J_r we can conclude that the biped robot consumes more energy for low velocities to generate one half step. Due to the limitations of the joint velocities we could not obtain superior values to 0.36 m/s. The energy consumption increases probably for higher velocity (Chevallereau & Aoustin 2001). The robot has been designed to be able to walk slowly, this walk require large torque and small joint velocities. Its design is also based on large feet in order to be able to use static walking, as a consequence the feet are heavy and bulky, thus the resulting optimal motion is close to the motion of a human with snowshoes.

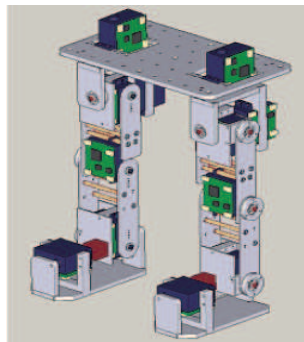


Fig. 2. Dimensional drawing of SPEJBL.

| Physical Parameters | Mass (kg) | Length (m) |
|---------------------|-----------|-----------------|
| Torso | 0.39 | 0.14 |
| Hip joints | 0.26 | linked to torso |
| Thigh | 0.12 | 0.12 |
| Shin | 0.05 | 0.12 |
| Ankle joints | 0.13 | 0.042 |
| Foot | 0.30 | 0.18x0.11 |

Table 2. Parameters of SPEJBL.

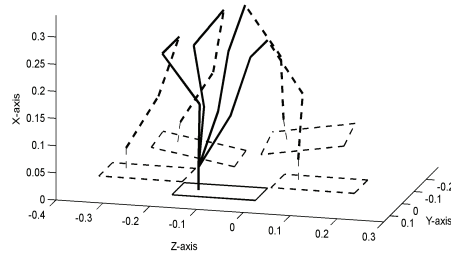


Fig. 3. Walking simulation for a half step.

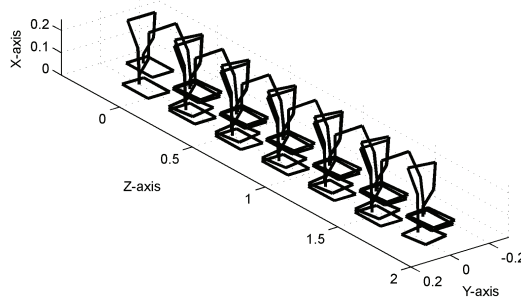


Fig. 4. Cyclic motion of biped SPEJBL.

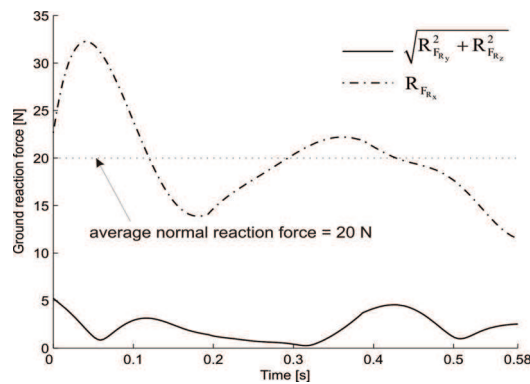


Fig. 5. The ground reaction force during the single support phase.

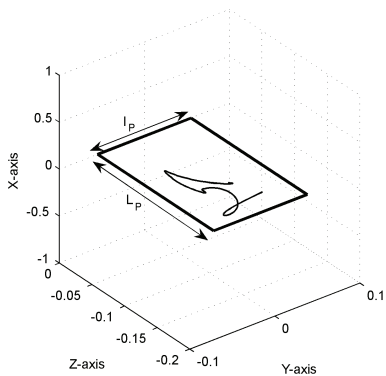
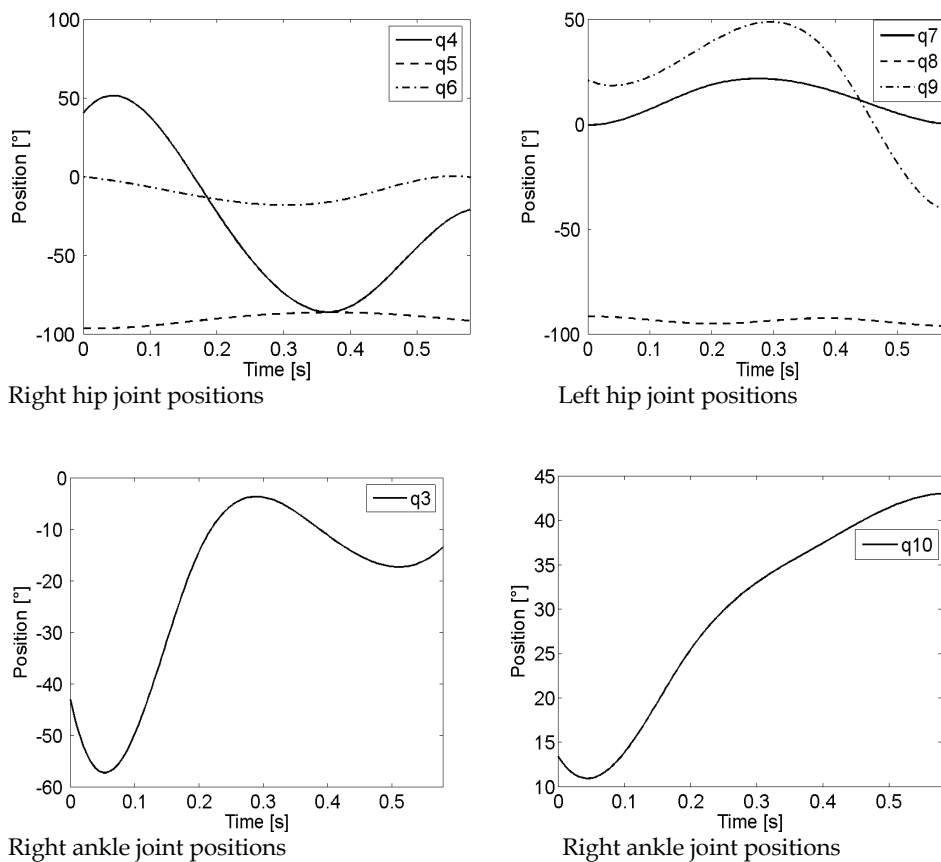


Fig. 6. The evolution of the CoP trajectory during a half step.



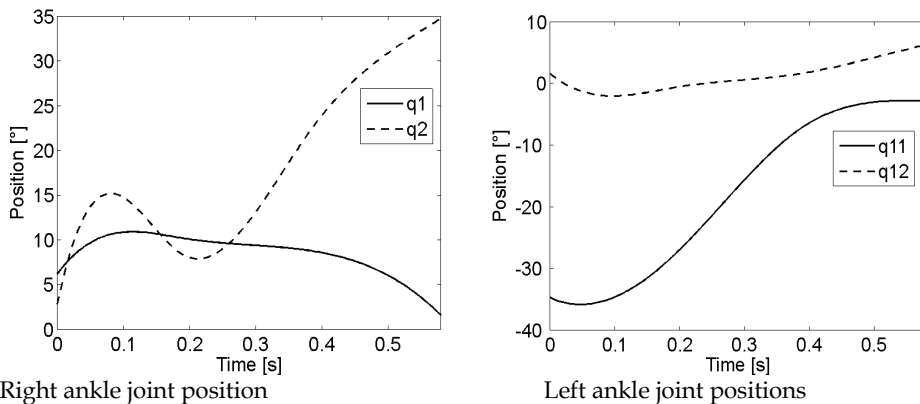


Fig. 7. Evolution of joint positions.

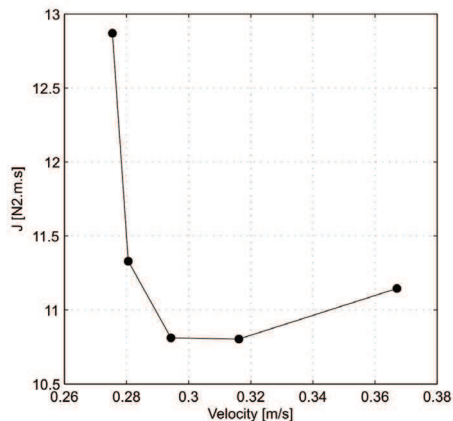


Fig. 8. J_r in function of several motion velocities for SPEJBL.

7. Conclusion

Optimal joint reference trajectories for cyclic walking gaits of a 3D experimental biped, SPEJBL are found. A methodology to design such optimal trajectories is developed. The definition of optimal trajectories is useful to test a robot design. In order to use classical optimization technique, the optimal trajectory is described by a set of parameters: we choose to define the evolution of the actuated relative angle as spline functions. A cyclic solution is desired. The number of the optimization variables is reduced by taking into account of the cyclicity condition explicitly.

Some inequality constraints such as the limits on the torques and the velocities, the condition of no sliding during motion and impact, some limits on the motion of the free leg are taken into account. The cost functional is calculated from the integral of the torques

norm. The torques are computed for sampling times using the inverse dynamic model. This model is obtained with the recursive Newton-Euler algorithm. The reference frame is connected to the stance foot. Optimal motions for a given duration of the half step have been obtained. The half step length and the advance velocity are the result of the optimization process. The numerical results obtained are realistic with respect to the size of the robot under study. Optimal motion for a given motion velocity can also be studied, in this case the motion velocity is considered as a constraint.

The proposed method to define optimal motion will be tested, considering a sub-phase of rotation of the supporting phase about the toe, closer to human. Another perspective is to evaluate the gradient of the cost functional and of the constraints with respect to the optimization parameters.

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