

# Efficient Data Collection with an Automated Robotic Helicopter Using Bayesian Adaptive Sampling Algorithms for Control

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## 1. Introduction

Traditional methods of data collection are often expensive and time consuming. We propose a novel data collection technique, called Bayesian Adaptive Sampling (BAS), which enables us to capture maximum information from minimal sample size. In this technique, the information available at any given point is used to direct future data collection from locations that are likely to provide the most useful observations in terms of gaining the most accuracy in the estimation of quantities of interest. We apply this approach to the problem of estimating the amount of carbon sequestered by trees. Data may be collected by an autonomous helicopter with onboard instrumentation and computing capability, which after taking measurements, would then analyze currently available data and determine the next best informative location at which a measurement should be taken. We quantify the errors in estimation and work towards achieving maximal information from minimal sample sizes. We conclude by presenting experimental results that suggest our approach towards biomass estimation is more accurate and efficient as compared to random sampling.

Bayesian Adaptive Sampling (BAS) is a methodology that allows a system to examine currently available data in order to determine new locations at which to take new readings. This procedure leads to the identification of locations where new observations are likely to yield the most information about a process, thus minimizing the required data that must be collected. As an example of the application of this methodology, we examine the question of standing woods in the United States. In order to estimate the amount of carbon sequestered by trees in the United States, the amount of standing woods must be estimated with quantifiable uncertainty (Wheeler, 2006). Such estimates come from either satellite images or near ground measurements. The amounts of error in the estimates from these two approaches are currently unknown. To this end, an autonomous helicopter with differential GPS (Global Positioning System), LIDAR (Light Detection and Ranging), stereo imagers, and spectrometers has been developed as a testing platform for conducting further studies (Wheeler, 2006). These instruments are capable of measuring the reflectance data and the location of the Sun and helicopter in terms of the zenith and the azimuth angles (Figure 1). The objective is to develop a controlling software system for this robotic helicopter, which optimizes the required ground

Source: *Advances in Robotics, Automation and Control*, Book edited by: Jesús Arámburo and Antonio Ramírez Treviño, ISBN 78-953-7619-16-9, pp. 472, October 2008, I-Tech, Vienna, Austria

sampling. There are a number of methods by which data may be collected. The first simplistic data collection method is to conduct an exhaustive ground sampling, that is, to send the helicopter to every possible location. The second approach is to perform random sampling until the estimates have acceptable standard errors. Although random sampling presents a possibility that the helicopter will take samples from the locations that offer the greatest amount of information, and therefore reduce the needed sample size, there is no guarantee that such a sample set will be chosen every time. The third and more efficient method is to take only a few samples from “key” locations that are expected to offer the greatest amount of information. The focus of this paper is to develop a methodology that will identify such key locations from which the helicopter should gather data.

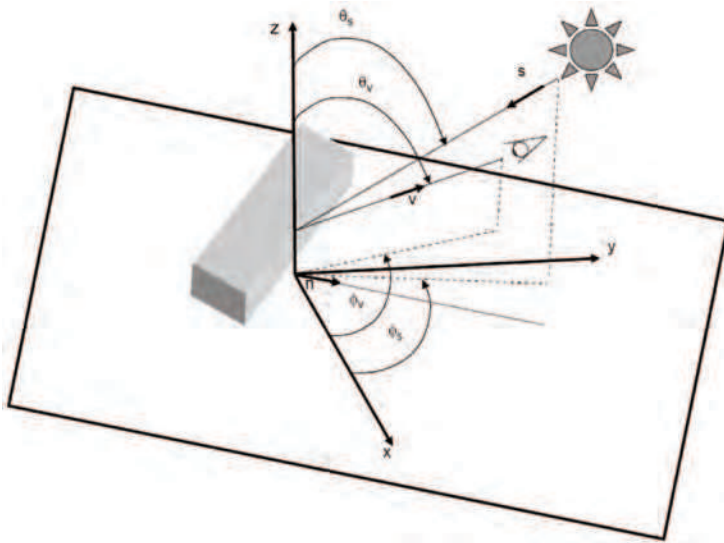


Fig. 1. Viewpoint and Position of the Sun.  $\theta_s, \phi_s$  are the zenith and the azimuth angles of the Sun, and  $\theta_v, \phi_v$  are the zenith and the azimuth angles of the view, respectively (Wheeler, 2006).

In the work described here, the key locations are identified using current and previously collected data. The software works in tandem with the sampling hardware to control the helicopter's position. Once a sample has been obtained, the data are fed into the system, which then calculates the next best location to gather further data. Initially, the system assumes an empirical model for the ground being examined. With each addition of data from the instruments, the parameter estimates of the model are updated, and the BAS methodology is used to calculate the helicopter's next position. This process is repeated until the estimated uncertainties of the parameters are within a satisfactory range. This method allows the system to be adaptive during the sampling process and ensures adequate ground coverage.

The application employs a bi-directional reflectance distribution function (BRDF), in which the calculation of the amount of reflection is based on the observed reflectance values of the object, and the positions of the Sun and the viewer (Nicodemus, 1970). The advantage of using this function is that it enables the system to compensate for different positions of the

Sun during sampling. Once the reflectance parameters are estimated, BAS uses the principle of maximum entropy to identify the next location where new observations are likely to yield the most information.

In summary, the BAS methodology allows the system to examine currently available data in order to determine new locations at which to take new reflectance readings. This procedure leads to the identification of locations where new observations are likely to yield the most information.

## 2. Background and related work

Computing view points based on maximum entropy using prior information has been demonstrated by Arbel & Ferrie (1970), where this technique was used to create entropy maps for object recognition. Vazquez et al. (2001) also demonstrated a technique for computing good viewpoints based on Information Theory. Whaite & Ferrie (1994) developed an autonomous explorer that seeks out those locations that give maximum information without using a priori knowledge of the environment. Makay (1992) used Shannon’s entropy to obtain optimal sample points that would yield maximum information. The sample points are taken from the locations that have largest error bars on the interpolation function. In our work, the optimal locations that offer the maximum amount of information are identified using the principle of maximum entropy, where the maximization is performed using techniques suggested by Sebastiani and Wynn (2000).

A simplified but elucidating version of an example given in Sebastiani and Wynn (2000) is as follows: Suppose that the relationship between a dependent variable  $y$  and an independent variable  $x$  is known to be linear over the domain of interest  $x \in [1,4]$ , and is known to be one of two equations with additive noise,  $y_i = 5 + 7x_i + \epsilon_i$  or  $y_i = 17 + 3x_i + \epsilon_i$ , each with a Bayesian prior belief to update of  $1/2$ . Figure 2 shows the two possibilities over

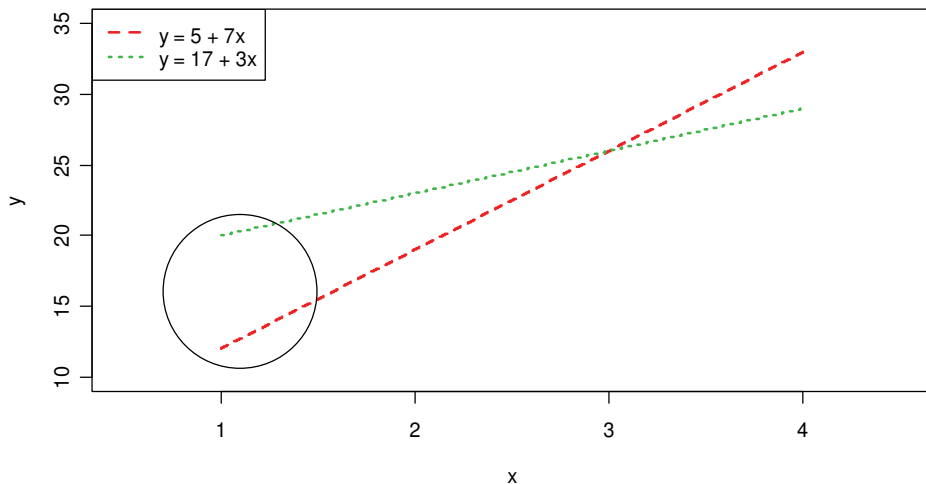


Fig. 2. Two lines, with circle indicating the region where the two models have the greatest difference, and thus where the maximum entropy sampling procedure indicates a new observation should be taken in order to best determine which model is more likely correct.

the domain of interest. Our intuition tells us that an observation taken at  $x = 3$  will not help us to discriminate between the two possible models, as the expected response,  $E(y | x=3)$  is the same for each of the two models. Our intuition further tells us that an observation taken at  $x = 1$  will yield a response,  $y | x=1$ , providing the most "information" as to the correct model, since the two models are most distinct at  $x = 1$ . An application of the theorems of Sebastiani and Wynn show that the optimal location of the independent variable at which to take an observation that maximizes the entropy (expected information as defined by Shannon (1948)) is at  $x^* = \arg_x \min\{(5 + 7x) - (17 + 3x)\}^2 = \arg_x \min\{144 - 96x + 16x^2\} = 1$ , which agrees with our intuition.

### 3. Application and model

Surface BRDF is often used to measure vegetation or other attributes of a surface. By its very nature, BRDF requires measurements taken from a variety of viewing angles and sun positions (and hence different times of day), as solar radiation reflected by the surface is not uniform in all directions (Zhang et al., 1997), nor is it uniform from the same direction when the sun is shining on the location from different directions, as can be seen in Figure 3. This shows the importance of including not only the viewpoint position but also the position of the sun when taking reflectance measurements in order to accurately measure ground cover. The model for the data used in our framework is based on the semi-empirical MISR (multi-angle imaging spectrometer) BRDF Rahman model (Rahman et al., 1993):

$$r(\theta_s, \theta_v, \phi_s, \phi_v) = \rho [\cos(\theta_s) \cos(\theta_v) \{\cos(\theta_s) + \cos(\theta_v)\}]^{k-1} \times \exp(-b \cdot p(\Omega)) h(\theta_s, \theta_v, \phi_s, \phi_v) \quad (1)$$

where

$$h(\theta_s, \theta_v, \phi_s, \phi_v) = 1 + \frac{1 - \rho}{1 + G(\theta_s, \theta_v, \phi_s, \phi_v)} \quad (2)$$

$$G(\theta_s, \theta_v, \phi_s, \phi_v) = \sqrt{\tan^2(\theta_s) + \tan^2(\theta_v) - 2 \tan(\theta_s) \tan(\theta_v) \cos(\phi_s - \phi_v)} \quad (3)$$

$$p(\Omega) = \cos(\theta_s) \cos(\theta_v) + \sin(\theta_s) \sin(\theta_v) \cos(\phi_s - \phi_v) \quad (4)$$

where  $r = r(\theta_s, \theta_v, \phi_s, \phi_v)$  is the measured reflectance,  $\rho$  is the surface reflectance at zenith,  $k$  is the surface slope of reflectance,  $b$  is a constant associated with the hotspot, or "antisolar point" (the point of maximum reflectivity, which is the position where the sensor is in direct alignment between the Sun and the ground target),  $\theta_s, \phi_s$  are the zenith and the azimuth angles of the Sun, respectively (Fig. 1), and  $\theta_v, \phi_v$  are the zenith and the azimuth angles of the view, respectively (Fig. 1), where in each case the zenith angle  $\theta$  ranges from 0 to  $\pi/2$  (horizontal to vertical) and the azimuth angle ranges from 0 to  $2\pi$  (all the way around a circle), thus the viewpoint (and the sun) are assumed on a half-hemisphere centered over the object or in this case region of land to be studied.

With this application, the goal of this study is to estimate the parameters  $\rho$ ,  $k$  and  $b$  to a predefined level of accuracy, or as some in the engineering field say, to “minimize the length of the error bars of the parameter estimates”, with as few observations as is reasonably possible.

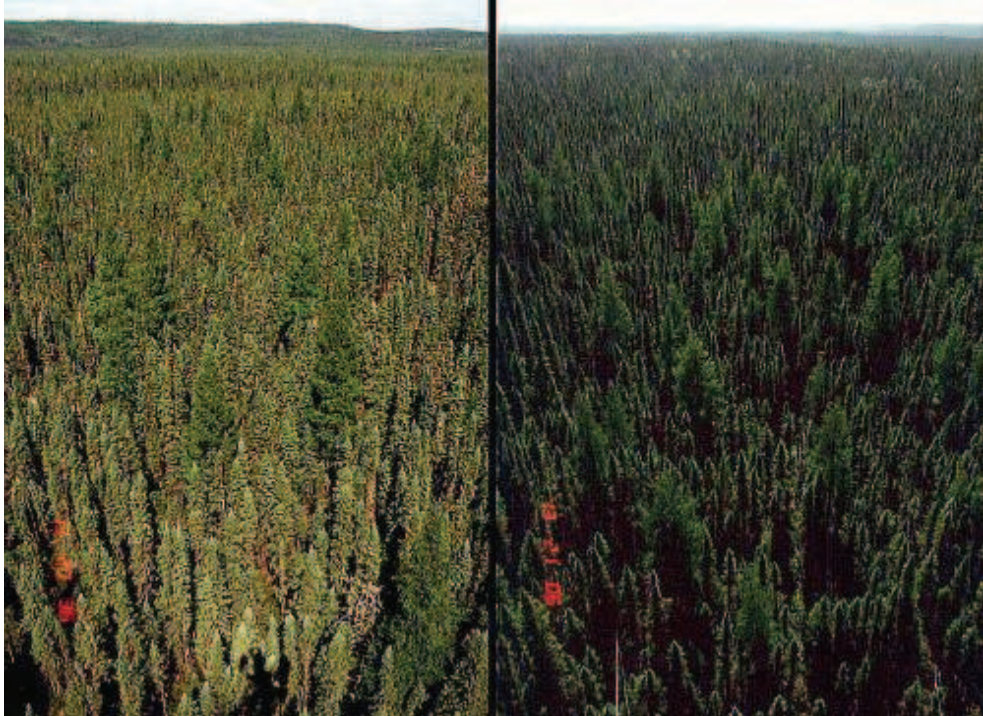


Fig. 3. The effect of forward scattering of light (on the left) and backward scattering of light (on the right) in two photographs of the same location taken from the same position at different times of day (Lucht & Schaaf, 2006).

#### 4. Methodology

Our framework consists of the following two steps:

1. Parameter Estimation: In this step, we estimate the values of the parameters ( $\rho$ ,  $k$  and  $b$ ), and their covariance matrix and standard errors, given currently available data of the amount of observed reflected light, and the zenith and azimuth angles of the Sun and the observer.
2. Bayesian Adaptive Sampling (Optimal Location Identification): In this step, we use the principle of maximum entropy to identify the key locations from which to collect the data.

Once the key location is identified, the helicopter goes to that location, and the instruments on the helicopter measure the reflectance information. These data are then fed into the Parameter Estimation stage and the new values of the parameters ( $\rho$ ,  $k$  and  $b$ ) are

calculated. This process is repeated (Fig. 4) until the standard errors of the parameter estimates achieve some predefined small value, ensuring adequacy of the estimated parameters. This process minimizes the required number of observations to be taken by ensuring that each new observation is taken from a location which maximizes the expected information obtained from the forthcoming observation, and using the then observed data in order to determine the location from which to take the subsequent observation.

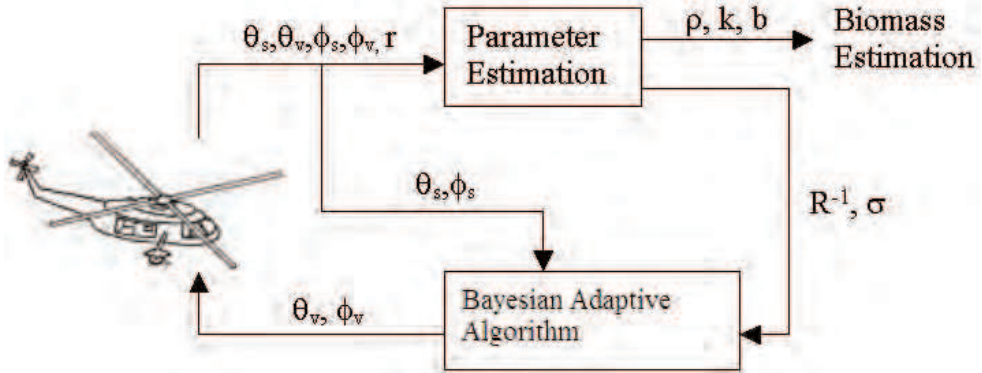


Fig. 4. Overview of Bayesian Adaptive Sampling.

### 5. Implementation

#### 5.1 Parameter estimation

The input to this step is the observed reflectance value ( $r$ ), zenith and azimuth angles of the Sun ( $\theta_s, \phi_s$ ), and zenith and azimuth angles of the observer/helicopter ( $\theta_v, \phi_v$ ). The parameters ( $\rho, k$ , and  $b$ ) are estimated using the following iterated linear regression algorithm: Taking the natural logarithm of  $r = r(\theta_s, \theta_v, \phi_s, \phi_v)$  results in the following near linear version of this model:

$$\ln r = \ln \rho + (k - 1) \ln [\cos(\theta_s) \cos(\theta_v) \{ \cos(\theta_s) + \cos(\theta_v) \}] - b p(\Omega) + \ln h(\theta_s, \theta_v, \phi_s, \phi_v) \tag{5}$$

Note that aside from the term  $\ln h(\theta_s, \theta_v, \phi_s, \phi_v)$ , which contains a nonlinear  $\rho$ , the function  $\ln(r)$  is linear in all three parameters,  $\ln(\rho), k$ , and  $b$ . "Linearization" of  $\ln(h)$  is accomplished by using the estimate of  $\rho$  from the previous iteration, where at iteration  $n$  in the linear least-squares fit the value of  $h(\theta_s, \theta_v, \phi_s, \phi_v)$  from (2) is modified and is taken to be the constant in (6) where  $\rho^{(0)}$  is set equal to zero.

$$h^{(n)}(\theta_s, \theta_v, \phi_s, \phi_v) = 1 + \frac{1 - \rho^{(n-1)}}{1 + G(\theta_s, \theta_v, \phi_s, \phi_v)} \tag{6}$$

We thus obtain as our model, for observation  $i$ , the linear model

$$y_i = \beta_0 + k x_{i1} + b x_{i2} + \varepsilon_i \tag{7}$$

where we assume that the errors in the model,  $\varepsilon_i$ , are independent and identically distributed (*iid*) random variables from a Normal distribution with mean zero and variance  $\sigma^2$ ;  $\beta_0 = \ln \rho$  is the regression constant/intercept to be estimated;  $x_{i1}, x_{i2}$  are the independent/ explanatory variables given in (8) and (9), which are functions of the viewpoint and the position of the sun as described above and are measured at observation time  $i$ ; and  $y_i$  is a function of the response and functions which are known/fixed constants at the given time  $i$  as shown in (10).

$$x_{i1} = \ln[\cos(\theta_{si})\cos(\theta_{vi})\{\cos(\theta_{si}) + \cos(\theta_{vi})\}] \tag{8}$$

$$x_{i2} = -\rho(\Omega) = -[\cos(\theta_{si})\cos(\theta_{vi}) + \sin(\theta_{si})\sin(\theta_{vi})\cos(\phi_{si} - \phi_{vi})] \tag{9}$$

$$y_i = \ln(r_i) + \ln[\cos(\theta_{si})\cos(\theta_{vi})\{\cos(\theta_{si}) + \cos(\theta_{vi})\}] - \ln h^{(n)}(\theta_{si}, \theta_{vi}, \phi_{si}, \phi_{vi}) \tag{10}$$

Next, linear regression is performed on the model in (7) (with iteration until the convergence of the parameters due to the nonlinear term  $h(\theta_s, \theta_v, \phi_s, \phi_v)$  being treated as the constant  $h^{(n)}(\theta_s, \theta_v, \phi_s, \phi_v)$ ). From the regression, we find estimates of the following quantities: Estimates of the parameters  $\beta_0, k,$  and  $b$ , an estimate of  $\sigma^2$  (the variance of the errors, as discussed in and around (7)), and an estimate of the covariance matrix,  $R^{-1}$ , of the parameter estimates, i.e., the estimate of the matrix given in (11). Recall that  $\beta_0 = \ln \rho$  so that  $\rho = \exp(\beta_0)$ , and through use of the delta method (Casella and Berger, 2002) an estimate of  $\rho$  is  $\hat{\rho} = \exp(\hat{\beta}_0)$  and  $\hat{V}(\hat{\rho}) \approx (\exp(\hat{\beta}_0))^2 \hat{V}(\hat{\beta}_0) = \hat{\rho}^2 \hat{V}(\hat{\beta}_0)$ .

$$R^{-1} = \begin{bmatrix} V(\hat{\beta}_0) & Cov(\hat{\beta}_0, \hat{k}) & Cov(\hat{\beta}_0, \hat{b}) \\ Cov(\hat{k}, \hat{\beta}_0) & V(\hat{k}) & Cov(\hat{k}, \hat{b}) \\ Cov(\hat{b}, \hat{\beta}_0) & Cov(\hat{b}, \hat{k}) & V(\hat{b}) \end{bmatrix} \tag{11}$$

**5.2 Bayesian adaptive sampling**

This step identifies the most informative location  $(\theta_v, \phi_v)$  to which to send the helicopter to take a new reflectance reading. We employ the principle of maximum entropy, in which the available information is analyzed in order to determine a unique epistemic probability distribution. The maximization is performed as per techniques suggested by Sebastiani and Wynn (2000), where in order to maximize the amount of information about the posterior parameters ( $\rho, k$  and  $b$ ), we should maximize the entropy of the distribution function. Mathematically, maximizing the entropy is achieved by maximizing  $\ln |X' \Sigma^{-1} X + R|$  where  $|A|$  is the determinant of the matrix A,  $A'$  indicates the transpose of the matrix A,  $\Sigma$  is the covariance matrix of the error terms, and  $X$  is the regression design matrix where the  $i^{th}$  row is associated with the  $i^{th}$  observation (either a future or past observation, depending on the context)

$$X = \begin{bmatrix} 1 & x_{11} & x_{12} \\ 1 & x_{21} & x_{22} \\ \vdots & \vdots & \vdots \\ 1 & x_{k1} & x_{k2} \end{bmatrix}.$$

As the natural logarithm is a monotone increasing function, and we have assumed that the errors are *iid*  $N(0, \sigma^2)$  and so  $\Sigma = \frac{1}{\sigma^2}I$  where  $I$  is the identity matrix, our criteria reduces to maximizing the quantity given in (12).

$$\left| \frac{1}{\sigma^2} X'X + R \right| \quad (12)$$

Note that  $\sigma^2$  and  $R$  are estimated in the Parameter Estimation step and are thus at this stage assumed to be known quantities. As described above, the matrix  $X$  contains functions of the zenith and azimuth angles of the Sun,  $(\theta_s, \phi_s)$ , and the viewpoint/helicopter,  $(\theta_v, \phi_v)$  at which future observations are to be taken (i.e., for each time  $i$  at which future observations are to be taken). Since the times at which future observations are to be taken are known, and thus the positions of the sun at these times are known, the only remaining unknown quantities in (12) are the values of  $(\theta_s, \phi_s)$ . Thus, the new location(s) to which the helicopter will be sent are the values of  $(\theta_v, \phi_v)$  in the rows of  $X$  associated with new observations that maximize (12).

### 5.3 Initialization

Before data have been collected, one of course cannot calculate parameter estimates as described in section 5.1. Before parameter estimates are collected, one does not have an estimate of  $R$  or  $\sigma^2$  to use in (12) from section 5.2. In order to initialize the procedure, one can use a Bayesian prior for these quantities, or equivalently, any estimate based upon prior knowledge of the quantities. In the absence of any prior knowledge, one may take an uninformative prior for  $R$  to be the identity matrix  $I$ , and any arbitrary estimate of  $\sigma^2$ . See the appendix for a more thorough discussion of the quantity in (12) to be maximized.

## 6. Simulation

We conduct two simulated experiments in which the estimates of the model parameters are calculated. In the first experiment, "Estimation Using Random Observations", the data are collected by sending the helicopter to random locations. In the second experiment, "Estimation using BAS", the data are collected using BAS.

We note that the Sun moves through  $2\pi$  radians in a 24-hour period, i.e., at the rate of  $2\pi/(24*60) \approx 0.004363323$  or slightly less than 0.005 radians per minute. We will assume it takes about 2 minutes for the helicopter to move to a new location. Thus, the position of the Sun changes by approximately 0.01 radians between measurements.

In our simulation, the true values of the parameters  $\rho, k$  and  $b$  are 0.1, 0.9, and -0.1, respectively. For the purpose of this paper, the observed values were simulated with added



noise from the process with known parameters. This allows us to measure the efficacy of the algorithm in estimating the parameters and minimizing the standard errors of our estimates. In actual practice, the parameters would be unknown, and we would have no way of knowing how close our estimates are to the truth, that is, if the estimates are as accurate as implied by the error bars.

**6.1 Estimation using observations taken at random locations**

In this experiment, we send the helicopter to 20 randomly chosen locations to collect data. Starting with the fifth observation, we use the regression-fitting algorithm on the collected input data set (the observed reflectance information, and the positions of the Sun and the helicopter), to estimate the values of the parameters  $\rho, k, b$  as well as their standard errors. Table 1 shows the results of this experiment.

Obs #	$\theta_v$	$\phi_v$	r	Estimate (se) of $\rho$	Estimate (se) of $k$	Estimate (se) of $b$
1	0.114	1.673	0.157552			
2	0.882	6.013	0.156616			
3	0.761	0.917	0.192889			
4	0.678	1.308	0.180404			
5	0.260	0.114	0.152558	0.0683 (0.1172)	0.8497 (0.0607)	-0.5958 (0.1413)
6	1.195	2.367	0.146659	0.0767 (0.0932)	0.7906 (0.0476)	-0.4506 (0.1040)
7	0.237	2.805	0.149475	0.0830 (0.0746)	0.8268 (0.0404)	-0.3745 (0.0893)
8	0.166	1.700	0.155497	0.0832 (0.0641)	0.8286 (0.0345)	-0.3722 (0.0788)
9	0.320	2.012	0.154191	0.0831 (0.0572)	0.8277 (0.0307)	-0.3735 (0.0713)
10	1.224	4.085	0.129133	0.0917 (0.0465)	0.8369 (0.0381)	-0.2483 (0.0539)
11	1.409	3.442	0.135005	0.0917 (0.0431)	0.8380 (0.0309)	-0.2481 (0.0503)
12	0.092	1.559	0.154096	0.0920 (0.0394)	0.8398 (0.0285)	-0.2462 (0.0471)
13	0.806	0.891	0.200401	0.0888 (0.0402)	0.8129 (0.0284)	-0.2952 (0.0453)
14	1.256	5.467	0.147654	0.0891 (0.0385)	0.8181 (0.0259)	-0.2914 (0.0433)
15	0.227	1.284	0.155373	0.0889 (0.0368)	0.8169 (0.0248)	-0.2919 (0.0418)
16	1.129	5.522	0.148721	0.0889 (0.0354)	0.8174 (0.0236)	-0.2918 (0.0402)
17	0.507	5.696	0.150381	0.0891 (0.0333)	0.8183 (0.0225)	-0.2904 (0.0380)
18	0.119	4.363	0.142232	0.0890 (0.0302)	0.8181 (0.0207)	-0.2908 (0.0357)
19	0.245	0.524	0.151915	0.0889 (0.0299)	0.8172 (0.0205)	-0.2901 (0.0355)
20	0.446	2.408	0.144471	0.0884 (0.0297)	0.8149 (0.0204)	-0.2930 (0.0354)

Table 1. Observations and Estimates Using Random Sampling

**6.2 Estimation using BAS**

In this experiment, the first five locations of the helicopter are chosen simultaneously using an uninformative prior distribution (*i.e.*, as no estimate of  $R$  has yet been formed; it is taken to be  $\sigma^2 I$ , with  $\sigma^2$  taken to be  $10^{-6}$  based on information regarding the accuracy of the instrumentation) and an  $X$  matrix with five rows in which the positions of the Sun ( $\theta_{si}, \phi_{si}$ ) are known and (12) is maximized over five pairs of helicopter viewpoints ( $\theta_{vi}, \phi_{vi}$ ). Subsequently, we use BAS to calculate the next single best informative location for the helicopter to move to in order to take a new reflectance observation, in which case (12) is

maximized with an  $X$  matrix containing a single row in which the position of the Sun  $(\theta_s, \phi_s)$  is known,  $\sigma^2$  and  $R$  are taken to be their estimated values from the regression performed on the data collected to this point, and the only unknowns, to be calculated, are a single pair of helicopter viewpoint values,  $(\theta_v, \phi_v)$ . After the helicopter is sent to this location and takes a reflectance reading, a new regression is performed on all data accumulated to this point, and the procedure is repeated. In practice, a computer onboard the helicopter would perform the regression, calculate the new next-best location, direct the helicopter to that location, take the reflectance reading, and repeat until some predetermined stopping point, which might include some fixed number of observations, or until the error bars have reached some predefined level indicating an acceptable accuracy in the parameter estimates, or perhaps until the helicopter has nearly exhausted its fuel supply. In this simulation, we stop at the fixed number of 20 observations.

Table 2 shows the results from this experiment. In both experiments estimates of the parameters, along with their standard errors, cannot be formed until at least five observations have been taken.

Obs #	$\theta_v$	$\phi_v$	$r$	Estimate (se) of $\rho$	Estimate (se) of $k$	Estimate (se) of $b$
1	0.460	0.795	0.172364			
2	0.470	0.805	0.177412			
3	1.561	3.957	0.161359			
4	1.561	0.825	0.183571			
5	1.265	3.977	0.129712	0.1041 (0.0325)	0.90904 (0.00879)	-0.1249 (0.0290)
6	0.514	0.845	0.173072	0.1042 (0.0252)	0.90927 (0.00700)	-0.1255 (0.0233)
7	1.561	3.400	0.160130	0.1045 (0.0223)	0.90857 (0.00615)	-0.1220 (0.0199)
8	1.172	4.007	0.130101	0.1029 (0.0192)	0.90547 (0.00577)	-0.1329 (0.0180)
9	0.723	0.875	0.189697	0.1039 (0.0244)	0.90663 (0.00748)	-0.1428 (0.0228)
10	1.561	0.885	0.192543	0.1042 (0.0213)	0.90801 (0.00569)	-0.1394 (0.0185)
11	0.527	0.895	0.172811	0.1042 (0.0193)	0.90796 (0.00523)	-0.1392 (0.0172)
12	1.561	4.047	0.164530	0.1044 (0.0193)	0.90696 (0.00519)	-0.1343 (0.0167)
13	1.561	4.057	0.164822	0.1046 (0.0190)	0.90636 (0.00505)	-0.1314 (0.0161)
14	1.137	4.067	0.131443	0.1038 (0.0169)	0.90483 (0.00471)	-0.1365 (0.0148)
15	0.713	0.935	0.183894	0.1042 (0.0169)	0.90538 (0.00480)	-0.1397 (0.0149)
16	1.561	0.945	0.192280	0.1048 (0.0163)	0.90777 (0.00427)	-0.1333 (0.0136)
17	1.187	4.097	0.134701	0.1047 (0.0146)	0.90757 (0.00399)	-0.1340 (0.0125)
18	0.655	0.965	0.176841	0.1048 (0.0140)	0.90779 (0.00385)	-0.1349 (0.0120)
19	1.561	4.117	0.168819	0.1049 (0.0142)	0.90694 (0.00388)	-0.1321 (0.0120)
20	1.148	4.127	0.132199	0.1045 (0.0132)	0.90617 (0.00373)	-0.1349 (0.0114)

Table 2. Observations and Estimates Using BAS

### 6.3 Results

In this section, we compare and analyze the results of our two experiments. The comparison results shown graphically in Figures 5-7 show that the estimates using the data from the "well chosen" locations using BAS are closer to the true values,  $\rho = .1$ ,  $k = 0.9$  and  $b = -0.1$ , than the estimates based on data from the randomly chosen locations. Also, the error bars

using BAS are much shorter, indicating higher confidence in the estimates of the parameters based on the "well chosen locations", *e.g.*, the length of the error bar for the estimate calculated using data/observations from five well chosen locations is as short as the error bar based on data collected from 20 random locations.

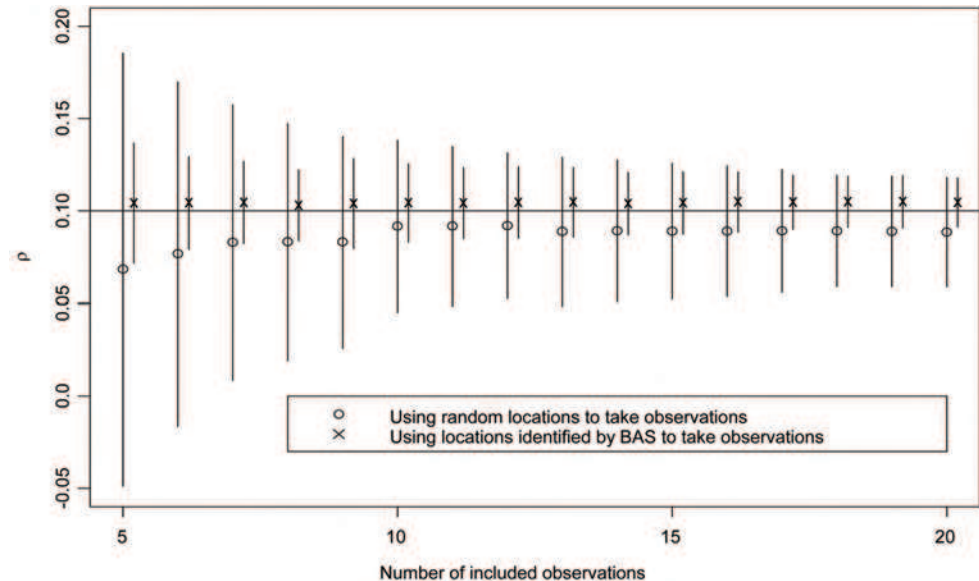


Fig. 5. Estimates and Error Bars for  $\rho$ .

Within each figure, the horizontal axis indicates the number of observations between five and twenty that were used in forming the estimates. The vertical axis is on the scale of the parameter being estimated. Above each observation number, an "o" represents the point estimate (using the data from the first observation through the observation number under consideration) of the parameter using the randomly chosen locations and the observations from those locations. The "x" represents the point estimate of the parameter using observations taken at locations chosen through BAS. The bars are "error bars" and extend one standard error above and below the estimated parameter, based on the data collected at locations from the appropriate experiment. The horizontal line represents the true value of the parameter in our simulation.

Note that in Figure 6 and Figure 7, the error bars rarely overlap the true value of the parameter. This can be attributed to two factors. In large part, this is due to the fact that they are engineering "error bars" with a length of one standard error beyond the point estimate. Traditional 95% statistical confidence intervals based on two standard errors would in virtually every case overlap the true values. Additionally, these are cumulative plots, in which the same data are used, adding observations to form the parameter estimates as one moves to the right in each figure. Thus the point estimates and error bars are dependent upon one another within a figure, and not independent point estimates and confidence intervals.

Finally, we see that the estimates using BAS (to select the points from which to take observation) are generally closer to the truth than when we use random points to take observations, and more importantly the standard errors associated with any given number of observations are much smaller.

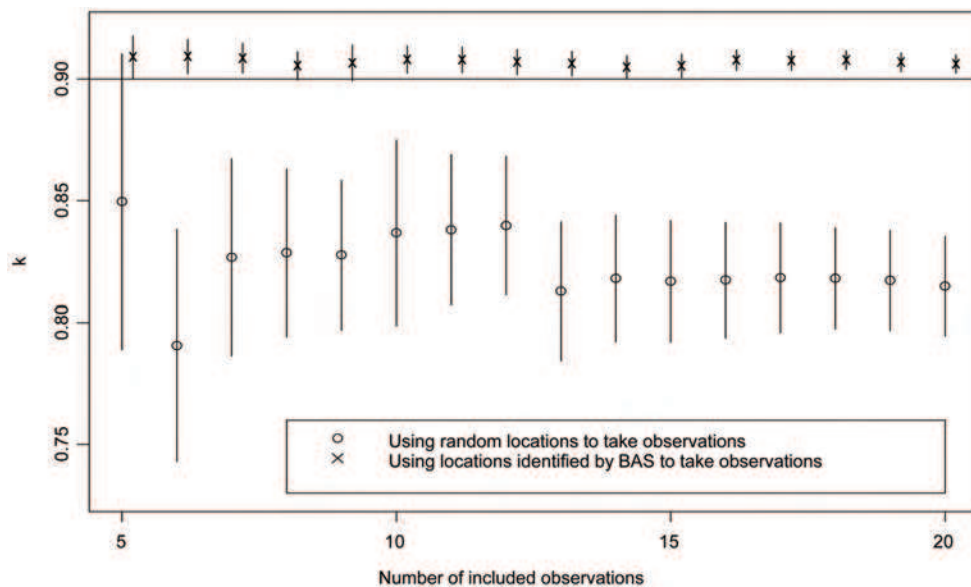


Fig. 6. Estimates and Error Bars for  $k$ .

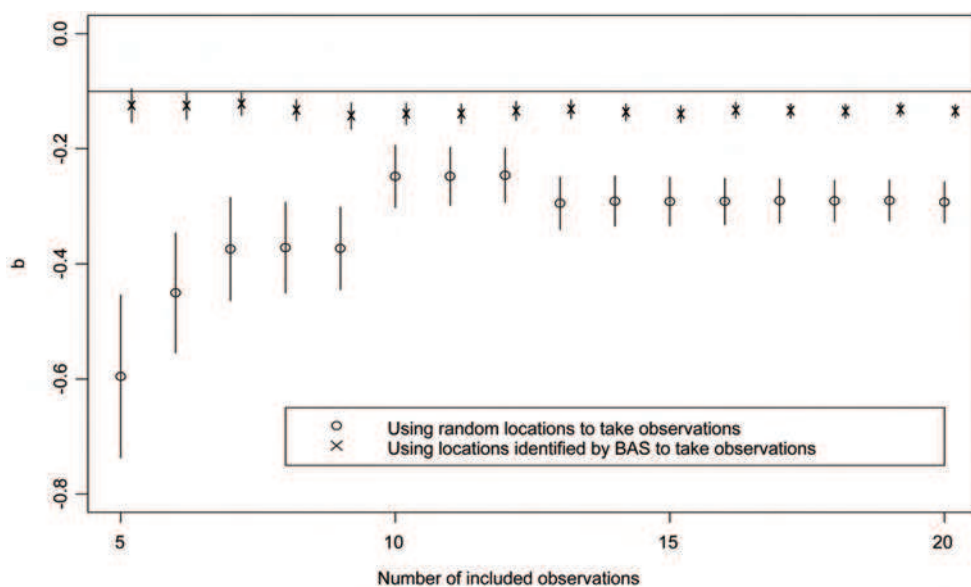


Fig. 7. Estimates and Error Bars for  $b$ .

## 7. Conclusions and future work

Our initial results have shown that BAS is highly efficient compared to random sampling. The rate at which the standard errors, or the error bars, are reduced is much quicker, and hence the significant amount of information is found more quickly compared to other traditional methods. We have also shown that this methodology performs well even in the absence of any preliminary observations. Further simulation has shown evidence that BAS can be three times as efficient as random sampling. This efficiency amounts to savings of time and money during actual data collection and analysis.

In addition to the application discussed in this paper, the theoretical framework presented here is generic and can be applied directly to other applications, such as, military, medical, computer vision, and robotics.

Our proposed framework is based on the multivariate normal distribution. The immediate extensions of this framework will be to accommodate non-normal parameter estimate distributions. As part of our future study, we intend to employ sampling methodologies using Bayesian Estimation Methods for non-normal parameter estimate distributions. We also intend to use cost effectiveness as an additional variable. In this initial work, the focus was to identify the viewpoints that would give us the most information. However, it is not always feasible or efficient to send the helicopter to this next "best" location. As part of our future work, we intend to identify the next "best efficient" location for the helicopter from which it should collect data.

## 8. Acknowledgements

Much of this work was performed as part of the CAMCOS (Center for Applied Mathematics, Computation, and Statistics) student industrial research program in the Department of Mathematics at San Jose State University, San Jose, California, 95192-0103, USA.

This work was supported in part by the NASA Ames Research Center under Grant NNA05CV42A. We are appreciative of the help of Kevin Wheeler, Kevin Knuth and Pat Castle, who while at Ames suggested these research ideas, provided background materials, and assisted in securing funding.<sup>1</sup>

## 9. Appendix

In section 5, we noted that maximizing the entropy of the experiment is equivalent to maximizing the quantity  $\left| \frac{1}{\sigma^2} X X + R \right|$  originally given in (12), where  $R$  and  $\sigma$  come from a

Bayesian prior belief which in this case could be interpreted as the estimates based upon a regression of the data collected up to the given point in time. The matrix  $X$  contains a row(s) with functions of known position(s) of the sun  $(\theta_s, \phi_s)$  and unknown positions of the

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<sup>1</sup> Before this research could be implemented in practice, two of the above mentioned liaisons left NASA and the then current funding expired.

viewpoint/helicopter  $(\theta_v, \phi_v)$  which are then the parameters over which the objective function is maximized. In this setting,  $X$  is based on future observations. If the  $X$  matrix is filled with the appropriate functions (as defined in (8) and (9)) from all of the observations already taken, each observation being associated with one row in the  $X$  matrix, where we might call this matrix  $X_{obs}$  where the subscript "obs" reminds us that this matrix is based on previous observations rather than future observations, then the quantity

$$\frac{1}{\hat{\sigma}^2} X'_{obs} X_{obs} \tag{13}$$

is known as the observed Fisher Information. In this setting, the observed Fisher Information matrix is the inverse of the estimated variance matrix of the parameters (Cassella and Berger, 2002; Weisberg, 2005; Montgomery et al., 2006), and thus (13) **IS** the matrix  $R$  that is used above after the initial set of observations are taken and estimates can be formed.

Any linear algebra textbook will show that stacking two conformable matrices, say

$$\aleph = \begin{bmatrix} X_{obs} \\ \dots \\ X \end{bmatrix},$$

will give the result  $\aleph' \aleph = X'_{obs} X_{obs} + X' X$  and thus, after obtaining initial estimates of the variances from the regression and using them for our prior  $R$ , and also estimating the error variance  $\sigma^2$ , our objective function given in (12) can be written as  $\left| \frac{1}{\hat{\sigma}^2} X X + R \right| =$

$$\left| \frac{1}{\hat{\sigma}^2} X X + \frac{1}{\hat{\sigma}^2} X'_{obs} X_{obs} \right| = \left| \frac{1}{\hat{\sigma}^2} (X X + X'_{obs} X_{obs}) \right| = \left( \frac{1}{\hat{\sigma}^2} \right)^3 \left| (X X + X'_{obs} X_{obs}) \right| = \frac{1}{\hat{\sigma}^6} \left| \aleph' \aleph \right|$$

where the third equality follows from the fact that for scalar  $a$  and a matrix  $A$  of dimension  $n \times n$ ,  $|aA| = a^n |A|$ , and that  $X X + X'_{obs} X_{obs}$  is a  $3 \times 3$  matrix since we are estimating three parameters. Noting that we have no control over  $\hat{\sigma}^2$ , the estimated variance of the error terms, we see that maximizing (12) is equivalent to maximizing  $|\aleph' \aleph|$ , where the first, say,  $n$  rows of  $\aleph$  are based on the functions of  $\theta_{si}, \theta_{vi}, \phi_{si}, \phi_{vi}$  in (8) and (9) for observations which have already been taken, and one final row in which the position of the sun,  $\theta_s, \phi_s$ , is known and the maximization occurs over the two parameters  $\theta_v, \phi_v$  which will indicate the new position to which the helicopter should move in order to take the next observation.<sup>2</sup>

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<sup>2</sup> Of course, this assumes that we are using prior data to define our Bayesian prior for  $R$ . If we have a strong belief for some other prior distribution of the parameters (and thus a different variance matrix  $R^{-1}$ ), then the above will not hold.

In summary, we find that maximizing the expected Shannon Information, i.e. the entropy, of an experiment, in which the experiment is associated with a regression fit where the errors are assumed to be Normally distributed, is equivalent to maximizing the determinant of the Fisher Information associated with that experiment. Further study is warranted in cases which are not regression based and/or where errors are not necessarily Normally distributed.

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## **Advances in Robotics, Automation and Control**

Edited by Jesus Aramburo and Antonio Ramirez Trevino

ISBN 978-953-7619-16-9

Hard cover, 472 pages

**Publisher** InTech

**Published online** 01, October, 2008

**Published in print edition** October, 2008

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Steven M. Crunk and Marian Farah (2008). Efficient Data Collection with an Automated Robotic Helicopter Using Bayesian Adaptive Sampling Algorithms for Control, *Advances in Robotics, Automation and Control*, Jesus Aramburo and Antonio Ramirez Trevino (Ed.), ISBN: 978-953-7619-16-9, InTech, Available from: [http://www.intechopen.com/books/advances\\_in\\_robotics\\_automation\\_and\\_control/efficient\\_data\\_collection\\_with\\_an\\_automated\\_robotic\\_helicopter\\_using\\_bayesian\\_adaptive\\_sampling\\_algo](http://www.intechopen.com/books/advances_in_robotics_automation_and_control/efficient_data_collection_with_an_automated_robotic_helicopter_using_bayesian_adaptive_sampling_algo)

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