

# Pareto Optimum Design of Robust Controllers for Systems with Parametric Uncertainties

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## 1. Introduction

The development of high-performance controllers for various complex problems has been a major research activity among the control engineering practitioners in recent years. In this way, synthesis of control policies have been regarded as optimization problems of certain performance measures of the controlled systems. A very effective means of solving such optimum controller design problems is genetic algorithms (GAs) and other evolutionary algorithms (EAs) (Porter & Jones, 1992; Goldberg, 1989). The robustness and global characteristics of such evolutionary methods have been the main reasons for their extensive applications in off-line optimum control system design. Such applications involve the design procedure for obtaining controller parameters and/or controller structures. In addition, the combination of EAs or GAs with fuzzy or neural controllers has been reported in literature which, in turn, constitutionally formed intelligent control scheme (Porter et al., 1994; Porter & Nariman-zadeh, 1995; Porter & Nariman-zadeh, 1997). The robustness and global characteristics of such evolutionary methods have been the main reasons for their extensive applications in off-line optimum control system design. Such applications involve the design procedure for obtaining controller parameters and/or controller structures. In addition to the most applications of EAs in the design of controllers for certain systems, there are also much research efforts in robust design of controllers for uncertain systems in which both structured or unstructured uncertainties may exist (Wolovich, 1994). Most of the robust design methods such as  $\mu$ -analysis,  $H_2$  or  $H_\infty$  design are based on different norm-bounded uncertainty (Crespo, 2003). As each norm has its particular features addressing different types of performance objectives, it may not be possible to achieve all the robustness issues and loop performance goals simultaneously. In fact, the difficult mixed norm-control methodology such as  $H_2/H_\infty$  has been proposed to alleviate some of the issue of meeting different robustness objectives (Baeyens & Khargonekar, 1994). However, these are based on the worst case scenario considering in the most possible pessimistic value of the performance for a particular member of the set of uncertain models (Savkin et al., 2000). Consequently, the performance characteristics of such norm-bounded uncertainties robust designs often degrades for the most likely cases of uncertain models as the likelihood of the

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worst-case design is unknown in practice (Smith et al., 2005). Recently, there have been many efforts for designing robust control methods. In these methods for reducing the conservatism or accounting more for the most likely plants with respect to uncertainties, the probabilistic uncertainty, as a weighting factor, propagates through the uncertain parameter of plants. In fact, probabilistic uncertainty specifies set of plants as the actual dynamic system to each of which a probability density function (PDF) is assigned (Crespo & Kenny, 2005). Therefore, such additional information regarding the likelihood of each plant allows a reliability-based design in which probability is incorporated in the robust design. In this method, robustness and performance are stochastic variables (Stengel & Ryan, 1989). Stochastic behavior of the system can be simulated by Monte- Carlo Simulation (Ray & Stengel, 1993). Robustness and performance can be considered as objective functions with respect to the controller parameters in optimization problem. GAs have also been recently deployed in an augmented scalar single objective optimization to minimize the probabilities of unsatisfactory stability and performance estimated by Monte Carlo simulation (Wang & Stengel, 2001), (Wang & Stengel, 2002). Since conflictions exist between robustness and performance metrics, choosing appropriate weighting factor in a cost function consisting of weighted quadratic sum of those non-commensurable objectives is inherently difficult and could be regarded as a subjective design concept. Moreover, trade-offs existed between some objectives cannot be derived and it would be, therefore, impossible to choose an appropriate optimum design reflecting the compromise of the designer's choice concerning the absolute values of objective functions. Therefore, this problem can be formulated as a multi objective optimization problem (MOP) so that trade-offs between objectives can be derived consequently.

In this chapter, a new simple algorithm in conjunction with the original Pareto ranking of non-dominated optimal solutions is first presented for MOPs in control systems design. In this Multi-objective Uniform-diversity Genetic Algorithm (MUGA), a  $\epsilon$ -elimination diversity approach is used such that all the clones and/or  $\epsilon$ -similar individuals based on normalized Euclidean norm of two vectors are recognized and simply eliminated from the current population. Such multi-objective Pareto genetic algorithm is then used in conjunction with Monte-Carlo simulation to obtain Pareto frontiers of various non-commensurable objective functions in the design of robust controllers for uncertain systems subject to probabilistic variations of model parameters. The methodology presented in this chapter simply allows the use of different non-commensurable objective functions both in frequency and time domains. The obtained results demonstrate that compromise can be readily accomplished using graphical representations of the achieved trade-offs among the conflicting objectives.

## 2. Stochastic robust analysis

In real control engineering practice, there exist a variety of typical sources of uncertainty which have to be compensated through robust control design approach. Those uncertainties include plant parameter variations due to environmental condition, incomplete knowledge of the parameters, age, un-modelled high frequency dynamics, and etc. Two categorical types of uncertainty, namely, structured uncertainty and unstructured uncertainty are generally used in classification. The structured uncertainty concerns about the model uncertainty due to unknown values of parameters in a known structure. In conventional optimum control system design, uncertainties are not addressed and the optimization process is accomplished deterministically. In fact, it has been shown that optimization

without considering uncertainty generally leads to non-optimal and potentially high risk solution (Lim et al., 2005). Therefore, it is very desirable to find robust design whose performance variation in the presence of uncertainties is not high. Generally, there exist two approaches addressing the stochastic robustness issue, namely, robust design optimization (RDO) and reliability-based design optimization (RBDO) (Papadrakakis et al., 2004). Both approaches represent non deterministic optimization formulations in which the probabilistic uncertainty is incorporated into the stochastic optimal design process. Therefore, the propagation of a priori knowledge regarding the uncertain parameters through the system provides some probabilistic metrics such as random variables (e.g., settling time, maximum overshoot, closed loop poles, ...), and random processes (e.g., step response, Bode or Nyquist diagram, ...) in a control system design (Smith et al., 2005). In RDO approach, the stochastic performance is required to be less sensitive to the random variation induced by uncertain parameters so that the performance degradation from ideal deterministic behaviour is minimized. In RBDO approach, some evaluated reliability metrics subjected to probabilistic constraints are satisfied so that the violation of design requirements is minimized. In this case, limit state functions are required to define the failure of the control system. Figure (1) depicts the concept of these two design approaches where  $f$  is to be minimized. Regardless the choice of any of these two approaches, random variables and random processes should be evaluated reflecting the effect of probabilistic nature of uncertain parameters in the performance of the control system.

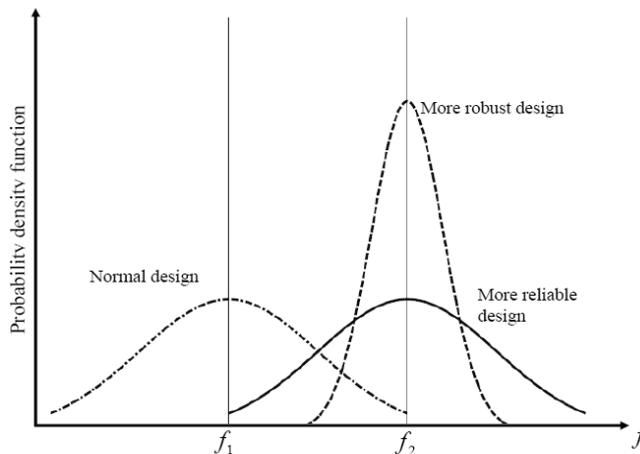


Fig. 1. Concepts of RDO and RBDO optimization

With the aid of ever increasing computational power, there have been a great amount of research activities in the field of robust analysis and design devoted to the use of Monte Carlo simulation (Crespo, 2003; Crespo & Kenny, 2005; Stengel, 1986; Stengel & Ryan, 1993; Papadrakakis et al., 2004; Kang, 2005). In fact, Monte Carlo simulation (MCS) has also been used to verify the results of other methods in RDO or RBDO problems when sufficient number of sampling is adopted (Wang & Stengel, 2001). Monte Carlo simulation (MCS) is a direct and simple numerical method but can be computationally expensive. In this method, random samples are generated assuming pre-defined probabilistic distributions for

uncertain parameters. The system is then simulated with each of these randomly generated samples and the percentage of cases produced in failure region defined by a limit state function approximately reflects the probability of failure.

Let  $X$  be a random variable, then the prevailing model for uncertainties in stochastic randomness is the probability density function (PDF),  $f_X(x)$  or equivalently by the cumulative distribution function (CDF),  $F_X(x)$ , where the subscript  $X$  refers to the random variable. This can be given by

$$F_X(x) = \Pr(X \leq x) = \int_{-\infty}^x f_X(x) dx \quad (1)$$

where  $\Pr(\cdot)$  is the probability that an event ( $X \leq x$ ) will occur. Some statistical moments such as the first and the second moment, generally known as mean value (also referred to as expected value) denoted by  $E(X)$  and variance denoted by  $\sigma^2(X)$ , respectively, are the most important ones. They can also be computed by

$$E(X) = \int_{-\infty}^{\infty} x dF_X(x) = \int_{-\infty}^{\infty} f_X(x) dx \quad (2)$$

and

$$\sigma^2(X) = \int_{-\infty}^{\infty} (x - E(X)) f_X(x) dx \quad (3)$$

In the case of discrete sampling, these equations can be readily represented as

$$E(X) \cong \frac{1}{N} \sum_{i=1}^N x_i \quad (4)$$

and

$$\sigma^2(X) \cong \frac{1}{N-1} \sum_{i=1}^N (x_i - E(X))^2 \quad (5)$$

where  $x_i$  is the  $i^{\text{th}}$  sample and  $N$  is the total number of samples.

In the reliability-based design, it is required to define reliability-based metrics via some inequality constraints (in time or frequency domain). Therefore, in the presence of uncertain parameters of plant ( $\mathbf{p}$ ) whose PDF or CDF can be given by  $f_{\mathbf{p}}(\mathbf{p})$  or  $F_{\mathbf{p}}(\mathbf{p})$ , respectively, the reliability requirements can be given as

$$P_f^i = \Pr(g_i(\mathbf{p}) \leq 0) \leq \varepsilon \quad i = 1, 2, \dots, k \quad (6)$$

In equation (6),  $P_f^i$  denotes the probability of failure (i.e.,  $g_i(\mathbf{p}) \leq 0$ ) of the  $i^{\text{th}}$  reliability measure and  $k$  is the number of inequality constraints (i.e., limit state functions) and  $\varepsilon$  is the highest value of desired admissible probability of failure. It is clear that the desirable value of each  $P_f^i$  is zero. Therefore, taking into consideration the stochastic distribution of

uncertain parameters ( $\mathbf{p}$ ) as  $f_p(\mathbf{p})$ , equation (6) can now be evaluated for each probability function as

$$P_f^i = \Pr(g_i(\mathbf{p}) \leq 0) = \int_{g_i(\mathbf{p}) \leq 0} f_p(\mathbf{p}) d\mathbf{p} \tag{7}$$

This integral is, in fact, very complicated particularly for systems with complex  $g(\mathbf{p})$  (Wang & Stengel, 2002) and Monte Carlo simulation is alternatively used to approximate equation (7). In this case, a binary indicator function  $I_{g(\mathbf{p})}$  is defined such that it has the value of 1 in the case of failure ( $g(\mathbf{p}) \leq 0$ ) and the value of zero otherwise,

$$I_{g(\mathbf{p})} = \begin{cases} 0 & g(\mathbf{p}) > 0 \\ 1 & g(\mathbf{p}) \leq 0 \end{cases} \tag{8}$$

Consequently, for each limit state function,  $g(\mathbf{p})$ , the integral of equation (7) can be rewritten as

$$P_f(\mathbf{p}) = \int_{-\infty}^{\infty} I_{g(\mathbf{p})}(G(\mathbf{p}), C(\mathbf{k})) f_p(\mathbf{p}) d\mathbf{p} \tag{9}$$

where  $G(\mathbf{p})$  is the uncertain plant model and  $C(\mathbf{k})$  is the controller to be designed in the case of control system design problems. Based on Monte Carlo simulation (Ray & Stengel, 1993; Wang & Stengel, 2001; Wang & Stengel, 2002; Kalos, 1986), the probability using sampling technique can be estimated using

$$P_f(\mathbf{p}) = \frac{1}{N} \sum_{i=1}^N I_{g_i(\mathbf{p})}(G(\mathbf{p}), C(\mathbf{k})) \tag{10}$$

where  $G_i$  is the  $i^{th}$  plant that is simulated by Monte Carlo Simulation. In other words, the probability of failure is equal to the number of samples in the failure region divided by the total number of samples. Evidently, such estimation of  $P_f$  approaches to the actual value in the limit as  $N \rightarrow \infty$  (Wang & Stengel, 2002). However, there have been many research activities on sampling techniques to reduce the number of samples keeping a high level of accuracy. Alternatively, the quasi-MCS has now been increasingly accepted as a better sampling technique which is also known as Hammersley Sequence Sampling (HSS) (Smith et al., 2005; Crespo & Kenny, 2005). In this paper, HSS has been used to generate samples for probability estimation of failures. In a RBDO problem, the probability of representing the reliability-based metrics given by equation (10) is minimized using an optimization method. In a multi-objective optimization of a RBDO problem presented in this paper, however, there are different conflicting reliability-based metrics that should be minimized simultaneously.

In the multi-objective RBDO of control system problems, such reliability-based metrics (objective functions) can be selected as closed-loop system stability, step response in time domain or Bode magnitude in frequency domain, etc. In the probabilistic approach, it is, therefore, desired to minimize both the probability of instability and probability of failure to a desired time or frequency response, respectively, subjected to assumed probability

distribution of uncertain parameters. In a RDO approach that is used in this work, the lower bound of degree of stability that is the distance from critical point -1 to the nearest point on the open loop Nyquist diagram, is maximized. The goal of this approach is to maximize the mean of the random variable (degree of stability) and to minimize its variance. This is in accordance with the fact that in the robust design the mean should be maximized and its variability should be minimized simultaneously (Kang, 2005). Figure (2) depicts the concept of this RDO approach where  $f_X(x)$  is a PDF of random variable,  $X$ . It is clear from figure (2) that if the lower bound of  $X$  is maximized, a robust optimum design can be obtained. Recently, a weighted-sum multi-objective approach has been applied to aggregate these objectives into a scalar single-objective optimization problem (Wang & Stengel, 2002; Kang, 2005).

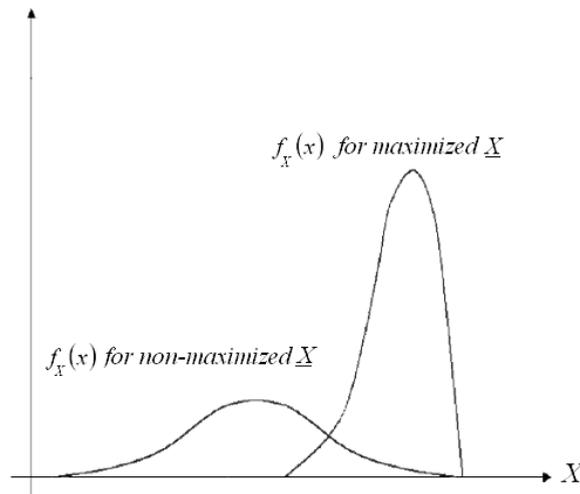


Fig. 2. Concept of RDO approach

However, the trade-offs among the objectives are not revealed unless a Pareto approach of the multi-objective optimization is applied. In the next section, a multi-objective Pareto genetic algorithm with a new diversity preserving mechanism recently reported by some of authors (Nariman-Zadeh et al., 2005; Atashkari et al., 2005) is briefly discussed for a combined robust and reliability-based design optimization of a control system.

### 3. Multi-objective Pareto optimization

Multi-objective optimization which is also called multi-criteria optimization or vector optimization has been defined as finding a vector of decision variables satisfying constraints to give optimal values to all objective functions (Atashkari et al., 2005; Coello Coello & Christiansen, 2000; Coello Coello et al., 2002; Pareto, 1896). In general, it can be mathematically defined as follows; find the vector  $X^* = [x_1^*, x_2^*, \dots, x_n^*]^T$  to optimize

$$F(X) = [f_1(X), f_2(X), \dots, f_k(X)]^T \quad (11)$$

subject to  $m$  inequality constraints

$$g_i(X) \leq 0 \quad i = 1 \dots m \tag{12}$$

and  $p$  equality constraints

$$h_j(X) \leq 0 \quad j = 1 \dots p \tag{13}$$

where,  $X^* \in \mathfrak{R}^n$  is the vector of decision or design variables, and  $F(X) \in \mathfrak{R}^k$  is the vector of objective functions. Without loss of generality, it is assumed that all objective functions are to be minimized. Such multi-objective minimization based on the Pareto approach can be conducted using some definitions.

**Pareto dominance**

A vector  $\mathbf{U} = [u_1, u_2, \dots, u_k] \in \mathfrak{R}^k$  dominates to vector  $\mathbf{V} = [v_1, v_2, \dots, v_k] \in \mathfrak{R}^k$  (denoted by  $\mathbf{U} < \mathbf{V}$ ) if and only if  $\forall i \in \{1, 2, \dots, k\}, u_i \leq v_i \wedge \exists j \in \{1, 2, \dots, k\}: u_j < v_j$ . It means that there is at least one  $u_j$  which is smaller than  $v_j$  whilst the rest  $u$ 's are either smaller or equal to corresponding  $v$ 's.

**Pareto optimality**

A point  $X^* \in \Omega$  ( $\Omega$  is a feasible region in  $\mathfrak{R}^n$ ) is said to be Pareto optimal (minimal) with respect to all  $X \in \Omega$  if and only if  $F(X^*) < F(X)$ . Alternatively, it can be readily restated as  $\forall i \in \{1, 2, \dots, k\}, \forall X \in \Omega - \{X^*\}, f_i(X^*) \leq f_i(X) \wedge \exists j \in \{1, 2, \dots, k\} : f_j(X^*) < f_j(X)$ . It means that the solution  $X^*$  is said to be Pareto optimal (minimal) if no other solution can be found to dominate  $X^*$  using the definition of Pareto dominance.

**Pareto Set**

For a given MOP, a Pareto set  $\mathcal{P}^*$  is a set in the decision variable space consisting of all the Pareto optimal vectors,  $\mathcal{P}^* = \{X \in \Omega \mid \nexists X' \in \Omega: F(X') < F(X)\}$ . In other words, there is no other  $X'$  in  $\Omega$  that dominates any  $X \in \mathcal{P}^*$

**Pareto front**

For a given MOP, the Pareto front  $\mathcal{PF}^*$  is a set of vectors of objective functions which are obtained using the vectors of decision variables in the Pareto set  $\mathcal{P}^*$ , that is,  $\mathcal{PF}^* = \{F(X) = (f_1(X), f_2(X), \dots, f_k(X)): X \in \mathcal{P}^*\}$ . Therefore, the Pareto front  $\mathcal{PF}^*$  is a set of the vectors of objective functions mapped from  $\mathcal{P}^*$ .

Evolutionary algorithms have been widely used for multi-objective optimization because of their natural properties suited for these types of problems. This is mostly because of their parallel or population-based search approach. Therefore, most difficulties and deficiencies within the classical methods in solving multi-objective optimization problems are eliminated. For example, there is no need for either several runs to find the Pareto front or quantification of the importance of each objective using numerical weights. It is very important in evolutionary algorithms that the genetic diversity within the population be preserved sufficiently (Osyczka, 1985). This main issue in MOPs has been addressed by

much related research work (Nariman-zadeh et al., 2005; Atashkari et al., 2005; Coello Coello & Christiansen, 2000; Coello Coello et al., 2002; Pareto, 1896; Osyezka, 1985; Toffolo & Benini, 2002; Deb et al., 2002; Coello Coello & Becerra, 2003; Nariman-zadeh et al., 2005). Consequently, the premature convergence of MOEAs is prevented and the solutions are directed and distributed along the true Pareto front if such genetic diversity is well provided. The Pareto-based approach of NSGA-II (Osyezka, 1985) has been recently used in a wide range of engineering MOPs because of its simple yet efficient non-dominance ranking procedure in yielding different levels of Pareto frontiers. However, the crowding approach in such a state-of-the-art MOEA (Coello Coello & Becerra, 2003) works efficiently for two-objective optimization problems as a diversity-preserving operator which is not the case for problems with more than two objective functions. The reason is that the sorting procedure of individuals based on each objective in this algorithm will cause different enclosing hyper-boxes. It must be noted that, in a two-objective Pareto optimization, if the solutions of a Pareto front are sorted in a decreasing order of importance to one objective, these solutions are then automatically ordered in an increasing order of importance to the second objective. Thus, the hyper-boxes surrounding an individual solution remain unchanged in the objective-wise sorting procedure of the crowding distance of NSGA-II in the two-objective Pareto optimization problem. However, in multi-objective Pareto optimization problem with more than two objectives, such sorting procedure of individuals based on each objective in this algorithm will cause different enclosing hyper boxes. Thus, the overall crowding distance of an individual computed in this way may not exactly reflect the true measure of diversity or crowding property for the multi-objective Pareto optimization problems with more than two objectives.

In our work, a new method is presented to modify NSGA-II so that it can be safely used for any number of objective functions (particularly for more than two objectives) in MOPs. Such a modified MOEA is then used for multi-objective robust desing of linear controllers for systems with parametric uncertainties.

#### **4. Multi-objective Uniform-diversity Genetic Algorithm (MUGA)**

The multi-objective uniform-diversity genetic algorithm (MUGA) uses non-dominated sorting mechanism together with a  $\epsilon$ -elimination diversity preserving algorithm to get Pareto optimal solutions of MOPs more precisely and uniformly (Jamali et.al., 2008.)

##### **4.1 The non-dominated sorting method**

The basic idea of sorting of non-dominated solutions originally proposed by Goldberg (Goldberg, 1989) used in different evolutionary multi-objective optimization algorithms such as in NSGA-II by Deb (Deb et al., 2002) has been adopted here. The algorithm simply compares each individual in the population with others to determine its non-dominancy. Once the first front has been found, all its non-dominated individuals are removed from the main population and the procedure is repeated for the subsequent fronts until the entire population is sorted and non-dominately divided into different fronts.

A sorting procedure to constitute a front could be simply accomplished by comparing all the individuals of the population and including the non-dominated individuals in the front. Such procedure can be simply represented as following steps:

- 1-Get the population (pop)
- 2-Include the first individual {ind(1)} in the front P\* as P\*(1), let P\*\_size=1;
- 3-Compare other individuals {ind (j), j=2, Pop\_size} of the pop with { P\*(K), K=1, P\*\_size} of the P\*;  
 If ind(j)<P\*(K) replace the P\*(K) with ind(j)  
 If P\*(K)<ind(K), j=j+1, continue comparison;  
 Else include ind(j) in P\*, P\*\_size= P\*\_size+1, j=j+1, continue comparison;
- 4-End of front P\*;

It can be easily seen that the number of non-dominated solutions in P\* grows until no further one is found. At this stage, all the non-dominated individuals so far found in P\* are removed from the main population and the whole procedure of finding another front may be accomplished again. This procedure is repeated until the whole population is divided into different ranked fronts. It should be noted that the first rank front of the final generation constitute the final Pareto optimal solution of the multi-objective optimization problem.

**4.2 The ε-elimination diversity preserving approach**

In the ε-elimination diversity approach that is used to replaced the crowding distance assignment approach in NSGA-II (Deb et al., 2002), all the clones and ε-similar individuals are recognized and simply eliminated from the current population. Therefore, based on a value of ε as the elimination threshold, all the individuals in a front within this limit of a particular individual are eliminated. It should be noted that such ε-similarity must exist both in the space of objectives and in the space of the associated design variables. This will ensure that very different individuals in the space of design variables having ε-similarity in the space of objectives will not be eliminated from the population. The pseudo-code of the ε-elimination approach is depicted in figure (3). Evidently, the clones and ε-similar

```

ε-elim= ε-elimination(pop)           // pop includes design variables and
                                     // objective function
i=1; j=1;
get K (K=1 for the first front);
While i,j < pop_size
    e(i,j)= || X(i,:),X(j,:) || / || X(i,:) || ; X(i),X(j) ∈ P*_k ∪ PF*_k //finding mean value of ε
                                                within pop.
end
ε=mean(e);
i=1;
until i+1<pop_size;
j=i+1
    until j<pop_size
        if e(i,j)<ε
            then {pop}={pop}/ {pop(j)} //remove the ε-similar individual
            j=j+1
        end
    end
i=i+1
end
    
```

Fig. 3. The ε-elimination diversity preserving pseudo-code

individuals are replaced from the population by the same number of new randomly generated individuals. Meanwhile, this will additionally help to explore the search space of the given MOP more effectively. It is clear that such replacement does not appear when a front rather than the entire population is truncated for  $\varepsilon$ -similar individual.

### 4.3 The main algorithm of MUGA

It is now possible to present the main algorithm of MUGA which uses both non-dominated sorting procedure and  $\varepsilon$ -elimination diversity preserving approach and is given in figure (4).

```

Get N                //population size
t=1 ;               //set generation number
Random_N(Pt);      //generate the first population (P1) randomly
Qt=Recomb(Pt)    //generate population Qt from Pt by genetic operators
Rt=Pt ∪ Qt      //union of both parent and offspring population
R't= $\varepsilon$ -elimination (Rt) //remove  $\varepsilon$ -similar individuals in Rt
R''= R't ∪ Random_(Rtsize-R'tsize) (P't) //add random individuals to fill Rt to 2N

Do non-dominate sorting procedure (R'') //R''=P*1∪ P*2∪...∪P*k where k is total
                                         number of fronts

i=1
Pt+1=∅
While not Pt+1size>N //includes fronts into new population
    Pt+1= Pt+1∪ P*i
    i=i+1
end
N'=N- Pt+1size
While not (0.9 N' < Pt+1size < 1.1 N') //remove the  $\varepsilon$ -similar individuals within
                                         the tolerance of  $\pm 10$  percent
    F'= $\varepsilon$ -elimination (P*i-1)
    If F'_size < N'
        e=1.1*e
    else
        e=0.9 * e //adjust the value of threshold to get the right population
                    size of the last front
    end
end
t=t+1 //Start next generation

```

Fig. 4. The pseudo-code of the main algorithm of MUGA

It first initiates a population randomly. Using genetic operators, another same size population is then created. Based on the  $\varepsilon$ -elimination algorithm, the whole population is then reduced by removing  $\varepsilon$ -similar individuals. At this stage, the population is re-filled by randomly generated individuals which helps to explore the search space more effectively. The whole population is then sorted using non-dominated sorting procedure. The obtained fronts are then used to constitute the main population. It must be noted that the front which must be truncated to match the size of the population is also evaluated by  $\varepsilon$ -elimination procedure to identify the  $\varepsilon$ -similar individuals. Such procedure is only performed to match

the size of population within  $\pm 10$  present deviation to prevent excessive computational effort to population size adjustment. Finally, unless the number of individuals in the first rank front is changing in certain number of generations, randomly created individuals are inserted in the main population occasionally (e.g. every 20 generations of having non-varying first rank front).

## 5. Process model and controller evaluation method

In this section, the process models and the robust PI/PID controller design methodologies are presented using some conflicting objective functions defined in both time and frequency domains.

### 5.1 The process model

Many industrial systems can be adequately presented by a first-order lag with time delay (Toscana, 2005) as

$$G(s) = \frac{ke^{-\tau s}}{1 + Ts} \quad (14)$$

In the case of stochastic robust design, parameters of the plant given by equation (14) vary according to *a priori* known probabilistic distribution functions around a nominal set of parameters. In this work, beta distributions with the coefficients of 2 and 2 with the limits of  $\pm 50\%$  of the nominal values of plant parameters,  $k = \tau = T = 1$  have been selected, respectively. Stochastic step response of the 10 samples that are simulated by Monte Carlo simulation is shown in figure (5). It is clear from figure (5) that the response of the uncertain system has a large variability and the performance of the system deteriorates significantly with parameters variation. Consequently, the controller design must be accomplished robustly.

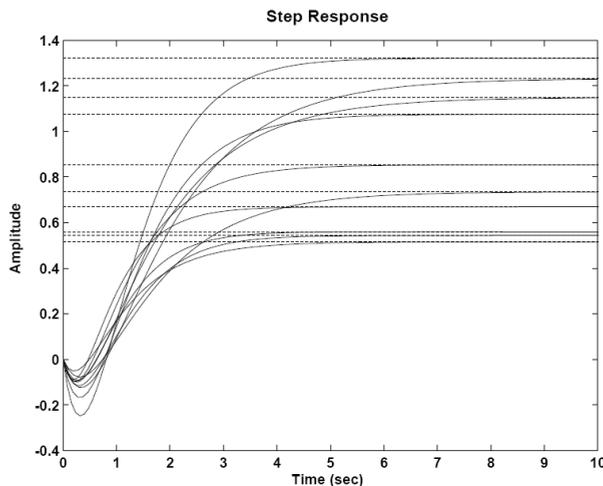


Fig. 5. Stochastic step response of the uncertain plant

### 5.2 The robust design of PI/PID controllers

Simple structure PI/PID Controllers are widely used for many industrial processes represented by the transfer function of equation (14). The transfer functions,  $C(s)$ , of the standard PI/PID Controllers of the feedback control system shown in figure (6) are

$$\begin{cases} C(s) = K_p + \frac{K_i}{s} \\ C(s) = K_p + \frac{K_i}{s} + K_d s \end{cases} \quad (15)$$

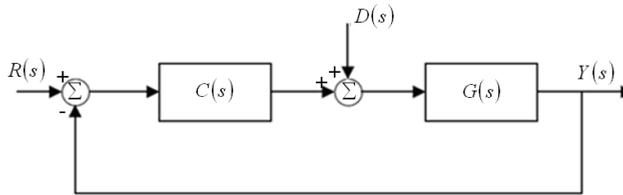


Fig. 6. Closed loop SISO system with plant  $G(s)$  and controller  $C(s)$

The design vector of the PI and PID controllers are  $\mathbf{k}_{PI} = [K_p, K_i]$  and  $\mathbf{k}_{PID} = [K_p, K_i, K_d]$ , respectively. They have to be optimally determined based on the mixed robust and reliability-based multi-objective Pareto approach for the uncertain first-order system using some stochastic evaluation metrics that are introduced as follows.

Two robust performance metrics have been proposed in this work, performance metrics in time domain and performance metrics in frequency domain. In this section, design vector of PI controller is obtained based on time domain performance metrics and design vector of PID controller is obtained based on frequency domain performance metrics.

The most important goal of robust controller design is the robust stability which implies that all the closed-loop poles of the system remain in the stable left half-plane ( $\Re(s_i) < 0$ ) in the presence of any uncertainty in the nominal plant's transfer function. Thus, in the case of stochastic robust design, the limit state function to define the probability of failure of robust stability will be represented by

$$g_{ins}(\mathbf{p}) = -\max\{\Re(s_1^i), \Re(s_2^i), \dots, \Re(s_n^i)\} \quad i = 1, 2, \dots, N \quad (16)$$

where,  $g_{ins}(\mathbf{p})$  is the limit state function of the instability,  $\Re(s^i)$  is the real part of the closed-loop poles of the  $i^{th}$  uncertain plants, and  $n$  is the order of the closed-loop plant.

The probability of failure of stochastic stability can now be computed using by equation (10)

$$Pr_{ins} = \frac{1}{N} \sum_{i=1}^N I_{g_{ins}}(G(\mathbf{p}_i), C(\mathbf{k})) \quad (17)$$

in association with equation (16) employing the quasi Monte Carlo Sampling or HSS for  $N$  samples. For obtaining the acceptable stability, such probability of instability should be minimized.

In addition to the minimizing the probability of instability, maximizing the stability margin in the frequency domain is another important measure of good performance of a robust

controller for uncertain systems. The inclusion of the stability margin (to be maximized) in the vector of the cost functions ensures that stable PI/PID controllers having the most stability margin are obtained. Such robust stability margin, also referred to as degree of stability  $S_{\infty}^{-1}$ , can be simply computed using the sensitivity transfer function

$$S(s) = \frac{1}{1 + C(s)G(s)} = \frac{1}{1 + L(s)} \tag{18}$$

for a unity feedback control system shown in figure (6). In frequency domain, the return difference  $|-1 - L(j\omega)| = |1 + L(j\omega)|$  simply represents the length of a vector drawn from critical point -1 to open-loop transfer function in the Nyquist diagram. Consequently, the inverse of  $\infty$ -norm of sensitivity transfer function given by equation (18)

$$S_{\infty}^{-1} = \|S\|_{\infty}^{-1} = \min_{\omega} |1 + L(j\omega)| \tag{19}$$

represents the minimum distance of the Nyquist diagram to point -1. In the case of stochastic robust design, the degree of stability for each stochastic system is a random variable. Therefore, in a RDO problem considered in this study the lower bound of interested random variable (degree of stability) is maximized using an optimization method. It should be noted that the degree of stability given by equation (19) also directly represents the additive disturbance rejection property as follows

$$Y(s) = \frac{G(s)}{1 + L(s)} D(s) = G(s)S(s)D(s) \tag{20}$$

where,  $D(s)$  is the load disturbance transfer function. It is evident from equation (20) that maximizing the minimum value of  $|1 + L(j\omega)|$  based on equation (19) will cause a better disturbance rejection according to equation (20). Therefore, systems with high degree of stability represent a good ability to reject the load disturbance (Toscana, 2005).

A good step response behavior of the system is one of the performance metrics in controller design procedure that illustrates how system acts in transient and steady state periods. Another method to obtain these properties of the step response is Bode magnitude of the close-loop or complementary transfer function. In the stochastic robust design both step response and Bode magnitude are random process.

In the reliability- based design approach, it is desired to minimize the probability of a failure of a random process as a function of  $w$  ( $w$  represents time or frequency) due to the uncertain probabilistic parameters. In this approach, let  $h(\mathbf{p}, w)$  is the random response (step response or Bode magnitude) of an uncertain plant due to uncertain parameters  $\mathbf{p}$ , and let define  $\bar{h}(w)$  and  $\underline{h}(w)$  as upper and lower failure boundary, respectively. Therefore, if the random process is held within these bounds, the uncertain system has a robust performance.

In this work, step response metrics are used to design PI controller and Bode magnitude metrics are used to design PID controllers.

The lower and upper failure boundaries to define the corresponding limit state function,  $g_{resp}(\mathbf{p}) \leq 0$ , in time domain is given using the Heaviside function

$$\underline{h} = -0.1H(t) + 0.8H(t - 3) + 0.25H(t - 7) \tag{21a}$$

$$\bar{h} = 1.2H(t) - 0.15H(t - 7) \tag{21b}$$

for a period of  $t \in [0, t_f]$ ,  $t_f = 15$ . If  $\underline{r}$  and  $\bar{r}$  are defined as

$$\begin{aligned} \underline{r}_i &= 1, h_i < \underline{h}_i \quad i = 1, 2, \dots, k_t \\ \underline{r}_i &= 0 \quad \text{otherwise} \end{aligned} \quad (22a)$$

$$\begin{aligned} \bar{r}_i &= 1, h_i > \bar{h}_i \quad i = 1, 2, \dots, k_t \\ \bar{r}_i &= 0 \quad \text{otherwise} \end{aligned} \quad (22b)$$

where  $h$  is the time response of the plant and  $k_t$  is the number of sample time, the limit state function indicator can then be computed as

$$I_{g_{resp}}(\mathbf{p}) = \frac{1}{k_t} = \sum_{i=1}^{k_t} (\underline{r}_i + \bar{r}_i) \quad (23)$$

which is used in equation (10) to obtain the probability of failure to the desired time response boundaries.

The complementary transfer function  $T(s)$  can be used to obtain closed-loop system response which is the transfer function of the reference input  $R(s)$  to the output  $Y(s)$  and is given as

$$T(s) = 1 - S(s) = \frac{L(s)}{1 + L(s)} \quad (24)$$

The quantity  $|T(j\omega)|$  represents the magnitude of the closed-loop frequency response. It is well known that the performance of the closed-loop system response is related to  $|T(j\omega)|$ . In order to select appropriate boundaries for such frequency response behavior, the relationship between peak value of the closed-loop magnitude response (Nise, 2004),  $M_p = \max_{\omega} |T(j\omega)|$ , and the damping ratio,  $\zeta$ , for a second order system with nominal parameters is considered here as a reference. Such relations are given by

$$M_p = \frac{1}{2\zeta\sqrt{1-\zeta^2}} \quad (25)$$

at a frequency of  $\omega_p$  given by

$$\omega_p = \omega_n \sqrt{1-2\zeta^2} \quad (26)$$

Since  $\zeta$  and  $\omega_n$  are related to maximum overshoot and settling time of step response, respectively, for a good transient and steady-state response it is required that  $\zeta \geq \zeta^+$  and  $\omega_n \geq \omega_n^+$ . The selected values of  $\zeta^+$  and  $\omega_n^+$  in this work are 0.7. In order to achieve a good closed-loop performance, the complementary transfer function  $T(s)$  given by equation (24) is used in frequency domain using lower and upper failure boundaries to define the corresponding limit state function  $g_{resp}(\mathbf{p}) \leq 0$ . If  $h \equiv |T(j\omega)|$  which is a random process having sets of CDFs varying with frequency (Crespo & Kenny, 2005, Crespo, 2003), both the upper failure boundary defined by  $\bar{h}(\omega)$  and the lower failure boundary  $\underline{h}(\omega)$  are used to compute the probability of failure to a good frequency response. Based on the previous discussion, the boundaries are defined as

$$\underline{h}(\omega) = 1 \quad \omega \in [10^{-2}, 10^{-1}] \tag{27a}$$

$$\bar{h}(\omega) = \frac{0.7}{\omega} \quad \omega \in [0.5, 10^2] \tag{27b}$$

If  $\underline{r}$  and  $\bar{r}$  are defined as

$$\begin{aligned} \underline{r}_i &= 1, \quad h_i < \underline{h}_i \quad i = 1, 2, \dots, k_\omega \\ \underline{r}_i &= 0 \quad \text{otherwise} \end{aligned} \tag{28a}$$

$$\begin{aligned} \bar{r}_i &= 1, \quad h_i > \bar{h}_i \quad i = 1, 2, \dots, k_\omega \\ \bar{r}_i &= 0 \quad \text{otherwise} \end{aligned} \tag{28b}$$

where  $h$  is the frequency response of the plant and  $k_\omega$  is the number of sample frequency, the limit state function indicator can then be computed as

$$I_{g_{resp}}(\mathbf{p}) = \frac{1}{k_\omega} = \sum_{i=1}^{k_\omega} (\underline{r}_i + \bar{r}_i) \tag{29}$$

which is used in equation (10) to obtain the probability of failure to the desired frequency response boundaries of the complementary transfer function.

## 6. Results

The objectives  $Pr_{ins}$ ,  $Pr_{resp}$  and  $S_\infty^{-1}$  are now considered simultaneously in a Pareto optimization process to obtain some important trade-offs among the conflicting objectives. In a mixed robust and reliability-based design approach, the vector of objective functions to be optimized in a Pareto sense is given as follows

$$\vec{F} = [Pr_{ins}, Pr_{resp}, S_\infty^{-1}] \tag{30}$$

which are computed using equations (17), (23), (29), and (19), respectively, in the quasi-Monte Carlo simulation process. The evolutionary process of the Pareto multi-objective optimization is accomplished using MUGA (Jamali, et al., 2008) where a population size of 45 has been chosen with crossover probability  $P_c$  and mutation probability  $P_m$  as 0.85 and 0.09, respectively. The optimization process of the robust PI/PID controllers given by equation (15) is accomplished by 250 Monte Carlo evaluations using HSS distribution for each candidate control law during the evolutionary process. The vector of objective functions given by equation (30) is used to obtain non-dominated optimum PI/PID controllers to represent the trade-offs among the objective functions.

### 6.1 Pareto optimum PI controllers

A total number of 80 non-dominated optimum design points have been obtained and shown in figure (7) in the plane of probability of failure to the desired time response ( $Pr_{resp}$ ) and the degree of stability ( $\cdot$ ). The value of probability of instability ( $Pr_{ins}$ ) of all the non-dominated optimum points has been obtained zero which demonstrates that all optimum controllers are stable in the Monte Carlo simulation (Hajiloo et al., 2007).

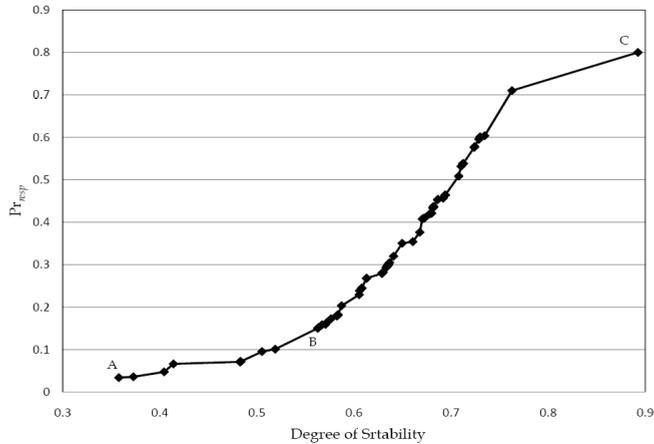


Fig. 7. Pareto fronts of  $Pr_{resp}$  and degree of stability ( $S_{\infty}^{-1}$ )

Since, the value of probability of instability ( $Pr_{ms}$ ) of all non-dominated optimum points has been found equal to zero, therefore, the result of the 3-objective optimization process corresponds to a 2-objective optimization process which is shown in figure (7). It can be observed from the Pareto front of figure (7) that improving one objective will cause another objective deteriorates accordingly.

The best point obtained for  $Pr_{resp}$  is point A which corresponds to the worst value of  $S_{\infty}^{-1}$ . These values for time response and degree of stability are 0.0338 and 0.3577, respectively. In other words, optimum design point A represents 3.38% probability of failure to the desired time response and its minimum distance to the critical point  $-1+0j$  in the Nyquist diagram is 0.3577, representing its degree of stability for 250 Monte Carlo evaluations. Alternatively, the best value of obtained  $S_{\infty}^{-1}$  is that of point C which corresponds to the worst value of  $Pr_{resp}$  and are 0.8 and 0.8923, respectively. Figure (8) shows the corresponding 1, 10, 30, 50, 70, 90, 99 percentiles of time responses of both design points A and C which demonstrates the stochastic behavior of the corresponding PI controllers for 250 Monte Carlo simulations of the plant subjected to the assumed probabilistic uncertainties. An  $m$  percentiles curve presents a confidence limit of  $m$  percent probability that the time response behavior would be below that curve.

By careful investigation of figure (7) an important trade-off can be observed from the Pareto front of objectives  $Pr_{resp}$  and  $S_{\infty}^{-1}$ . It is clear that the gradient of the Pareto front in section A-B increases noticeably in section B-C. Apparently, optimum design point B shows a significant improvement of degree of stability ( $S_{\infty}^{-1}$ ) in comparison with that of point A whilst its probability of failure to the desired time response does not degrades significantly in section A-B as much as it does in section B-C. Thus, optimum design point B representing a PI controller with  $K_p = 0.3$  and  $K_i = 0.31$  can be optimally chosen from a trade-off point of view for objectives  $Pr_{resp}$  and  $S_{\infty}^{-1}$ . Figure (9) shows percentiles of stochastic time response behavior of point B which can be compared to those of optimum design points A and C shown in figure (8).

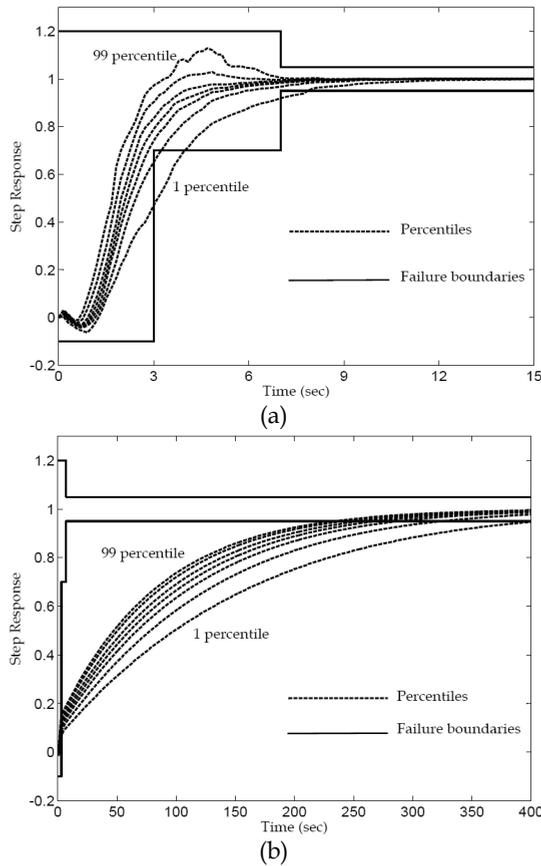


Fig. 8. Step response behaviors of optimum designs (a) point A (b) point C

Table 1 summarizes the values of those objectives together with the corresponding values of PI controller gains for three optimum design points A, B, and C shown in figure (7).

Design points	$K_p$	$K_i$	$Pr_{ins}$	$Pr_{resp}$	$S_{\infty}^{-1}$
A	0.516	0.454	0	0.0338	0.3577
B	0.3	0.31	0	0.1500	0.5624
C	0.8	0.892	0	0.8	0.8923

Table 1. Optimum values of objective functions and their gains for the PI controller obtained from 250 Monte Carlo simulations

The robust stability margins of all optimum points have been shown in figure (10). In this figure, the cumulative distribution functions (CDF) have been shown for all design points. It is evident that the optimum design point C exhibits the best stability robustness, because lower bound of its degree of stability is greater than other design points and variance of the degree of stability of design point C is very small.

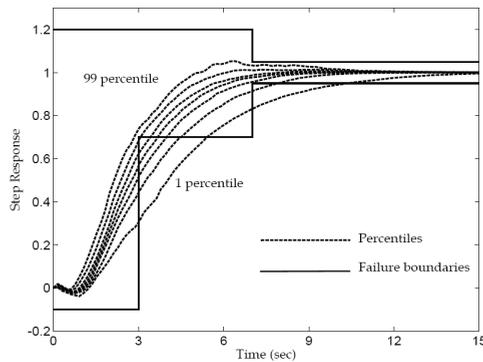


Fig. 9. Step response behaviors of optimum design B

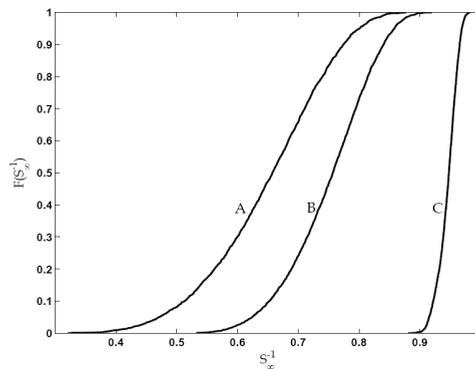


Fig. 10. CDFs for robust stability margins of different optimum designs

## 6.2 Pareto optimum PID controllers

A total number of 31 non-dominated optimum design points have been obtained and shown in figure(11) in the plane of probability of frequency response failure ( $Pr_{resp}$ ) and the degree of stability ( $S_\infty^{-1}$ ). The value of probability of instability ( $Pr_{ins}$ ) of all the non-dominated optimum points has been obtained zero which demonstrates that all obtained optimum controllers are stable in the Monte Carlo simulation. Therefore, the results of the 3-objective optimization process correspond to those of a 2-objective optimization process excluding the probability of instability. It can be observed from the Pareto front of figure (11) that improving one objective will cause another objective deteriorates accordingly. The best point obtained for  $Pr_{resp}$  is point A which corresponds to the worst value of  $S_\infty^{-1}$ . These values for the probability of frequency response failure and the degree of stability are 0.089 and 0.4815, respectively. In other words, optimum design point A represents 8.9% probability of frequency response failure and its minimum distance to the critical point  $-1+0j$  in the Nyquist diagram is 0.4815, representing its degree of stability in 250 Monte Carlo evaluations. Alternatively, the best value of obtained  $S_\infty^{-1}$  is that of point C which corresponds to the worst value of  $Pr_{resp}$  which are 0.1381 and 0.9798, respectively. In other words, optimum design point C represents 13.81% probability of frequency response failure while its minimum distance to the critical point  $-1+0j$  in the Nyquist diagram is 0.9788 representing its improved degree of stability.

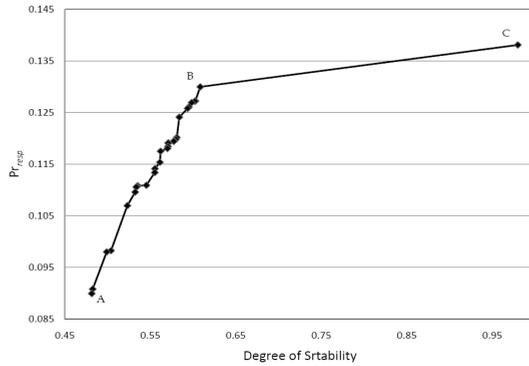


Fig. 11. Pareto fronts of  $Pr_{resp}$  and degree of stability ( $S_{\infty}^{-1}$ )

Figure (12) shows the corresponding 1, 10, 30, 50, 70, 90, 99 percentiles of step response of a non-dominated optimum design points B which demonstrate the stochastic behavior of the corresponding PID controllers in 250 Monte Carlo simulations of the plant subjected to the assumed probabilistic uncertainties in the plant. Figure (13) shows the Nyquist diagram for the design point B.

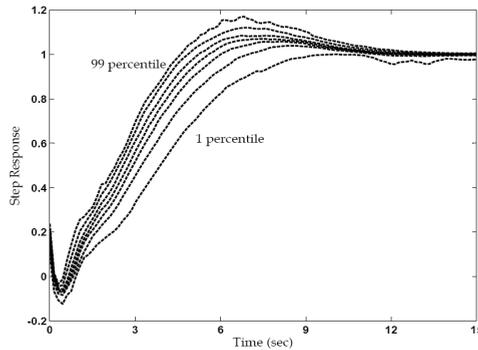


Fig. 12. Probabilistic step response behaviors of optimum design B

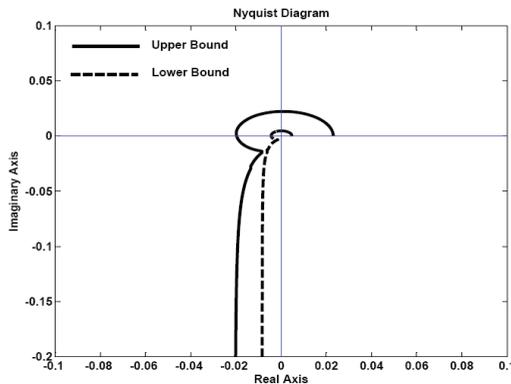


Fig. 13. Nyquist diagram of optimum design B

The robust stability margins of all optimum points have been also shown in figure (14). In this figure, the cumulative distribution functions (CDF) have been shown for all design points.

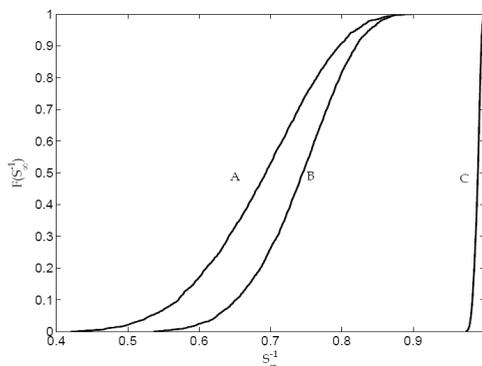


Fig. 14. CDFs for robust stability margins of different optimum designs

Table 2 summarizes the values of those objectives together with the corresponding values of PID controller gains for three optimum design points A, B, and C shown in figure (11).

Design points	$K_p$	$K_i$	$K_d$	$Pr_{ins}$	$Pr_{resp}$	$S_\infty^{-1}$
A	0.2132	0.4035	0.0572	0	0.0899	0.4815
B	0.2210	0.3879	0.2185	0	0.1299	0.6084
C	0.0130	0.0129	0.0119	0	0.1381	0.9798

Table 2. Optimum values of objective functions and their gains for the PID controller obtained from 250 Monte Carlo simulations

## 7. Conclusion

A multi-objective genetic algorithm with a recently developed diversity preserving mechanism was used to optimally design PI/PID controllers from a reliability-based point of view in a probabilistic approach. The objective functions which often conflict with each other were appropriately defined using some probabilistic metrics in time and frequency domain. The multi-objective optimization of robust PID controllers led to the discovering some important trade-offs among those objective functions. The framework of such hybrid application of multi-objective GAs and Monte Carlo Simulation of this work for the Pareto optimization of both robust and reliability-based approach using some non-commensurable stochastic objective functions is very promising and can be generally used in the optimum design of real-world complex control systems with probabilistic uncertainties

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## **Robotics Automation and Control**

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This book was conceived as a gathering place of new ideas from academia, industry, research and practice in the fields of robotics, automation and control. The aim of the book was to point out interactions among various fields of interests in spite of diversity and narrow specializations which prevail in the current research. The common denominator of all included chapters appears to be a synergy of various specializations. This synergy yields deeper understanding of the treated problems. Each new approach applied to a particular problem can enrich and inspire improvements of already established approaches to the problem.

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