

# Multi-Model Approaches for Bilinear Predictive Control

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## 1. Introduction

It is well known that linear controllers can exhibit serious performance limitations when applied to nonlinear systems since nominal linear models used during design cannot represent the nonlinear plant in its whole operating range (Arslan et al., 2004). For this reason, several researches has been proposed new techniques in order to supply a solution for this problem. The main alternative technique, proposed by academy, to resolve the referred problem is known as multi-model approach. The basic idea of multi-model approach consists in decompose the system's operating range into a number of operating regimes that completely cover the chosen trajectory as showed in (Foss et al., 1995). There are, basically, two approaches for multi-model. The first one consists of to design a set of suitable controllers (one for each operating regime) and to calculate weighting factors to them as showed in (Arslan et al., 2004) and (Cavalcanti et al., 2007a). The global control signal is a weighting sum of the contributions of each controller. The second one consists of to build a global model as a weighting sum of each local model as showed in (Foss et al., 1995) and (Cavalcanti et al., 2007b). In both cases, a way to measure distances between models is defined. Multivariable Model Predictive Control (MMPC) has been presented in this chapter. MPC is the an of the most important control technique used in industry. Multivariable Bilinear Generalized Predictive Control (MBGPC) is formulated and, its alternative solution, Multivariable Bilinear Generalized Predictive Control with Iterative Compensation (MBGPCIC) is presented. This chapter shows either proposed metrics in order to build multi-model based controllers (based in MBGPC and MBGPCIC) and presents simulation results applied in distillation columns.

## 2. Multivariable Bilinear Generalized Predictive Control (MBGPC)

MBGPC is a MPC technique based in the minimization of a objective function. This objective function considers the predicted output of a system. The prediction is obtained by a mathematical model of this system.

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Considering a multi-input, multi-output (MIMO) system, as showed in Fig.1, with  $p$ -inputs and  $q$ -outputs, being  $y$  the system's output and  $u$  the system's input.

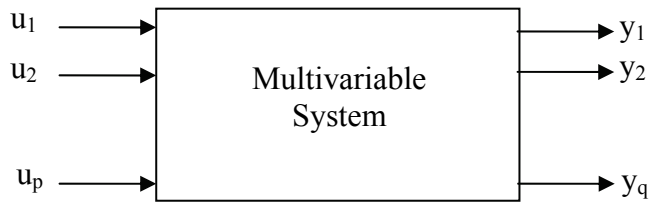


Fig. 1. Block diagram, Multivariable system

Considering that this system is described by the following matrix polynomial expression, showed in (Fontes, 2002):

$$\begin{aligned} A(q^{-1})\Delta_q(q^{-1})y(k) &= B(q^{-1})\Delta_p(q^{-1})u(k-1) + \\ D_e(q^{-1})D[u(k-1)]D_d(q^{-1})\Delta_q(q^{-1})y(k-1) &+ \\ C(q^{-1})e(k) \end{aligned} \quad (1)$$

where  $y(k) \in R^q$  is the process output vector,  $u(k) \in R^p$  is the process input vector and  $e(k) \in R^q$  is the Gaussian white noise with zero mean and covariance  $diag(\sigma^2)$ . The matrices  $A(q^{-1})$ ,  $B(q^{-1})$  and  $C(q^{-1})$  are polynomials matrices in shift operator  $q^{-1}$  and are defined by:

$$A(q^{-1}) = I_{q \times q} + A_1 q^{-1} + \dots + A_{na} q^{-na} \quad (2)$$

$$B(q^{-1}) = B_0 + B_1 q^{-1} + \dots + B_{nb} q^{-nb} \quad (3)$$

$$C(q^{-1}) = I_{p \times p} + C_1 q^{-1} + \dots + C_{nc} q^{-nc} \quad (4)$$

$$D_d(q^{-1}) = D_{d,0} + D_{d,1} q^{-1} + \dots + D_{d,nd_d} q^{-nd_d} \quad (5)$$

$$D_e(q^{-1}) = D_{e,0} + D_{e,1} q^{-1} + \dots + D_{e,nd_e} q^{-nd_e} \quad (6)$$

where  $A(q^{-1}) \in R^{q \times q}$ ,  $B(q^{-1}) \in R^{q \times p}$ ,  $C(q^{-1}) \in R^{q \times q}$ ,  $D_e(q^{-1}) \in R^{q \times p}$  and  $D_d(q^{-1}) \in R^{p \times q}$ . The matrix  $D[u(k-1)]$  is defined as:

$$D[u(k-1)] = \text{diag}[u_1(k-1) \quad u_2(k-1) \quad \dots \quad u_p(k-1)] \quad (7)$$

The nonlinear model presented in (1) is quasi-linearized to be used in MBGPC. The multivariable quasilinear multi-model must be obtained by rewriting the expression (1) of the following form:

$$A(q^{-1}, u)\Delta_q(q^{-1})y(k) = B(q^{-1})\Delta_p(q^{-1})u(k-1) + C(q^{-1})e(k) \quad (8)$$

where

$$A(q^{-1}, u) = A(q^{-1}) - q^{-1}D_e(q^{-1})D[u(k-1)]D_d(q^{-1}) \quad (9)$$

The polynomial matrix  $A(q^{-1}, u)$  is calculated considering its parameters as constant in prediction horizon. The polynomial matrix  $A(q^{-1}, u)$  is considered diagonal in this work. The output prediction  $i$ -step ahead may be obtained of the expression (8), such that:

$$\tilde{A}(q^{-1}, u)y(k+i) = B(q^{-1})\Delta_p(q^{-1})u(k+i-1) + C(q^{-1})e(k+i) \quad (10)$$

where  $\tilde{A}(q^{-1}, u) = A(q^{-1}, u)\Delta_q(q^{-1})$ . In this case, the polynomial matrix  $C(q^{-1})$  is equal to  $I_{p \times p}$  due the fact that the noise be supposed white. Considering the following Diophantine equation:

$$I_{p \times p} = E_i(q^{-1}, u)\tilde{A}(q^{-1}, u) + q^{-i}F_i(q^{-1}, u) \quad (11)$$

where

$$E_i(q^{-1}, u) = E_{i,o}(u) + \dots + E_{i,i-1}(u)q^{-(i-1)} \quad (12)$$

$$F_i(q^{-1}, u) = F_{i,o}(u) + \dots + F_{i,na}(u)q^{-na} \quad (13)$$

Pre-multiplying (10) for  $E_i(q^{-1}, u)$  we obtain:

$$E_i(q^{-1}, u)\tilde{A}(q^{-1}, u)y(k+i) = E_i(q^{-1}, u)B(q^{-1})\Delta_p(q^{-1})u(k+i-1) + E_i(q^{-1}, u)e(k+i) \quad (14)$$

Rewriting (9) of the following form:

$$E_i(q^{-1}, u)\tilde{A}(q^{-1}, u) = I_{p \times p} - q^{-i}F_i(q^{-1}, u) \quad (15)$$

Substituting (15) in (14) we obtain:

$$y(k+i) = F_i(q^{-1}, u)y(k) + E_i(q^{-1}, u)e(k+i) + E_i(q^{-1}, u)B(q^{-1})\Delta_p(q^{-1})u(k+i-1) \quad (16)$$

As the degree of  $E_i(q^{-1}, u)$  is  $i-1$ , then the sub-optimal prediction of  $y(k+i)$  is:

$$\hat{y}(k+i) = F_i(q^{-1}, u)y(k) + E_i(q^{-1}, u)B(q^{-1})\Delta_p(q^{-1})u(k+i-1) \quad (17)$$

In order to separate past and future values, we make:

$$E_i(q^{-1}, u)B(q^{-1}) = H_i(q^{-1}, u) + q^{-i}H_{ipa}(q^{-1}, u) \quad (18)$$

As the degree of  $H_i(q^{-1}, u)$  is less than  $i-1$ , the predictor may be written as:

$$\hat{y}(k+i) = F_i(q^{-1}, u)y(k) + H_{ipa}(q^{-1}, u)\Delta_p(q^{-1})u(k-1) + H_i(q^{-1}, u)\Delta_p(q^{-1})u(k+i-1) \quad (19)$$

The last term of (19) considers the future inputs (forced response) and the two first terms consider only past inputs (free response). So, we define:

$$\hat{y}(k+i) = H_i(q^{-1}, u)\Delta_p(q^{-1})u(k+i-1) + Y_{ii} \tag{20}$$

where

$$Y_{ii} = F_i(q^{-1}, u)y(k) + H_{ipa}(q^{-1}, u)\Delta_p(q^{-1})u(k-1) \tag{21}$$

The expression (21) is the free response. The objective function is given by:

$$J = \sum_{i=N_1}^{NY} \|r(k+i) - \hat{y}(k+i)\|_R^2 + \sum_{i=1}^{NU} \|\Delta u(k+i-1)\|_Q^2 \tag{22}$$

where  $N_1$  is minimum prediction horizon,  $NY$  is prediction horizon,  $NU$  is the control horizon,  $R$  and  $Q$  are weighting matrices of error signal and control effort in instant  $k$  in the chosen trajectory, respectively,  $\hat{y}(k+i)$  is the sub-optimum  $i$ -step ahead predicted output,  $r(k+i)$  is the future reference trajectory. The control effort is obtained, without constraints, by the minimization of the objective function (22).

Consider the predictions set:

$$y_{N_{1y}} = H_{N_{1y}} \Delta_p U_{NU} + Y_{IN_{1y}} \tag{23}$$

where

$$y_{N_{1y}} = [\hat{y}(k+N_1) \quad \hat{y}(k+N_1+1) \quad \dots \quad \hat{y}(k+NY)]^T \tag{24}$$

$$H_{N_{1y}} = \begin{bmatrix} H_{N_1-1} & H_{N_1-2} & \dots & H_{N_1-NU} \\ H_{N_1} & H_{N_1-1} & \dots & H_{N_1+1-NU} \\ \vdots & \vdots & \ddots & \vdots \\ H_{NY-1} & H_{NY-2} & \dots & H_{NY-NU} \end{bmatrix} \tag{25}$$

$$\Delta_p U_{NU} = \begin{bmatrix} \Delta_p(q^{-1})u(k) \\ \Delta_p(q^{-1})u(k+1) \\ \vdots \\ \Delta_p(q^{-1})u(k+NU-1) \end{bmatrix} \tag{26}$$

$$Y_{IN_{1y}} = \begin{bmatrix} Y_{IN_1} \\ Y_{IN_1+1} \\ \vdots \\ Y_{INY} \end{bmatrix} \tag{27}$$

The objective function (22) may be rewritten of the following form:

$$J = (H_{N_{1y}} \Delta_p U_{NU} + y_{IN_{1y}})^T \bar{R} (H_{N_{1y}} \Delta_p U_{NU} + y_{IN_{1y}}) + \Delta_p U_{NU}^T \bar{Q} \Delta_p U_{NU} \quad (28)$$

where

$$\bar{R} = \text{diag}[R_1, \dots, R_{q \times NY}] \quad (29)$$

$$\bar{Q} = \text{diag}[Q_1, \dots, Q_{p \times NU}] \quad (30)$$

This minimization is obtained by the calculation of its gradient (making it equals zero), of the following form:

$$\frac{\partial J}{\partial \Delta_p U_{NU}} = 0 \quad (31)$$

The minimization of (28) produces the following control law:

$$\Delta_p U_{NU} = (H_{N_{1y}}^T H_{N_{1y}} + \bar{Q})^{-1} H_{N_{1y}}^T \bar{R} (r - y_{IN_{1y}}) \quad (32)$$

Because of the receding control horizon, only the first  $p$  rows of (32) are computed.

### 3. Multivariable Bilinear Generalized Predictive Control With Iterative Compensation (MBGPCIC)

#### 3.1 Motivation

The quasi-linearization presented in (8) produces a prediction error that degrades the controller performance. This prediction error increases with the prediction horizon. In order to solve this problem, several algorithms has been proposed (Fontes et al., 2002), (Fontes et al. 2004), (Fontes and Ângelo, 2006) and (Fontes and Laurandi, 2006). This section presents the multivariable case showed in (Fontes and Laurandi, 2006).

The basic quasi-linear algorithm, presented by (Goodhart, 1994), calculates the output prediction  $i$ -step ahead considering the terms  $A_j(q^{-1}, u)$  with  $j=1, \dots, na$  depending only of known values of the input (until  $k-1$  step). The approximation of this approach generates a prediction error that increases with the prediction horizon and degrades the controller performance.

#### 3.2 The basic idea of iterative compensation algorithm

The idea of the iterative compensation algorithm consists of consider the effort control sequence (obtained by the classic quasi-linear algorithm) to correct the parameters of  $A(q^{-1}, u)$ . Considering the following sequence of effort control, calculated in  $k$  step:

$$\Delta_p U_{NU} = [\Delta u_1(k) \ \dots \ \Delta u_p(k) \ \dots \ \Delta u_1(k+NU) \ \dots \ \Delta u_p(k+NU)]^T \quad (33)$$

The classic quasi-linear algorithm, considering (30), calculates the sequence of future control efforts, that is given by:

$$U_{NU} = [u_1(k) \ \cdots \ u_p(k) \ \cdots \ u_1(k+NU) \ \cdots \ u_p(k+NU)]^T \quad (34)$$

where:

$$u_j(k+i) = u_j(k-1) + \sum_{t=0}^i \Delta u_j(k+t), \quad j=1, \dots, p \quad (35)$$

The iterative compensation algorithm uses the values of (34) to correct the polynomial matrix (9). This correction will produce a new prediction and a new sequence of control efforts. This process is repeated until a stop criterion is achieved. It is important to remember that this algorithm considers the receding horizon too. When the algorithm converges, only the first  $p$  rows of (34) are sent to the process.

### 3.3 Convergence and stop criterion

The stop criterion of the algorithm is based in the norm variation of the vector  $\Delta_p U_{NU}$ . The procedure of correction of  $A(q^{-1}, u)$  is repeated until that the norm of the variation calculated in the iteration  $r$  be less than a tolerance value  $\varepsilon$  established:

$$\sqrt{(\Delta_p U_{NU,r} - \Delta_p U_{NU,r-1})^T (\Delta_p U_{NU,r} - \Delta_p U_{NU,r-1})} < \varepsilon \quad (36)$$

It is important to remember that this algorithm may not converge. In this case, another stop criterion must be established. The second stop criterion is based in the maximum number of iterations of the algorithm:

$$r < N_{\max} \quad (37)$$

where  $N_{\max}$  is the maximum number of iterations of the algorithm. If the algorithm stops for the  $N_{\max}$  criterion, the control effort sent to the process is the control effort calculated by the classic quasi-linear algorithm.

## 4. Multi-models approaches for MPC

Considering a bilinear multivariable model showed in (1) that describes the system's behavior in a small region. This structure is valid around its operating regime and more or less invalid outside this regime. Considering yet that the process has been decomposed into  $NOR$  operating regimes, the first step to develop a multi-model structure is to identify  $NOR$  local models (in this case bilinear), of the following form:

$$\begin{aligned} A_{(j)}(q^{-1})\Delta_q(q^{-1})y(k) &= B_{(j)}(q^{-1})\Delta_p(q^{-1})u(k-1) + \\ D_{e(j)}(q^{-1})D[u(k-1)]D_{d(j)}(q^{-1})\Delta_q(q^{-1})y(k-1) + \\ C_{(j)}(q^{-1})e(k) \end{aligned} \quad (38)$$

where  $j = 1, \dots, NOR$ .

#### 4.1 Building a global model

The first multi-model approach consists in to obtain a global model from the local bilinear models showed in (38). To build the global model, we must consider that there is a validity function  $\delta_j(k)$  that is designed such that its value is close to one for operating points where the local model structure is a good description of the system and close to zero otherwise, in instant  $k$ . In this case, each polynomial matrix  $P^{(k)}(q^{-1})$  of (1) for each instant  $k$  would be calculated of the following form:

$$P^{(k)}(q^{-1}) = \sum_{j=1}^{NOR} P_{(j)}(q^{-1})w_{j,k} \quad (38)$$

where  $P^{(k)}(q^{-1})$  the global built polynomial matrix of the bilinear model,  $P_{(j)}(q^{-1})$  is the polynomial matrix of the  $j^{th}$  bilinear model, and:

$$w_{j,k} = \frac{\delta_j(k)}{\sum_{i=1}^{NOR} \delta_i(k)}; \quad j = 1, \dots, NOR \quad (39)$$

where  $w_{j,k}$  is a weighting factor to the  $j^{th}$  bilinear model in instant  $k$ . Theses approaches has the following defined property:

$$\sum_{j=1}^{NOR} w_{j,k} = 1 \quad (40)$$

#### 4.2 Building a global controller

The second multi-model approach consists of to build a global controller from a set of controllers. In this case, one controller is designed to each operation point. A validity function  $\delta_j(k)$  is designed too, in order to evaluate what controller must have a greater weighting factor. Considering the control effort  $u_{i,j}(k)$  with  $j = 1, \dots, NOR$  and  $i = 1, \dots, p$  of each controller (one for each operation point), the control effort sent to the process is given by:

$$u_i(k) = \sum_{j=1}^{NOR} w_{j,k} u_{i,j}(k) \quad (41)$$

where  $w_{j,k}$  is calculated as showed in (39).

#### 4.3 Metric based in norms

The validity function is usually called by *metric*. In multivariable case, in a process with  $p$ -inputs and  $q$ -outputs, the output is  $y(k) \in R^q$  and the input is  $u(k) \in R^p$ . In a known trajectory of process output, the distance from the first operation point to the last operation point is given by:

$$d_{1,NOR} = \|y_{NOR} - y_1\|_2 \quad (42)$$

To measure the distance from the current operation point to the operation point of  $j^{th}$  designed controller, we can use the expression:

$$\delta_j = \frac{d_{1,NOR}}{\|y(k) - y_j\|_2}; \quad i = 1, \dots, NOR \quad (43)$$

To this metric, only monotonic (increasing or decreasing) trajectories must be considered.

#### 4.4 Metric based in phase margin

Phase margin has been chosen as measurement parameter in order to quantitatively estimate the distance between two different models.

One of the most important techniques for measure the robustness of a dynamic system is the margin phase technique. In general terms, in a linear time-invariant system, phase margin is maximum phase angle that can be added, such as this system not becomes instable. In this chapter, a multivariable bilinear model is obtained for each operating regime, and this model is quasi-linearized. Each obtained model has a phase margin value (or multivariable equivalent). An interpolated model is calculated to obtain the current valid model (that depends of the current operation point). The difference between the interpolated model and models of the chosen operating regimes is calculated. This difference is considered to the computation of a set of weighting factors to the controllers. An equivalent method to the multivariable margin phase calculus is showed.

The model showed in (8) is time-step quasi-linear (linear at each time instant). We obtain a transfer function in  $z^{-1}$  operator, pre-multiplying (8) for  $A^{-1(i)}(z^{-1}, u)$  of the following form:

$$y(z^{-1}) = A_{(k)}^{-1}(z^{-1}, u) B_{(k)}(z^{-1}) z^{-1} u(z^{-1}) \quad (44)$$

where  $A_{(k)}^{-1}(z^{-1}, u)$  and  $B_{(k)}(z^{-1})$  are interpolated polynomial matrices in instant  $k$ . From (44), the matrix transfer function for the interpolated model in instant  $k$  is given by:

$$G_{(k)}(z^{-1}, u) = A_{(k)}^{-1}(z^{-1}, u) B_{(k)}(z^{-1}) z^{-1} \quad (45)$$

In this approach, we consider the  $p \times q$  transfer functions from (45), and calculate individually the phase margin from each one transfer function.

For each operating regime, the minor margin phase is chosen. Considering that  $MPM^{(j)}$  represents the minor phase margin of the matrix transfer function in the  $j^{th}$  operating regime and that  $\max(MPM^{(j)})$  is the maximum of phase margin for  $j = 1, \dots, NOR$  and  $\min(MPM^{(j)})$  is the minimum phase margin for  $j = 1, \dots, NOR$ .

Considering yet that  $G_{(k)}(z^{-1}, u)$  is an interpolated transfer function given in  $k$  instant, and  $MPM^{(k)}$  is its minor phase margin, the distance factor to the designed controller in the  $j^{th}$  operating regime, in instant  $k$  is given by:



$$\delta_j = 1 - \frac{|MPM^{(k)} - MPM^{(j)}|}{\max(MPM^{(j)}) - \min(MPM^{(j)})} \quad (46)$$

It is important to observe that the interpolated model's parameter (in this case, obtained by cubic spline interpolation method) is always in the bounds of the estimated model's parameter (it is guaranteed by the algorithm). The algorithm guarantees also that, if a calculated phase margin is out of bounds  $\min(MPM^{(i)})$  and  $\max(MPM^{(i)})$ , the designed controllers of these bounds have weighting factor equals 1 (maximum).

## 5. Applications of multi-model controllers based in proposed metrics

This section shows some applications of the combinations between:

- approaches showed in sections 4.1 and 4.2;
- controllers showed in sections 2 and 3, and;
- metrics showed in sections 4.3 and 4.4.

The application showed in this chapter is based in a simulated debutanizer distillation column. Debutanizer distillation column is usually used to remove the light components from the gasoline stream to produce Liquefied Petroleum Gas (LPG). The most common control strategy is to manipulate the reflux flow rate and the temperature in column's bottom and, to control the concentrations of any product in *butanes* stream and in C5+ stream as showed in (Almeida, *et al.*, 2000). The chosen process variables are: concentration of *i*-pentane in butanes stream ( $y_1$ ) and concentration of *i*-butene in C5+ stream ( $y_2$ ). The studied column is simulated in Hysys software and is showed in Figure 2.

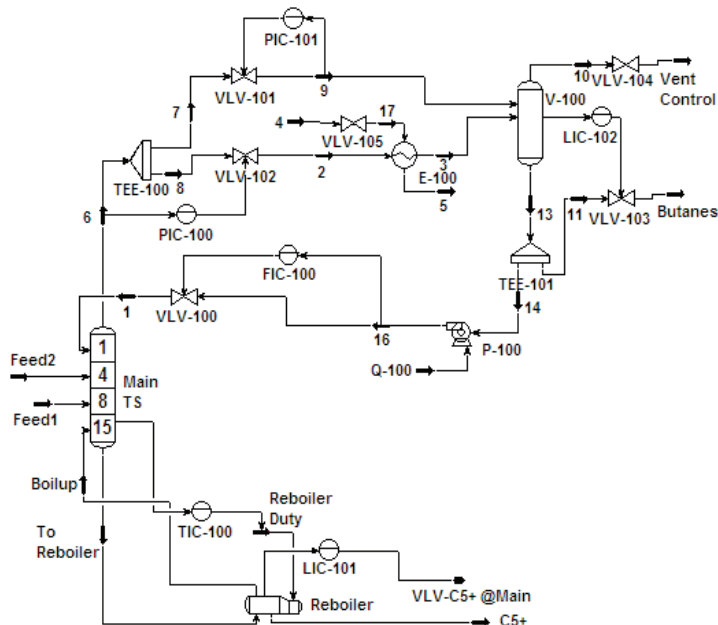


Fig. 2. Distillation Column simulated in Hysys Software

The reflux flow rate ( $u_1$ ) is manipulated through the FIC-100 controller and the temperature of column's bottom ( $u_2$ ) is manipulated through the TIC-100 controller. The reflux flow rate is measured in  $\text{m}^3/\text{h}$  and the temperature of column's bottom is measured in  $^\circ\text{C}$ .

In the applications showed in this chapter, three operation points were chosen, as showed in Table 1. The identified bilinear models were obtained using the multivariable recursive least squares algorithm and the model's structure has been chosen by using the Akaike criterion. In all points, the chosen sample rate is 4 minutes. Only monotonic trajectories are being considered. The trajectory of  $y_1$  is monotonically increasing and the trajectory of  $y_2$  is monotonically decreasing.

Operation Point	Input	Output (Mass Fractions)
1	$u_1 = 40 \text{ m}^3/\text{h}$	$Y_1 = 0.014413$
	$u_2 = 147 \text{ }^\circ\text{C}$	$Y_2 = 0.001339$
2	$u_1 = 37 \text{ m}^3/\text{h}$	$Y_1 = 0.017581$
	$u_2 = 147.5 \text{ }^\circ\text{C}$	$Y_2 = 0.001161$
3	$u_1 = 34 \text{ m}^3/\text{h}$	$Y_1 = 0.021994$
	$u_2 = 148 \text{ }^\circ\text{C}$	$Y_2 = 0.001004$

Table 1. Three operation points chosen in distillation column.

In order to quantitatively assess the performance of multi-model quasi-linear GPC, some indices like showed in (Goodhart, et al., 1994) are calculated. These indices may be extended to multivariable case, of the following form:

$$\varepsilon_{1,i} = \sum |u_i(k)| / N \quad (47)$$

where  $i = 1, \dots, p$  and  $N$  is the amount of control effort applied in the process to achieve the desired response. The index showed in (47) is the account of total control effort to achieve a given response. The variance of controlled actuators is:

$$\varepsilon_{2,i} = \sum (u_i(k) - \varepsilon_{1,i})^2 / N \quad (48)$$

The deviation of the process of integral of absolute error (IAE) is:

$$\varepsilon_{3,j} = \sum |r_j(k) - y_j| / N \quad (49)$$

where  $j = 1, \dots, q$ . The overall measure of effectiveness is defined as:

$$\varepsilon_j = \sum_{i=1}^p (\alpha_i \varepsilon_{1,i} + \beta_i \varepsilon_{2,i}) + \rho_j \varepsilon_{3,j} \quad (50)$$

The factors  $\alpha_i$ ,  $\beta_i$  and  $\rho_j$  are weightings chosen to reflect the actual financial cost of energy usage, actuator wear and product quality, respectively. In this case, we consider  $\alpha_i = 0.1$ ,  $\beta_i = 0.15$  and  $\rho_j = 0.5$  because we have established as priority the product quality.

### 5.1 Application 1 - Controller based in global model, norm-2 metric and MBGPC

In this simulation, the process is in the 3<sup>rd</sup> operating regime and a deviation in reference is applied in the proposed controller. With this reference deviation, the process will come to close to the 1<sup>st</sup> operating regime. The proposed quasi-linear multi-model is compared with quasi-linear single-model (using the model of the 3<sup>rd</sup> operating regime). Figures 3 and 4 show the process's output and Figures 4 and 6 show the control effort.

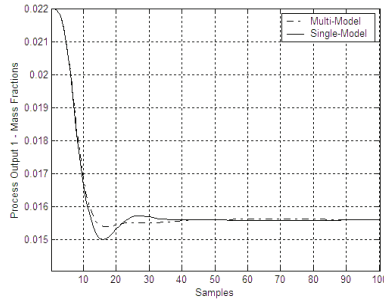


Fig. 3. Process Output 1. Comparison between single-model and multi-model approach (application 1).

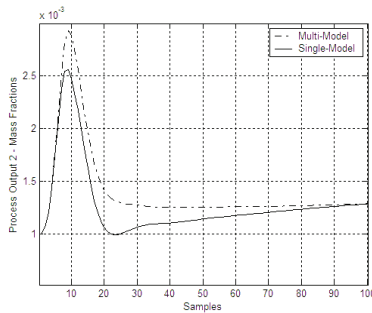


Fig. 4. Process Output 2. Comparison between single-model and multi-model approach (application 1).

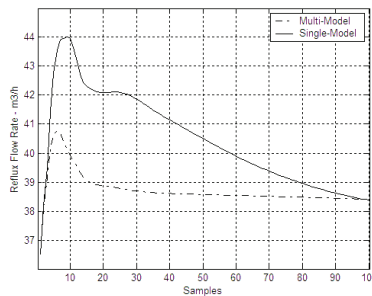


Fig. 5. Reflux Flow rate. Comparison between single-model and multi-model approach (application 1).

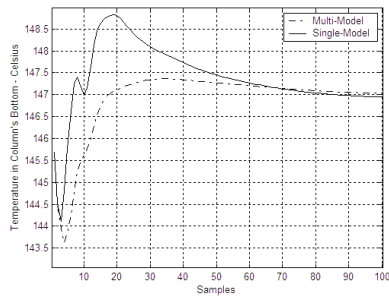


Fig. 6. Temperature in column's bottom. Comparison between single-model and multi-model approach (application 1).

Figures 3, 4, 5 and 6 shows the better performance of multi-model approach, so much of point view of the process response as of the control effort.

## 5.2 Application 2 - Controller based in global controller, norm-2 metric and MBGPC

In this simulation, the same reference deviation as in section 5.1 is applied. The proposed quasi-linear multi-model is compared with quasi-linear single-model (using the model of the 3<sup>rd</sup> operating regime). Figures 7 and 8 show the process's output and Figures 9 and 10 show the control effort.

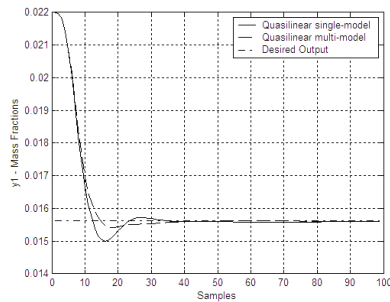


Fig. 7. Process Output 1. Comparison between single-model and multi-model approach (application 2).

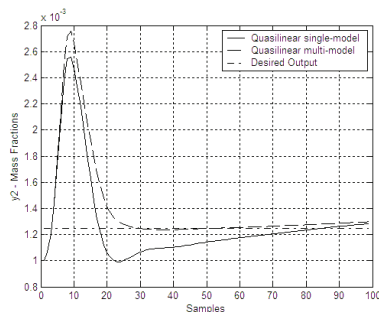


Fig. 8. Process Output 2. Comparison between single-model and multi-model approach (application 1).

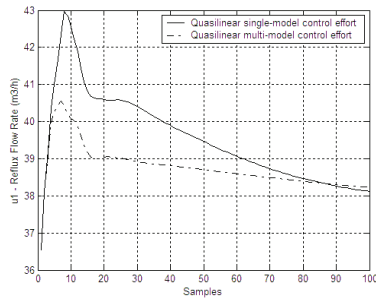


Fig. 9. Reflux Flow rate. Comparison between single-model and multi-model approach (application 2).

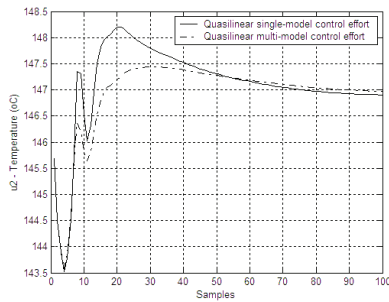


Fig. 10. Temperature in column's bottom. Comparison between single-model and multi-model approach (application 2).

**5.3 Application 3 - controller based in global controller, phase margin metric and MBGPC**

In this simulation, the same reference deviation as in section 5.1 and 5.2 is applied. The proposed quasi-linear multi-model is compared with quasi-linear single-model (using the model of the 3<sup>rd</sup> operating regime). Figures 11 and 12 show the process's output and Figures 13 and 14 show the control effort.

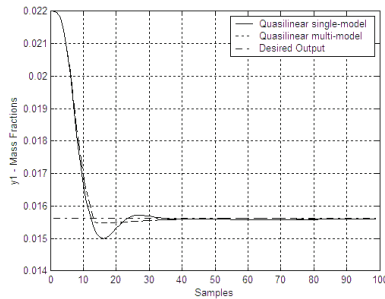


Fig. 11. Process Output 1. Comparison between single-model and multi-model approach (application 3).

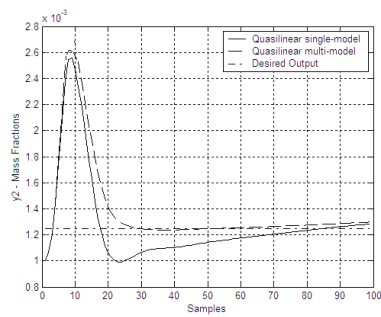


Fig. 12. Process Output 2. Comparison between single-model and multi-model approach (application 3).

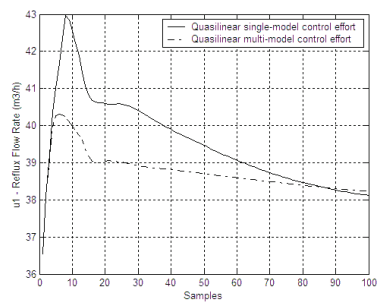


Fig. 13. Reflux Flow rate. Comparison between single-model and multi-model approach (application 3).

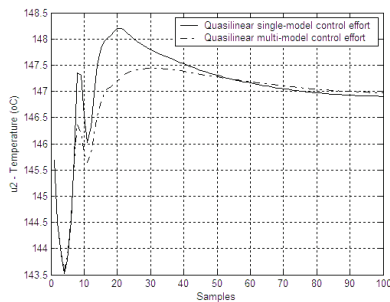


Fig. 14. Temperature in column's bottom. Comparison between single-model and multi-model approach (application 2).

#### 5.4 Qualitative comparison between applications 1,2 and 3

This section shows the qualitative comparison between application 1,2 and 3. The comparison is based in the indices showed in (47), (48), (49) and (50) with  $N=100$ .

The table 2 shows, in all the cases, that multi-model approach has better performance in relation of single-model approach in terms of less energy usage, less actuator wear and better product quality. Application 2 has presented better performance in relation of the other multi-model approaches.

I/O	Approach	$\varepsilon_1$	$\varepsilon_2$	$\varepsilon_3$	$\varepsilon$
1	Single	40.47	2.61	287.46	163.00
2	Single	147.38	0.63	142.40	90.47
1	Application 1	38.72	0.31	255.20	146.26
2	Application 1	146.88	0.36	117.71	77.52
1	Application 2	38.38	0.32	248.41	142.83
2	Application 2	146.94	0.29	103.48	70.36
1	Application 3	38.55	0.33	253.41	145.36
2	Application 3	147.01	0.31	113.71	75.51

Table 2. Qualitative comparison between applications 1,2 and 3.

#### 5.5 Application 4 - controller based in global controller, norm-2 metric and MBGPCIC

In this simulation, the process is the following operation point:  $u_1 = 31 \text{ m}^3/\text{h}$ ,  $u_2 = 148.5 \text{ }^\circ\text{C}$ ,  $y_1=0.028125$  e  $y_2=0.000874$ . The reference deviation 0.01371 and 0.000465 are used in this application. This approach has been compared with application 2. Figures 15 and 16 show the process's output and Figures 17 and 18 show the control effort.

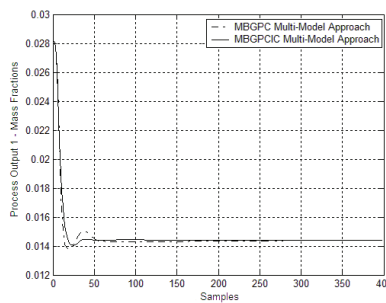


Fig. 15. Process Output 1. Comparison between MBGPC and multi-model approach and MBGPCIC multi-model approach (application 4).

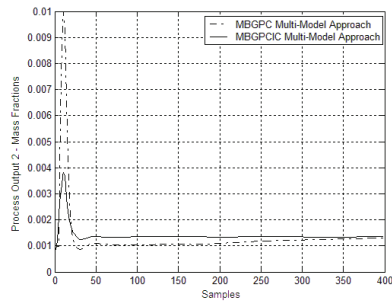


Fig. 16. Process Output 2. Comparison between MBGPC and multi-model approach and MBGPCIC multi-model approach (application 4).

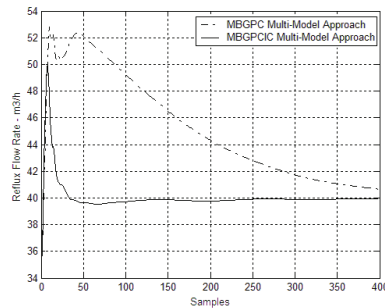


Fig. 17. Reflux Flow rate. Comparison between MBGPC and multi-model approach and MBGPCIC multi-model approach (application 4).

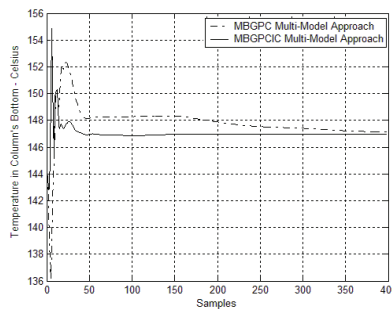


Fig. 18. Temperature in column's bottom. Comparison between MBGPC and multi-model approach and MBGPCIC multi-model approach (application 4).

Figures 15, 16, 17 and 18 show the better performance of MBGPCIC multi-model approach, so much of point view of the process response as of the control effort. The justification to this significant improvement of MBGPCIC multi-model approach in relation to MBGPC



multi-model approach is the wide variation in control effort. So, the iterative compensation procedure finds more space to minimize the prediction error.

Due to the large operating range, this approach has not been compared with single-model approach, because its performance was very poor.

Table 3 shows the performance indices of this comparison for  $N=400$ .

I/O	Approach	$\varepsilon_1$	$\varepsilon_2$	$\varepsilon_3$	$\varepsilon$
1	MBGPCIC Approach	50.47	6.61	1219.90	632.02
2	MBGPCIC Approach	40.71	5.78	1151.12	595.48
1	MBGPC Approach	148.27	8.00	952.00	498.07
2	MBGPC Approach	147.21	1.73	260.98	150.41

Table 3. Qualitative comparison between MBGPC and multi-model approach and MBGPCIC multi-model approach (application 4).

## 6. Conclusion

This chapter showed the importance and the relevance of multi-model approaches. Several researches has been proposed in order to solve design problems in process that operates in a large range (like batch processes). Some proposals of multi-model have been presented in this chapter and its comparison with classic approaches. All multi-model approaches presented better performance when compared to single-model approach. The next step of this research is to adjust these approaches to a robust and stable algorithm of multi-model.

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The book presents an excellent overview of the recent developments in the different areas of Robotics, Automation and Control. Through its 24 chapters, this book presents topics related to control and robot design; it also introduces new mathematical tools and techniques devoted to improve the system modeling and control. An important point is the use of rational agents and heuristic techniques to cope with the computational complexity required for controlling complex systems. Through this book, we also find navigation and vision algorithms, automatic handwritten comprehension and speech recognition systems that will be included in the next generation of productive systems developed by man.

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