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# Complex System Reliability Analysis Method: Goal-Oriented Methodology

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Additional information is available at the end of the chapter

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## Abstract

Goal-oriented (GO) methodology is a success-oriented method for complex system reliability analysis based on modeling the normal operating sequence of a system and all possible system states. Recently, GO method has been applied in reliability and safety analysis of a number of systems, spanning defense, transportation, and power systems. This chapter provides a new approach for reliability analysis of complex systems, first, by providing its development history, its engineering applications, and the future directions. Then, the basic theory of GO method is expounded. Finally, the comparison of GO method, fault tree analysis and Monte-Carlo simulation is discussed.

**Keywords:** system reliability analysis, complex system, GO method, reliability model

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## 1. Introduction

Quality and reliability are key attributes of economic success of a system because they result in an increase in productivity at low cost and vital for business growth and enhanced competitive position. The recent advances in electronics, computing, communication, control, and networking have resulted in integrated systems that are: (i) complex in structure, (ii) large in scope and scale, (iii) characterized by multimode operation, (iii) capable of working in varied working conditions, and (iv) hierarchically organized. The reliability of such complex systems is a critical factor of their fitness for their intended use and hence is vital for in design and manufacturing. Reliability analysis method of complex systems is conducive to prevent defects in the first place in all aspects when they do occur in operation, in order to improve their reliability and reduce their life-cycle cost.

Fault tree analysis (FTA) and Monte-Carlo simulation (MCS) are now the standard reliability and safety analysis methods. Different from them, goal-oriented (GO) methodology [1] is a

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success-oriented method for system reliability analysis based on modeling the normal operating sequence of a system and all possible system states. It is especially suitable for complex systems having time-sequence of operation, multiple states, and so on, and the quantitative analysis and the qualitative analysis of the GO method are conducted by the GO operation according to the GO model. The keys of the GO method are the GO model and the GO operation. Although the GO method was introduced in 1976 [1], it was largely unknown until recently. The GO method has become increasingly popular in recent years because of its advantages in terms of its ease of creating a model and of its representational and analysis power [2, 3]. This chapter is an attempt at providing a basic theory of GO method in terms of GO model, GO operation, and comparison with FTA and MCS.

### 1.1. Development of GO method

The major application of GO method is in establishing a system reliability model, and its quantitative and qualitative analysis. The chronological development of the GO method can be broadly divided into two periods of growth: (1) 1970s~the early 1990s, the basic model and theory of the GO method, its comparison with FTA method, and its GO operator type and function were explained in the research reports by the Electric Power Research Institute in US [4–8]; (2) After the late 1990s, the GO method has attracted more attention again, in particular in the People's Republic of China. Perhaps, the initial application of the GO method was in reliability and safety analysis of missile and weapons systems. Recently, the GO method has also been applied in the reliability analysis of defense systems, water, oil, and gas supply systems, manufacturing systems, transportation systems, power systems, and logistics management systems [2, 3]. Furthermore, the theory of GO method for a complex system with complex correlations (dependencies), closed-loop feedback, multiple functions, multiple fault modes, etc., has been developed from three aspects, which are GO model, basic GO algorithm, and GO method for complex systems with various characteristics [9–17]. Literature [2, 3] gave the overview of development of GO method in detail.

### 1.2. Further of GO method

The GO method has been widely used. There are a number of areas where the powerful advantages of the GO model and its reliability analysis methodology can be exploited.

Although there is a great deal of interest in the design of complex systems for reliability, research on designing these systems for both functional and structural reliability and life-cycle cost is needed. To go further, there is still a substantial research gap in the optimal design of systems for reliability and life-cycle cost, taking into account the issues of structure, function, behavior, and other characteristics, such as active or cold standby redundancy, and fault-tolerant mechanisms. In terms of modeling, analysis, and software tools, there are a number of research issues to consider. These include: (i) How to integrate the product structure, behavior, and functions in a reliability model; (ii) How to conduct the reliability analysis of complex systems accurately, thoroughly, and quickly; (iii) How to optimally allocate reliability among subsystems, taking into consideration the structural and functional hierarchy, as well as redundancy management techniques under resource constraints; and (iv) How to develop

software tools to support design of complex systems for reliability that are intuitive and support collaborative design. Considering the advantages of GO model and its reliability analysis method, the GO method not only can solve the existing problems above, but also it can further develop system reliability theory and application. Meanwhile, the corresponding software will also have a more extensive application prospects and important value.

Another important direction for research is the application of the GO methodology to related areas of quality control, fault diagnosis and prognosis, and condition-based maintenance.

## 2. GO model

The GO model is a key element of GO method. It is developed directly using product schematic diagrams, its structure, and its functional hierarchy. According to the GO model, the reliability analysis is conducted by GO operation. The GO model composed of GO operator and signal flow.

### 2.1. GO operator

GO operator contains function operator and logical operator, which represents the unit itself or logical relationship, respectively. Its data, type, and GO operation formula are the basic attributes of GO operator. There are 17 standard GO operators in basic GO theory, and their signs and description are shown in **Figure 1** and **Table 1**, respectively. In **Figure 1**,  $S$ ,  $C$ , and  $R$  are the input signal, GO operator itself, and output signal, respectively.

#### 2.1.1. Standard GO operator

In this section, six frequently-used standard GO operators are illustrated from aspects of description, operation rule table, and GO operation formula, respectively. In operation rule table,  $V_S$ ,  $V_C$ , and  $V_R$  are the state of the input signal, GO operator itself, and output signal, respectively, and  $0, \dots, N$  represent their state values. In GO operation formula,  $P_S(i)$ ,  $P_R(i)$ , and  $P_C(i)$  are the state probability of the input signal, GO operator itself, and output signal, respectively. The state cumulative probability of input signal  $A_S(i)$  and output signal  $A_R(i)$  are defined as

$$\begin{cases} A_S(i) = \sum_{j=0}^i P_S(j) & i = 0, \dots, N - 1; A_S(N) = 1 \\ A_R(i) = \sum_{j=0}^i P_R(j) & i = 0, \dots, N - 1; A_R(N) = 1 \end{cases} \quad (1)$$

### 1. Type 1 operator

- **Description:** It describes the unit with two states, which are success state (enable signal flow pass) and failure state (stop signal flow passing). For example, electric resistance, switch, valve, and pipeline.

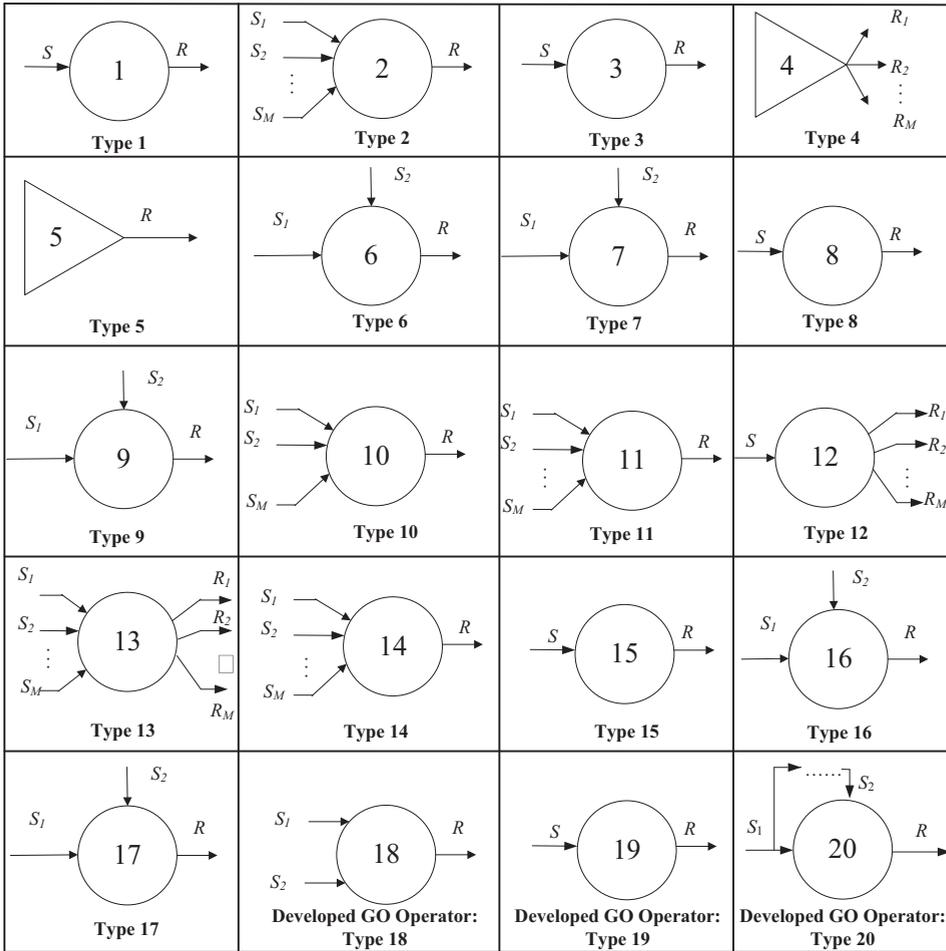


Figure 1. Signs of standard GO operators and three developed GO operators.

• **Operation rule table:**

$V_S$	$V_C$	$V_R$
$0, \dots, N-1$	1	$0, \dots, N-1$
$N$	1	$N$
$0, \dots, N$	2	$N$

• **GO operation formula:**

$$\begin{cases} P_R(i) = P_S(i) \cdot P_C(1), & i = 0, 1, \dots, N-1 \\ P_R(N) = P_S(N) \cdot P_C(1) + P_C(2) \end{cases} \quad (2)$$

## 2. Type 2 operator

- **Description:** It describes the logical relationship OR among some inputs signal and one output signal.

- **Operation rule table:**

$V_{S1}$	$V_{S2}$	...	$V_{SM}$	$V_R$
$0, \dots, N$	$0, \dots, N$	$0, \dots, N$	$0, \dots, N$	$\text{MIN}\{V_{S1}, V_{S1}, \dots, V_{SM}\}$

- **GO operation formula:**

$$\begin{cases} P_R(0) = A_R(0) \\ P_R(i) = A_R(i) - A_R(i-1), i = 1, \dots, N \end{cases} \quad (3)$$

## 3. Type 3 operator

- **Description:** It describes unit with failure state, operating state, and operating ahead state. For example, control system, contactor coil, and so on.

- **Operation rule table:**

$V_S$	$V_C$	$V_R$
$0, \dots, N$	0	0
$0, \dots, N-1$	1	$0, \dots, N-1$
$N$	1	$N$
$0, \dots, N$	2	$N$

- **GO operation formula:**

$$\begin{cases} P_R(0) = P_C(0) + P_S(0) \cdot P_C(1) \\ P_R(i) = P_S(i) \cdot P_C(1), i = 1, \dots, N-1 \\ P_R(N) = P_S(N) \cdot P_C(1) + P_C(2) \end{cases} \quad (4)$$

## 4. Type 5 operator

- **Description:** It describes the single input unit, which as system input. For example, battery, water source, and so on.

- **GO operation formula:**

$$P_R(i) = \begin{cases} P_j, i = I_j \\ 0, i \neq I_j \end{cases}, j = 1, \dots, L, i = 0, \dots, N \quad (5)$$

## 5. Type 6 operator

- **Description:** It describes the unit receiving signal to turn on. For example, electric water pump, contactor, and so on.

Operator type	Functional description
Type 1	Unit with failure state and operating state
Type 2	Logical relation "OR"
Type 3	Unit with failure state, operating state, and operating ahead state
Type 4	Multi-signal input unit
Type 5	Single input unit
Type 6	Unit receiving signal to turn on
Type 7	Unit receiving signal to turn off
Type 8	Unit with delayed response
Type 9	Output signal decided by the state of two input signal
Type 10	Logical relation "AND"
Type 11	Logical relation "k-out-of-m"
Type 12	Input signal can choose the output path
Type 13	Unit with multiple input signals and output signals
Type 14	Linear relation between multiple input signals and one output signal
Type 15	Logical relation of output signal affected by the probability event of input signal
Type 16	Unit requested to resume OFF-state
Type 17	Unit requested to resume ON-state

**Table 1.** Functional description of standard GO operators.

- **Operation rule table:**

$V_{S1}$	$V_{S2}$	$V_C$	$V_R$
$I_1(0, \dots, N)$	$I_2(0, \dots, N)$	0	$I_1$
$I_1(0, \dots, N)$	$I_2(0, \dots, N)$	1	$\text{MAX}\{I_1, I_2\}$
$I_1(0, \dots, N)$	$I_2(0, \dots, N)$	2	$N$

- **GO operation formula:**

$$\begin{cases} A_R(i) = A_{S1}(i)[P_C(0) + A_{S2}(i) \cdot P_C(1)], i = 0, \dots, N - 1 \\ P_R(N) = P_{S1}(N) + A_{S1}(N - 1)[P_C(2) + P_{S2}(N) \cdot P_C(1)] \\ P_R(0) = A_R(0) \\ P_R(i) = A_R(i) - A_R(i - 1) i = 1, \dots, N - 1 \end{cases} \quad (6)$$

## 6. Type 10 operator

- **Description:** It describes the logical relationship AND among some inputs signal and one output signal.

- **Operation rule table:**

$V_{S1}$	$V_{S2}$	...	$V_{SM}$	$V_R$
$0, \dots, N$	$0, \dots, N$	$0, \dots, N$	$0, \dots, N$	$\text{MAX}\{V_{S1}, V_{S1}, \dots, V_{SM}\}$

- **GO operation formula:**

$$\begin{cases} A_R(i) = \prod_{j=1}^M A_{Sj}(i), \quad i = 0, \dots, N - 1 \\ P_R(N) = 1 - A_R(N - 1) \\ P_R(0) = A_R(0) \\ P_R(i) = A_R(i) - A_R(i - 1) \quad i = 1, \dots, N - 1 \end{cases} \quad (7)$$

### 2.1.2. Developed GO operator

In this section, three developed GO operators are illustrated from aspects of description, operation rule table, and GO operation formula, respectively.

#### 1. Type 18 operator

- **Description:** It describes the logical relation of standby mode, which is combination of primary equipment group CG and standby equipment group CBG working under condition of primary equipment group faulting. The input signals and output signal of Type 18 operator are denoted as  $L_1, L_2,$  and  $R,$  respectively. The signal flow  $L_1$  represents primary equipment group working. The signal flow  $L_2$  represents standby equipment group working under condition of primary equipment group faulting, and the signal flow  $L_2$  is also the output signal of GO operator, which represents the standby equipment group. And the signal flow  $R$  represents standby structure working.  $L_1, L_2,$  and  $R$  have two states, which are state 1: success state and state 2: fault state.
- **Operation rule table:** The combination composed of a Type 18 operator and a Type 20 operator is often used to represent standby structure at any place.  $S_1, S_2,$  and  $S_0$  represent the input signal flows and output signal flow of Type 20 operator, respectively, and  $S_2$  represents also the input signal flow of Type 18 operator, i.e.,  $L_1$ .

$V_{CG}$	$V_{CBG}$	$V_{S1}$	$V_{S0}$	$V_{S2}$	$V_{L2}$	$V_R$
1	1	1	2	1	2	1
1	1	2	2	2	2	2
1	2	1	2	1	2	1
1	2	2	2	2	2	2
2	1	1	1	2	1	1
2	1	2	2	2	2	2
2	2	1	1	2	2	2
2	2	2	2	2	2	2

- **GO operation formula:**

$$\begin{cases} P_R(1) = P_{L1}(1) + P_{L2}(1) \\ P_R(2) = 1 - P_R(1) \\ P_{L1}(1) = P_{S1}(1) \cdot P_{CG}(1) = P_{S1}(1) - P_{S0}(1) \\ P_{L2}(1) = P_{S0}(1) \cdot P_{CBG}(1) \end{cases} \quad (8)$$

where  $P_{S0}(1)$  and  $P_{CBG}(1)$  are the success probability of  $S_0$  and  $CBG$ , respectively.

## 2. Type 19 operator

- **Description:** It describes the unit turning unstable operation into normal operating. The input signals and output signals of Type 19 operator are denoted as  $S$  and  $R$ , respectively.  $S$  is a multistate signal flow, which contains an operating state, a faulting state, and  $m$  unstable operation states. And  $m$  unstable operation states are divided into two kinds, which are  $q$  unstable operation states turned into operation state by Type 19 operator, and  $m-q$  unstable operation states.  $R$  is also a two-state signal flow, which contains operating state and faulting state.  $C$  is the unit itself, i.e., Type 19 operator, which contains operating state and faulting state.

- **Operation rule table:**

$V_S$	$V_C$	$V_R$
0	0	0
0	$N$	$N$
$1, \dots, q$	0	0
$1, \dots, q$	$N$	$N$
$q+1, \dots, m$	$0, N$	$N$
$N$	$0, N$	$N$

- **GO operation formula:**

$$\begin{cases} P_R(0) = P_S(0) \cdot P_C(0) + \sum_{j=1}^q P_S(j) \cdot P_C(0) = \sum_{j=0}^q P_S(j) \cdot P_C(0) \\ P_R(N) = P_S(0) \cdot P_C(N) + \sum_{j=1}^q P_S(j) \cdot P_C(N) + \sum_{j=q+1}^m P_S(j) \cdot P_C(0) + \sum_{j=q+1}^m P_S(j) \cdot P_C(N) \\ \quad + P_S(N) \cdot P_C(0) + P_S(N) \cdot P_C(N) \\ = \sum_{j=0}^q P_S(j) \cdot P_C(N) + \sum_{j=q+1}^m P_S(j) + P_S(N) \end{cases} \quad (9)$$

where  $P_S(j)$  is state probability of unstable operation states for  $S$ ,  $j = 1, 2, \dots, m$ ,  $P_S(0)$  and  $P_S(N)$  are state probability of operating state and faulting state for  $S$ , respectively,

$P_S(0) + P_S(N) + \sum_{j=1}^q P_S(j) + \sum_{j=q+1}^m P_S(j) = 1$ ;  $P_C(0)$  and  $P_C(N)$  are state probability of operating state and faulting state for C,  $P_C(0) + P_C(N) = 1$ .

### 3. Type 20 operator

- **Description:** It describes the signal flow of conditional operating mode, which is one of its input signal faults under the condition of another input signal success. The input signals and output signal of Type 20 operator are denoted as  $S_1$ ,  $S_2$ , and  $R$ , respectively.  $S_1$ ,  $S_2$ , and  $R$  have two states, which are state 1: success state and state 2: fault state. It is often used in standby structure.
- **Operation rule table:**

$V_{S1}$	$V_{CG}$	$V_{S2}$	$V_R$
1	1	1	2
1	2	2	1
2	1, 2	2	2

- **GO operation formula:**

$$\begin{cases} P_R(1) = P_{S1}(1) \cdot [1 - P_{CG}(1)] = P_{S1}(1) - P_{S2}(1) \\ P_R(2) = 1 - P_R(1) \end{cases} \quad (10)$$

where  $P_{S\alpha}(1)$ ,  $P_{CG}(1)$ , and  $P_R(1)$  are the success probability of  $S_\alpha$ , CG, and  $R$ , respectively,  $\alpha = 1, 2$ ;  $P_R(2)$  is the fault probability of  $R$ .

### 2.2. Signal flow

Signal flow represents specific fluid flow, such as oil, gas, electricity, and so on, or a logical process. It describes the relationships among the GO operator, its inputs, and outputs. And its attribution includes state value and state probability. The signal flow is used to connect the GO operator and as the direction of GO operation.

### 2.3. GO model

GO model is developed by using signal flows to connect GO operators according to the system principle diagram, engineering drawing, and function constitute directly. And it reflects the system characteristics visually. The proper GO model should be satisfied with:

- The operator in GO model must be labeled with type and number, especially the number which is unique. The first number in the GO operators represents the type of operator and the second number represents the numbering of the operators.
- There is at least one input GO operator (such as Type 4 or Type 5). Generally, the number of GO operator begins with the input GO operator.

- For each GO operator in GO model, its input signal flow must be the output signal flow of the other GO operators. Each signal flow must be labeled with the unique number, and the numbers on the signal line represent the signal flow numbering.
- The signal flow sequence must start with any input GO operator and end with the output signal flow of system. The GO model does not allow being a cyclic model. Generally, the number of signal flow should be labeled with the output signal flow of an input GO operator.

**Example:**

The structure diagram of pressurized water reactor (PWR) purification system and its GO model are, respectively, shown in **Figures 2** and **3**, and the GO operator type of component is presented in **Table 2**.

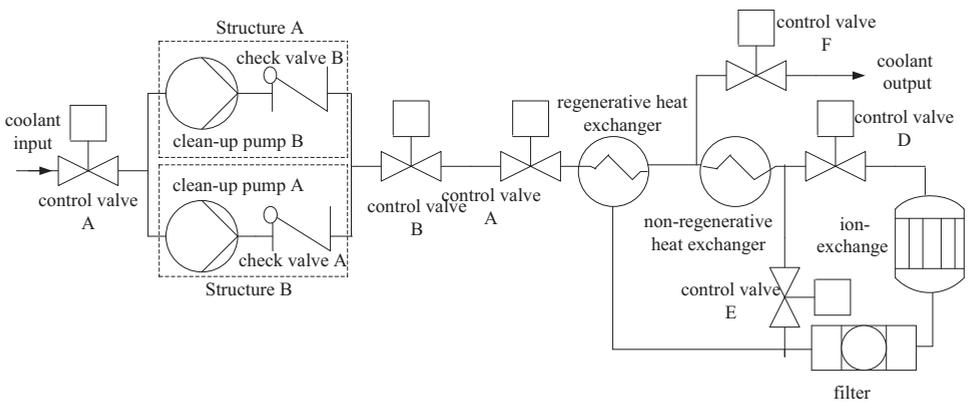


Figure 2. Structure diagram of PWR purification system.

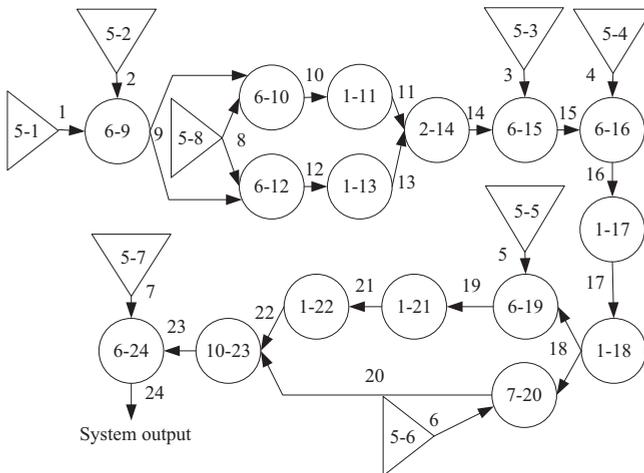


Figure 3. GO model of PWR purification system.

Operator number	Component	Operator type
1	Coolant input	5
2, 3, 4, 5, 6, 7	Control signal	5
8	Control power	5
9	Control valve A	6
10	Clean-up pump A	6
11	Check valve A	1
12	Clean-up pump B	6
13	Check valve B	1
15	Control valve B	6
16	Control valve C	6
17	Regenerative heat exchanger	1
18	Nonregenerative heat exchanger	1
19	Control valve D	6
20	Control valve E	7
21	Ion exchange	1
22	Filter	1
24	Control valve F	6

Table 2. GO operator type of component for Figure 2.

### 3. GO operation

GO operation is defined that it begins with output signal flow of input GO operators in GO model to calculate the state probability and state value of output signal flow for the next GO operator, and it will be finished until the system output signal flow is calculated after the sequence of signal flow. GO operation contains the quantitative and qualitative analyses. And the quantitative and qualitative analyses are conducted using the GO operation based on the GO algorithm, following the reliability analysis process of GO method. GO algorithm and GO analysis process are the key elements of GO operation.

#### 3.1. GO algorithm

The operational efficiency and accuracy of analysis are thus affected by the GO algorithm. The GO algorithm is comprised of a state combination algorithm [4] and a probability formula algorithm [18–21]. The number of state combinations for a complex system is very large, and the probability of a combination state cannot be easily computed. The probability formula algorithm is faster and easier than the state combination algorithm, and it is the mainstream GO algorithm. Thus, this section illustrates two kinds of probability formula algorithms, which are direct GO algorithm and GO algorithm with shared signals.

### 3.1.1. Direct GO algorithm

Direct GO algorithm is based on the calculation of state probability for signal flow, and the proper direct GO algorithm should be satisfied with:

- The state probability of input GO operator is the state probability of its output signal flow. The output signal flow of input GO operator is the input signal flow of the next GO operator.
- The state probability of output signal flow for the next GO operator is calculated based on its GO operation formula and data, and this output signal flow will be the next GO operator's input signal flow.
- Based on the above rules, following the signal flow sequence, the output signal flow of every GO operator in GO model can be obtained, and the GO operation will be finished until the state probability of output signal flow represented system output is calculated.
- When GO operator is executed with quantitative analysis, it is not necessary to list the corresponding state combination.

### 3.1.2. GO algorithm with shared signals

In GO model, the output signal flow of a GO operator often connects multiple GO operators, and they are the input signal flows of more than one GO operators, such output signal flow is called shared signal. If the GO operation adopts the direct GO algorithm, the quantitative analysis results will have biases. Thus, GO algorithm with shared signals was proposed in order to obtain more accurate result.

#### 1. Shared signal

There are two situations in GO model on shared signal, which are as follows:

- Completely contain. It means all items in formula of a signal flow's state probability have state probability of shared signal.
- Not completely contain. It means some items in formula of a signal flow's state probability have not state probability of shared signal.

The rules of the processing shared signals in GO algorithm with shared signals are as follows:

- Behind a shared signal, all of signal flows in sequence of GO model still have this shared signal, and there are two situations, which are completely contain and not completely contain.
- For multiple signal flows with the same shared signal, their joint state probability cannot be obtained by using the product of their state probabilities directly.
- For two signal flows completely contain the same one shared signal, their joint state probability can be obtained by using the product of their state probabilities to divide the state probability of shared signal.
- For multiple signal flows with the same one shared signal, their joint state probability formula can be obtained by turning a high stage term of shared signal in the formula into a one stage term of shared signal.

## 2. Probability formula

For a system with  $L$  shared signals, the probability formula of GO algorithm with shared signals is given by Eq. (10).

$$P_R = \sum_{K_1=0}^1 \sum_{K_2=0}^1 \cdots \sum_{K_L=0}^1 P_{RK_1K_2 \dots K_L} \prod_{l=1}^L [(1 - P_{Sl})(1 - K_l) + P_{Sl}K_l] \quad (11)$$

where  $P_{RK_1K_2 \dots K_L}$  is the cumulative probability of system output under a combination of all shared signals, and  $Sl$  is a shared signal in the system.  $K_l = 0$  and  $K_l = 1$  are the failure and success states of the shared signal  $l$ , respectively.  $P_{Sl}$  and  $P_R$  are success probability of shared signal  $l$  and system output, respectively. The item of  $\prod_{l=1}^L [(1 - P_{Sl})(1 - K_l) + P_{Sl}K_l]$  is the state probability for each combination of shared signals.

## 3. Calculating form

It is difficult and complex to derive mathematical formulae for a complex system with a large number of shared signals. A new form of Eq. (10) involves probabilistic weighting of shared signals. The probabilistic weighting improves operation efficiency greatly and avoids the need for complex mathematical formulae. The calculation process is shown in **Table 3**. In **Table 3**, numbers 1 and 0 represent success state and failure state of a shared signal  $Sl$ , respectively.

The success probability of system can be obtained by Eq. (11).

$$P_R = \sum_{j=1}^{2^L} A_j B_j \quad (12)$$

where  $A_j$  is the state probability for each combination of shared signals, and  $B_j$  can be obtained by the GO operation of the system, which sets success and failure probability of a shared signal to 1 and 0 depending on the state of shared signal for each combination of shared signals.

### 3.2. Reliability analysis process of GO method

The reliability analysis process of GO method is the criterion and prerequisite for conducting quantitative analysis and qualitative analysis. Generally, the steps of GO analysis process are as follows:

State combination of shared signal				State probability of combination	Success probability of system
$S_1$	$S_2$	...	$S_L$		
0	0	0	0	$A_1$	$B_1$
0	0	0	1	$A_2$	$B_2$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
1	1	1	1	$A_2^L$	$B_2^L$
Success probability of system				$P_R$	

**Table 3.** Calculation form of GO algorithm with shared signal.

**Step 1.** Conducting system analysis. The system analysis is the base of GO method, and it directly affects the developing GO model and conducting GO operation. First, to analyze system structure and system function constitutes according to the principle diagram, engineering drawing, or function flowchart of system. Second, to determine system characteristics, such as correlations, multistate, and so on. Then, to determine the interfaces, inputs, and output of system. Finally, to define the success rule of system according to system analysis result.

**Step 2.** Developing GO model. First, to select GO operator according to system analysis results, and then to establish GO model through the signal flow to connect GO operator.

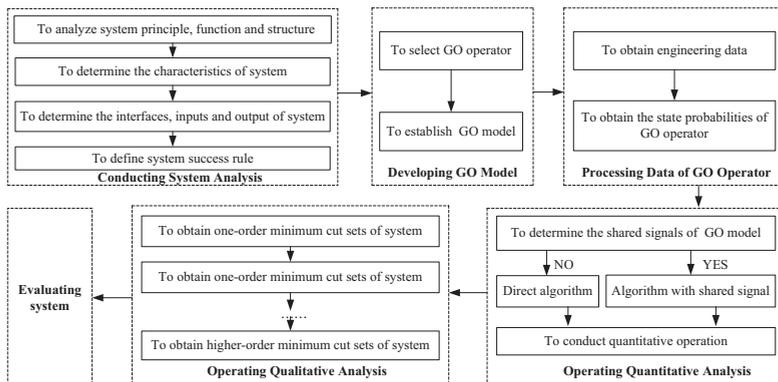
**Step 3.** Processing data of GO operator. According to engineering practice, to obtain the state probabilities of GO operator.

**Step 4.** Operating quantitative analysis. If the GO model does not contain shared signal, the direct algorithm can be selected to conduct GO operation. If the GO model contains shared signal, the GO algorithm with shared signal should be selected to conduct GO operation. Otherwise, it will cause a big error.

**Step 5.** Operating qualitative analysis. Setting the reliability of a function GO operator in GO model is 0, and the reliabilities of other GO operators are kept constant; in this case, if system reliability is 0 by GO operation, this GO operator will be a one-order minimum cut set. Setting the reliabilities of two function GO operators in GO model is 0 except one-order minimum cut set, and the reliabilities of other GO operators are kept constant; in this case, if system reliability is 0 by GO operation, the two GO operators will be a two-order minimum cut set. In the same way, the higher-order minimum cut sets of system can be obtained.

**Step 6.** Evaluating system. The quantitative analysis result and qualitative analysis result can be used as a guidance and theoretical basis for improving system and fault diagnosis of system, and so on.

Above all, the reliability analysis of GO method is formulated, as shown in **Figure 4**.



**Figure 4.** Reliability analysis process of GO method.

## 4. Example

In this section, taken a hydraulic oil supply system (HOSS) of an armored vehicle as a case, its reliability analysis is respectively conducted by GO method, FTA, and MCS in order to illustrate the usage and characteristics of GO method.

### 4.1. Reliability analysis of HOSS based on GO method, FTA, and MCS

#### 4.1.1. Reliability analysis of HOSS based on GO method

##### 1. Conducting system analysis

- To analyze system principle, function, and structure:** HOSS supplies oil for an armored vehicle, pump-motor system, pump-motor control system, hydraulic torque converter, and lubrication system. HOSS consists of pressure oil tank, pump P1, P2, P3, and P4, oil filter LF1, LF2, and LF3, pressure relay, bypass valve LF2B and LF3B, one-way valve CV1 and CV2, constant pressure valve RV1, RV2, and RV3, hydraulic torque converter TC, radiator HE, and so on, as shown in **Figure 5**. Oil is extracted by P1 from oil pan via LF1, and then oil is injected into pressure oil tank via LF2 and case inner passage. When LF2

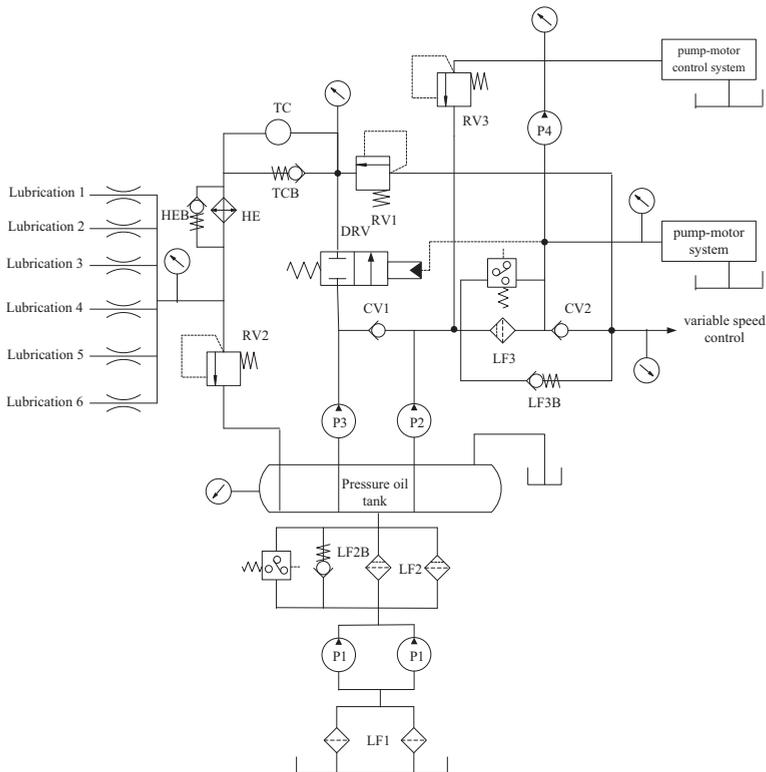


Figure 5. Diagram of HOSS.

group is obstructed and pressure between input and output becomes more than 0.5 mega Pascal, oil will be injected into pressure oil tank via LF2B. Oil is extracted by P2 from pressure oil tank; then oil is injected into CV2 via LF3 and then it is injected into hydraulic manifold block as the pressure oil provided for oil cylinder of variable speed control system and pump-motor control system by P4. LF3 and LF3B are another parallel structure, and they are same as LF2 group and LF2B. Because of the pressure of control oil which decreases a little at the situation of high speed, ingress oil of P2 can meet requirements of system. In addition, oil is extracted by P3 from pressure oil tank via DRV to TC. Then, ingress of oil is injected into lubrication system via HE. TC and TCB, and HE and HEB are same as LF2 group and LF2B. RV1, RV2, and RV3 are constant pressure valves of variable speed control and pump-motor system, lubrication system, and pump-motor control system, respectively.

- **To determine characteristics of system:** According to analysis of HOSS, the LF2 group and LF2B, LF3 and LF3B, TC and TCB, and HE and HEB are standby structures in HOSS. The standby equipments haven't the change-over switch.
- **To determine interfaces, input, and output of system:** According to analysis of HOSS, the oil from oil pan and pump group power are system inputs, and oil supply of variable speed control system, pump-motor system, pump-motor control system, hydraulic torque converter, and lubrication system are system outputs.
- **To define system success rule:** According to analysis of HOSS, success rule can be defined as a system that can provide oil to variable speed control system, pump-motor system, pump-motor control system, hydraulic torque converter, and lubrication system of an armored vehicle under high speed condition at steering situation without considering overload protection.

## 2. Developing GO model

- **To select GO operator:** According to the system analysis result and the types of GO operator, the functional GO operators and logical GO operators are selected to describe the units itself and logical relationships in HOSS, respectively, as presented in **Table 4**.
- **To establish GO model:** According to diagram of HOSS and analysis result of HOSS, the GO model of HOSS is developed from system input to system output, as shown in **Figure 6**.

## 3. Processing data of GO operator

According to statistical results of data from engineering, the success state probabilities of component in HOSS are presented in **Table 5**.

## 4. Operating quantitative analysis

It is shown in **Figure 6** that signal flow S1, S4, S7, S8, S15, S16, S17, and S22 are shared signals; the GO algorithm with shared signals should be adopted to conduct GO operation, and the calculation form is as presented in **Table 6**.

No.	Component	Operator type	No.	Component	Operator type
1	Oil pan	5	24	RV3	1
2, 3	LF1	1	25	P3	6
5, 6	P1	6	26	DRV	1
7	Pump group power	5	28	TC	1
9, 10	LF2	1	30	TCB	1
13	LF2B	1	32	HE	1
15	Pressure oil tank	1	34	HEB	1
16	P2	6	36	RV2	1
17	LF3	1	4, 8, 11, 27		2
18	CV2	1	37		10
20	LF3B	1	14, 21, 31, 35		18A
22	RV1	1	12, 19, 29, 33		20
23	P4	6			

Table 4. GO operator in GO model of HOSS.

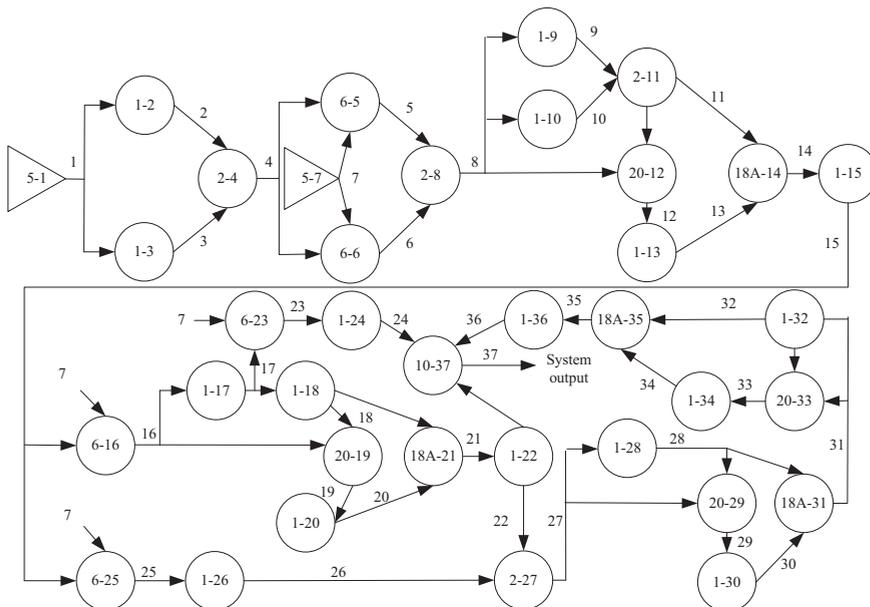


Figure 6. GO model of HOSS.

No.	Success state probability	No.	Success state probability	No.	Success state probability
1	0.99975006256	13	0.99867457268	24	0.99949819446
2	0.99413847887	15	0.99990821563	25	0.99867457268
3	0.99413847887	16	0.99950052524	26	0.99923302636
5	0.99950052524	17	0.99987532520	28	0.97865595028
6	0.99950052524	18	0.99891901058	30	0.99867457268
7	0.98676571976	20	0.99867457268	32	0.99548611722
9	0.99987532520	22	0.99949819446	34	0.99966858815
10	0.99987532520	23	0.99950052524	36	0.99949819446

Table 5. GO model of HOSS.

State of shared signal								State probability of combination	Success probability of system
S1	S4	S7	S8	S15	S16	S17	S22		
0	0	0	0	0	0	0	0	8.154e-32	0
0	0	0	0	0	0	0	1	1.620e-28	0
0	0	0	0	0	0	1	1	1.290e-24	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	0
1	1	1	1	1	1	1	1	0.98528272861	0.99846792597
Success probability of HOSS ( $P_{37}$ )								0.9837732025	

Table 6. Quantitative analysis result by calculation form of GO algorithm with shared signal for Figure 6.

### 5. Operating qualitative analysis

According to the Step 5 in Section 3.2, all minimum cut sets of HOSS can be obtained by multiple GO operations, as presented in Table 7.

#### 4.1.2. Reliability analysis of HOSS based on FTA and MCS

##### 1. Reliability analysis of HOSS based on FTA

The reliability analysis process of FTA mainly contains developing fault tree model of system, obtaining all minimum cut sets of system by using Fussell-Vesely method, and obtaining the system success probability according to the minimum cut sets of system. In this case, the brief fault tree model of HOSS is shown in Figure 7, and the brief analysis processes are presented in Table 8.

##### 2. Reliability analysis of HOSS based on MCS

The reliability analysis process of MCS mainly contains generating random numbers of success probability of GO operators, establishing simulation model, and obtaining success probability of system by operating a specified number of simulation times according to simulation model.

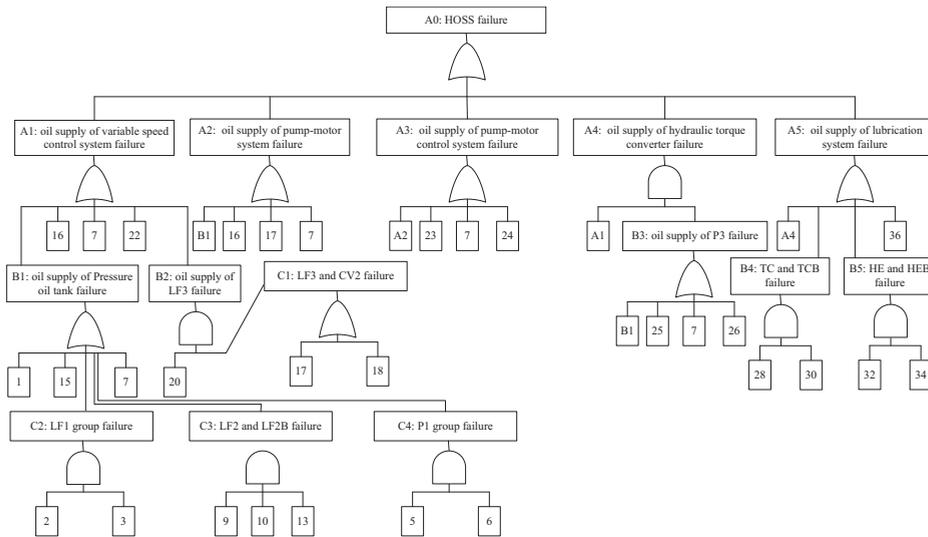


Figure 7. Brief fault tree model of HOSS.

Order	No.	Minimum cut sets	Order	No.	Minimum cut sets
1	1	Oil pan	2	2, 3	LF1 group
	7	Pump group power		5, 6	P1 group
	15	Pressure oil tank		18, 20	CV2, LF3B
	16	P2		28, 30	TC, TCB
	22	RV1		32, 34	HE, HEB
	23	P4	3	9, 10, 13	LF2 group, LF2B
	24	RV3			
	36	RV2			
	17	LF3			

Table 7. Qualitative analysis result by GO method for Figure 6.

In this case, the quantitative analysis result of HOSS by simulating 1 million times is 0.9838136000.

#### 4.2. Comparison with FTA and MCS

According to reliability analysis process and analysis results of GO method, FTA, and MCS, the qualitative analysis results by the GO method are consistent with FTA, and the quantitative analysis result by the GO method is very close to the result by MCS. It illustrates that both the accurate quantitative analysis result and qualitative analysis result can be obtained by multiple GO operations based on GO method, and the comparisons of GO method, FTA, and MCS are presented in Table 9.

Brief analysis processes					Minimum cut sets	Order
A0	A1	B1	1	1	1	1
			C2	2, 3	2, 3	2
			C3	9, 10, 13	9, 10, 13	3
			15	15	15	1
			C4	5, 6	5, 6	2
		7	7	7	1	
		16	16	16	1	
		7	7	7	1	
		B2	C1, 20	17, 20	-	-
				18, 20	18, 20	2
	22		22	22	1	
	A2		B1	B1	B1	-
			16	16	16	1
		7	7	7	1	
		17	17	17	1	
	A3	A2	A2	<b>A2</b>	-	
		23	23	23	1	
		7	7	7	1	
		24	24	24	1	
	A4	A1, B3	A1, B1	A1, B1	-	
A1, 25			A1, 25	-		
A1, 7			A1, 7	-		
A1, 26			A1, 26	-		
A5	A4	A4	A4	-		
	B4	28, 30	28, 30	2		
	B5	32, 34	32, 34	2		
	36	36	36	1		

**Table 8.** Brief analysis processes of FTA for HOSS.

Feature	GO	FTA	MCS
Modeling oriented	Success	Failure	Logical
Modeling method	Decision-making tree	Fault tree	Logical language description
Model consistency	High	Poor	High
Model structure	Similar with schematic diagram	Hierarchy logical graph	Logical relationship
Model size	Small and compact	Hierarchy and large	Large

Feature	GO	FTA	MCS
Model element	Components and logical gate, characteristics	Failure event and logical gate	Components and logical language
Model description	Reflect system natural	Reflect cause and effect of failure	Reflect logical relation in system
Model sign	Various types and typical	Few types and not typical	Few types and not typical
Quantitative analysis	Accurate, stable	Approximative	Accurate, unstable
Qualitative analysis	Easy	Complex	-

**Table 9.** Comparisons of GO method, FTA, and MCS.

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