Unsteady Axial Viscoelastic Pipe Flows of an Oldroyd B Fluid

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1. Introduction

The unsteady flow of a fluid in cylindrical pipes of uniform circular cross-section has applications in medicine, chemical and petroleum industries [3,4,5]. For viscoelastic fluids, the unsteady axial decay problem for UCM fluid is considered by Rahman et al. [6]; and for Newtonian fluids as a special case. Rajagopal [7] has studied exact solutions for a class of unsteady unidirectional flows of a second-order fluid under four different flow situations. Atalik et al. [8] furnished a strong numerical evidence that non-linear Poiseuille flow is unstable for UCM, Oldroyd-B and Giesekus models. This fact is supported experimentally by Yesilata, [9]. The unsteady flow of a blood, considered as Oldroyd-B fluid, in tubes of rigid walls under specific APGs is concerned by Pontrelli, [10, 11].

Flow of a polymer solution in a circular tube under a pulsatile APG was investigated by Barnes et al. [12, 13]. The same problem for a White-Metzner fluid is performed by Davies et al. [14] and Phan-Thien [15]. Recently, periodic APG for a second-order fluid has been studied by Hayat et al. [16]. Numerical simulation based on the role of the pulsatile wall shear stress in blood flow, is investigated by Grigioni et al. [1].

The present paper is concerned with the unsteady flow of a viscoelastic Oldroyd-B fluid along the axis of an infinite tube of circular cross-section. The driving force is assumed to be a time-dependent APG in the following three cases:

- i. APG varies exponentially with time,
- ii. Pulsating APG,
- iii. A starting flow under a constant APG.

2. Formulation of the problem

The momentum and continuity equations for an incompressible and homogenous fluid are given by

$$\rho \frac{dq}{dt} = -\nabla P + \nabla \cdot \underline{\underline{S}}, \quad (1)$$

and

$$\nabla \cdot \underline{\underline{q}} = 0, \quad (2)$$

where ρ is the material density, $\underline{\underline{q}}$ is the velocity field, p is the isotropic pressure and $\underline{\underline{S}}$ is the Cauchy or extra-stress tensor. The constitutive equation of Oldroyd-B fluid is written as

$$\underline{\underline{T}} = -p\underline{\underline{I}} + \underline{\underline{S}}; \underline{\underline{S}} + \lambda_1 \overset{\nabla}{\underline{\underline{S}}} = \mu \{ \underline{\underline{A}}_1 + \lambda_2 \overset{\nabla}{\underline{\underline{A}}_1} \} \quad (3)$$

where $\underline{\underline{T}}$ is the total stress, $\underline{\underline{I}}$ is the unit tensor, μ is a constant viscosity, λ_1 and λ_2 , ($0 \leq \lambda_2 \leq \lambda_1$) are the material time constants, termed as relaxation and retardation times; respectively. The deformation tensor $\underline{\underline{A}}_1$ is defined by

$$\underline{\underline{A}}_1 = \underline{\underline{L}} + \overset{\nabla}{\underline{\underline{L}}}; \underline{\underline{L}} = \nabla \underline{\underline{q}}. \quad (4)$$

and “ $\overset{\nabla}{\underline{\underline{\cdot}}}$ ” denotes the upper convected derivative ; i.e. for a symmetric tensor $\underline{\underline{G}}$ we get,

$$\overset{\nabla}{\underline{\underline{G}}} = \frac{\partial \underline{\underline{G}}}{\partial t} + \underline{\underline{q}} \cdot \nabla \underline{\underline{G}} - \underline{\underline{G}} \cdot \underline{\underline{L}} - \overset{\nabla}{\underline{\underline{L}}} \cdot \underline{\underline{G}}. \quad (5)$$

The symmetry of the problem implies that $\underline{\underline{S}}$ and $\underline{\underline{q}}$ depend only on the radial coordinate r in the cylindrical polar coordinates (r, θ, z) where the z -axis is chosen to coincide with the axis of the cylinder. Moreover, the velocity field is assumed to have only a z -component, i.e.

$$\underline{\underline{q}} = (0, 0, \underline{\underline{w}}), \quad (6)$$

which satisfies the continuity equation (2) identically. The substitution of Eq. (6), into Eqs. (1) and (3) yields the set of equations

$$S_{rz} + \lambda_1 \frac{\partial S_{rz}}{\partial t} = \mu \left(\frac{\partial w}{\partial r} + \lambda_2 \frac{\partial^2 w}{\partial r \partial t} \right), \quad (7)$$

$$\frac{\partial p}{\partial z} = \frac{\partial S_{rz}}{\partial r} + \frac{1}{r} S_{rz} - \rho \frac{\partial w}{\partial t}, \quad (8)$$

$$\frac{\partial p}{\partial r} = \frac{\partial p}{\partial \theta} = 0. \quad (9)$$

Equations (8) and (9) imply that the pressure function takes the form; $p = z f(t) + c$, so that

$$\frac{\partial p}{\partial z} = f(t). \quad (10)$$

The elimination of S_{rz} from (7) and (8) shows that velocity field $w(r, t)$ is governed by:

$$\rho \left(\lambda_1 \frac{\partial^2 w}{\partial t^2} + \frac{\partial w}{\partial t} \right) - \mu \left(1 + \lambda_2 \frac{\partial}{\partial t} \right) \left(\frac{\partial^2 w}{\partial t^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right) = - \left(1 + \lambda_1 \frac{\partial}{\partial t} \right) \frac{\partial p}{\partial z}. \quad (11)$$

The non-slip condition on the wall and the finiteness of w on the axis give

$$w(r, t)|_{r=R} = 0 \text{ and } \frac{\partial w}{\partial r} \Big|_{r=0} = 0. \quad (12)$$

Introducing the dimensionless quantities

$$\eta = \frac{r}{R}, \tau = \frac{\mu t}{\rho R^2}, \varphi = \frac{\mu L}{\Delta P R^2} w, \lambda = \frac{\lambda_2}{\lambda_1} \text{ and } H = \frac{\lambda_1 \mu}{\rho R^2} = \frac{We}{Re}, \quad (13)$$

where R is the radius of the pipe, ΔP a characteristic pressure difference, L is a characteristic length, We and Re are the Weissenberg and Reynolds numbers; respectively, into Eqs. (10), (11) and (12) we get

$$H \frac{\partial^2 \varphi}{\partial \tau^2} + \frac{\partial \varphi}{\partial \tau} - [1 + \lambda H \frac{\partial}{\partial \tau}] \left[\frac{\partial^2 \varphi}{\partial \eta^2} + \frac{1}{\eta} \frac{\partial \varphi}{\partial \eta} \right] = [1 + H \frac{\partial}{\partial \tau}] \Psi(\tau), \quad (14)$$

with the BCs.

$$\varphi(1, \tau) = 0 \text{ and } \frac{\partial \varphi(0, \tau)}{\partial \eta} = 0, \quad (15)$$

and

$$\Psi(\tau) = -\frac{L}{\Delta p} \frac{\partial p}{\partial z} = -\frac{L}{\Delta p} f(t). \quad (16)$$

Equation (14) subject to BCs. (15) is to be solved for different types of APGs; i.e. different forms of the function $\Psi(\tau)$.

3. Pressure gradient varying exponentially with time

We consider the two cases of exponentially increasing and decreasing with time APGs separately.

3.1. Pressure gradient increasing exponentially with time

Let,

$$\Psi(\tau) = -\frac{L}{\Delta p} \frac{\partial p}{\partial z} = K e^{\alpha^2 \tau}, \quad (17)$$

and assume that

$$\varphi(\eta, \tau) = g(\eta) e^{\alpha^2 \tau}, \quad (18)$$

where K and α are constants. The substitution of Eqs. (17) and (18) into Eq. (14) leads to

$$g'' + \frac{1}{\eta} g' - \frac{\alpha^2 (H\alpha^2 + 1)}{\lambda H\alpha^2 + 1} g = -K \frac{H\alpha^2 + 1}{\lambda H\alpha^2 + 1}, \quad (19)$$

while the BCs. (15) reduce to

$$g(1) = 0, g'(0) = 0 \quad (20)$$

A solution of Eq. (19) subject to the BCs. (20) is

$$g(\eta) = \frac{K}{\alpha^2} \left[1 - \frac{I_0(\beta\eta)}{I_0(\beta)} \right], \quad (21)$$

where $I_0(x)$ is the modified Bessel-functions of zero-order, and

$$\beta^2 = \frac{\alpha^2(1 + H\alpha^2)}{1 + \lambda H\alpha^2}. \quad (22)$$

Therefore, the velocity field is given by

$$\phi(\eta, \tau) = \frac{K}{\alpha^2} \left[1 - \frac{I_0(\beta\eta)}{I_0(\beta)} \right] e^{\alpha^2 \tau}. \quad (23)$$

The solution given by Eq. (23) processes the following properties:

- i. The time dependence is exponentially increasing such that for $\eta \neq 1$ $\lim_{\tau \rightarrow \infty} \phi(\eta, \tau) \rightarrow \infty$. It may be recommendable to choose another APG which increases up to a certain finite limit in order to keep $\phi(\eta, \tau)$ finite.
- ii. The present solution depends on the parameter β in the same form as the solution for the UCM [6]. For any value of β the Oldroyd-B fluid exhibits the same form as the UCM- fluid. However, in the present case β depends on λ in addition to H and α^2 . A close inspection show that $\lim_{\lambda \rightarrow 0} \beta^2 = \beta^2$ for the UCM-fluid while the $\lim_{\lambda \rightarrow 1} \beta^2 = \alpha^2$ which coincides with the case of the Newtonian fluid, [8].
- iii. The parameter β is inversely proportional to λ where the decay rate increases by increasing the value of H . However, as mentioned above, as λ approaches the value $\lambda = 1$ all the curves matches together approaching the value $\beta^2 = \alpha^2$ asymptotically. The behavior of β as a function of λ , where H is taken as a parameter is shown in Fig. (1).

For small values of $|\beta|$ and by using the asymptotic expansion of $I_0(x)$,

it can be shown that the velocity profiles approaches the parabolic distribution;

$$\lim_{\beta \rightarrow 0} \phi(\eta, \tau) = \frac{K(H\alpha^2 + 1)}{4(\lambda H\alpha^2 + 1)} (1 - \eta^2) e^{\alpha^2 \tau} \quad (24)$$

Figure 1. The $\lambda\beta$ - relation $H=2, 3, 5, 7, 9$, (Bottom to top)

For the case of large $|\beta|$ the velocity distribution is given as;

$$\lim_{\beta \rightarrow \infty} \varphi(\eta, \tau) = \frac{K}{\alpha^2} \left[1 - \frac{1}{\sqrt{\eta}} e^{-\beta(1-\eta)} \right] e^{\alpha^2 \tau} \quad (25)$$

This solution is completely different from the parabolic distribution and it depends on η only in the neighborhood of the wall. Therefore, such a fluid exhibits boundary effects.

The rising-APG velocity field $\phi(\eta, \tau)$ is plotted in Figs. (2a) and (2b) as a function of η at different values of β for $\alpha = 2$ and $\alpha = 5$.

3.2. Pressure gradient decreasing exponentially with time

The solution at present is obtained from the previous case by changing α^2 by $-\alpha^{-2}$. Therefore,

$$\varphi(\eta, \tau) = -\frac{K}{\alpha^2} \left[1 - \frac{j_0(\beta_1 \eta)}{j_0(\beta_1)} \right] e^{-\alpha^2 \tau}. \quad (26)$$

where

Figure 2. Rising – (a,b) APG velocity filed ; $\beta=5.2, 3.5, 2.5, 2.1$ (Bottom to top) Fig. (c) : Decreasing – APG velocity filed ; $\beta=8.7, 3.9, 2.6, 2.1$ (Top to Bottom) Fig. (d) : Decreasing – APG velocity filed ; $\beta=16.4, 9.2, 6.5, 5.3$ (Top to Bottom)

$$\beta_1^2 = \frac{\alpha^2(1-H\alpha^2)}{1-\lambda H\alpha^2} \quad (27)$$

The discussion of this solution is similar to the case of increasing APG except that the velocity decays exponentially with time and the value $\alpha^2 = 1/\lambda H$ is not permissible as it leads to infinite β_1^2 ; i.e.

$$\lim_{\alpha^2 \rightarrow 1/\lambda H} \beta_1^2 \rightarrow \infty \quad (28)$$

The two cases of small and large $|\beta_1|$ produce similar results as the previous solution. Thus

$$\lim_{\beta_1 \rightarrow 0} \phi(\eta, \tau) = -\frac{K}{4\alpha^2} \beta_1^2 (1-\eta^2) e^{-\alpha\tau}, \quad (29)$$

and

$$\lim_{\beta_1 \rightarrow \infty} \phi(\eta, \tau) = -\frac{K}{\alpha^2} \left[1 - \frac{1}{\sqrt{\eta}} \frac{\cos(\beta_1 \eta - \frac{\pi}{4})}{\cos(\beta_1 - \frac{\pi}{4})} \right] e^{-\alpha^2 \tau}. \quad (30)$$

4. Pulsating pressure gradient

The present case requires the solution of Eq. (14) subject to BCs. (15) in the form

$$\Psi(\tau) = -\frac{L}{\Delta P} \frac{\partial p}{\partial z} = K e^{in\tau}; \quad i = \sqrt{-1}, \quad (31)$$

K and n are constants. Assuming the velocity function has the form

$$\phi(\eta, \tau) = re \left[f(\eta) e^{in\tau} \right], \quad (32)$$

$$\therefore f'' + \frac{1}{\eta} f' - in \frac{(1+inH)}{(1+in\lambda H)} f = -K \frac{(1+inH)}{(1+in\lambda H)}. \quad (33)$$

The solution of this equation satisfying the BCs. (15) is :

$$f(\eta) = \frac{k}{in} \left[1 - \frac{I_0(\beta\eta)}{I_0(\beta)} \right], \quad \beta^2 = in \frac{(1+inH)}{(1+in\lambda H)}. \quad (34)$$

Hence, the velocity distribution is given by:

$$\phi(\eta, \tau) = re \left\{ \frac{k}{in} e^{in\tau} \left[1 - \frac{I_0(\beta\eta)}{I_0(\beta)} \right] \right\}. \quad (35)$$

Obviously; for small $|\beta|$,

$$\lim_{\beta \rightarrow 0} f(\eta) = \frac{K}{in} \left(\frac{\beta^2(1-\eta^2)}{4} \right). \quad (36)$$

and for large $|\beta|$

$$\lim_{\beta \rightarrow \infty} \frac{I_0(\beta\eta)}{I_0(\beta)} = \frac{1}{\sqrt{\eta}} e^{-\beta(1-\eta)}, \quad (37)$$

So that,

$$\varphi(\eta, \tau) = re \left\{ \frac{K}{in} \left[1 - \frac{1}{\sqrt{\eta}} e^{-\beta(1-\eta)} e^{in\tau} \right] \right\}, \quad (38)$$

where,

$$\beta^2 = \frac{in(1+inH)}{(1+in\lambda H)} = \frac{1}{1+n^2\lambda^2 H^2} [n^2 H(\lambda-1) + in(1+n^2\lambda H^2)], \quad (39)$$

or simply,

$$\beta = \sqrt{\Re} e^{i\theta/2}, \quad (40)$$

$$\Re = \frac{n}{1+n^2\lambda^2 H^2} \sqrt{n^2 H^2 (1-\lambda)^2 + (1+n^2\lambda H^2)^2}, \quad (41)$$

$$\frac{\theta}{2} = \frac{1}{2} \tan^{-1} \left[\frac{1+n^2\lambda H}{nH(1-\lambda)} \right]. \quad (42)$$

Substituting from Eqs.(40,41,42) into Eq. (37), we get:

$$\lim_{\beta \rightarrow \infty} \varphi(\eta, \tau) = \frac{k}{n} \left\{ \sin n\tau - \frac{1}{\sqrt{\eta}} e^{-(1-\eta)\sqrt{\Re} \cos(\theta/2)} \sin \left[n\tau - (1-\eta)\sqrt{\Re} \sin \frac{\theta}{2} \right] \right\}. \quad (43)$$

As $\lambda \rightarrow 0$, [6], $\Re \rightarrow r_1 = n\sqrt{1+n^2 H^2}$ and $\frac{\theta}{2} \rightarrow \frac{\theta_1}{2} = -\frac{1}{2} \tan^{-1} \left(\frac{1}{nH} \right)$.

Then

$$\varphi(\eta, \tau) = \frac{k}{n} \left\{ \sin n\tau - \frac{1}{\sqrt{\eta}} e^{-(1-\eta)\sqrt{r_1} (\cos \theta_1/2)} \sin \left[n\tau - (1-\eta)\sqrt{r_1} \left(\sin \frac{\theta_1}{2} \right) \right] \right\} \quad (44)$$

The velocity field $\varphi(\eta, \tau)$ is plotted in Figs. (3a) and (3b); respectively, against η for different values of β . The two limiting cases for small and large $|\beta|$ are represented in three-dimensional Figs. (4a) and (4b) in order to emphasize the oscillating properties of the solution.

Figure 3. a) : Pulsating – APG ; $n=2$, $H=5$, $\beta=3.7, 2.5, 1.8, 1.5$ (b) : Pulsating – APG ; $n=5$, $H=5$, $\beta=6.8, 4.1, 2.9, 2.4$ [Top to Bottom for all]

Figure 4. a): Pulsating-APG, $n = 2$, $H = 5$, at small $|\beta|$; $\beta=3.7$ (b): Pulsating-APG, $n = 3$, $H = 5$, at large $|\beta|$; $\beta=6.8$

5. Constant pressure gradient

Here we consider the flow to be initially at rest and then set in motion by a constant ABG “-K”. Hence, $\Psi(\tau)$; Eq.(14), subject to BCs. (15) reduces to

$$\frac{L}{\Delta P} \frac{\partial P}{\partial z} = -K. \quad (45)$$

Therefore, we need to solve the equation

$$H \frac{\partial^2 \Phi}{\partial \tau^2} + \frac{\partial \Phi}{\partial \tau} - [1 + \lambda H \frac{\partial}{\partial \tau}] [\frac{1}{\eta} \frac{\partial \Phi}{\partial \eta} + \frac{\partial^2 \Phi}{\partial \eta^2}] = K, \quad (46)$$

subject to the boundary and initial conditions

$$\phi(1, \tau) = 0, \text{ for } \tau \geq 0,$$

$$\phi(\eta, 0) = 0, \text{ for } 0 \leq \eta \leq 1 \quad (47)$$

Equation (46) can be transformed to a homogenous equation by the assumption

$$\Phi(\eta, \tau) = \frac{K}{4}(1 - \eta^2) - \psi(\eta, \tau), \quad (48)$$

where $\Psi(\eta, \tau)$ represents the deviation from the steady state solution. Hence,

$$[\frac{\partial}{\partial \tau} (1 + H \frac{\partial}{\partial \tau}) - (1 + \lambda H \frac{\partial}{\partial \tau}) (\frac{\partial^2}{\partial \eta^2} + \frac{1}{\eta} \frac{\partial}{\partial \eta})] \psi = 0, \quad (49)$$

subject to the boundary and initial conditions

$$\Psi(1, \tau) = 0 \text{ for } \tau \geq 0. \quad (50)$$

$$\psi(\eta, 0) = \frac{K}{4}(1 - \eta^2) \text{ for } 0 \leq \eta \leq 1. \quad (51)$$

Assuming that $\psi(\eta, \tau) = F(\eta) \cdot G(\tau)$, Eq.(49) separates to

$$HG'' + (1 + \lambda H \alpha^2)G' + \alpha^2 G = 0, \quad (52)$$

$$F'' + \eta^{-1}(1 + \lambda H \alpha^2)F' + \alpha^2 F = 0. \quad (53)$$

Equation (52) has the solution,

$$G(\tau) = Ae^{\gamma_1 \tau} + Be^{\gamma_2 \tau} \quad (54)$$

where γ_1 and γ_2 are the roots of the Eq.(52). On the other hand, Eq. (53) has the solution

$$F(\eta) = J_0(\alpha_m \eta). \quad (55)$$

Therefore,

$$\gamma_{1,2} = \frac{-(1 + \lambda H \alpha_m^2) \pm \sqrt{(1 + \lambda H \alpha_m^2)^2 - 4 \alpha_m^2 H}}{2H}. \quad (56)$$

The BCs. (50,51) implies that the constant α_m takes all zeros of the Bessel-function J_0 ($\alpha_1, \alpha_2, \dots$). Hence,

$$\Psi(\eta, \tau) = \sum_{m=1}^{\infty} J_0(\alpha_m \eta) G(\tau), \quad (57)$$

$$\therefore \Psi(\eta, \tau) = \sum_{m=1}^{\infty} J_0(\alpha_m \eta) (A_m e^{\gamma_{1m} \tau} + B_m e^{\gamma_{2m} \tau}). \quad (58)$$

The initial condition (50) and BCs. (51) will not be sufficient to evaluate the constants A_m and B_m . Hence, it is required to employ another condition. We assume that $G(\tau)$ is smooth about the value $\tau = 0$ and can be expanded in a power series about $\tau = 0$. Assuming $G(\tau)$ to be linear function of τ in the domain about $\tau = 0$, then $G'' = 0$ in Eq. (52). Hence

$$(1 + \lambda H \alpha_m^2) G_m'(0) + \alpha_m^2 G_m(0) = 0, \quad (59)$$

$$G_m(\tau) = A_m e^{\gamma_{1m} \tau} + B_m e^{\gamma_{2m} \tau}, \quad (60)$$

From which we obtain

$$A_m [(1 + \lambda H \alpha_m^2) \gamma_{1m} + \alpha_m^2] + B_m [(1 + \lambda H \alpha_m^2) \gamma_{2m} + \alpha_m^2] = 0. \quad (61)$$

To determine the constants A_m and B_m we firstly satisfy the remaining condition (51). Owing to Eq. (58) and the initial condition, Eq. (51), we notice that,

$$\Psi(\eta, 0) = \sum_{m=1}^{\infty} (A_m + B_m) J_0(\alpha_m \eta) = \frac{K}{4} (1 - \eta^2). \quad (62)$$

Via the Fourier–Bessel series, Eq. (62) leads to,

$$A_m + B_m = \frac{K}{2J_1^2(\alpha_m)} \int_0^1 \eta (1 - \eta^2) J_0(\alpha_m \eta) d\eta. \quad (63)$$

Performing this integration we get

$$A_m + B_m = \frac{2K}{\alpha_m^3 J_1(\alpha_m)} - \frac{K}{\alpha_m^2} \frac{J_0(\alpha_m)}{J_1^2(\alpha_m)}. \quad (64)$$

From Eqs. (61) and (64) we obtain :

$$A_m = \frac{[(1 + \lambda H \alpha_m^2) \gamma_{2m} + \alpha_m^2]}{(1 + \lambda H \alpha_m^2)(\gamma_{2m} - \gamma_{1m})} \left[\frac{2K}{\alpha_m^3 J_1(\alpha_m)} - \frac{K}{\alpha_m^2} \frac{J_0(\alpha_m)}{J_1^2(\alpha_m)} \right], \quad (65)$$

$$B_m = \frac{[(1 + \lambda H \alpha_m^2) \gamma_{1m} + \alpha_m^2]}{(1 + \lambda H \alpha_m^2)(\gamma_{1m} - \gamma_{2m})} \left[\frac{2K}{\alpha_m^3 J_1(\alpha_m)} - \frac{K}{\alpha_m^2} \frac{J_0(\alpha_m)}{J_1^2(\alpha_m)} \right]. \quad (66)$$

Finally, the velocity field has the series representation

$$\begin{aligned} \phi(\eta, \tau) = & \frac{K}{4} (1 - \eta^2) - \sum_{m=1}^{\infty} \frac{J_0(\alpha_m \eta)}{(1 + \lambda H \alpha_m^2)(\gamma_{2m} - \gamma_{1m})} \left\{ [(1 + \lambda H \alpha_m^2) \gamma_{2m} + \alpha_m^2] e^{\gamma_{1m} \tau} \right. \\ & \left. - [(1 + \lambda H \alpha_m^2) \gamma_{1m} + \alpha_m^2] e^{\gamma_{2m} \tau} \right\} \left[\frac{2K}{\alpha_m^3 J_1(\alpha_m)} - \frac{K}{\alpha_m^2} \frac{J_0(\alpha_m)}{J_1^2(\alpha_m)} \right]. \end{aligned} \quad (67)$$

The constant-APG velocity field $\phi(\eta, \tau)$ as a function of η shown in Fig. (5).

Figure 5. The velocity distribution for constant – APG taking $H=0.2$, $\tau=0.1$ where the summation is taken for $\alpha_1=2.4$, $\alpha_2=5.8$, $\alpha_3=8.4$

6. Results and discussion

The behavior of $|\beta|$ as a function of λ where H is taken as a parameter is shown in Fig. (1). The behavior of β is inversely proportional to λ while it is fast-decreasing for higher H -values. For any β -value, the Oldroyd-B fluid exhibits the same form as the UCM-fluid. A close inspection of $\beta^2 = \alpha^2(1+\alpha^2H)/(1+\lambda\alpha^2H)$ shows that UCM-fluid is obtained by $\lim_{\lambda \rightarrow 0} \beta^2 = \beta^2$ while $\lim_{\lambda \rightarrow 0} \beta^2 = \alpha^2$ leads to the case of Newtonian fluid. For small values of $|\beta|$ as well as $|\beta\eta|$ and by using the asymptotic expansion of $I_0(x)$, it can be shown that the velocity profiles approaches the parabolic distribution.

For decay-APGs, Figs. (2a) and (2b) show that the velocity profiles of Oldroyd-B and UCM fluids are parabolic for small values of $|\beta\eta|$ while for large $|\beta\eta|$ they are completely different from this situation. The solutions depend on η only in the neighboring of the wall. Therefore, such fluids exhibit boundary layer effects [17].

For pulsating-APG, the velocity distribution is represented in Figs. (3a) and (3b). The smallest value of β in both curves is almost parabolic as shown by Eq. (36) while the largest

value exhibits boundary effect as revealed by Eq.(43). To emphasize the oscillating nature of the solution a three-dimensional diagrams (4a) and (4b) for the smallest and largest values of $|\beta|$ are respectively sketched.

Grigioni, et al [1], studied the behavior of blood as a viscoelastic fluid using the Oldroyd-B model. The results obtained for the velocity distribution stands in agreement with the obtained results in the present work.

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