Chapter 4

# Compressed Sensing: Ultra-Wideband Channel Estimation Based on FIR Filtering Matrix

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## 1. Introduction

Ultra-wideband (UWB) communication (Win & Scholtz, 1998; Yang & Giannakis, 2004a) is a fast emerging technology since the Federal Communication Commission released a spectral mask in the spring of 2002. The major reason for UWB technology to receive much attention is its promising ability to provide low-power consumption, high bit rate and multipath resolution, and coexist with the narrow-band system by trading bandwidth for a reduced transmits power. In the impulse radio UWB (IR-UWB) systems, the duration of pulse is ultra-short, typically on the order of nanoseconds. On one hand, the ultra-short impulses make it possible to resolve and combine signal echoes with path length differential down to 1 ft exploiting the diversity inherent in the multipath channel and improving the position accuracy. On the other hand, the new technical (Witrisal et al., 2009) challenges are posed: (1) analog-to-digital converters (ADCs) working at the Nyquist rate are in general very expansive and power demanding; (2) the synchronization which is accomplished at the scale of sub nanosecond duration is extremely complex; (3) capture a sufficient amount of the rich multipath diversity need accuracy channel estimation. Compare to the transmitter easily implement, the IR-UWB receiver are too complex.

The emerging theory of compressed sensing (CS) (Candès, et al., 2006; Donoho, 2006) provides new approaches for practical UWB receiver design. When the short duration pulses in the UWB system propagate through the multipath channels, the received signals remain sparse in time domain. The sampling rate can be reduced to sub-Nyqusit rate and the receiver can reconstruct the initial signal with high probability. Accordingly, there has been a growing interest in applying the CS theory to sparse channel estimation (Bajwa et al., 2010; Berger et al., 2010). The recent literature on sparse channel estimation can be found in (Bajwa et al., 2010; Berger et al., 2010) and in their references. It is proved that conventional channel estimation methods provide higher errors because they ignore the prior knowledge of the sparseness (Wan et al., 2010). The sparse channel estimation problem is faced in (Paredes et al., 2007) under a time domain sparse model point of view. In (Paredes et al., 2007) a suitable dictionary formed by delayed versions of the UWB transmitted pulse is defined in order to better match the UWB signal. However, the spike basis achieves maximal incoherence with the Fourier basis (Candès & Wakin, 2008) and is for that reason that seems more convenient to work with frequency domain. To ensure that every measurement counts, they propose to pre-modulate the input signal with a spread spectrum sequence before the Fourier transformation. As the IR-UWB signals have resolvable multipath with a sparse structure at the receiver, the application of CS theory to UWB channel estimation has also found wide interest in the UWB community. For the CS based UWB channel estimation, the main goal has been to estimate the sparse channel with reduced number of observations (Paredes et al., 2007; Liu & Lu, 2009; Naini et al., 2009).That is equivalent to reducing the sampling rate at the receiver. In (Paredes et al., 2007), a channel detection method based on the Matching Pursuit algorithm is proposed, where the path delays and gains are calculated iteratively. In (Liu & Lu, 2009), the authors combine the maximum likelihood (ML) approach with the CS theory. In (Naini et al., 2009), a spread spectrum modulation structure is placed before the measurement matrix to enhance the estimation performance. The common assumption of the studies in (Paredes et al., 2007; Liu & Lu, 2009; Naini et al., 2009) is that the UWB channels are sparse. However, depending on the environment (e.g., an industrial environment may have dense multipath), the sparsity assumption of the channels may not hold. And the receiver may be a little complex for the compressed sensing framework.

In this context a Finite Impulse Response (FIR) filtering matrix estimator for UWB channel based on the theory of CS is advanced. An FIR filter is introduced at the transmitter to get a quasi-Toeplitz measurement matrix. So the reconstruction accuracy using the CS framework is improved. Also, the receiver is simplified since a filter at the transmitter has been adopted in place of the measurement matrix at the receiver. The key point is to avoid the magnification of noise by the measurement matrix. Unlike the Generalized Likelihood Ratio Test (GLRT) detector design, the correlation detector for UWB signals employing the channel parameters estimated in this chapter needs no prior knowledge about the channel noise. In addition, the desired receiver performance calls for fewer measurements. Then both the Orthogonal Matching Pursuit (OMP) and the Basis Pursuit De-noising (BPDN) are compared to the Dantzig Selector (DS) for different signal noise ratio (SNR) to give the opinions for choosing suitable reconstruction algorithms. Realistic channel estimation is considered. Simulations discussed later indicate the efficiency of the proposed method.

This chapter is organized as follows. In section 1, the motivation and research status are introduced. In section 2, a brief description of compressed sensing and its application for UWB channels is introduced. In section 3, the FIR filtering matrix method for UWB channel estimation based on the CS theory is proposed. In section 4, the estimation results are used in the UWB signals detection. In section 5, the simulation results together with the analysis are given. In section 6, we offer the conclusions and discussions. The references are given in section 7.

## 2. Compressed sensing for UWB channel estimation

In essence, CS theory has shown that a sparse signal can be recovered with high probability from a set of random linear projections using nonlinear reconstruction algorithms. The sparsity of the signal can be in any domain (time domain, frequency domain, wavelet domain, etc.) and the number of random measurements, in general, is much smaller than the number of samples in the original signal, which leads to a reduced sampling rate and, hence, reduced use of ADCs resources.

In UWB impulse radio communications, an ultra-short duration pulse, typically on the order of nanoseconds, is used as the elementary pulse-shaping to carry information (Reed, 2005). Transmitting ultra-short pulses leads to several desirable characteristics: (1) simplicity is attained in the transmitter since a carry-less baseband signal is used for conveying information (Lottici et al., 2002); (2) the transmitted signal power is spread broadly in frequency having little or not impact on other narrowband radio systems operating on the same frequency (Qiu et al., 2005); (3) the received UWB signal is rich in multipath diversity introduced by the large umber of propagation paths existing in a UWB channel. For the most important fact that transmitting an ultra-short pulse through a multipath UWB channel leads to a received UWB signal that can be approximated by a linear combination of a few elements from a pre-defined basis, yielding thus a sparse representation of received UWB signal. Next, we briefly describe the CS framework in (Candès et al., 2006) and (Donoho, 2006), and apply this framework into the UWB channel.

### 2.1. CS overview

Consider the problem of reconstructing an $N$×1 discrete-time signal vector $x \in \mathbb{R}^N$ . It can be shown that if $x$ is sparse, in the sense that $x$ can be represented as a superposition of a small number of vector taken from a dictionary $D$=[$D_1$,…,$D_N$] of tight-frames, which provides a $K$-sparse representation of $x$, that is

$$x = \sum_{n=0}^{N\text{-}1} D_n u_n = \sum_{l=1}^{K} D_{n_l} u_{n_l} \tag{1}$$

Where $x$ is a linear combination of $K$ vector chosen from the arbitrary basis $D$, and $K$<<$N$; {$n_l$} are the indices of those vectors; {$u_l$} are the weighting coefficients; $\alpha$=$K$/$N$ is called sparse-ratio. Alternatively, in this chapter we write the signal vector in matrix notation

$$x = Du \tag{2}$$

Where $u$=[$u_0$, $u_1$, ⋯,$u_{N\text{-}1}$]$^T$ which has $K$ nonzero coefficients, where $K$<<$N$. In CS, signal $x$ can be represented by $K$ entries of $u$ in place of $N$ entries of $x$, that reduces dimension of the signal of interest. We need to estimate only $K$ real-parameters not $N$ to reconstruct $x$ from a channel realization. When sparse-ratio $\alpha$ is very small, the compressive gain becomes high.

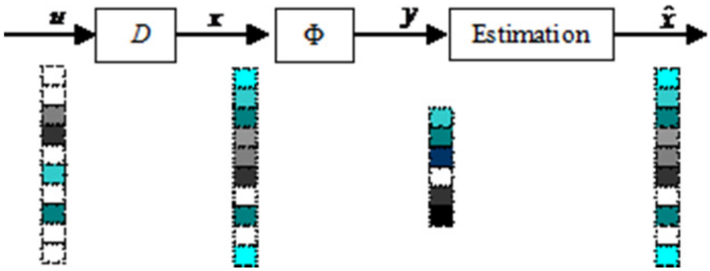



**Figure 1.** Compressive sensing model

In the viewpoint of the CS theory, a sparse signal can be reconstructed successfully from far fewer data/measurements than what is usually considered necessary (Donoho, 2006). Figure 1 shows the CS theory frame. It gives the whole course of the signal projection and reconstruction. The sampling from $x$ becomes a linear transformation, that is $x$ can be reconstructed from $M$ measurements and $M<<N$. By projecting $x$ onto a random measurement matrix $\Phi \in \mathbb{R}^{M\times N}$, a set of measurements $y \in \mathbb{R}^{M}$ can be obtained as

$$y = \Phi x = \Phi D u \tag{3}$$

where Φ is called measurement matrix, which is incoherent with $D$; and $y$ is the signal we received in receiver, who has $M$ entries, each becomes a measurement of $x$. Instead of using the $N$-sample $x$ to find the weighting coefficients $u$, $M$-sample measurement vector $y$ can be used. Accordingly, $u$ can be estimated as

$$\hat{u} = \min \|u\|_1 \text{ s.t. } y = \Phi D u \tag{4}$$

Where $l_p$ -norm is defined as $\|u\|_p = \left(\sum_{n-1}^{N} |u_n|^P\right)^{1/p}$. Note that, the advantage of estimating $u$ from the vector y instead of $x$ is that the former having much fewer samples corresponds to a much lower sampling rate at the receiver. If the dictionary $D$ and measurement matrix Φ are acquired, and they satisfy $M$=$CK$log$N$<<$N$, signal $x$ can be recovered from measurements $y$ using reconstruction algorithms with overwhelmingly high probability, even we don't know the sparse pattern of the unknown signal $u$ (Candès & Tao, 2006). $C$ ≥1 is then called the oversampling factor.

In short, sampling and processing signals in the CS framework can be concluded just like this: First, we must design tight-frames $D$ according to the character of signal of interest. That is to design a overcomplete dictionary to get the sparse representation of $x$; after the first stage, one should design a $M$×$N$ sensing matrix Φ, through which measurement $y$ can be achieved. Finally, $x$ can be recovered with $y$, $D$ and Φ employing reconstruction algorithm. In next section, we will present how this concept can be used for UWB channel estimation.

### 2.2. CS for UWB channel estimation

The CS theory explained in (2)–(4) can be applied to UWB channel estimation for the fact that the Gaussian pulse response of the UWB channel is sparse. We show the simulation results in section 5. Suppose that $r \in \mathbb{R}^N$ is the discrete-time representation of the received signal given as

$$r = Ph + n \tag{5}$$

Where $P \in \mathbb{R}^{N\times N}$ is a scalar matrix representing the time-shifted pulses, $h$= [$\alpha_1$, $\alpha_2$, , $\alpha_3$ ]T are the channel gain coefficients, and $n$ are the AWGN terms. Since the UWB channel structure is sparse, $h$ has only $K$nonzero coefficients. Similar to (3), the received signal $r$ can be projected onto a random measurement matrix $\Phi \in \mathbb{R}^{M\times N}$ so as to obtain $y \in \mathbb{R}^M$ as

$$y = \Phi Ph + \Phi n = Ah + v \tag{6}$$

Due to the presence of the noise term $v$, the channel $h$ can be estimated as

$$\hat{h} = \min\|h\|_1 \text{ s.t. } \|Ah - y\|_2 \le \varepsilon \tag{7}$$

Where $\varepsilon$ is related t o the noise term as $\varepsilon \ge \|v\|_2$. Considering (7), the channel estimation performance depends on the sparsity of $h$(i.e., the value of $K$), as well as the number of observations $M$. It is therefore necessary to understand the discrete-time equivalent structure of $h$ and the effects of standardized channel models.

## 3. UWB channel estimator based on CS

While CS research has focused primarily on signal reconstruction and approximation, the CS framework can be extended to a much broader rang of statistical inference tasks, well suited for applications in wireless UWB communications. UWB channel estimation is one of those applications which will be used extensively in this section. Next we will investigate the effect of the IEEE 80.15.4a UWB channel models (Molisch et al., 2006) on the channel estimation performance from a practical implementation point of view. Then a new sparse channel estimation method is proposed by improving the random measurement method based on CS for discrete time signals in (Paredes et al., 2007). According to the amplification of channel noises as well as measurement signals, we designed a new channel estimation method with FIR filtering matrix.

### 3.1. UWB channel

In the following, we initially present the discrete-time equivalent channel $h$ followed by the UWB channel models. In order to obtain $h$, the general channel impulse response (CIR) should be presented first. Accordingly, the continuous-time channel $h(t)$ can be modeled as

$$h(t)=\sum_{l=0}^{L-1}\alpha_l\delta(t-\tau_l) \quad (8)$$

Where $\alpha_l$ is the $l$-th multipath gain coefficient, $\tau_l$ is the delay of the $l$-th multipath component, $\delta(\cdot)$ is the Dirac delta function and $L$ is the number of resolvable multipath.

The continuous-time CIR given in (8) assumes that the multipath may arrive any time. This is referred to as the $\tau$-spaced channel model (Erküçük et al., 2007). Suppose that two consecutive multipath with delays $\tau_k$ and $\tau_{k+1}$ arrive very close to each other. Further suppose that a pulse of duration $T_s$ is to be transmitted through this channel. If $T_s>|\tau_{k+1}-\tau_k|$, then the pulse at the receiver cannot be resolved individually for each path, and experiences the combined channel response of the $k$th and ($k$+1)th paths. Let us define an approximate $T_s$-spaced channel model that combines multipath arriving in the same time bin, $[(n-1)T_s, nT_s]$, $\forall n$. Accordingly, for $[(n-1)T_s, nT_s]$, $\forall n$, the delays $\{\tau_l|0, 2, \ldots, L\text{-}1\}$ that arrive in the corresponding quantized time bins can be determined, and the associated $\{\alpha_l|0, 2, \ldots, L\text{-}1\}$ gains can be linearly combined to give the new channel coefficients $\{\alpha_n|1, 2, \ldots, N\}$. Note that some of the $\{\alpha_n\}$ values may be zero due to no arrival during that time bin, hence, the number of nonzero coefficients $K$ satisfies the condition $K\leq L\leq N$. The equivalent $T_s$-spaced channel model can be expressed as

$$h(t)=\sum_{n=1}^{N}\alpha_n\delta(t-nT_s) \quad (9)$$

Where $T_c = NT_s$ is the channel length. Using (9), the discrete-time equivalent channel can be written as

$$h=[\alpha_1,\alpha_2,\ldots,\alpha_N]^T \quad (10)$$

where the channel resolution is $T_s$. Then the discrete-time equivalent channel vector obtained above can be used in (5)–(7) in the context of CS theory. Next, we consider the UWB channel models to be used with the channel vector $h$.

The CS based UWB channel estimation studies assume that the UWB channel vectordefined above is sparse. However, this is a vague assumption. In order to classify a channel as sparse, initially the channel environment should be examined. In (IEEE Std 802.15.4a, 2007), members of the IEEE 802.15.4a standardization committee have developed a comprehensive standardized model for UWB propagation channels. Accordingly, they have considered different environments and have conducted measurement campaigns in order to model the UWB channels for each environment. The channel environments that they have parameterized include indoor residential, indoor office, outdoor, industrial environments, agricultural areas and body area networks. The details of the related channel models and their associated parameters can be found in (Molisch et al., 2006). We motivate our study with the selection of a variety of environments either having a line-of-sight (LOS) or a non-LOS (NLOS) transmitter-receiver connection. Accordingly, the CM1 (indoor residential LOS), CM2 (indoor residential NLOS), CM5 (outdoor LOS) and CM8 (industrial NLOS) channel models are widely used in UWB research. We now summarize the characteristics of channel models CM1, CM2, CM5 and CM8 in the following.

**CM1:** This is by-far the most commonly used channel model in order to assess the system performance. It models an LOS connection in an indoor residential environment. It is the most sparse channel model where few Rake fingers can collect considerable amount of signal energy.

**CM2:** This is a channel model with an NLOS connection in an indoor residential environment. It complements CM1. It is a sparse channel model but usually contains more multipath compared to CM1.

**CM5:** This is a channel model with an LOS connection in an outdoor environment. Typically, the multipath arrive in a few clusters.

**CM8:** This is a channel model with an NLOS connection in an industrial environment. The multipath arrive densely so that the channel does not have a sparse structure.

Using the $T_s$-spaced channel model in (9) and the parameters for channel models CM1, CM2, CM5 and CM8 in (Molisch et al., 2006), a realization for CM1 channel model is plotted in Figure 4 when the channel resolution is $T_s$=0.66ns. It can be observed that the typical channel properties listed above can be observed. The impulse response of the UWB channel is sparse.

## 3.2. Random measurement estimation

According to the models proposed by the IEEE 802.15.4a working group, the impulse response of the UWB channel is modeled as function (8) follow in time domain. Consider the simple communications model of transmitting a pulse $p(t)$ throughout a noiseless UWB communication channel $h(t)$. The received UWB signal can be modeled as

$$g(t) = p(t) * h(t) = \sum_{l=0}^{L-1} p_l \delta(t - \tau_l) \tag{11}$$

In this chapter, we suppose $p(t)$ is a first derivative of the Gaussian pulse with unit power. Then the estimate value of $g(t)$ is $\hat{g}(t)$, which represents a referent template for subsequent correlation detection of UWB signals. The received UWB signal given by (11) has been sampled to define the discrete-time vector $g$, which is taken as a signal targeted for estimation. It is available to get the estimate of $g$ by sampling directly from $g(t)$. However, the extremely high bandwidth of the received UWB signal requires high-speed A/D converters. Some approaches for UWB receivers are needed to attain the required sampling rates. The random measurement method focuses on this goal by sensing data at the receiver using a $M$×$N$ measurement matrix $\Phi_1$, which is obeying the restricted isometry property (RIP) (Bajwa et al., 2007), leading to measurements $y_1$

$$y_1 = \Phi_1 g = \Phi_1 P h \tag{12}$$

The referent template $\hat{g}(t)$ access to correlation detection can be reconstructed successfully with only $M$ measurements at the receiver, provided that $g$ is sparse in some space. The principle architecture of the random measurement estimation is given in Figure 2.

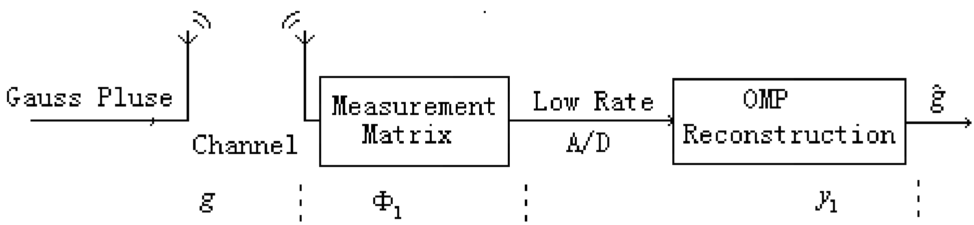



**Figure 2.** The random measurement estimation

In (Paredes et al, 2007), a over-complete dictionary is designed, in which the signal $g$ has concise representations when expressed. Then a better performance of the channel estimation was guaranteed. The results, however, are based on a premise that there is no channel noise on pilot symbols. It is not true in actual channel. When the noise is introduced, we have

$$g_n(t) = p(t) * h(t) + n(t) = g(t) + n(t) \tag{13}$$

Where $n(t)$ denotes additive white Gaussion noise (AWGN) in UWB channel, follow the $N(0, \delta^2)$ distribution. We restrict our attention to discrete signals, then the measurement process on the signal itself exploiting measurements matrix $\Phi_1$ is described as this

$$y_1 = \Phi_1(g + n) = \Phi_1 Ph + \Phi_1 n_1 \tag{14}$$

Note that, the random measurement method processes the noise from $N$-dimension to $M$-dimension via the projection. In terms of the conversion in dimension, the noise power translates into $(N/M)^2\delta^2$ versus $\delta^2$ before projecting. Because of $M<<N$, that means the sampling rate reduces at the expense of magnification of the channel noise.

### 3.3. FIR filtering matrix estimation

In this chapter, we propose a new method based on filtering matrix for UWB channel estimation with CS framework. In order to improve the estimation performance, some implements should be taken to suppress the magnification of the noise from measurements matrix. The concrete step is illustrated in Figure 3, which gives the architecture of the proposed method. The processing flows for both the transmitter and the receiver have been adjusted leading to higher accuracy and lower complexity in receiver.

As can been seen from Figure 3 (Yu Huanan & Guo Shuxu, 2010), a UWB signal is transmitted by a UWB pulse generator and through an FIR filter. Then, the received signal is directly sampled through a low-rate A/D conversion after the propagation paths. Finally, the estimation of the impulse response $\hat{h}$ can be reconstructed via OMP algorithm (Pati et al., 1993).

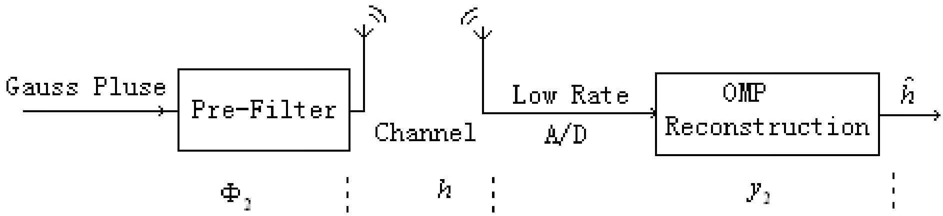



**Figure 3.** The FIR filtering matrix estimation

According to function (1), the transmitting signal $x(t)$, which is $K$-sparse over some overcomplete dictionary $P$,

$$x(t) = \sum_{n=0}^{N-1} P_n(t)h_n = P(t)h \tag{15}$$

Where

$$P(t) = [P_0(t), P_1(t), \ldots, P_{N-1}(t)] \tag{16}$$

$$h = [h_0, h_1, \ldots, h_{N-1}]^T \tag{17}$$

Note that there are only $K$ non-zeros in $h$. $x(t)$ is then fed into a $L$-length FIR filter. Suppose that $m(t)$ is the impulse response of the FIR filter, the received signal for the UWB communication is given by

$$y_2(t) = p(t) * m(t) * h(t) + n(t) \tag{18}$$

Since the UWB channel is sparse, the impulse response of the UWB channel $h(t)$ can be viewed as a sparse signal. Let $h$ be the discrete-time representation of $h(t)$, which is set up as the estimation target in this section. In addition the identity matrix is used as an overcomplete dictionary because of the sparsity of $h$. Define $c(t) = p(t) * m(t)$, then (18) becomes

$$y_2(t) = c(t) * h(t) + n(t) \tag{19}$$

Where $c(t)$ and $h(t)$ are processed using a low-rate A/D, which is $M$-dimension. The output $y_2(t)$ is then uniformly sampled with sampling period $T_s$. $T_s$ follows the relation $T_s/T_h = q$, where $q$ is a positive integer, and $T_h$ denotes the time delay between each adjacent channel. $M$ samples are collected so that $M \cdot T_s = \lfloor L \cdot T_h + T_x \rfloor$, which is the duration of $y(t)$. Now we have the down-sampled output signal $y_2(mT_s)$, $m$=0, 1,…, $M-1$

$$
\begin{aligned}
y_2(mT_s) &= c(mT_s) * h(mT_s) + n(mT_s) \\
&= \int_0^T c(mT_s - \tau)h(\tau)d\tau + n(mT_s) \\
&= \sum_{l=0}^{L-1} c_i h(mT_s - iT_h) + n(mT_s)
\end{aligned}
\tag{20}
$$

The output $y_2(t)$ is uniformly sampled with sampling period $T_s$. Now we rewrite (20) in matrix notation

$$
y_2 = \Phi_2 h + n \tag{21}
$$

where $\Phi_2$ is a quasi-Toeplitz matrix. It has such property: each row of $\Phi_2$ has $L$ non-zero entries and each row is a copy of the row above, shifted by $q$ places. Following (Bajwa et al., 2007), it is illustrated that the quasi-Toeplitz matrix obeys the RIP.

According to (20) and (21), let $y_2$ be the random projected signal where $\Phi_2=p(t)*m(t)$ is the measurement matrix, and identity matrix is used as over-complete dictionary. The random projected signal $y_2$ can be acquired with $M$-dimension low-rate A/D converters, and the OMP algorithm is then applied on y2 to recover $\hat{h}$. While the convolution process is following (21), associated with the reconstruction results above, the referent template $\hat{g}$ for correlation detection is acquired.

The whole process above accounts to a filtering action on the Gaussian pulse, which we selected as the transmitted pulse waveform. The receiver becomes very simple, with only one $M$-dimension low-rate A/D to collect measurement samples after the filter and channel. It can be seen that the noise does not go through the projection from $N$-dimension to $M$-dimension, thus the noise has not been magnified in the CS framework. Furthermore the proposed method has a better measurement matrix compared with the random measurement method. Hence, a better performance of the estimation accuracy can be achieved.

## 4. Correlation detection for the UWB signals

The random measurements method is focused on CS reconstruction of noiseless UWB signals, which relies on the assumption that the noiseless composite pulse-multipath waveform is sparse in a pre-designed dictionary. In a more realistic UWB communication scenario, however, the received signal is contaminated with noise and interferences, and the challenges fall in the design of a UWB receiver with the ultimate goal of signal detection.

### 4.1. Correlation detection

Suppose that the impulse response of the UWB channel $h(t)$ is invariant in each data frame, including $N_p$ pilot symbols and $N_s$ data modulated symbols. The total number of symbols in one burst is $N_p$+$N_s$ And $N_f$ first derivative of the Gaussian pulse $p(t)$ are repeated over consecutive frames to transmit one pilot or binary symbol. For the sake of damping the effect of AWGN, we average the received signal during a data frame. The maximum excess delay of the dense multi-path channel is given by $T_{med}$. $p(t)$ is of unit energy and has time duration $T_p$, and also the duration of a frame is given by $T_f$. In order to avoid inter-symbol interference (ISI) and intra-symbol interference (Yang & Giannakis, 2004b), it is assumed that $T_f > T_p + T_{med}$.

In the UWB correlation detector, if there exits a module to time precisely, the pilot and data symbols can be exactly separated. When the pilot is canceled, and also the referent template $\hat{g}$ estimated above is employed into the correlation detection of the received signals, the transmitted signal during a data frame for UWB communication is shown as follows

$$s(t) = \sum_{j=0}^{N_s-1}\sum_{n=0}^{N_f-1} b_j p(t - jN_f T_f - nT_f - N_p N_f T_f) \tag{22}$$

here, $b_j \in \{\pm 1\}$ are the $j$-th information bits. Signal $s(t)$ propagates through an $L$-path fading channel whose response to $p(t)$ is $\sum_{l=1}^{L} h_l p(t-\tau_l)$ such that the received signal at the receiver is

$$r_b(t) = \sum_{j=0}^{N_s-1}\sum_{n=0}^{N_f-1}\sum_{l=1}^{L} b_j \alpha_l p(t - jN_f T_f - nT_f - N_p N_f T_f - \tau_l) + n(t) \tag{23}$$

Where $n(t)$ is thermal noise with two-sided power spectral density $N_0/2$. The integral term implements the correlation operation between the received UWB signal $r_b(t)$ and the estimate template $\hat{g}(t)$, and then the information bits can be acquired as

$$\hat{b}_j = \text{sign}(\sum_{j=0}^{N_f-1} \int_{jT_f+kT_s}^{(j+1)T_f+kT} r_b(t)\hat{g}(t - jT_f - kT_s)dt \tag{24}$$

It can be seen from (24), since $T_s$ is the sampling period, only the $M$-dimension low-rate A/D is needed.

## 4.2. Signal reconstruction algorithms

Then, the Orthogonal Matching Pursuit, the Basis Pursuit De-noising and the Dantzig Selector are used to detect original signal to give the opinions for choosing suitable reconstruction algorithms.

When $M \ll N$, the (8) is an uncertain function, so the search for the most sparse solution becomes an NP-hard problem. The literature (Donoho, 2006) proved that this problem can be inverted to the problem of answering a programming problem. In this chapter we study the three algorithms: the BPDN algorithm and the DS algorithm in $\ell_1$-norm and the OMP algorithm in the greedy algorithm.

The BPDN algorithm derived from the Basis Pursuit (BP) algorithm, so it is an optimizing strategy. The BPDN algorithm tries to deal with such problems:

$$\min_{x} \frac{1}{2}\|y - \Phi x\|_2^2 + \lambda \|x\|_1 \tag{25}$$

It can be rewrite as follow

$$\min_{x} \|x\|_1 \text{ s.t. } \|y - \Phi x\|_2^2 \le \sigma \tag{26}$$

The other is the DS algorithm based on $\ell_1$-norm, which can deal with the following problem

$$\min_{x} \|x\|_1 \text{ s.t. } \left\|\Phi^{\mathrm{T}}(y - \Phi x)\right\|_\infty \le \sigma \tag{27}$$

Comparing (26) with (27), we see that the DS algorithm is similar with the BPDN algorithm. The main difference is that the BPDN algorithm relies on punishing residuals using the $\ell_2$-norm to realize the optimization, while the DS algorithm relies on minimizing correlation between residuals and all atoms.

OMP is a kind of greedy algorithm, deviating from the $\ell_1$-norm shrinkage strategy. One of the most important properties of the algorithm is that it does not choose the same atom twice, so the estimation value satisfies $\|\hat{x}\|_0 = K$ , after $K$ iterations.

Literature (Pati et al., 1993) proved that all of the BPDN algorithm, DS algorithm and OMP algorithm can obtain $E\log m$ times of the mean square error about the Oracle estimator, and $E$ is a constant here.

## 5. Simulations and results

In this section, the performance of the CS based on FIR filter matrix meuof the new method and random measurement estimation are made. Then three experimentations have been designed as follows.

The simulation parameters are set as follows: the transmitted UWB signal pulse $p(t)$ is the first-order derivative of the Gaussian pulse and is normalized to have unit energy. The duration of the time resolution of the channel is $T_s$=0.66$ns$, $T_f$ =110$ns$. The UWB channel model CM1 (LOS) proposed by IEEE working group are adopted in our simulation. Table 1 shows the principal parameters of the channel models.

Simulation 1: consider the pulse propagating through a noiseless propagation scenarios. We adopt a UWB channel that models an indoor residential environment with line-of-sight IEEE 802.15.4a channel model. Figure 4 shows the impulse response of the UWB channel. Then the first derivative of the Gaussian pulse is selected as the transmitted pulse waveform, according to which the response of the channel is shown in Figure 5. It is just the real value of the referent template for the subsequent correlation detection.

| | |
|---|---|
| Average cluster arrival rate (ns) | 0.0265 |
| Multipath component delay factor (ns) | 6.7 |
| Average pulse arrival rate (1/ns) | 3.2 |
| Cluster delay factor (ns) | 7.8 |
| Standard deviation of the channel gain (dB) | 4.0 |
| Standard deviation of the channel coefficient in cluster (dB) | 4.2243 |
| Standard deviation of the channel coefficient between clusters (dB) | 4.2243 |

**Table 1.** Parameters in UWB channel models proposed by IEEE

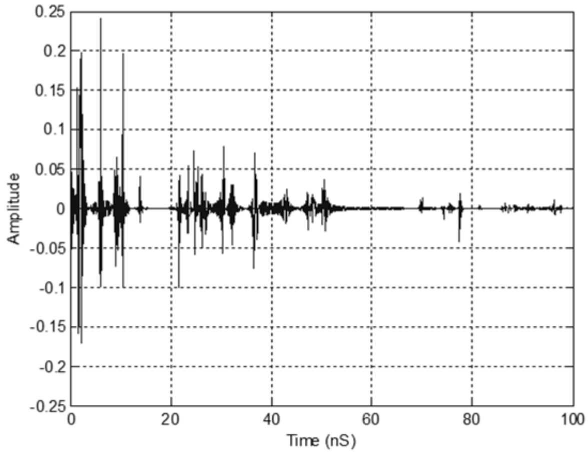



**Figure 4.** Impulse response of the UWB channel

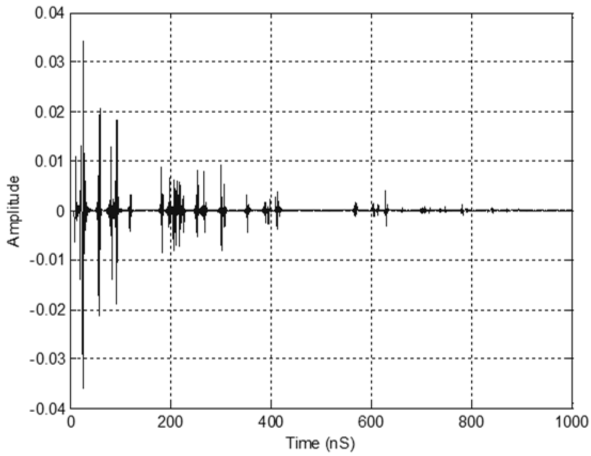
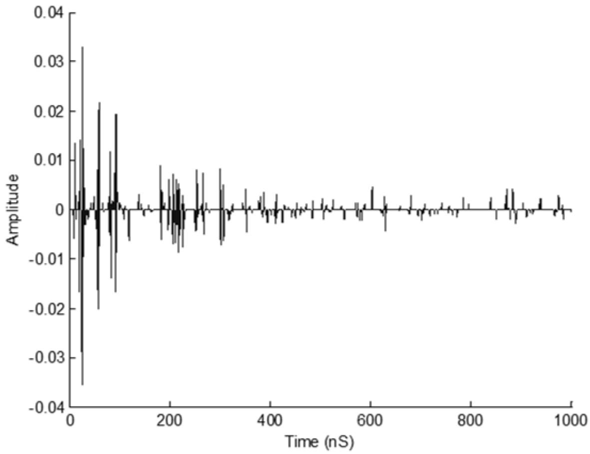
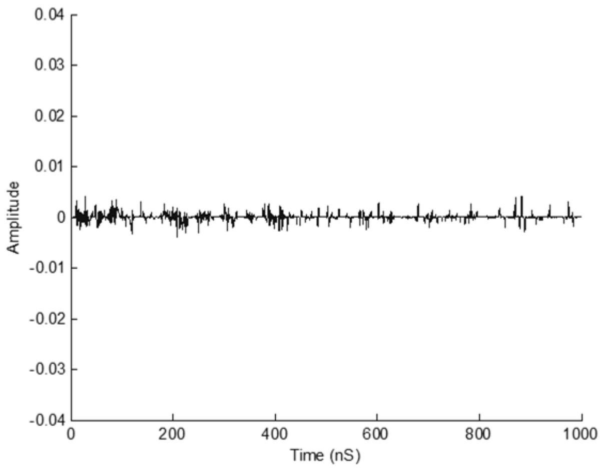



**Figure 5.** Gaussian pulse response of the UWB channel

Simulation 2: $\hat{g}$ is reconstructed via OMP algorithm in the absence of noise. The simulation parameters are set as follows: $N$=1000, $M$=360, $K$=180. Figure 6 and Figure 7 show the simulation results and reconstruction error respectively for both random measurement method and FIR filtering matrix method.



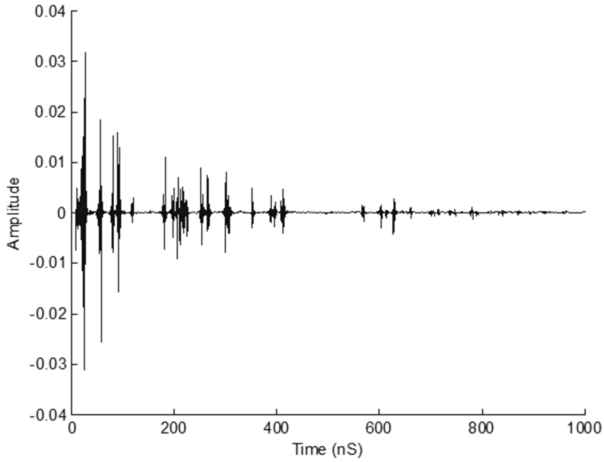

(a) The reconstruction result



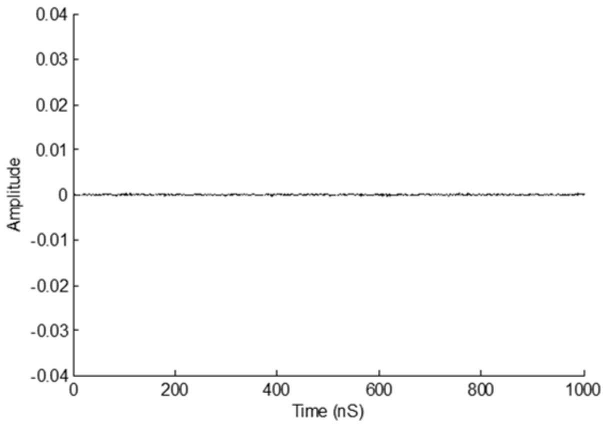

(b) The reconstruction error

**Figure 6.** The reconstruction performance of the random measurement method

As Figure 6 and Figure 7 shown, both the method proposed in this chapter and the random measurement method can successfully implement channel estimation for UWB communication. Moreover, both the methods sample at a reduced sampling rate, which is only $M/N$=1/3 of the sampling directly rate. Since a better measurement matrix is used in this chapter, better performance of the estimation accuracy can be achieved. As depicted in the second picture of Figure 7, reconstruction error approximate to zero or negligible values. In the CS theory, more measurements can improve the advancement of the estimation precision at the expense of increasing the sampling rate of the A/D conversion. This work can be advanced by designing better over-complete dictionary or better measurements matrix, which is an ongoing research.




(a) The reconstruction result




(b) The reconstruction error

**Figure 7.** The reconstruction performance of the FIR filtering matrix method

Simulation 3: UWB signals have been detected via correlation detection method and the reconstructed referent template has been acquired via three estimation approaches: the random measurement method, FIR filtering matrix method and direct-sampling method. Further, the parameters in (22) are set to $N_p$=25 and $N_s$=10 When 1000 information bit is transmitted through the UWB channel, the effect of AWGN is taken into consideration.

Figure 8 illustrates the BER performance on the assumption that the pilot symbols are transmitted in the absence of noise, whereas Figure 9 illustrates the BER performance when the pilot symbols are affected by the AWGN.

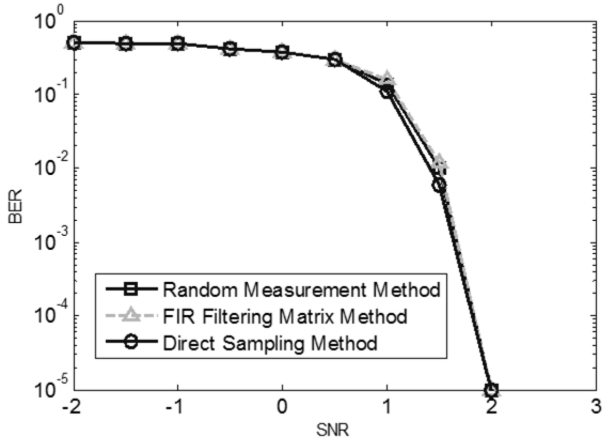



**Figure 8.** The BER performances of three estimation methods with noiseless UWB signals

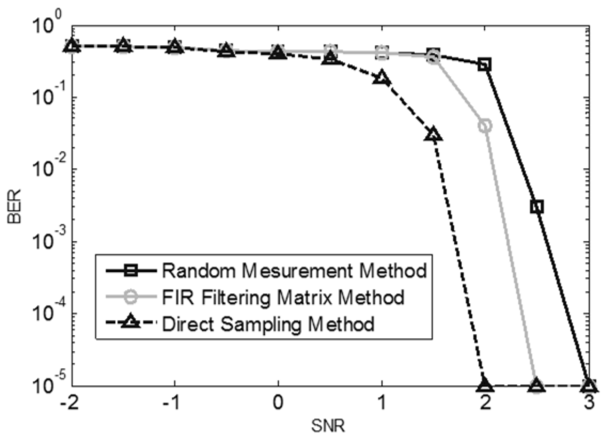



**Figure 9.** The BER performances of three estimation methods with noiseless UWB signals

In addition, the results of simulation 2 illustrates that FIR filtering matrix method has a better performance in channel estimation than random measurement method under the same simulation conditions. As Figure 8 indicated, when the pilot symbols go through the noiseless channel, the results via direct-sampling estimation are the optimized-template signals, however, it require an A/D converter with much higher sampling rate. Moreover, the BER curves shown above illustrate that both the random measurement method and the method proposed in this chapter can estimate the template signals precisely. When they are compared with the direct-sampling estimation, all the BER curves are close to each other terribly. It is obvious that, the accuracy of the channel estimation has little impact on the BER of the correlation detector. So far as referent template can be reconstructed successfully through the reconstruction method based on the CS theory, the BER curves approach each other.

As Figure 9 demonstrated, when taking into account of the effect of AWGN, the FIR filtering matrix method based on CS has an obvious advantage of the BER performance. While that of the random measurement estimation is the worst one comparing with the others. This performance is expected since the noise has been magnified through measurement matrix. That is, the unsuccessful result of the reconstruction algorithm at low signal-to-noise ratio (SNR) is inevitable. Note that the BER of the correlation detector increases horribly as the referent template has not been estimated efficiently.

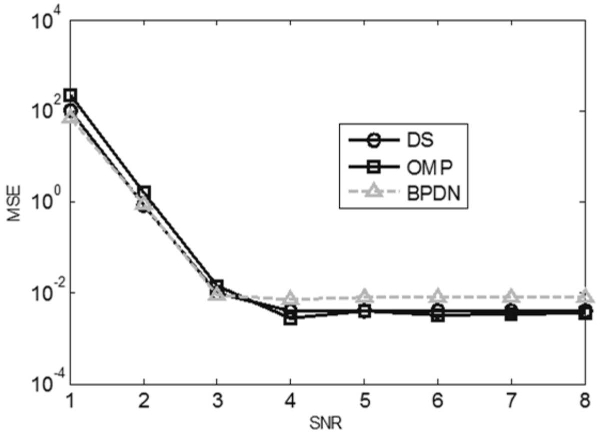



**Figure 10.** The comparison of the mean square error for the three reconstruction algorithms

Simulation 4: The performance test applies the OMP, the DS and the BPDN to reconstruct the original signals. The main parameters for these methods are set as follows. The maximum number of OMP iterations is set to 100 and the target residual energy is set to 0.3% of the energy for the projected signal, i.e. $\sigma= 3\times10^{-3}$. The target residual energy $\sigma$ is also used in the BPDN and the DS method. For the BPDN, the relaxation parameter is supposed to be 0.05, i.e. $\varepsilon= 0.05$. For the DS, the tolerance for primal-dual algorithm is $\xi=10^{-3}$ and the max value of primal-dual iterations is set to 50. Thus, we use the mean squared error (MSE) as the performance criterion, so the tests results are achieved by 50 operations for average.

Figure 10 shows the MSE performance of the CS-based channel estimation for the three reconstruction methods. We observe that: (1) the MSE performance of the DS is slightly better than that of the BPDN. (2) In the higher operating SNR, the OMP has strong competitive advantages to the other two methods, however, turn into worse under lower operating SNR. This simulation shows that the FIR filtering matrix method is indeed leading to the improved performance for the CS reconstruction.

## 6. Conclusion

In this chapter, we proposed a pre-filtering method for UWB channel estimation based on the theory of CS, whose measurement matrix is just a Toeplitz matrix, and the channel estimation accuracy is improved. The method proposed in this paper avoided the magnification to the noise. Thus when the reconstructed signal is used as a referent template at the receiver in the noise realization, a better BER performance can be achieved.

The correlation detector for UWB communication discussed in this paper employs the channel estimates to the conventional correlation detection directly, while the design of the whole system combining the channel estimation and signals detection will be a further research. Moreover, it is the key point of improving the BER performance of the correlation detector to search for a CS reconstruction method, which can successfully recover the referent template under the noise realization and fewer measurements with overwhelming probability. In addition, we analyze the choices of reconstruction algorithms using several simulations. Both the OMP and the BPDN algorithms are compared to the Dantzig selector for different signal noise ratio to give the opinions for choosing suitable reconstruction algorithms.

Admittedly, there are several other theoretical and practical aspects of UWB channel estimation methods based on compressed sensing that need discussing in future. Below, however, we briefly comment on some of these aspects. First, the different types of measurement matrix according to the UWB channels should be in further study. In this paper, we do some attempts to construct the quasi-Toeplitz matrix developing the model of UWB channel estimator. Somewhat similar theoretical arguments can be made to argue the other type of measurement matrix to get better estimation performance. Second, extensive numerical simulations carried out in literatures for a number of CS estimators have established that the performance of CS estimation methods is markedly superior to that of traditional methods based on LS criterion. However, the nontraditional methods based on MUSIC and ESPRIT algorithms are not optimal for estimating sparse channels. This is because it is possible for a channel to have a small number of resolvable paths but still have a very large number of underlying physical paths, especially in the case of diffuse scattering. So the two algorithms can be employed combining with the compressed sensing framework. Third, one expects the representation of real-world multipath channels in certain bases to be only effectively sparse. The channel model and channel parameters are localized with the perfect channel model in this paper. Finally, and perhaps most importantly for the success of the envisioned wireless systems, the CS can be leveraged to design efficient overcomplete dictionary for estimating sparse UWB channels.

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