The Nuclear Mean Field Theory and Its Application to Nuclear Physics

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1. Introduction

Our universe consists of substance. Atoms and molecules are basic components of material. Each atom contains a nucleus which is spread in a small area of atom, and electrons. Also, a nucleus contains Neutrons and protons. It is well known today that electrons in atom and Neutrons and protons in the nucleus are interacting together through different forces. It is clear today that the source of different interactions are composed of four basic forces of the universe, namely gravitational, coulomb, strong and weak nuclear interactions.

In quantum mechanics, to study a particle, it is necessary to have knowledge about its interaction with the surrounding media. The Schrödinger equation is a second-order differential equation that is solved to obtain energy spectrum and wave functions of a particle in quantum mechanics. For a many-body system such as atom or nucleus, it is not possible to solve a set of Schrödinger equations to obtain energy spectrum and wave functions analytically. Therefore in such situations, it is necessary to use an average potential which is a mean potential of all interacting forces acting upon a single particle. Then the Schrödinger equation is should be solve for a single particle. This procedure is called the mean field method [1, 2, 3].

To review this method consider a system consisting of N identical interacting particles. The Hamiltonian of system composed of kinetic energy, T, and potential energy , V, is defined as

$$H = T + V = \sum_{i=1}^{N} t(r_i) + \sum_{\substack{i,j<1 \\ i<j}}^{N} v(r_i, r_j) = \sum_{i=1}^{N} -\frac{\hbar^2}{2m_N} \nabla_i^2 + \sum_{\substack{i,j<1 \\ i<j}}^{N} v(r_i, r_j) \tag{1}$$

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where m_N is the mass of each particle, and r_i denotes the coordinates of particle i. A summed single particle potential energy, so far undefined, can be added and subtracted of the Hamiltonian to obtain the following relation,

$$H = \left[T + \sum_{i=1}^{N} v(r_i) \right] + \left[V - \sum_{i=1}^{N} v(r_i) \right] = H_{MF} + V_{RES} \tag{2}$$

where

$$H_{MF} = T + \sum_{i=1}^{N} v\left(r_i\right) \equiv T + V_{MF} =$$
$$\sum_{i=1}^{N} \left[t\left(r_i\right) + v(r_i) \right] \equiv \sum_{i=1}^{N} h\left(r_i\right) \tag{3}$$

is the mean field Hamiltonian of the system and

$$V_{RES} = V - \sum_{i=1}^{N} v(r_i) = \sum_{\substack{i,j<1 \\ i<j}}^{N} v(r_i, r_j) - \sum_{i=1}^{N} v\left(r_i\right), \tag{4}$$

is the mean residual interaction. It should be noted that the residual interaction is related to the strength of the actual interaction and can be reduced if the mean field potential is close to the actual potential of the system.

Actually, the mean field method is an approximation in which each particle of system moves under an external field generated by the remaining $N-1$ particles. This mean potential, V_{MF}, can be considered as an average of all possible interactions of nucleons during the short time interval ΔT, between the selected nucleon and its surrenders,

$$V_{MF} = \sum_{i=1}^{N} v(r_i), \; v\left(r_i\right) = \frac{1}{\Delta T} \int_{T}^{T+\Delta T} dt \sum_{\substack{j=i \\ j \neq i}}^{N} v\left(r_i\left(t\right), \; r_j(t)\right). \tag{5}$$

It is important to know that the time average idea was considered only for clearance of the subject and not applicable in practice unless one studies the thermo-dynamical behavior of nucleus.

Therefore the idea of using mean field theory capable of reducing many particles interacting system in to a system of non-interacting (quasi-particles) considered in an external field , VMF which is the mean potential of possible forces of interaction. The mean field potential is

considered such that the stationary Schrödinger equation is solved simply to obtain single particle states and their related energy spectrum. These single-particle states are used to construct the N particle wave function as follows.

The corresponding N-particle Schrödinger equation is used to obtain solutions of the mean-field Hamiltonian H_{MF}

$$H_{MF}\Psi_0\left(r_1,r_2,....r_N\right) = E\Psi_0\left(r_1,r_2,....r_N\right).$$
(6)

The wave function $\Psi_0(r_1, r_2, ...r_N)$ can be separated by using the ansatz single particle wave functions

$$\Psi_0\left(r_1,r_2,....r_N\right) = \phi_{\alpha_1}\left(r_1\right)\phi_{\alpha_2}\left(r_2\right)....\phi_{\alpha_N}\left(r_N\right).$$
(7)

Substituting this ansatz in to the Schrödinger equation (6) yields N identical one-particle Schrödinger equations

$$h(r)\phi_\alpha\left(r\right) = \varepsilon_\alpha\phi_\alpha\left(r\right),$$
$$h(r) = t(r) + v(r) = -\frac{\hbar^2}{2m_N}\nabla^2 + v(r).$$
(8)

With the quasi-particle energy, ε_α, that is satisfies the following condition

$$E = \sum_{i=1}^{N}\epsilon_\alpha.$$
(9)

The wave function of the many-body system is thus an anti symmetric product of single-particle wave functions which are one-particle wave functions of an external potential well. In summary the mean field theory reduces the complicated many-body problem in to a simple one-particle system.

The main idea in this approach is to determine the mean field potential or in particular, an appropriate mean field potential in which the residual interactions between the quasi-particles should be optimal. To do so, one may seek an optimal set $\{\phi_\alpha(r)\}$ of one-quasi-particle states. This is a Rayleigh-Ritz variational approximation in which the variation $\phi_\alpha(r) \to \phi_\alpha(r) + \delta\phi_\alpha(r)$ of the single-particle orbital is minimized

$$E_{gs} = \left\langle\Psi_0|H|\Psi_0\right\rangle$$
$$H = T + V_{MF} + V_{RES}$$
(10)

As a starting point, one may construct an ansatz wave function. It is customary to use a product of single particle wave functions as Eigen function of the system,

$$\Psi_0\left(r_1, r_2, \ldots r_N\right) = \prod_{i=1}^{N} \phi_{\alpha_i}\left(r_i\right) \tag{11}$$

It is an anti-symmetrized product ansatz wave function following the Hartree-Fock method and is called the Slater determinant of the given single particle states

$$\Psi_0\left(r_1, r_2, \ldots r_N\right) = C\left[\prod_{i=1}^{N} \phi_{\alpha_i}\left(r_i\right)\right]. \tag{12}$$

Here $\Psi_0(r_1, r_2, \ldots r_N)$ is an anti-symmetric wave function. Also C is the normalization constant. For instance, consider a three-particles system with its single-particle Eigen states labeled 1, 2, and 3. Then the normalized anti-symmetric state, or the Slater determinant, is

$$\Psi_0\left(r_1, r_2, \ldots r_N\right) = \frac{1}{\sqrt{6}} \begin{vmatrix} \phi_{\alpha_1}\left(r_1\right) & \phi_{\alpha_1}\left(r_2\right) & \phi_{\alpha_1}\left(r_3\right) \\ \phi_{\alpha_2}\left(r_1\right) & \phi_{\alpha_2}\left(r_2\right) & \phi_{\alpha_2}\left(r_3\right) \\ \phi_{\alpha_3}\left(r_1\right) & \phi_{\alpha_3}\left(r_2\right) & \phi_{\alpha_3}\left(r_3\right) \end{vmatrix}. \tag{13}$$

The energy E of the system has to be varied under the constraint that the normalization of Ψ_0 is preserved, i.e. $\langle\Psi_0|\Psi_0\rangle$=1. This leads to the constrained variational problem,

$$\delta\left(\frac{\langle\Psi_0|H|\Psi_0\rangle}{\Psi_0|\Psi_0}\right) = 0,$$

which can be transformed in to an unconstrained one by minimizing the energy for normalized wave function, $\Psi_0(r_1, r_2, \ldots r_N)$. After performing the variation, the single-particle energy, ε_α, is can also be obtained.

One powerful method to address such uncertainties is the following Hartree consistent equation [4,5],

$$-\frac{\hbar^2}{2m_N}\nabla^2\phi_\alpha\left(r\right) + V_{H(F)}\left(\{\phi_i\}\right)\phi_\alpha\left(r\right) = \epsilon_\alpha\phi_\alpha\left(r\right),$$
$$i = 1, 2, \ldots\ldots N, \ \alpha = 1, 2, \ldots\ldots\infty \tag{14}$$

This equation is like the Schrödinger equation except that the simple potential term, $V\left(r\right)$, is replaced with a function of unknown wave function,

$$V\left(r\right) = V_{H(F)}\left(\left\{\phi_i(r)\right\}\right)$$

Here, the Hartree mean field potential, $V_{H(F)}$, is different from, V_{HF}, Hartree-Fock mean field.

The differential equation (14) is nonlinear and therefore, much more difficult to solve than the regular Schrödinger equation. The solution can only be carried using consistent iteration method. In this procedure, one can start using a complete set of guessed single-particle states $\left\{\phi_i^0\ (r)\right\}$, $i = 1, ...N$ to calculate the initial potential term, $V_{H(F)}^{(0)}$. In the next step, the equation for a complete set of new wave functions $\left\{\phi_\alpha^{(1)}\ (r)\right\}$ $\alpha = 1, ...\infty$ is solved to obtain Eigen energies $\varepsilon_\alpha^{(1)}$. The procedure is then repeated with new Eigen function $\phi_\alpha^{(1)}\ (r)$ to obtain the new potential , $V_{H(F)}^{(1)}$. This approach can be depicted through the following schematic diagram,

$$\phi_i^0\left(r\right) \rightarrow V_{H(F)}^{(0)} \rightarrow \epsilon_\alpha^{(1)},$$
$$\epsilon_\alpha^{(1)} \xrightarrow{\phi_i^{(1)}} V_{H(F)}^{(1)} \rightarrow ... \rightarrow \phi_\alpha^{(n)}, \epsilon_\alpha^{(n)}.$$

(15)

This procedure is repeated to achieve self-consistency for wave functions (or Eigen energies). This means that after each loop the resultant wave function or Eigen energies compared with the starting wave function or Eigen energies and when their difference becomes less than a given preset limit, i.e.

$$\left\|\phi_\alpha^{(n-1)} - \phi_\alpha^{(n)}\right\| < preset\ limit,$$

the procedure is repeated, otherwise, it will be automatically terminated. Where the ∥ ...∥ denotes the norm.

The results of each run, contain a self-consistent mean field, $V_{H(F)}(r)$, the Eigen state, $\phi_\alpha\ (r)$, and its associated Eigen energies , ε_α, are all simultaneously generated. We may also note that for a finite potential-well, there will be, in addition to the bound states, an infinite number of unbound states.

In our discussions, the generated mean-field potential is a central one, that is only a function of r. Central mean field potentials describe only systems with spherical symmetry such as spherical nuclei or atoms. This is because of natural real forces that are conservative and satisfy the conservation of energy.

In some convenient way to avoid self-consistency loops, a phenomenological potential like a simple square well with finite depth, simple harmonic oscillator well and complicated Woods-Saxon with considerable parameters that can be determined using the fit of potential with experimental data, is introduced.

2. Applications of mean-field theory in nuclear physics

Over the years after Rutherford's valuable experiments that suggest nuclei for atom, many theoretical and experimental attempts have been done to obtain knowledge about the stability of nuclei. It is clear today that a nucleus of mass number A, Neutron number N and proton number (atomic number) Z, consists of A strongly interacting nucleons (protons and neutrons in the nucleus without considering their different properties called nucleon.). In addition to the strong nuclear force that is responsible for nuclear stability, the protons also sense the attractive coulomb potential because of their charge. In regular nuclear physics, the protons and neutrons are considered the point particles without any internal structure. This is an excellent approximation when the aim is to study nuclear structure at low energies. In such approach, the nuclear forces are considered a central attractive force with proper specifications like independence of charge and low range. Note that in advance models of nuclear physics such as the Yukawa Meson exchange model, it is believed that nucleons constructed quarks and interact together through the meson exchange mechanism in the base of the particle physics lows. The lightest nucleus is Deuterium with one neutron and one proton. The interaction of nucleons in the nucleus can be studied both theoretically and practically using simple Deuteron nucleus. This two-nucleon system is described by two-body interaction matrix elements, without a detailed account of the methods used to obtain them. On the other hand, the A - nucleons nucleus in quantum mechanics using the Schrödinger equation is not a solvable problem analytically at least for $A>10$. Therefore, one has to look for a reasonable approximate method to solve this many-body problem consisting of strongly interacting nucleons. A powerful approximation is to convert such many-body system in to a non-interacting system of quasi-particles using a suitable external mean field potential. The remaining interactions, called residual interaction, can be treated as a perturbation potential in the base of perturbation approximation. As discussed earlier, the transformation of system of particles in to quasi-particles is not simple, and its success depends on the nuclear system under consideration.

As mentioned above, a conventional approach is to select a particular type of mean field potential to avoid the steps leading to self-consistency. The selected mean field potential and considered remaining residual interactions as approximations produce the preciseness of the obtained results. The simplest custom potential is the three-dimensional harmonic oscillator potential well

$$V_{HO}(r) = -V_1 + kr^2 = -V_1 + \frac{1}{2}m\omega^2 r^2 \tag{16}$$

where V_1 and k are the parameters to be fitted to the practical data for best result. A common, more realistic choice is the Woods–Saxon potential [6]

$$v_{WS}(r) = \frac{-V_0}{1+e^{(r-R)/a}}.$$

where V_0, R and a are the nuclear potential depth, the nuclear radius, and the surface diffuse-ness, respectively. They are parameterized as follows,

$$V_0 = \left(51 \pm 33 \frac{N-Z}{A} \right) MeV , \ R = r_0 A^{\frac{1}{3}} = 1.27 \, A^{\frac{1}{3}} \, fm, \ a = 0.67 \ fm.$$

The + and – signs are considered for protons and neutrons, respectively. In the case when there is no distinction between protons and neutrons a suitable average value of $V_0 = 57 \, MeV$ can be used for nucleons.

The Woods–Saxon potential, v_{WS}, is a suitable choice for the mean field potential however it is a complicated function of, r, and it is not an analytically solvable one. To overcome this problem, it is possible to select the proper three- dimensional oscillator potential with energy quantum, $\hbar\omega$ and depth, V_1. The energy difference of levels, $\hbar\omega$, and depth, V_1, can be obtained with a best fit to the Woods–Saxon potential, v_{WS}, as a function of, V_0, R and a as the nuclear potential depth, nuclear radius, and surface diffuseness of the Woods–Saxon potential, respectively. The wave functions and energy spectrum of equivalent harmonic oscillator potential agreed well with the Woods–Saxon potential ones especially near the bottom of the wells in low energies. The difference of these potentials increases when the potential ap-proaches zero. Actually the major difference of these potentials is that the harmonic oscillator potential varies more sharply than the Woods–Saxon one near the surface of the nucleus.

2.1. The spin–orbit interaction

Sometimes in 1949, Meyer and independently, Haxel, Jensen, and Swees showed that if in addition to mean field central potential, V_{MF}, a non-central potential is included in the Schrödinger equation, all closed shell nucleon numbers can be obtained successfully. These numbers 2, 8, 20, 28, 50, 82, and 126 are called magic numbers because the origin of these numbers was not known at that time. The Woods-Saxon or its equivalent harmonic oscillator central potential is not able to reproduce experimentally observed precise data of the single-particle structure energies of the nucleus using the mean field approach.

The non-central potential due to the interaction between the spin of nucleons with the angular momentum of orbital that nucleons located on it, is called spin-orbit interaction. As a result of spin-orbit interaction [7, 8], the nuclear energy level for a given l (except for $l = 0$) is split in to two sublevels. The sublevels are characterized by total angular momentum numbers equal to $\left(l + \frac{1}{2} \right)$ and $\left(l - \frac{1}{2} \right)$ corresponding to whether the spin is parallel or anti-parallel to the orbital angular momentum. Each sublevel with spin j accommodates $(2j+1)$ neutrons or protons. The same interaction with a different structure is observed in atoms with a different sign as in the nucleus.

Consider that the harmonic oscillator central potential is produced only for the first three observed magic numbers 2, 8, and 20. To obtain the remaining numbers 28, 50, 82 and 126, it is necessary to add a spin-orbit interaction potential to the Schrödinger equation.

The origin of the spin–orbit interaction is not the same in atoms and nucleus. The atomic spin–orbit force is due to a well-known electromagnetic interaction, and the scale of energy separation is in the order of milli-electronvolts, while the energy difference of sublevels separated because of the nuclear spin-orbit interaction is in the order of million electronvolts and its origin is not well understood yet. In most cases, this force is considered phenomenologically. For the spin–orbit term, we use [9]

$$v_{LS}(r) = v_{LS}^0 \left(\frac{r_0}{\hbar}\right)^2 \frac{1}{r}\frac{d}{dr}\left[\frac{1}{1+e^{(r-R)/a}}\right] \tag{17}$$

The second pair of parentheses guarantees that the derivative does not operate on the wave function when substituted in the radial Schrödinger equation. The r dependence of this interaction arises from its central nature.

The derivative part of this potential is often neglected for simplicity and $v_{LS}(r)$ is replaced by a constant; however, to obtain precise results, the radial part should be considered. We have

$$v_{LS}^0 = 0.44V0.$$

To obtain the strength of the spin-orbit part, we use

$$J^2 = (L+S)^2$$
$$J^2 = L^2 + S^2 + 2L.S,$$

and its expectation value for the nuclear wave equation made

$$< L.S > = \frac{\hbar^2}{2}\left[j(j+1) - l(l+1) - \frac{1}{2}(\frac{1}{2}+1)\right]$$
$$< L.S > = \frac{\hbar^2}{2}l \quad for \quad j = l+\frac{1}{2},$$
$$< L.S > = \frac{\hbar^2}{2}(-l-1) \quad for \quad j = l-\frac{1}{2}.$$

In addition to the mean field plus spin–orbit interaction, protons in nuclei interact together via the coulomb force, which is defined by the following relation, considering nuclei as a sphere with a constant charge density [10]

$$V_C(r) = \frac{Ze^2}{4\pi\varepsilon_0}\begin{cases}\dfrac{3-\left(\dfrac{r}{R}\right)^2}{2R} & r \le R, \\[2ex] \dfrac{1}{r} & r \ge R\end{cases} \tag{18}$$

To obtain the energy spectrum and wave functions for neutrons, one needs to solve the radial Schrödinger equation for the Woods-Saxon and spin-orbit potentials. Such second-order differential equation cannot be solved analytically. To solve this complicated differential equation, it is necessary to introduce some new variables and use reasonable approximations. By introducing new variable [11] $y = \frac{1}{1+\exp\frac{(r-R)}{a}}$, the Woods–Saxon potential reduces to its simple form $V_{WS} = V_{0y}$ while the spin-orbit term changes to $V_{LS} = \frac{(y-y^2)}{R_0 + a\ln\left(\frac{1}{y}-1\right)}$. For orbits with small l, the Taylor expansion of the $\frac{1}{r}$ near $r = r_m$, is reasonable. According to the definition of variable y we have, $f(y) \equiv \frac{1}{1+\exp\frac{(r-R)}{a}}$ hence by expanding $f(y)$ around $y_m \equiv \frac{1}{1+\exp\frac{(r_m-R_0)}{a}}$ with $0 < y_m < 1$, since $0 < y < 1$, y^3 and the higher terms are negligible, the radial part of the spin-orbit term can be approximated using [12],

$$-\frac{a}{r}\frac{d}{dr}\left(\frac{1}{1+\exp\frac{(r-R)}{a}}\right) = \frac{1}{r_m}\left(C_0 + C_1 y + C_2 y^2\right), \tag{19}$$

where C_0, C_1, and C_2 are dimensionless coefficients and evaluated as

$$C_0 = \frac{-ar_m y_m + 2a^2 y_m}{2r_m^2(1-y_m)},$$

$$C_1 = 1 + \frac{ar_m y_m + 2a^2}{r_m^2(1-y_m)},$$

$$C_2 = -1 + \frac{ar_m(1+2y_m) + 2a^2}{r_m^2(1-y_m)2y_m}.$$

Likewise, the Taylor expansion is applicable for $\frac{1}{r^2}$,

$$\frac{1}{r^2} \approx \frac{1}{r_m^2}\left(D_0 + D_1 y + D_2 y^2\right),$$

where D_i's, similar to the C_i', are obtained through

$$D_0 = 1 - \frac{4a}{r_m(1-y_m)} + \frac{3a^2 + ar_m}{r_m^2(1-y_m)^2},$$

$$D_1 = \frac{6a}{r_m y_m(1-y_m)} - \frac{6a^2 + 2ar_m}{r_m^2 y_m(1-y_m)^2},$$

$$D_2 = \frac{3a^2 - ar_m(1-2y_m)}{r_m^2(1-y_m)2y_m^2}.$$

This type of expansion has been widely used for differential equations resulting from the Schrödinger equation with different potentials [12].

By means of these expansions, the spin-orbit term transforms into

$$VLS = (C0 + C1y + C2y2),$$

and the centrifugal term is obtained

$$Vc.f=(D0+D1y+D2y2).$$

By using these expansions, the spin-orbit term transforms into $V_{LS} \propto (C_o + C_1y + C_2y^2)$ and the centrifugal term is changed to the favorable type $V_{CF} \propto (D_o + D_1y + D_2y^2)$. The substitution of V_{LS} and V_{CF} as a function of variable y into the Schrödinger equation transforms this equation in to the following analytically solvable differential equation

$$y(1-y)\left[y(1-y)\frac{d^2R(y)}{d\,y^2} + (1-2y)\frac{dR(y)}{d\,y^2} \right] +$$
$$+\frac{2ma^2}{\hbar^2}[V_{0y} + V_{LS}^{'(0)}(C_o + C_1y + C_2y^2)$$
$$-\frac{\hbar^2}{2m}\frac{l(l+1)}{r_m^2}(D_o + D_1y + D_2y^2)\,R(y)=0$$
$$V_{LS}^{'(0)} = \frac{r_0^2V_{LS}^{(0)}}{2ar_m}(j(j+1)-l(l+1)-3/4)$$

This equation can be transformed into the following simple form,

$$y(1-y)\frac{d^2R(y)}{dy^2} + (1-2y)\frac{dR(y)}{dy} + \frac{-\varepsilon^2 + \beta^2y - \gamma^2y^2}{y(1-y)}R(y)=0 \qquad (20)$$

Equation (20) can be transformed into the well-known form of hypergeometric differential equation or, alternatively Nikiforo-Avorono (NU) type [13]. The obtained results using the NU method are

$$-\varepsilon^2 = \frac{2ma^2}{\hbar^2}\left(E + V_{LS}^{'(0)}\right) - \frac{l(l+1)}{r_m^2}D_0$$

$$\beta^2 = \frac{2ma^2}{\hbar^2}\left(V_0 + V_{LS}^{'(0)}C_1\right) - \frac{l(l+1)}{r_m^2}D_1$$

$$\gamma^2 = -\frac{2ma^2}{\hbar^2}\left(E + V_{LS}^{'(0)}C_2\right) - \frac{l(l+1)}{r_m^2}D_2$$

$$R = C\frac{\Gamma(2\varepsilon-1)\Gamma(-2i\lambda)}{\Gamma(\varepsilon+\eta'+1-i\lambda)\Gamma(\varepsilon-\eta'-i\lambda)}\left[(1-y)^{i\lambda} + \frac{\Gamma(2i\lambda)\Gamma(\varepsilon-\eta'+1-i\lambda)\Gamma(\varepsilon-\eta'-i\lambda)}{\Gamma(-2i\lambda)\Gamma(\varepsilon-\eta'+1+i\lambda)\Gamma(\varepsilon-\eta'+i\lambda)}(1-y)^{-i\lambda}\right]$$

where Γ is the well-known gamma function, and C is the normalization constant. λ, μ, and η' are defined as follows

$$\lambda = \sqrt{\beta^2 + \varepsilon^2 + \gamma^2}$$

$$\mu = i\lambda$$

$$\eta' = \sqrt{\gamma^2 - 1/4} - 1/2$$

Note that λ is valid only for the $\beta^2 > \varepsilon^2 + \gamma^2$ condition. In a special case where $l=0$, the solution reduces to its simple form. Also, the energy eigenvalues are obtained as a function of z satisfying the following relation

$$-\frac{z}{\sqrt{1-z^2}} = \tan\varnothing(z), \; z = \sqrt{1 - \frac{|E|}{V_0}}$$

which Φ (z) can be evaluated using a graphical method.

$$\varnothing(z) = k_0 R_0 z +$$

$$+\tan^{-1}\left(\frac{z}{\sqrt{1-z^2+\left(\frac{1}{k_0a}\right)}}\right) - \sum_{n=0}^{\infty}\left[\tan^{-1}\left(\frac{2k_0a}{n}z\right) - \tan^{-1}\left(\frac{z}{\sqrt{1-z^2+\left(\frac{n}{k_0a}\right)}}\right) - \tan^{-1}\left(\frac{z}{\sqrt{1-z^2+\left(\frac{n+1}{k_0a}\right)}}\right)\right].$$

Note that

$$k = \frac{\lambda}{a} = \left[\frac{2m}{\hbar^2}\left(V_0' - |E|\right)\right]^{1/2},$$

and

$$k_0 = \frac{2m}{\hbar^2}(V_0^{'})^{1/2} \, , \, z = \frac{k}{k_0} \, .$$

Finally,

$$\left| E \right| = V_0 \left(1 - Z^2 \right)$$

The results obtained in this special case are in agreement with the results obtained using other methods [14].

3. Conclusions

In this chapter we briefly discussed the idea of mean field theory as an improvable approximation method for many-body problems of identical particles like atoms and nucleus that cannot be solved analytically. We have shown that for a system of A - nucleons nucleus by considering a suitable potential using this model, one is able to obtain energy spectrum and wave equations. However, the obtained results cannot reproduce the measured nuclear spectroscopy, but one may hope to become successful by considering an accurate potential in the Schrödinger equation.

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