Chapter 1

Wind Turbine Power: The Betz Limit and Beyond

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Additional information is available at the end of the chapter

http://dx.doi.org/10.5772/52580

1. Introduction

With a severe energy crisis facing the modern world, the production and utilization of energy has become a vital issue, and the conservation of energy has acquired prime importance. Energy production and consumption are directly related to everyday life in much of human society, and issues of energy research are extremely important and highly sensitive. Being aware of the global warming problem, humans tend to rely more on renewable energy (RE) resources.

1.1. Benefits of wind energy

In [1], scientists and researchers have tried to accelerate solutions for wind energy generation design parameters. Researchers claim that a short time, society, industry, and politics will welcome the use of wind energy as a clean, practical, economical, and environmentally friendly alternative. In an effort to approach a more sustainable world, after the 1973 oil crisis RE sources began to appear on the agenda, and wind energy attracted significant interest. Because of extensive studies on this topic, wind energy has recently been applied in various industries, where it has begun to compete with other energy resources [1].

Among the various renewable energy types as highlighted by [2], wind provides an intermittent but environmentally friendly energy source that does not pollute atmosphere. Wind power calculations are initiated from the kinetic energy definition, and wind power is found to be proportional to half the air density multiplied by the cube of the wind velocity. When seeking to determine the potential usage of wind energy, wind power formulation is derived first by use of kinetic energy definition and then by basic physical definitions of power as the ratio of work over time, work as the force multiplied by the distance, and force as the change of momentum. [2].

1.2. Aerodynamics aspects of wind turbines

Reviews about many of the most important aerodynamic research topics in the field of wind energy are shown in the report of a different study [3] Wind turbine aerodynamics concerns the modeling and prediction of aerodynamic forces, such as performance predictions of wind farms, as well as the design of specific parts of wind turbines, such as rotor-blade geometry. The basics of blade-element momentum theory were presented along with guidelines for the construction of airfoil data. Various theories for aerodynamically optimum rotors were discussed, and recent results on classical models were presented. State-of-the-art advanced numerical simulation tools for wind turbine rotors and wakes were reviewed, including rotor predictions as well as models for simulating wind turbine wakes and flows in wind farms [3].

1.3. Wind power density

Concerning power density and its relation to wind speed, the report given in [4] presented the features of wind power distributions that were analytically obtained from wind distribution functions. Simple equations establishing a relationship between mean power density and wind speed have been obtained for a given location and wind turbine. Different concepts relating to wind power distribution functions were shown—among them the power transported by the wind and the theoretical maximum convertible power from wind, according to the Betz' law. Maximum convertible power from the wind was explained within more realistic limits, including an approximate limit to the maximum power from a wind turbine, was obtained. In addition, different equations were obtained establishing relationships between mean power density and mean wind speed. These equations are simple and useful when discarding locations for wind turbine installation [4].

1.4. Wind power applications

The range of wind power usage is scarce. One of the most important usages is electricity. Hubbard and Shepherd [5] considered wind turbine generators, ranging in size from a few kilowatts to several megawatts, for producing electricity both singly and in wind power stations that encompass hundreds of machines. According to the researchers' claims, there are many installations in uninhabited areas far from established residences, and therefore there are no apparent environmental impacts in terms of noise. The researchers do point out, however, situations in which radiated noise can be heard by residents of adjacent neighborhoods, particularly those who live in neighborhoods with low ambient noise levels [5].

Wind power is used worldwide, not only in developed countries. Specific studies [6, 7] presented a detailed study of a Manchegan windmill while considering the technological conditions of the original Manchegan windmills. In addition, a wind evaluation of the region was carried out, the power and momentum of the windmills were calculated, and the results obtained were discussed, along with a comparison with the type of

Southern Spanish windmill. These windmills were important for wheat milling and had been an important factor in the socio-economic development of rural Spain for centuries [6, 7].

Another example is considered in [8]. This study, conducted with reference to land in Syria, evaluated both wind energy potential and the electricity that could be generated by the wind. An appropriate computer program was especially prepared and designed to perform the required calculations, using the available meteorological data provided by the Syrian Atlas. The program is capable of processing the wind data for any specific area that is in accordance with the needed requirements in fields of researches and applications. Calculations in the study show that a significant energy potential is available for direct exploitation. The study also shows that approximately twice the current electricity consumption in Syria can be generated by wind resources [8].

The potential usage of wind power at Kudat and Labuan for small-scale energy demand was given in [9]. According to their statement, the acquisition of detailed knowledge about wind characteristics at a site is a crucial step in planning and estimating performance for a wind energy project. From this study, the researchers concluded that sites at Kudat and Labuan that they had considered during the study years were unsuitable for large-scale wind energy generation. However, they did confirm that small-scale wind energy could be generated at a turbine height of 100 meters [9]. In light of their findings, James and others [10] reported that the potential impact of the UK's latest policy instrument, the 2010 micro-generation tariffs, is considered applicable to both micro-wind and photovoltaics.As the researchers observed, building-mounted micro-wind turbines and photovoltaics have the potential to provide widely applicable carbon-free electricity generation at the building level. Because photovoltaic systems are well understood it is easy to predict performance using software tools or widely accepted yield estimates. Micro-wind research, however, is far more complex, and in comparison, it is poorly understood [10].

Abdeen [11] addresses another example of wind power usage. As the researcher observed, the imminent exhaustion of fossil energy resources and the increasing demand for energy were the motives for Sudanese authorities to put into practice an energy policy based on rational use of energy. The authorities also based their conclusions on exploitation of new and renewable energy sources. It was pointed out that after 1980, as the supply of conventional energy has not been able to follow the tremendous increase in production demand in rural areas of Sudan; a renewed interest for the application of wind energy has been shown in many places. Therefore, the Sudanese government began to pay more attention to wind energy utilization in rural areas. Because the wind energy resource in many rural areas is sufficient for attractive application of wind pumps, although as fuel it is insufficient, the wind pumps will be spread on a rather large scale in the near future. Wind is a form of renewable energy that is always in a non-steady state due to the wide temporal and spatial variations of wind velocity. Results suggested that wind power would be more profitably used for local and small-scale applications, especially for remote rural areas. The study finds that Sudan has abundant wind energy [11]. Another recent study [12] considered the wind power in Iran. According to the study's claims, climate change, global warming, and the recent worldwide economic crisis have emphasized the need for low carbon emissions while also ensuring economic feasibility. In their paper, the researchers investigated the status and wind power potential of the city of Shahrbabak in Kerman province in Iran. The technical and economic feasibility of wind turbine installation was presented, and the potential of wind power generation was statistically analyzed [12].

1.5. Types of wind turbines

There are different types of wind turbines: bare wind turbines, augmented wind turbines, horizontal axis wind turbines, and vertical axis wind turbines, just to mention a few.

1.5.1. Bare wind turbines

According to research findings as given by [13], the derivation of the efficiency of an ideal wind turbine is attributed to the three prominent scientists associated with the three principal aerodynamic research schools in Europe during the first decades of the previous century: Lanchester, Betz, and Joukowsky. According to this study, detailed reading of their classical papers had shown that Lanchester did not accept that the velocity through the disc is the average of the velocities far upstream and far downstream, by which his solution is not determined. Betz and Joukowsky used vortex theory to support Froude's result and derived the ideal efficiency of a wind turbine at the same time. This efficiency has been known as the Joukowsky limit in Russia and as the Betz limit everywhere else. As the researchers suggested, because of the contribution of both scientists, this result should be called the Betz-Joukowsky limit everywhere [13]. The maximal achievable efficiency of a wind turbine is found to be given by the Betz number $B = 16/27$. Derivation of the classical Betz limit could be followed as given by [14] and [15].

The question of the maximum wind kinetic energy that can be utilized by a wind turbine, which is of fundamental importance for employment of wind energy, was reconsidered in [16]. According to their study, the researchers observed that in previous studies, an answer to this question was obtained only for the case of an infinite number of turbine-rotor blades, in the framework of application of the one-dimensional theory of an ideal loaded disk without loss for friction and turbulence taken into account. This implies that for an ideal wind turbine, the maximum energy that can be extracted from the wind kinetic energy, or the power coefficient, does not exceed the Betz limit. Based on the exact calculation of the Goldstein function, the researchers determined the maximum power coefficient of an ideal wind turbine having a finite number of blades. As was expected, the maximum turned out to be always lower than the absolute Lanchester-Betz-Joukowski limit. According to their findings, with an increase in the number of blades, the power coefficient rises approaching the estimate of Glauert for a rotor with an infinite number of blades, only if by taking wake flow twisting into account [16].

In a different proposal, Cuerva and Sanz-Andre´s presented an extended formulation of the power coefficient of a wind turbine [17]. Their formulation was a generalization of the Betz-

Lanchester expression for the power coefficient as a function of the axial deceleration of the wind speed provoked by the wind turbine in operation. The extended power coefficient took into account the benefits of the power produced and the cost associated to the production of this energy. By means of the proposed simple model, the researchers evidenced that the purely energetic optimum operation condition giving rise to the Betz-Lanchester limit (maximum energy produced) does not coincide with the global optimum operational condition (maximum benefit generated) if cost of energy and degradation of the wind turbine during operation is considered. The new extended power coefficient, according to the researchers claim, is a general parameter useful to define global optimum operation conditions for wind turbines, considering not only the energy production but also the maintenance cost and the economic cost associated to the life reduction of the machine [17].

1.5.2. Augmented wind turbines

It was suggested in [18] that one could extract more power from the wind by directing the wind by a diffuser that could be incorporated into the system. The benefit of such a device is to decrease the size of the system and thus decrease its cost [18].

According to [19], the performance of a diffuser-augmented wind turbine has been established by matching the forces acting on the blade element to overall momentum and energy balances. Good agreement with experimental data was obtained [19]. Based on computational fluid dynamics (CFD), an actuator disc CFD model of the flow through a wind turbine in a diffuser was developed and validated [20, 21]. Their research presumed a flow increase could be induced by a diffuser. They showed that from a one-dimensional analysis the Betz limit could be exceeded by a factor that is proportional to the relative increase in mass flow through the rotor. This result was verified by theoretical one-dimensional analysis by the CFD model [20, 21]. Supporting the same idea, [22] Sharpe stated that it is theoretically possible to exceed the Lanchester-Betz limit. His study presented a general momentum theory for an energy-extracting actuator disc that modeled a rotor with blades having radially uniform circulation. The study included the effects of wake rotation. Although the study reports that the general momentum theory is well known, the fall in pressure that is caused by the rotation of the wake that the theory predicts, is not usually recognized. Accounting for the wake rotational pressure drop changes some of the established conclusions of the momentum theory that appear in the literature. The conclusion from the study is that the theory establishes no loss of efficiency associated with the rotating wake [22]. Experimental and numerical investigations for flow fields of a small wind turbine with a flanged diffuser were carried out in [23].

The considered wind-turbine system gave a power coefficient higher than the Betz limit, which they attributed to the effect of the flanged diffuser. The experimental and numerical results gave useful information about the flow mechanism behind a wind turbine with a flanged diffuser. In particular, a considerable difference was seen in the destruction process of the tip vortex between the bare wind turbine and the wind turbine with a flanged diffuser [23]. According to the findings given in [24], suitable techniques to convert a country's wind availability (mostly in the low-speed regimes) as a renewable energy source must be scrutinized in order to achieve effective and efficient conversion. In this study, the researchers described efforts to step up the potential power augmentation offered by the Diffuser Augmented Wind Turbine (DAWT). Modification of the internal profile of the diffuser occurred by replacing the interior profile of the diffuser with an optimized airfoil shape as the interior profile of the diffuser. Additional velocity augmentation of approximately 66% could be achieved with the optimized profile when compared to a diffuser with an original flat interior [24]. As was pointed out in [25, 26], although there is an increase in maximum performance of a DAWT that is proportional to the mass flow of air, with application of simple momentum theory, the amount of energy extracted per unit of volume with a DAWT is the same as for an ordinary bare wind turbine [25, 26].

1.6. Modeling controversy

Debate is ongoing around the issue of DAWT. In a recent study, a general momentum theory to study the behavior of the classical free vortex wake model of Joukowsky was used [27]. This model, as the researchers reported, has attained considerable attention as it shows the possibility of achieving a power performance that greatly exceeds the Lanchester-Betz limit for rotors running at low tip speed ratios. This behavior was confirmed even when including the effect of a center vortex, which, without any simplifying assumptions, allowed azimuthal velocities and the associated radial pressure gradient to be taken into account in the axial momentum balance. In addition, a refined model that remedies the problem of using the axial momentum theorem was proposed. Using this model the power coefficient never exceeds the Lanchester-Betz limit, but rather tends to zero at a zero tip speed ratio [27]. As asserted in [28], for reasons of energy and momentum conservation a conventional diffuser system, as it is commonly used in water turbines cannot augment the power of a wind turbine beyond the Betz limit. However, if the propeller of the turbine is embedded into an external flow of air from which by means of its static structure energy can be transferred to the internal flow through the propeller, the propeller can supersede the Betz limit with respect to this internal flow [28]. Features of such practical methods toward achieving such improvements in wind power are discussed in [29].

1.7. Blades design

According to the report given by [30], the main problem of a wind turbine generator design project is the design of blades capable of satisfying, with optimum performance, the specific energy requirement of an electric system [30]. Simulations are very important to facilitate engineering and design of wind turbines for many reasons, especially those that concentrate upon reducing cost and saving human time. With regard to the designing the rotor blades, a CFD model for the evaluation of energy performance and aerodynamic forces acting on a straight-bladed vertical-axis Darrieus wind turbine was presented [31]. A modified blade element momentum theory for the counter-rotating wind turbine was developed [32]. This enabled the investigation of the effects of design parameters such as the combinations of the pitch angles, rotating speeds, and radii of the rotors on the aerodynamic performance of the counter-rotating wind turbine [32].

1.8. Wind farms

Vermeer and others surveyed wind farms. The focus of this study was on standalone turbines and wind farm effects. The survey group suggested that when assembling many wind turbines together, several issues should be considered [33]. Other research studies discussed the issue of optimizing the placement of wind turbines in wind farms [34]. Factors considered included multidirectional winds and variable wind speeds, the effect of ambient turbulence in the wake recovery, the effect of ground, variable hub height of the wind turbines, and different terrains [34]. A review of the state of the art and present status of active aeroelastic rotor control research for wind turbines was presented in [35]. A wind farm controller was reported in [36]. That controller distributes power references among wind turbines while it reduces their structural loads.

In this study the effect of losses are considered and discussed for bare wind turbines and for shrouded wind turbines.

2. One-dimensional fluid dynamics models

In this section, one-dimensional fluid dynamics models are analyzed and formulated based on the extended Bernoulli equation [37], accounting for losses that are assumed to be proportional to the square of the velocity of the air crossing the rotor blades of the wind turbine. The performance characteristics of the wind turbine are given by the power and thrust coefficients. The efficiency of the wind turbine is addressed and its relation to the power coefficient is discussed.

While developing a model to describe the performance of a wind turbine, common assumptions regarding the fluid flow are as follows [18]:

1. The entire field is one-dimensional.
2. The fluid considered is not compressible.
3. The flow field in the proximity of the turbine is a pure axial flow.

Other assumptions are given for the specific models.

2.1. Bare wind turbine

Consider a wind turbine that intercepts the flow of air moving with velocity V_0. The different quantities involved in the physics of the moving air are the pressure, p, the velocity, V, the cross section area, A (at different locations along the stream lines), and the thrust on the blades of wind turbine, T. The upstream condition is identified with a subscript of zero, the downstream condition is identified with a subscript of 3, and the turbine locations are identified by subscripts of 1, 2, and t (see Figure 1).

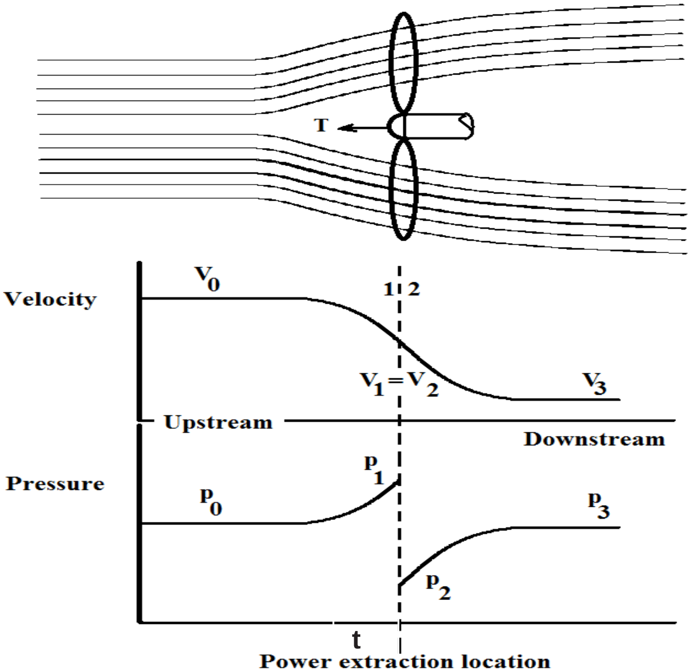

Figure 1. Schematics of the bare wind turbine. In the upper part, airstream lines are shown crossing the turbine's rotor. The velocity of the air at the rotor is the same based on the mass flow rate: $V_1=V_2=V_t$.The pressure drop is determined by $\Delta p=p_1-p_2$.

The steady state mass flow rate is given by:

$$\dot{m} = \rho V A = const. \tag{1}$$

The modified Bernoulli equation (with reference to the turbine head), which describes the energy balance through the wind turbine, is written between the entrance and exit sections, and is given by:

$$\frac{p_0}{\gamma} + \frac{1}{2}\frac{{V_0}^2}{g} = \frac{p_0}{\gamma} + \frac{1}{2}\frac{{V_3}^2}{g} + h_t + h_{loss} \tag{2}$$

In this equation h_t is the head of the turbine (related to the amount of power extraction), h_{loss} represents the head losses, and γ is the specific weight. The loss term in the energy equation accounts for friction (mechanical and fluid) and is assumed proportional to the kinetic energy of the rotor blades, expressed as follows:

$$h_{loss} = C_{loss} \frac{V_t^2}{2g} \tag{3}$$

Equations (2) and (3) are rearranged and the head of the turbine is given by:

$$h_t = \frac{1}{2}\frac{V_0{}^2}{g} - \frac{1}{2}\frac{V_3{}^2}{g} - C_{loss}\frac{V_t^2}{2g} \tag{4}$$

The power output from the turbine, P, is given by:

$$P = \gamma V_t A_t h_t = \frac{1}{2}\rho V_0^3 A_t \frac{V_t}{V_0}\left(1 - \left(\frac{V_3}{V_0}\right)^2 - C_{loss}\left(\frac{V_t}{V_0}\right)^2\right) \tag{5}$$

From equation (5) we can determine the power coefficient, $C_{P,}$ by:

$$C_P = \frac{P}{\frac{1}{2}\rho V_0^3 A_t} = \frac{V_t}{V_0}\left(1 - \left(\frac{V_3}{V_0}\right)^2 - C_{loss}\left(\frac{V_t}{V_0}\right)^2\right) \tag{6}$$

The developed thrust, T, on the turbine blades is given by the linear moment equation (see [15] for more details), as follows:

$$T = \rho V_t A_t (V_0 - V_3) \tag{7}$$

The thrust coefficient, $C_{T,}$ based on equation (7), is given by:

$$C_T = \frac{T}{\frac{1}{2}\rho V_0^2 A_t} = 2\frac{V_t}{V_0}\left(1 - \frac{V_3}{V_0}\right) \tag{8}$$

Equations (5) and (7) are governed by the following relationship:

$$P = TV_t \tag{9}$$

By observation, the velocity of air decreases toward the downstream. Therefore, we can simplify calculations by introducing the parameter, *a*, to express the velocity at the cross section of the turbine, and the upstream velocity can be determined by:

$$V_t = (1-a)V_0 \qquad 0 < a < 1 \tag{10}$$

Equating equations (5) and (9) gives the velocity of the air, V_3, at the downstream, and after some algebraic manipulation, it could be shown to be given by:

$$V_3 = \left(1 - a - \sqrt{\left(a^2 - C^*_{loss}\left(1-a\right)^2\right)}\right)V_0 \tag{11}$$

The normalized loss coefficient is defined by:

$$C^*_{loss} = C_{loss}V_0^2 \tag{12}$$

Equations (11) and (12) are useful to calculate the power coefficient (equation (6)) and the thrust coefficient (equation (8)). For the case where the losses are negligible, the known results in the literature are reproduced and given by the following equations:

The downstream velocity is given by:

$$V_3 = 1 - 2a \tag{13}$$

The power coefficient is given by:

$$C_P = 4a\left(1-a\right)^2 \tag{14}$$

The thrust coefficient is given by:

$$C_T = 4a\left(1-a\right) \tag{15}$$

Inversing the relation given in equation (15), the parameter, *a,* is given as a function of C_T by:

$$a = \frac{1 - \sqrt{1 - C_T}}{2} \tag{16}$$

Finally, the power coefficient as a function of the thrust coefficient is given by:

$$C_P = \frac{C_T + C_T\sqrt{1-C_t}}{2} \tag{17}$$

2.2. Augmented wind turbine

In order to exploit wind power as economically as possible, it was suggested that the wind turbine should be enclosed inside a specifically designed shroud [38, 39]. Several models were reported in the literature to analyze wind turbine rotors surrounded by a device (shroud), which was usually a diffuser [18, 25, and 26]. Others suggested different approaches [28].

In this section, the extended Bernoulli equation and mass and momentum balance equations are used to analyze the augmented wind turbine. The power coefficient and the thrust coefficients are derived, accounting for losses in the same manner as was done for the bare turbine case. The efficiency of the wind turbine could be defined as the ratio of the net power output to the energy input to the system. The efficiency based on this definition agrees with the Betz limit.

The schematics of the shrouded wind turbine are shown in Figure 2.

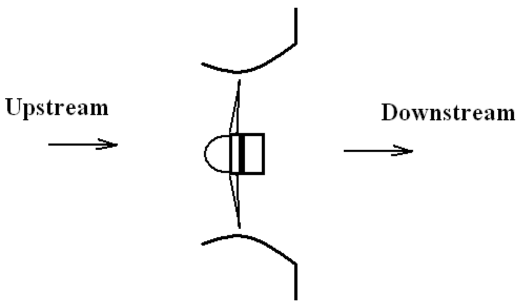

Figure 2. Schematics of the shrouded wind turbine. There is a vertical element at the exit of the wind turbine. This element contributes to reducing the power at the downstream side of the turbine, an effect that extracts more air through the wind turbine. (Idea reproduced similar to the description given by Ohya [40].)

This type of design has been recently reported [40], and it was shown that the power coefficient is about 2-5 times greater when compared to the performance of the bare wind turbine. The vertical part at the exit of the shroud reduces the pressure and therefore, the wind turbine draws more mass.

The balance equations are followed in the same manner as for the bare wind turbine. The modified Bernoulli equation differs by the pressure at the exit and is given by:

$$\frac{p_0}{\gamma}+\frac{1}{2}\frac{V_0^{\ 2}}{g}=\frac{p_3}{\gamma}+\frac{1}{2}\frac{V_3^{\ 2}}{g}+h_t+h_{loss} \tag{18}$$

The pressure drop between inlet and outlet (p_0-p_3) is rewritten as proportional (with C_F proportionality coefficient) to the difference in kinetic energies and it is given by:

$$\Delta p=p_0-p_3=\frac{1}{2}\rho\left(V_0^2-V_3^2\right)C_F \tag{19}$$

The power coefficient for the shrouded wind turbine is given by:

$$C_P=\frac{P}{\frac{1}{2}\rho V_0^3 A_t}=\frac{V_t}{V_0}\left(\left(C_F+1\right)\left(1-\left(\frac{V_3}{V_0}\right)^2\right)-C_{loss}\left(\frac{V_t}{V_0}\right)^2\right) \tag{20}$$

The thrust coefficient is given by:

$$C_T=\frac{T}{\frac{1}{2}\rho V_0^2 A_t}=2\left(C_F+1\right)\frac{V_t}{V_0}\left(1-\frac{V_3}{V_0}\right) \tag{21}$$

Manipulating equations (9)-(12) makes it possible to produce sample plots to consider in the next section.

2.3. Wind turbine efficiency

Usually, efficiency is defined as the ratio between two terms: the amount of net work, w, to the input, $q_{in,}$ energy to the device. Efficiency can be alternatively defined as the ratio between the derived power, $P_{out,}$ and the rate of energy flowing to the system, P_{in}. Based on these definitions, the efficiency of power generating machine is given by:

$$\eta=\frac{w_{net}}{q_{in}}=\frac{P_{out}}{P_{in}} \tag{22}$$

As was observed by Betz, the maximal achievable efficiency of the bare wind turbine is given by the Betz number B = 16/27. In section 2.1, the power coefficient of the bare turbine was considered under the assumption of frictional losses. In this case, the power coefficient can also be identified as the efficiency of the wind turbine in this case. However, the power coefficient for the shrouded wind turbine as considered in section 2.2 is not efficiency. Based on the definition of efficiency, one could observe that if we divide the power coefficient by the factor (CF + 1) (taking into account the increased mass flow to the system due to pressure drop), a similar expression of the shrouded wind turbine could be given by equation (6). Thus, according to the modeling assumptions and with special care in treatment of the loss coefficient, one could conclude that the efficiency of the wind turbine could not exceed the Betz limit, although the power coefficient in general could exceed the Betz limit, as was observed previously by others.

2.4. Maximum windmill efficiency in finite time

In a different approach, a model to estimate the efficiency of a wind turbine was introduced [41] and the efficiency at maximum power output η_{mp} was derived. Although the power developed in a wind turbine derives from kinetic energy rather than from heat, it was possible to view the basic model of the wind turbine in a schematic way, which is similar to the heat engine picture. After the wind turbine accepts energy input in its upstream side, it extracts power at the turbine blades and ejects energy at the downstream. Details of this approach are given elsewhere [41].

The derived value for the efficiency at maximum power operation was shown to be a function of the Betz number, *B*, and is given by the following formula:

$$\eta_{mp} = 1 - \sqrt{1 - B} \tag{23}$$

This value is 36.2%, which agrees well with those for actually operating wind turbines.

With an algebraic manipulation this expression could be approximated by:

$$\eta_{mp} = \frac{B}{2 - \frac{B}{2}} \tag{24}$$

The expression given in equation (24) differs by only a small percentage when compared to equation (23). With the aid of equation (24), one could estimate the efficiency as 8/23.

As compared to the efficiency of heat engines, the system efficiency could be defined as the mean value between the maximum efficiency (Carnot efficiency) and the efficiency at maximum power point (Curzon-Ahlborn efficiency). Thus, the efficiency of the wind turbine system η_{ts} could be given by:

$$\eta_{ts} = \frac{\left(B + 1 - \sqrt{1 - B}\right)}{2} \tag{25}$$

If equation (23) is used while neglecting the contribution of the Number $B/2$ compared to 2, the turbine efficiency given by equation (24) would be approximated by:

$$\eta_{ts} = \frac{3}{4}B \tag{26}$$

2.5. Wind turbine efficiency and the golden section

The golden section has been considered in different disciplines as a measure of beauty [42-49]. The schematics of the golden section are given in Figure 3.

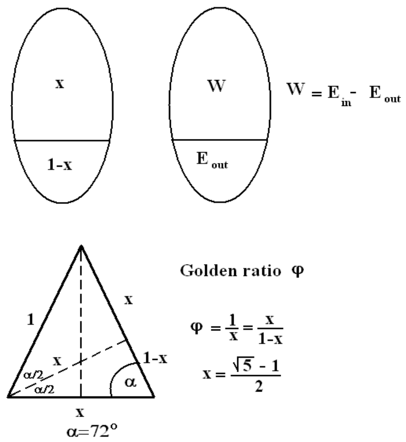

Figure 3. Schematics of the golden ratio. In the upper part to the left, the oval shape is divided into two parts, *x* and 1-*x*. in the upper part to the right, the same oval is divided into two parts with the following engine parameters: *W* represents the net work output; E_{out} represents the energy left the machine. E_{in} represents the energy that was put into the machine. In the lower part, a construction of the golden section is depicted by the isosceles triangle with a base angle of 72°. Comparing the different parts, *x* is defined as W/E_{in} or E_{out}/E_{in}, depending upon parts that have been produced and rejected.

In the upper part of the figure, the oval shape is divided into two parts, x and 1-x (as can be seen in the left side of the figure). In the right side, the same oval shape depicts the relation to quantities considered in engine machines. In the lower part of Figure 3, a golden section construction is depicted using the isosceles triangle with sides of unity and base triangle of 72°. If we apply the result of the golden section ratio (the golden section ratio is related to the sine of the angle of 18º, thus $2\sin(18) = \frac{\sqrt{5}-1}{2}$) to wind power efficiency, which is usually defined as the work gained divided by the energy input to the system, we can observe that the Betz limit agrees very well to the golden section (0.593 compared to 0.618). We can also see that the practical efficiencies of wind turbines in the proximity of 40% are comparable with the smaller part of the golden section 38.2%.

The golden section beauty can be related to wind turbines if we recall that the kinetic energy of the wind usually splits into two parts: useful and rejected. According to the Betz limit, about 60% of the energy is used to produce electric power and the rest is rejected. On the other hand, the real wind turbines extract about 40% and the rest is wasted. These findings are in match with the golden section division (61.8% and 38.2%). One could conclude by asking: Is this just a fortuitous result or is there something more deep and inherent in the beauty of nature?

2.6. Factors that affect the efficiency of the wind turbine

In terms of wind turbine efficiency, it is possible to highlight different parts of the turbine when estimating its value. Most of the studies discussed above considered extracting power from the kinetic energy of the wind. This could be defined as kinetic energy efficiencyη_{KE}. Other considerations should be accounted for, such as the mechanical efficiency, η_{me}(when power is decreased due to mechanical friction), the conversion to electricity efficiency,η_{con}, and the blockage efficiency, η_{bl} (which is defined as the amount of air blocked by the turbine blades as depicted by Figure 4). The overall turbine efficiency is given by:

$$\eta_{net} = \eta_{bl}\eta_{con}\eta_{me}\eta_{KE} \tag{27}$$

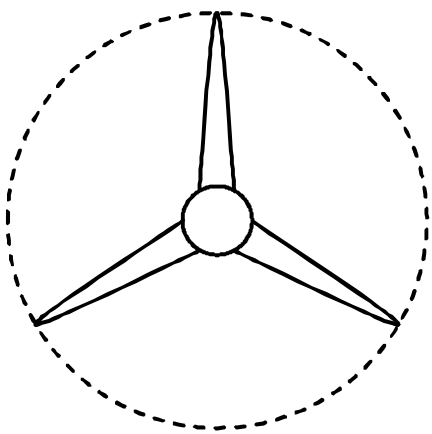

Figure 4. Schematic of the cross section of the rotor blades. The cross section illustrates the fact that the physical body of the rotor blades blocks some of the air particles, reducing potential power production from the air. The blocking efficiency can be defined as$\eta_{bl} = 1 - \frac{A_{rotor}}{A_{cross}}$. In this equation, A_{rotor} is the projected cross section of the rotor blades and A_{cross} is the cross section without the rotor blades.

3. Numerical considerations

In this section sample, plots of the results are considered.

3.1. The ideal bare wind turbine model

The one-dimensional bare wind turbine model without losses has been treated extensively and is well documented in textbooks [15]. The velocity of the air crossing the wind turbine velocity is assumed to be a fraction of the upstream air velocity. This fraction is introduced as a parameter, *a*, which expresses the ratio between the latter and the former. Parameter *a* is assigned a value in the range of numbers between zero and one. It is important to note that the physical range of this parameter is limited, for example, $0<a<0.5$, otherwise the velocity at the downstream becomes negative. To help clarify this point, calculations were performed covering the full range of parameter *a*. Equations (6) and (8) give the power coefficient and the thrust coefficient, respectively. Figure 5 shows these coefficients as functions of parameter *a* (a very well-known result in the professional literature in the field).

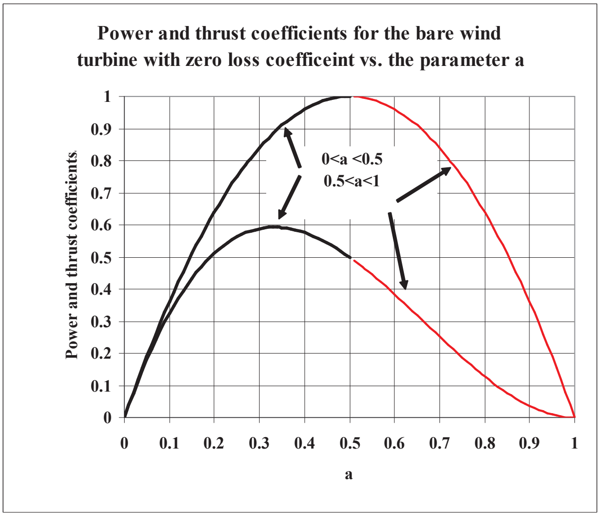

Figure 5. Power and thrust coefficients for the ideal bare wind turbine as a function of the parameter a (the ratio between the air velocity crossing the turbine blades and the upstream velocity of the air). The plot is reproduced similar to what is known in the literature, but highlighting the physical region ($0<a<0.5$) with thicker black color and the non-physical region ($0.5<a<1$) with thinner red color.

For explicit presentation, the physical range of parameter *a* (up to the value of 1/2) was drawn in a thick black color, while the rest of the plot was prepared using a thinner red color. It is clear from the figure that the coefficients vanish at the zero and one values of the parameter. In between, the maximum thrust coefficient (with a value of unity) occurs at the value of a=1/2. On the other hand, the maximum value of the power coefficient occurs at the value of a=1/3, for which the Betz limit is given ($C_P = B = 16/27$). If we plot the power coefficient as a function of the thrust coefficient, as Figure 6 shows, a loop shape would be produced. Again, the physical range was highlighted using a thick black color.

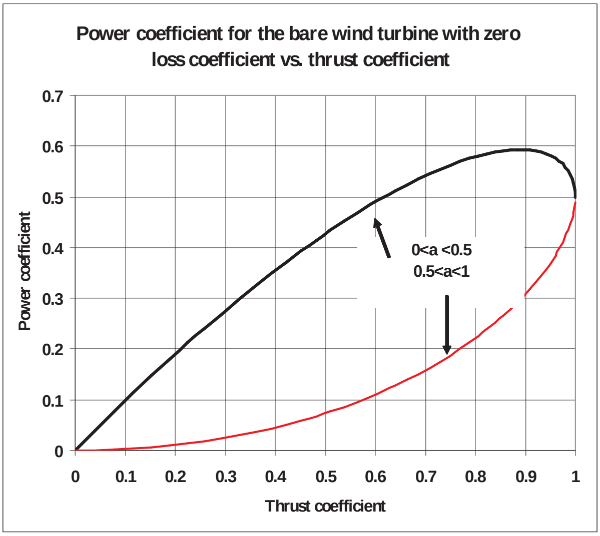

Figure 6. Power coefficient for the ideal bare wind turbine as a function of the thrust coefficient. The plot has been extended to include the non-physical region as was done in Figure 5 for reasons of consistency.

Two important points must be noted on such a plot: the maximum power coefficient (the Betz limit) for which the thrust coefficient receives a value of 8/9; and the maximum thrust (with the value of unity), for which the power coefficient gets a value of 1/2. These relations can be checked using equation (9), or more explicitly by using equation (17).

3.2. The bare wind turbine with losses

In this section, sample plots are given to demonstrate the effect of the losses as modeled in section 2.2. The losses are due to friction and are modeled as proportional to the velocity of the square of the velocity of the air flowing through the wind turbine. The plots are prepared for three different values of the non-dimensional loss coefficient C^*_{loss}: 0, 0.05, and 0.1. Figure 7 shows a plot of the power coefficient and of the thrust coefficient as a function of parameter a, covering the physical range while accounting for losses.

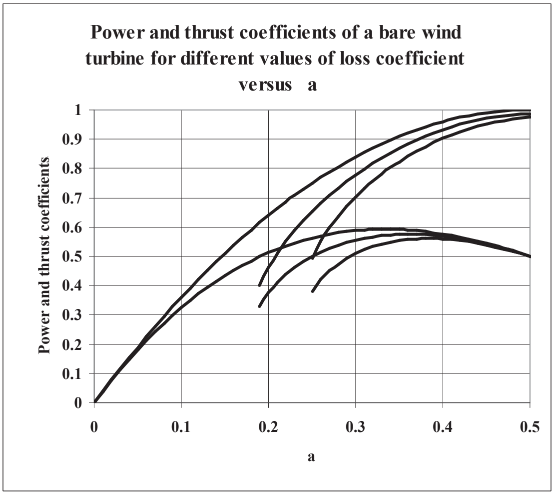

Figure 7. Power and thrust coefficients for the bare wind turbine as a function of parameter *a*, accounting for frictional losses, with different values.

It is clear from the figure that there is degradation in both coefficients in the order of a few percentage points. The two points (maximum power coefficient and maximum thrust coefficient) can be better visualized as illustrated by Figure 8.

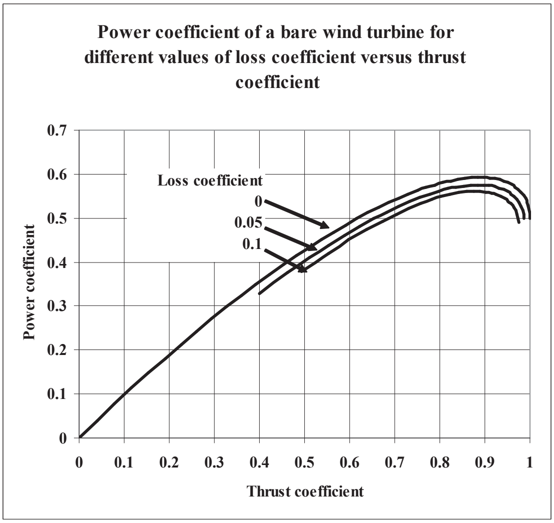

Figure 8. Power coefficient for the bare wind turbine as a function of the thrust coefficient, accounting for frictional losses, with different values.

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3.3. The shrouded wind turbine

The shrouded wind turbine was analyzed based on the extended Bernoulli equation, while accounting for frictional losses in the same manner as was done for the bare wind turbine. The increased air mass flow due to the larger drop in pressure was modeled as proportional to the kinetic energy difference, using the coefficient C_F (pressure drop coefficient—see equation (17)). As can be seen from Figure 9 and Figure 10 (equations (18) and (19) respectively), the power coefficient and the thrust coefficient are increased proportionally to the pressure coefficient, which is in agreement with the findings in the literature.

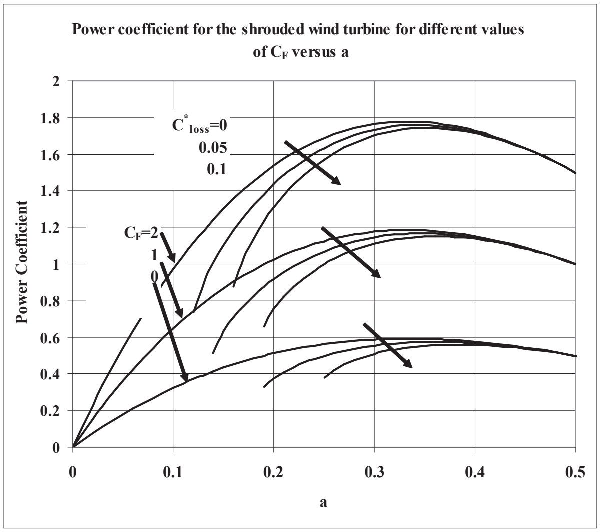

Figure 9. Power coefficient for the shrouded wind turbine as a function of parameter *a*, accounting for frictional losses and for augmentation coefficient C_F.

The maximum power coefficient and the maximum thrust points are illustrated in Figure 11.

By consulting Figure 11, one can observe that both maximum points are degraded by an increasing loss coefficient.

3.4. Efficiency of the wind turbine

As was considered by Betz, the power coefficient as originally defined agrees with the definition of efficiency for a device that extracts work from a given amount of energy. Thus, for the bare wind turbine, the maximum efficiency that could be extracted is actually given by the Betz limit. The effect of friction on wind turbine efficiency, as was expressed through the power coefficient, decreases with friction. A similar observation could be stated for the shrouded wind turbine if we use the definition as given by equation (22). Accordingly, the Betz limit is exceeded, that is, the shrouded wind turbine produces more power, but the amount of energy extracted per unit of volume with a shroud is the same as for an ordinary bare wind turbine. These results were found to be in agreement with results observed in [Van Bussel, 2007]. The efficiency of maximum power output that was observed by the finite time analysis was approximately 36%, which is comparable to experimental findings [14]. When compared to heat engines, the efficiency of the wind turbine could be expressed in terms of the Betz number by using equation (23). If the Betz number is substituted, the efficiency could be approximated as 47%, but if the other factors are taken into account, the practical efficiency could reach much lower values.

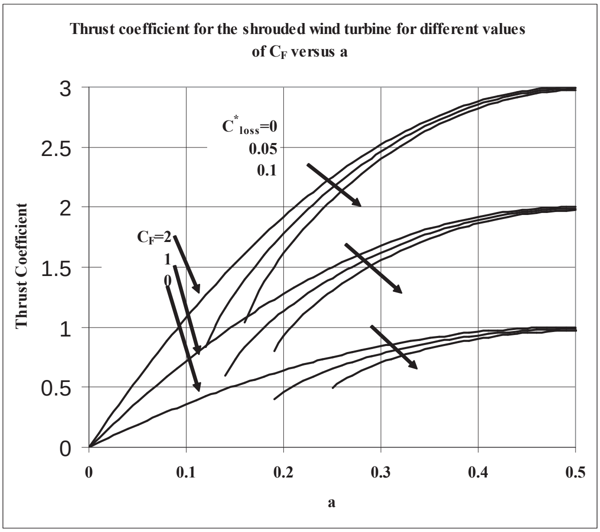

Figure 10. Thrust coefficient for the shrouded wind turbine as a function of parameter *a,* accounting for frictional losses and for augmentation coefficient $C_{F,}$ with different values.

4. Wind turbine arrangements

One could suggest ideas to increase power extraction from the wind, thus decreasing the overall cost. One suggestion is the bottoming wind turbine; another is the *flower leaves* arrangement of wind turbines. Both are discussed in the following sections.

4.1. Bottoming wind turbines

According to Betz, the maximal power extraction efficiency is 16/27. If it is possible to extract energy from the downstream expelled air, assuming the same limit exists, one could estimate an extra amount of 11/27*16/27 which is approximately 24%. This estimate suggests adding a smaller bottoming rotor behind the main larger rotor. The idea of the bottoming wind turbine is depicted schematically in Figure 12.

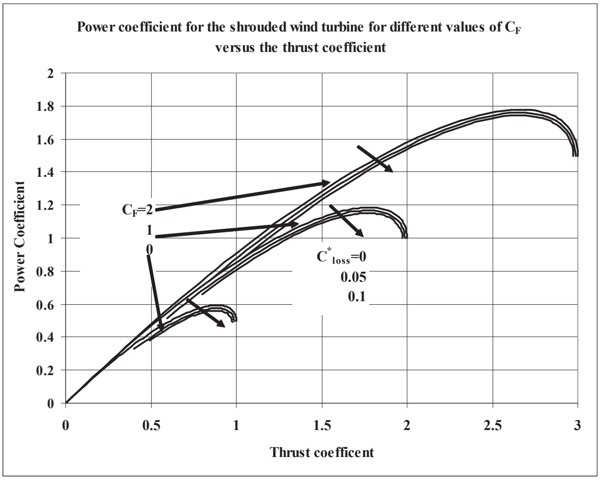

Figure 11. Power and coefficients for the shrouded wind turbine as a function of the thrust coefficient, accounting for frictional losses and for augmentation coefficient $C_{F,}$ with different values.

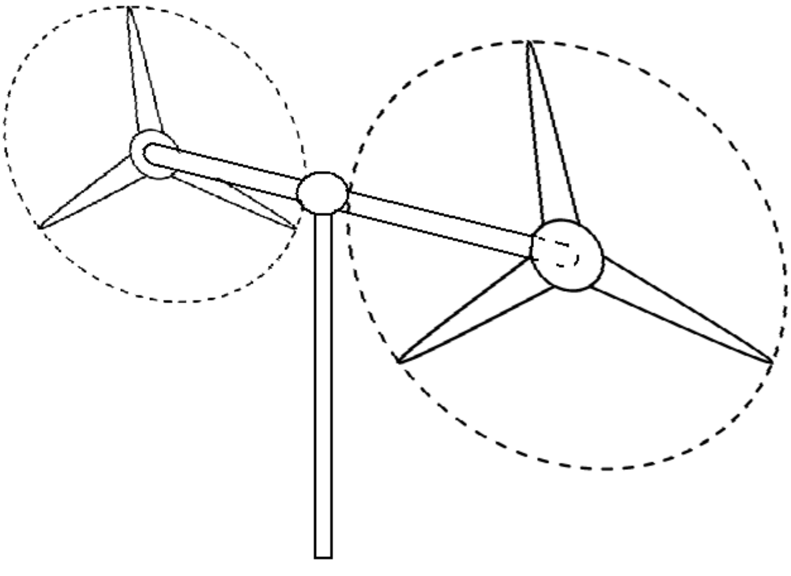

Figure 12. Schematics of the bottoming wind turbine idea. The main rotor is the first to intercept the airflow. Outlet air is then directed to the secondary rotor. The attractiveness of this idea is to gain more output with the same tower installation, reducing the inherently larger cost of erecting multiple towers.

4.2. Flower leaves arrangement of wind turbines

Considering the shrouded wind turbine as being a relatively small device is given (sometimes called flower power while searching the web), one could suggest installing different devices on the same tower. Such an arrangement reduces the cost of the installation. The idea is depicted schematically in Figure 13.

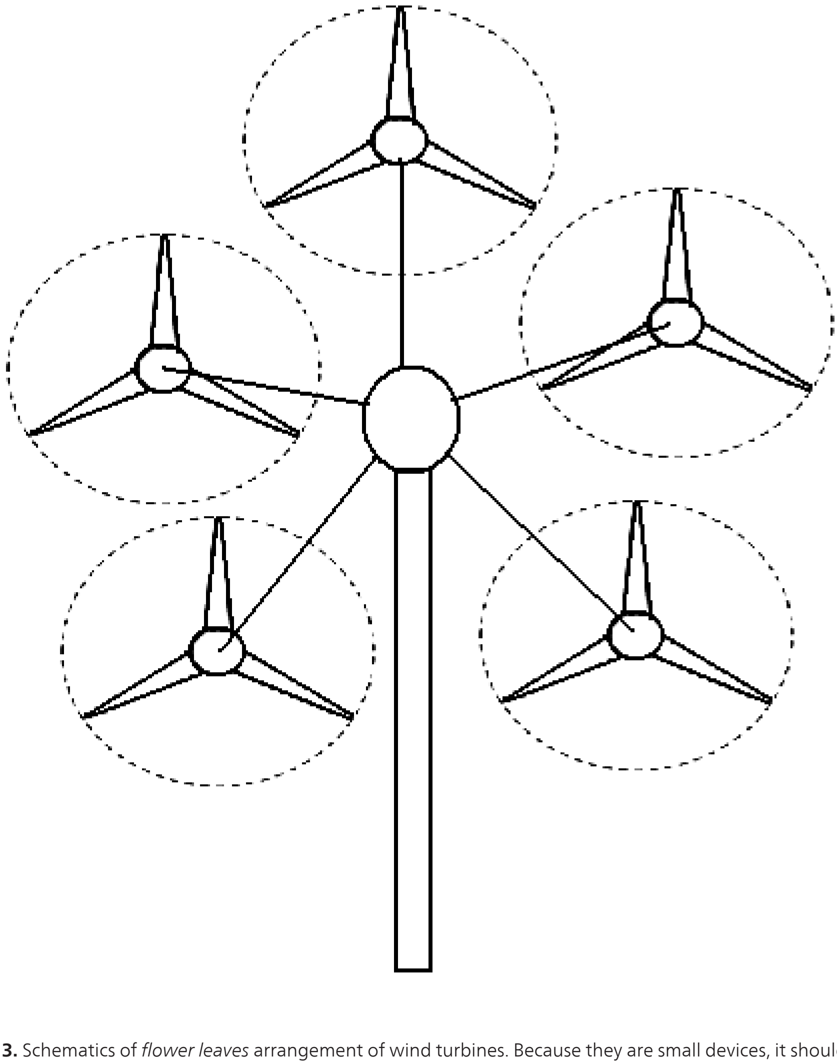

Figure 13. Schematics of *flower leaves* arrangement of wind turbines. Because they are small devices, it should be possible to install more of these shrouded systems on a single tower. While the concept of reducing installation costs is

similar to that of the bottoming wind turbine, the attractiveness of the flower leaves configuration is that many smaller turbines can be accommodated on a single tower, with significant cost reductions.

5. Summary and conclusions

In this study, wind turbine power was reconsidered. At the beginning, a literature review was given with relation to the potentiality of wind power, worldwide applications of wind power, and different factors that affect the performance of wind turbines, especially those related to one-dimensional modeling of the flow through the wind turbine. Later, different models were addressed, taking into account the effect of friction, which is usually neglected in the literature. In this study, friction is modeled to be proportional to the square of the velocity of the air crossing the wind turbine blades. The bare wind turbine model and the shrouded wind turbine model were analyzed based on the following balance equations: the mass balance equation, the momentum balance equation, and the energy balance equation that is exposed in the form of an extended (modified) Bernoulli equation. Through the analysis it was observed that both the power coefficient and the thrust coefficient degrade with friction. As was noticed in previous studies, the power coefficient given by the Betz number is the efficiency of the wind turbine (originally derived for the bare wind turbine). Following the same type of definition, a similar expression for the shrouded wind turbine could be derived. In a different approach, the wind turbine could be analyzed using finite time methods, as was given by [14]. In this study, the results were briefly summarized.

The well-known golden ratio usually is considered as a measure of beauty. It is interesting to notice that the Betz number differs from the golden ratio by only 4% (0.618 compared to 16/27=0.593).

In an effort to explain the discrepancy between theoretical efficiency and practical or measured efficiencies, different factors that affect the extraction of wind power are considered. These include mechanical friction, conversion efficiency to electricity, and blockage efficiency, which accounts for the blocked amount of air (usually is not mentioned in the literature), thus reducing the power output.

Finally, plots were given to suggest ways of assembling wind turbines to gain more of wind power for each tower installation.

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