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Optimal Production Decision in the Closed-Loop Supply Chain Considering Risk-Management and Incentives for Recycling

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1. Introduction

Production companies in the real world face many decision making situations, such as logistics, scheduling, and data mining in the supply chain. The optimal production decision is also major and important when deciding the optimal rate of product allocation, taking into consideration customers' demands, resource costs, and many conditions surrounding the real market. Many researchers have considered optimal production decision models with the total cost-minimization or the total profit-miaximization arising from the production processes of firms, and recently many mathematical models have been proposed (for example Letmathe & Balakrishnan, 2005; Li & Tirupati, 1997; Morgan & Daniels, 2001; Mula et al., 2006a).

In production processes, there are many uncertain factors in terms of total volume and customer's demands, such as the occurrence probability of machine breakdown and human error based on historical data, and the ambiguity that derives from the quality of information received and decision makers' intuition. Some recent articles have elaborated on studies of production planning problems including such ambiguous situations (for instance, Mula et al., 2006b, 2007; Vasant, 2000). We (Hasuike & Ishii, 2009a, 2009b) considered optimal production decision models, under both randomness for the return of each product and fuzziness for coefficients of constraints in production processes.

Particularly, in a forward supply chain, the customer is typically the end of the process. However, a closed loop supply chain includes the returns processes and the manufacturer has the intent of capturing additional value and further integrating all supply chain activities. Therefore, closed-loop supply chains include traditional forward supply-chain activities and the additional activities of the reverse supply chain such as product acquisition and reverse logistics. Then, in terms of reverse logistics and closed loop supply chain management, it is important to improve the sustainability of production companies' business considering the reintegration of their returned products into their own production network. The increase of the reintegration rate can be achieved at multiple levels also called recovery paths: reuse, repair, remanufacturing, recycling, etc. (in detail, Lebreton, 2007). Therefore, an incentive, which is the leverage for reintegrating valuable products, is a key factor in the closed roop supply chain management. If the buyback incentive is set higher than the market price, the production company will pay more than it. If the incentive is too low, customers will prefer to discard old products or resell on the secondary market rather than to return to the production company. Thus, it is most important how high incentives the production company provides to customers.

On the other hand, Corporate Social Responsibility (CSR) is currently the most important measure to sustain continuous developments of companies by performing environment-friendliness and suitable social activity, and it is also essential for avoiding the latent risk. Most recently, the CSR is not only a prominent research theme but also it can also be found in corporate missions and value statements (Svendsen et al., 2001). Companies increasingly realize that their actions in purchasing and supply chain management strongly affect their reputation and long-term success (Castka & Balzatova, 2008). Recently, many famous companies have faced damaging media reports, external pressure from activists, and internal pressure from investors. Consequently, companies start expanding their responsibility for their products, and managing the CSR of their partner within the supply chain (Bloemhof-Ruwaard et al., 1995; Emmelhainz & Adams, 1999; Kolk & Tudder, 2002).

CSR has been considered as a theme of many researchers since Carroll's study (1979). Wartick and Coghran (1985) traced the evolution of the corporate social performance model by focusing on three concepts of CSR: economic responsibility, public responsibility, and social responsiveness. They examined the management of social issues as a dimension of corporate social performance and concluded that the corporate social performance model is valuable for business and society.

CSR issues surrounding supply chains have only recently come to the fore, in the context of conceptual and survey studies (for instance, Carter & Jennings, 2002, 2004). Murphy and Poist (2002) stated that although supply chain practitioners have been slow to adopt CSR considerations, social responsibility concepts in the supply chain are increasing in importance. Carter and Jennings (2002, 2004) empirically established primary supply chain CSR categories of environment, diversity, human rights, philanthropy, and safety. Some researchers have examined individual elements of CSR in the supply chain. In response to growing CSR concerns, researchers have begun to deal with environmental risks, labor practices, procurement, and affirmative action purchasing. Moreover, some researchers have consider various types of practical models with CSR such as forest industry (Panwer et al., 2010), hospitality management (Kang et al., 2010), fashion industry (Perry & Tower, 2009), etc..

Nevertheless, decision support models to integrate CSR into supply chain management are surely needed. Within recent business and social environments, trade-offs between various objectives while providing resources to CSR activities are becoming increasingly complex. Most recently, Cruz and Wakolbinger (2008) and Cruz (2009) have considered various impacts and effects of CSR in supply chain management, and proposed the multi-criteria decision making approach. However, this research field of supply chain management with CSR is now developing, and there are few studies, particularly under uncertainty.

In this paper, we develop a multi-criteria model in the supply chain framework that captures the economic and CSR activities of manufacturing, retailer, and demand market under uncertainty. Particularly, we consider the evaluation of environmental conservation activities and resource recycling as CSR activities, and propose a new optimal production policy as well as the maximization of total profit and effects of CSR activities considering the minimization of latent risk using the advanced risk measure Conditional Value at Risk (CVaR) proposed by Rockafellar and Uryasev (2002).

CSR are generally presented as linguistic and ambiguous information including several types of subjectivities. Therefore, it is hard to consider the value of CSR as a fixed value. In this paper, in terms of linguistic property and subjectivity, we formulate effect of CSR activities as fuzzy numbers, and so our proposed model is formulated as an uncertainty programming problem. By performing deterministic equivalent transformations and considering the application in practice, the analytical and effective solution algorithm is developed.

This paper is organized as follows. In Section 2, we introduce and formulate the proposed optimal production model considering the maximization of total profit, recycling rate, and cost for CSR. In Section 3, we construct the analytical solution algorithm using the equivalent transformations. Finally, in Section 4, we conclude this paper and discuss future research problems.

2. Mathematical formulation and notation of parameters

In this section, we introduce the notation used in our proposed models integrating optimal production decision and CSR activities considering the closed-loop supply chain. Furthermore, as a risk measure, we introduce Conditional Value-at-Risk proposed by Rockafellar and Uryasev (2002). CVaR is known as a useful risk measure which is coherent, consistent with the second (or higher) order stochastic dominance, and the consistency with the stochastic dominance implies that minimizing the CVaR never conflicts with maximizing the expectation of any risk-averse utility function.

Through the whole paper, to simplify, we consider the following cases:

1. All products are produced in one manufacture and these products are delivered to p retailers.
2. We assume that the leadtime of production is larger than that of consumption, and so the production company must predict the demand in the next period and the amount of production items previously.
3. The shortage of each product at the retailers is forbidden as much as possible.

2.1 Notation of parameters

Sets

$i \in \{1,2,...,p\}$: index of retailers

$j \in \{1,2,...,n\}$: index of products

$t \in \{1,2,...,T\}$: index of production periods

$k \in \{1,2,...,m\}$: index of resource constraints

Demand and cost parameters

$d_{ij}^{(t)}$: demand volume for item *j* at destination *i* in period *t*

$r_j^{(t)}$: profit for item *j* in period *t*

$c_j^{(t)}$: production cost for item *j* in period *t*

$a_{kj}^{(t)}$: necessary resource volume for item *j* at *k*th resource constraint in period *t*

h_j : inventory holding cost per unit of item *j* at the manufacture, which is constant value to each period *t*

$u_{ij}^{(t)}$: transportation cost for item *j* from the manufacture to destination *i* in period *t*

v_j : recycling cost for item *j*, which is constant value to each period *t*

$R_C(t)$: expenses for CSR activities in period *t*

$b_k^{(t)}$: maximum volume of *k*th resource in period *t*

W : maximum total level of inventory

C : maximum value of total fund cost.

$g_j\left(p_j^{(t)}\right)$: recycling volume of item *j* in period *t* characterized by a continuous function on promotion cost $p_j^{(t)}$ for recycling

Decision variables

$x_j^{(t)}$: production volume of item j at the manufacture in period t

$y_{ij}^{(t)}$: transportation volume for item j from the manufacture to destination i in period t

$p_j^{(t)}$: promotion cost for recycling item j in period t

$w_j^{(t)}$: inventory level at end of period t for item j at the manufacture.

With respect to the relation between demand $d_{ij}^{(t)}$ and expenses of CSR activities $C_{CSR}^{(t)}$, consumers are generally well-affected in the case that production companies actively perform CSR activities. Then, their purchasing powers are certainly increasing but ambiguity due to including consumer's subjectivity. Therefore, we assume that this relation as the following linear relation:

$$d_{ij}^{(t)} = D_{ij}^{(t)} + \tilde{\alpha} C_{CSR}^{(t)} \tag{1}$$

where $D_{ij}^{(t)}$ is the random demands independent of CSR activities and $\tilde{\alpha}$ is assumed to be a fuzzy number characterized by the L-shape membership function $\mu_{\tilde{\alpha}} = (\bar{\alpha}, \delta)_L$ where $\bar{\alpha}$ is the center value and δ is the spread. Therefore, demand $d_{ij}^{(t)}$ is presented as a hybrid variable with both randomness and fuzziness.

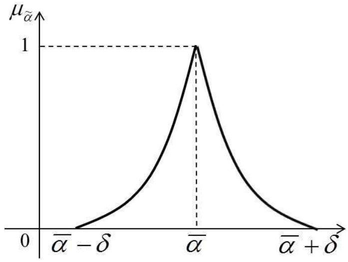

Fig. 1. L-fuzzy number $\tilde{\alpha}$

2.2 Identification of cost function

2.2.1 Identification of cost function

The objects of our proposed model are to maximize the total cost and recycling rate derived from customers' demands, the recycling volume, and the expenses of CSR activities. The total profit function is associated with total profit, transportation costs, production costs, and holding costs. Each profit or cost function is formulated as follows in the multi-item integrated model using the above-listed parameters:

a. Profit: $\sum_{t=1}^{T}\sum_{i=1}^{p}\sum_{j=1}^{n} r_j^{(t)} d_{ij}^{(t)}$

b. Production cost: $\sum_{t=1}^{T}\sum_{j=1}^{n} c_j^{(t)} x_j^{(t)}$

c. Transportation cost: $\sum_{t=1}^{T}\sum_{i=1}^{p}\sum_{j=1}^{n} u_{ij}^{(t)} y_{ij}^{(t)}$

d. Holding cost: $\sum_{t=1}^{T}\sum_{j=1}^{n} h_j w_j^{(t)}$

e. Recycling cost: $\sum_{t=1}^{T}\sum_{j=1}^{n} v_j g_j^{(t)}\left(p_j^{(t)}\right)$

f. Expenses of CSR activities: $\sum_{t=1}^{T} C_{CSR}^{(t)}$

Therefore, the total profit function $f\left(\boldsymbol{x},\boldsymbol{y},\boldsymbol{w},\boldsymbol{p},\boldsymbol{D}\right)$ is obtained as the following form:

$$f\left(\boldsymbol{x},\boldsymbol{y},\boldsymbol{w},\boldsymbol{p},\boldsymbol{D}\right)=\sum_{t=1}^{T}\sum_{i=1}^{p}\sum_{j=1}^{n} r_j^{(t)} d_{ij}^{(t)}-\sum_{t=1}^{T}\sum_{j=1}^{n} c_j^{(t)} x_j^{(t)}-\sum_{t=1}^{T}\sum_{i=1}^{p}\sum_{j=1}^{n} u_{ij}^{(t)} y_{ij}^{(t)}$$
$$-\sum_{t=1}^{T}\sum_{j=1}^{n} h_j w_j^{(t)}-\sum_{t=1}^{T}\sum_{j=1}^{n} v_j g_j^{(t)}\left(p_j^{(t)}\right)-\sum_{t=1}^{T}\sum_{j=1}^{n} p_j^{(t)}-\sum_{t=1}^{T} C_{CSR}^{(t)}$$

Furthermore, each constraint with respect to inventory, shortage, recycling volume, and general production process is defined as follows:

i. Inventory constraint:

$$x_j^{(t)} + w_j^{(t-1)} - \sum_{i=1}^{p}\left(y_{ij}^{(t)} - d_{ij}^{(t)}\right) = w_j^{(t)},\ \sum_{j=1}^{n} w_j^{(t)} \le W,\ \left(j = 1,2,\dots,n,\ t = 1,2,\dots,T\right)$$

ii. Shortage constraint:

$$y_{ij}^{(t)} \ge d_{ij}^{(t)},\ \left(i = 1,2,\dots,p,\ j = 1,2,\dots,n,\ t = 1,2,\dots,T\right)$$

iii. Recycling volume constraint:

$$g_j^{(t)}\left(p_j^{(t)}\right) \le \sum_{l=1}^{t}\sum_{i=1}^{p} d_{ij}^{(l)} - \sum_{l=1}^{t-1} g_j^{(l)}\left(p_j^{(l)}\right),\ \left(j = 1,2,\dots,n,\ t = 1,2,\dots,T\right)$$

iv. General production process constraint:

$$\sum_{j=1}^{n} a_{ij}^{(t)} x_j^{(t)} \le b_k^{(t)} + \sum_{j=1}^{n} a_{ij}^{(t)} g_j^{(t-1)}\left(p_j^{(t-1)}\right),\ \left(k = 1,2,\dots,m,\ t = 1,2,\dots,T\right)$$

v. Limited total fund cost constraint:

$$\sum_{t=1}^{T}\sum_{i=1}^{p}\sum_{j=1}^{n} r_j^{(t)} d_{ij}^{(t)} - f\left(\boldsymbol{x},\boldsymbol{y},\boldsymbol{w},\boldsymbol{p},\boldsymbol{D}\right) \le C$$

Consequently, the proposed integrated production decision and CSR model considering the closed-loop formulated as the following problem:

$$\begin{aligned}
&\text{Maximize } f\left(\boldsymbol{x},\boldsymbol{y},\boldsymbol{w},\boldsymbol{p},\boldsymbol{D}\right)\\
&\text{Maximize } R\left(\boldsymbol{p},\boldsymbol{D}\right) = \frac{\sum_{t=1}^{T}\sum_{j=1}^{n} g_j^{(t)}\left(p_j^{(t)}\right)}{\sum_{t=1}^{T}\sum_{j=1}^{n} d_j^{(t)}}\\
&\text{subject to } y_{ij}^{(t)} \ge d_{ij}^{(t)},\ \left(i = 1,2,\dots,p,\ j = 1,2,\dots,n,\ t = 1,2,\dots,T\right)\\
&\qquad x_j^{(t)} + w_j^{(t-1)} - \sum_{i=1}^{p}\left(y_{ij}^{(t)} - d_{ij}^{(t)}\right) = w_j^{(t)},\ \sum_{j=1}^{n} w_j^{(t)} \le W,\ \left(j = 1,2,\dots,n,\ t = 1,2,\dots,T\right)\\
&\qquad \sum_{j=1}^{n} a_{ij}^{(t)} x_j^{(t)} \le b_k^{(t)} + \sum_{j=1}^{n} a_{ij}^{(t)} g_j^{(t-1)}\left(p_j^{(t-1)}\right),\ \left(k = 1,2,\dots,m,\ t = 1,2,\dots,T\right)\\
&\qquad g_j^{(t)}\left(p_j^{(t)}\right) \le \sum_{l=1}^{t}\sum_{i=1}^{p} d_{ij}^{(l)} - \sum_{l=1}^{t-1} g_j^{(l)}\left(p_j^{(l)}\right),\ \left(j = 1,2,\dots,n,\ t = 1,2,\dots,T\right)\\
&\qquad \sum_{t=1}^{T}\sum_{i=1}^{p}\sum_{j=1}^{n} r_j^{(t)} d_{ij}^{(t)} - f\left(\boldsymbol{x},\boldsymbol{y},\boldsymbol{w},\boldsymbol{p},\boldsymbol{D}\right) \le C\\
&\qquad x_j^{(t)} \ge 0,\ y_{ij}^{(t)} \ge 0,\ w_j^{(t)} \ge 0,\ p_j^{(t)} \ge 0,\ \left(i = 1,2,\dots,p,\ j = 1,2,\dots,n,\ t = 1,2,\dots,T\right)
\end{aligned} \tag{2}$$

where we assume that initial inventories $w_j^{(0)}$ and recycling volume $g_j^{(0)}$ are given as constant values. Then, we set the following feasible solutions set as Φ.

$$\boldsymbol{x},\boldsymbol{y},\boldsymbol{w},\boldsymbol{p}\in\Phi=\left\{\boldsymbol{x},\boldsymbol{y},\boldsymbol{w},\boldsymbol{p}\left|\begin{array}{l}
y_{ij}^{(t)}\ge d_{ij}^{(t)},\ \left(i=1,2,\dots,p,\ j=1,2,\dots,n,\ t=1,2,\dots,T\right)\\
x_j^{(t)}+w_j^{(t-1)}-\sum_{i=1}^{p}\left(y_{ij}^{(t)}-d_{ij}^{(t)}\right)=w_j^{(t)},\ \sum_{j=1}^{n}w_j^{(t)}\le W,\ \left(j=1,2,\dots,n,\ t=1,2,\dots,T\right)\\
\sum_{j=1}^{n}a_{ij}^{(t)}x_j^{(t)}\le b_k^{(t)}+\sum_{j=1}^{n}a_{ij}^{(t)}g_j^{(t-1)}\left(p_j^{(t-1)}\right),\ \left(k=1,2,\dots,m,\ t=1,2,\dots,T\right)\\
g_j^{(t)}\left(p_j^{(t)}\right)\le\sum_{l=1}^{t}\sum_{i=1}^{p}d_{ij}^{(l)}-\sum_{l=1}^{t-1}g_j^{(l)}\left(p_j^{(l)}\right),\ \left(j=1,2,\dots,n,\ t=1,2,\dots,T\right)\\
\sum_{t=1}^{T}\sum_{i=1}^{p}\sum_{j=1}^{n}r_j^{(t)}d_{ij}^{(t)}-f\left(\boldsymbol{x},\boldsymbol{y},\boldsymbol{w},\boldsymbol{p},\boldsymbol{D}\right)\le C\\
x_j^{(t)}\ge 0,\ y_{ij}^{(t)}\ge 0,\ w_j^{(t)}\ge 0,\ p_j^{(t)}\ge 0,\ \left(i=1,2,\dots,p,\ j=1,2,\dots,n,\ t=1,2,\dots,T\right)\\
d_{ij}^{(t)}=D_{ij}^{(t)}+\tilde{\alpha}C_{CSR}^{(t)}
\end{array}\right.\right\}\tag{3}$$

Substituting demand relation (1) and performing the equivalent transformation, the feasible solutions set Φ is equivalently transformed into the following set:

$$\boldsymbol{x},\boldsymbol{y},\boldsymbol{w},\boldsymbol{p}\in\Phi=\left\{\boldsymbol{x},\boldsymbol{y},\boldsymbol{w},\boldsymbol{p}\left|\begin{array}{l}
y_{ij}^{(t)}\ge D_{ij}^{(t)}+\tilde{\alpha}C_{CSR}^{(t)},\ \left(i=1,2,\dots,p,\ j=1,2,\dots,n,\ t=1,2,\dots,T\right)\\
\sum_{j=1}^{n}a_{ij}^{(t)}x_j^{(t)}\le b_k^{(t)}+\sum_{j=1}^{n}a_{ij}^{(t)}g_j^{(t-1)}\left(p_j^{(t-1)}\right),\ \left(k=1,2,\dots,m,\ t=1,2,\dots,T\right)\\
\sum_{l=1}^{t}g_j^{(l)}\left(p_j^{(l)}\right)\le\sum_{l=1}^{t}\left(\sum_{i=1}^{p}y_{ij}^{(l)}-x_j^{(l)}+w_j^{(l)}-w_j^{(l-1)}\right),\ \left(j=1,2,\dots,n,\ t=1,2,\dots,T\right)\\
\sum_{j=1}^{n}w_j^{(t)}\le W,\ \left(j=1,2,\dots,n,\ t=1,2,\dots,T\right)\\
\sum_{t=1}^{T}\left(\sum_{j=1}^{n}\left(c_j^{(t)}x_j^{(t)}+\sum_{i=1}^{p}u_{ij}^{(t)}y_{ij}^{(t)}+h_jw_j^{(t)}+v_jg_j^{(t)}\left(p_j^{(t)}\right)+p_j^{(t)}\right)+C_{CSR}^{(t)}\right)\le C,\\
x_j^{(t)}\ge 0,\ y_{ij}^{(t)}\ge 0,\ w_j^{(t)}\ge 0,\ p_j^{(t)}\ge 0,\ \left(i=1,2,\dots,p,\ j=1,2,\dots,n,\ t=1,2,\dots,T\right)
\end{array}\right.\right\}\tag{4}$$

Then, the main problem (2) is also equivalently transformed into the following problem:

$$\begin{array}{ll}
\text{Minimize} & \sum_{t=1}^{T}\left(\sum_{j=1}^{n}\left(c_j^{(t)}x_j^{(t)}+\sum_{i=1}^{p}u_{ij}^{(t)}y_{ij}^{(t)}+h_jw_j^{(t)}+v_jg_j^{(t)}\left(p_j^{(t)}\right)-p_j^{(t)}\right)\right.\\
& \left.\quad+\left(1-\tilde{\alpha}\sum_{i=1}^{p}\sum_{j=1}^{n}r_j^{(t)}\right)C_{CSR}^{(t)}-\sum_{i=1}^{p}\sum_{j=1}^{n}r_j^{(t)}D_{ij}^{(t)}\right)\\
\text{Maximize} & \dfrac{\sum_{t=1}^{T}\sum_{j=1}^{n}g_j^{(t)}\left(p_j^{(t)}\right)}{\sum_{t=1}^{T}\sum_{j=1}^{n}D_{ij}^{(t)}+\tilde{\alpha}C_{CSR}^{(t)}}\\
\text{subject to} & \boldsymbol{x},\boldsymbol{y},\boldsymbol{w},\boldsymbol{p}\in\Phi
\end{array}\tag{5}$$

This problem is equivalent to a multi-criteria linear programming problem if each demand $d_{ij}^{(t)}$ is fixed or the expected value $E\left(d_{ij}^{(t)}\right)$ of random variable. However, in reality, the wild swing of demands associated with drastic changes of social and economic conditions often happens. In this case, it is obviously impossible that each demand is generally not fixed, and it is rough to degenerate the random distribution into the expected value by neglecting important factors considering fluctuation ranges such as the variance. Therefore, we must consider more advanced risk management approaches to avoid futile supply chain disruptions, particularly, focusing on the downside risk to decrease uncertainty up to the high-cost as much as possible.

3. Analytical and efficient solution algorithm based on CVaR

First, we introduce the standard CVaR in the stochastic programming problem. Standard CVaR for randomness was proposed by Rockafellar and Uryasev (2002) as follows:

$$\begin{aligned} & CVaR_{\beta}(\chi) = E\left[L(\chi,\xi) \middle| L(\chi,\xi) \geq VaR_{\beta}(\chi)\right] = \frac{1}{1-\beta}\int_{L(\chi,y)\geq VaR_{\beta}(y)} L(\chi,y)f(y)dy \\ & \left(VaR_{\beta}(y) = \inf\left\{r \middle| \Pr\left\{L(\chi,\xi) \leq r\right\} \geq \beta\right\}\right) \end{aligned} \tag{6}$$

where $L(\chi,\xi)$ is the loss function with fixed value χ and random variable ξ, and β is the confidence level. Then, $f(y)$ is the density function of random variable ξ. $VaR_{\beta}(\chi)$ is Value at Risk (VaR) and the standard risk measure in economic and financial fields and used in many practical risk management. Furthermore, we introduce the following function:

$$F_{\beta}(\chi,u) = r + \frac{1}{1-\beta}E\left[\left[L(\chi,\xi)-r\right]^{+}\right] \tag{7}$$

Rockafellar and Uryasev (2002) proved that the CVaR (10) can be minimized by minimizing the auxiliary function (7). Since the loss function $L(\chi,\xi)$ is convex due to the linearity for fixed demand ξ, we can obtain the optimal order quantity of our proposed model by solving the following CVaR minimization problem:

$$\begin{aligned} & \underset{\chi,r}{\text{minimize}} \;\; r + \frac{1}{1-\beta}E\left[\left[L(\chi,\xi)-r\right]^{+}\right] \\ \Leftrightarrow \;\; & \underset{\chi,r}{\text{minimize}} \;\; r + \frac{1}{1-\beta}\sum_{s=1}^{S} p_s\left[L(\chi,\xi_s)-r\right]^{+} \end{aligned} \tag{8}$$

where probability ξ is assumed to be a finite discrete random variable like assumptions in this paper, i.e.,

$$\begin{aligned} & \Pr\{\boldsymbol{\xi} = \boldsymbol{\xi}_s\} = p_s,\; \sum_{s=1}^{S} p_s = 1 \\ & \boldsymbol{\xi}_s = \left\{\xi_{sij}^{(t)},\; (i=1,\dots,p,\; j=1,\dots,n,\; t=1,\dots,T)\right\},\; (s=1,\dots,S) \end{aligned}$$

we can perform the transformation from second problem to third problem in (6). Furthermore, introducing parameters ω_k, $(k=1,2,\dots,K)$, this optimization problem is equivalently transformed into the following problem:

$$\begin{aligned}&\underset{\chi,r}{\text{minimize}} \quad r+\frac{1}{1-\beta}\sum_{s=1}^{S}p_s\omega_s\\&\text{subject to} \quad \omega_s \ge L(\chi,\xi_s)-r,\ \omega_s\ge 0,\ (s=1,2,\dots,S)\end{aligned} \tag{9}$$

Using CVaR model (9) for randomness and setting loss functions for both objective functions as $L_1(\chi,\xi)=-f(\boldsymbol{x},\boldsymbol{y},\boldsymbol{w},\boldsymbol{p},\boldsymbol{D})$ and $L_2(\chi,\xi)=-R(\boldsymbol{p},\boldsymbol{D})$, we reformulate our proposed closed-loop supply chain model under hybrid CVaR as follows:

$$\begin{aligned}&\underset{\boldsymbol{x},\boldsymbol{y},\boldsymbol{w},\boldsymbol{p},\gamma_1}{\text{minimize}} \quad \gamma_1+\frac{1}{1-\beta_1}\sum_{s=1}^{S}p_s\left[-f(\boldsymbol{x},\boldsymbol{y},\boldsymbol{w},\boldsymbol{p},\boldsymbol{D}_s)-\gamma_1\right]^+\\&\underset{\boldsymbol{p},\gamma_2}{\text{minimize}} \quad \gamma_2+\frac{1}{1-\beta_2}\sum_{s=1}^{S}p_s\left[-R(\boldsymbol{p},\boldsymbol{D}_s)-\gamma_2\right]^+\\&\text{subject to } \boldsymbol{x},\boldsymbol{y},\boldsymbol{w},\boldsymbol{p}\in\Phi_d\end{aligned} \tag{10}$$

where Φ_d is re-defined as the following feasible solutions set using the demand scenario of deterministic values $\boldsymbol{D}_s=\left\{D_{sij}^{(t)},(i=1,\dots,p,\ j=1,\dots,n,t=1,\dots,T)\right\},(s=1,\dots,S)$:

$$\boldsymbol{x},\boldsymbol{y},\boldsymbol{w},\boldsymbol{p}\in\Phi_d=\left\{\boldsymbol{x},\boldsymbol{y},\boldsymbol{w},\boldsymbol{p}\left|\begin{array}{l}y_{ij}^{(t)}\ge D_{sij}^{(t)}+\tilde{\alpha}C_{CSR}^{(t)},\ (i=1,2,\dots,p,\ j=1,2,\dots,n,\ t=1,2,\dots,T,s=1,2,\dots,S)\\ \sum_{j=1}^{n}a_{ij}^{(t)}x_j^{(t)}\le b_k^{(t)}+\sum_{j=1}^{n}a_{ij}^{(t)}g_j^{(t-1)}\left(p_j^{(t-1)}\right),\ (k=1,2,\dots,m,\ t=1,2,\dots,T)\\ \sum_{l=1}^{t}g_j^{(l)}\left(p_j^{(l)}\right)\le\sum_{l=1}^{t}\left(\sum_{i=1}^{p}y_{ij}^{(l)}-x_j^{(l)}+w_j^{(l)}-w_j^{(l-1)}\right),\ (j=1,2,\dots,n,\ t=1,2,\dots,T)\\ \sum_{j=1}^{n}w_j^{(t)}\le W,\ (j=1,2,\dots,n,\ t=1,2,\dots,T)\\ \sum_{t=1}^{T}\left(\sum_{j=1}^{n}\left(c_j^{(t)}x_j^{(t)}+\sum_{i=1}^{p}u_{ij}^{(t)}y_{ij}^{(t)}+h_jw_j^{(t)}+v_jg_j^{(t)}\left(p_j^{(t)}\right)+p_j^{(t)}\right)+C_{CSR}^{(t)}\right)\le C,\\ x_j^{(t)}\ge 0,\ y_{ij}^{(t)}\ge 0,\ w_j^{(t)}\ge 0,\ p_j^{(t)}\ge 0,\ \ (i=1,2,\dots,p,\ j=1,2,\dots,n,\ t=1,2,\dots,T)\end{array}\right.\right\} \tag{11}$$

Furthermore, introducing parameters η_s,ξ_s, problem (10) is equivalently transformed into the following problem:

$$\begin{aligned}&\underset{\boldsymbol{x},\boldsymbol{y},\boldsymbol{w},\boldsymbol{p},\gamma_1}{\text{minimize}} \quad \gamma_1+\frac{1}{1-\beta_1}\sum_{s=1}^{S}p_s\eta_s\\&\underset{\boldsymbol{p},\gamma_2}{\text{minimize}} \quad \gamma_2+\frac{1}{1-\beta_2}\sum_{s=1}^{S}p_s\xi_s\\&\text{subject to } \eta_s\ge\sum_{t=1}^{T}\left(\sum_{j=1}^{n}\left(c_j^{(t)}x_j^{(t)}+\sum_{i=1}^{p}u_{ij}^{(t)}y_{ij}^{(t)}+h_jw_j^{(t)}+v_jg_j^{(t)}\left(p_j^{(t)}\right)-p_j^{(t)}\right)+\left(1-\tilde{\alpha}\sum_{i=1}^{p}\sum_{j=1}^{n}r_j^{(t)}\right)C_{CSR}^{(t)}-\sum_{i=1}^{p}\sum_{j=1}^{n}r_j^{(t)}D_{ij}^{(t)}\right)-\gamma_1,\\&\eta_s\ge 0,\\&\xi_s\ge-\frac{\sum_{t=1}^{T}\sum_{j=1}^{n}g_j^{(t)}\left(p_j^{(t)}\right)}{\sum_{t=1}^{T}\sum_{j=1}^{n}D_{sij}^{(t)}+\tilde{\alpha}C_{CSR}^{(t)}}-\gamma_2,\ \xi_s\ge 0,\\&\boldsymbol{x},\boldsymbol{y},\boldsymbol{w},\boldsymbol{p}\in\Phi_d\end{aligned} \tag{12}$$

This problem includes only linear constraints, but it is hard to find an unique optimal decision for problem (12) analytically due to fuzzy number $\tilde{\alpha}$ and multi-criteria. Therefore, in order to solve this problem analytically, we introduce fuzzy goals as the fuzzy programming approach.

Let $Z_{CVaR}^{1} = \left(\gamma_1 + \frac{1}{1-\beta_1}\sum_{s=1}^{S} p_s \eta_s\right)$, $Z_{CVaR}^{2} = \left(\gamma_2 + \frac{1}{1-\beta_2}\sum_{s=1}^{S} p_s \xi_s\right)$. We consider the necessity measure for fuzzy constraints including $\tilde{\alpha}$, and the following linear fuzzy goals $\mu_{G_1}(\omega)$ and $\mu_{G_2}(\omega)$ for multi-criteria Z_{CVaR}^{1} and Z_{CVaR}^{2}, respectivly:

$$\mu_{G_i}(\omega) = \frac{Z_{CVaR}^{i,\max} - \omega}{Z_{CVaR}^{i,\max} - Z_{CVaR}^{i,\min}},\ i = 1,2 \tag{13}$$

where all max and min values are constant determined by the decision maker. Using these fuzzy goals and necessity measure $\inf \max\left\{1 - \mu_{\tilde{\alpha}}(\omega), \mu_{G_i}(\omega)\right\}$, problem (12) is transformed into the following model maximizing satisfaction level h:

$$\begin{aligned}
&\text{maximize } h \\
&\text{subject to } \gamma_1 + \frac{1}{1-\beta_1}\sum_{s=1}^{S} p_s \eta_s \le (1-h) Z_{CVaR}^{1,\max} + h Z_{CVaR}^{1,\min}, \\
&\gamma_2 + \frac{1}{1-\beta_2}\sum_{s=1}^{S} p_s \xi_s \le (1-h) Z_{CVaR}^{2,\max} + h Z_{CVaR}^{2,\min}, \\
&\eta_s \ge \sum_{t=1}^{T}\left(\sum_{j=1}^{n}\left(c_j^{(t)} x_j^{(t)} + \sum_{i=1}^{p} u_{ij}^{(t)} y_{ij}^{(t)} + h_j w_j^{(t)} + v_j g_j^{(t)}\left(p_j^{(t)}\right) - p_j^{(t)}\right)\right. \\
&\left. + \left(1 - \left(\bar{\alpha} - L^*(1-h)\right)\sum_{i=1}^{p}\sum_{j=1}^{n} r_j^{(t)}\right) C_{CSR}^{(t)} - \sum_{i=1}^{p}\sum_{j=1}^{n} r_j^{(t)} D_{ij}^{(t)}\right) - \gamma_1, \\
&\eta_s \ge 0, \\
&\xi_s \ge -\frac{\sum_{t=1}^{T}\sum_{j=1}^{n} g_j^{(t)}\left(p_j^{(t)}\right)}{\sum_{t=1}^{T}\sum_{j=1}^{n} D_{sij}^{(t)} + \left(\left(\bar{\alpha} - L^*(1-h)\right)\right) C_{CSR}^{(t)}} - \gamma_2,\ \xi_s \ge 0, \\
&\qquad \boldsymbol{x}, \boldsymbol{y}, \boldsymbol{w}, \boldsymbol{p} \in \Phi_d
\end{aligned} \tag{14}$$

where $L^*(h)$ is the inverse function of menbership function $L(\omega)$. This problem is equivalent to a standard linear programming problem if parameter h is fixed. Therefore, it is easy to solve this problem with fixed value $\bar{h}$ analytically and efficiently using linear programming approaches such as the Simplex method and Interior point method. Furthermore, using the bisection algorithm on h, we obtain the strict optimal production volume of the main problem (10). Consequently, we have proposed the versatile model for the closed-loop supply chain management considering the CSR activities, and developed the analytical solution algorithm based on the standard linear programming approaches.

4. Numerical example

In order to compare our proposed closed-loop supply chain model considering CSR with the previous standard models, we introduce the following simple numerical example. We assume that two periods (t=2), three products (n=3), and three retailers (p=3). To simplify, these products consists of one resource (k=1). Tables 1 and 2 show data of parameters with respect to returns, production cost, expenses for CSR, and demands in all retailers in each period which are assumed to be uniform distributions. Transportation, holding, and recycling costs are assumed to be same in two periods. Furthermore, we assume that $\tilde{\alpha}$ is a triangle fuzzy number characterized by $\mu_{\tilde{\alpha}} = (0.05, 0.02)_L$.

	Product A	Product B	Product C
$r_j^{(1)}$	20	40	20
$c_j^{(1)}$	3	10	5
$D_{1j}^{(1)}$	[25,35]	[50,60]	[37,43]
$D_{2j}^{(1)}$	[24,26]	[54,56]	[39,41]
$D_{3j}^{(1)}$	[32,38]	[40,60]	[39,41]
$r_j^{(2)}$	40	20	20
$c_j^{(2)}$	10	3	5
$D_{1j}^{(2)}$	[49,51]	[25,35]	[35,45]
$D_{2j}^{(2)}$	[50,70]	[19,21]	[35,45]
$D_{3j}^{(2)}$	[49,51]	[19,21]	[30,50]

Table 1. Data of parameters in each period

	Product A	Product B	Product C
u_j	0.5	1	0.5
h_j	4	1	3
v_j	10	8	8
a_j	1.5	2	4

Table 2. Data of parameters for common costs

We set b=2000, W=50, C=10000, and the following fuzzy goals:

$$\mu_{G_1}(\omega)=\frac{12000-\omega}{12000-10000},\mu_{G_2}(\omega)=\frac{-0.5-\omega}{-0.5-(-0.7)}$$

Then, we assume function of recycling volume is a linear function, i.e., $g_j^{(t)}\left(p_j^{(t)}\right)=2p_j^{(t)}$, and solve the proposed model (14) and the standard model without CSR object, and obtain the following optimal solution.

	Product A		Product B		Product C	
Period	1	2	1	2	1	2
With CSR	149	124.5	176	79.5	125	142.5
Not CSR	149	122	176	77	125	140

Table 3. Optimal solution of production volume

From this result, we find that the whole production volume tends to be larger than the standard model due to consideration of CSR. Furthermore, volumes of all products in period 2 of proposed model are much larger than standard model. Results of CSR activities in our proposed model $C_{CSR}^{(1)}$ and $C_{CSR}^{(2)}$ are 0 and 16.4, respectively. This means that the CSR activity for promotion and recycling is mainly performed in the second period.

Furthermore, we consider some cases of $g_j^{(t)}\left(p_j^{(t)}\right)$; linear function $g_j^{(t)}\left(p_j^{(t)}\right)=2p_j^{(t)}$, convex functions $g_j^{(t)}\left(p_j^{(t)}\right)=\left(p_j^{(t)}\right)^2,\exp\left(p_j^{(t)}\right)$, and concave function $g_j^{(t)}\left(p_j^{(t)}\right)=\sqrt{p_j^{(t)}}$, respectively. Then, we solve the proposed model (14) considering CSR for each function, and obtain the following optimal production volume and the optimal profit as Table 4.

$g_j^{(t)}\left(p_j^{(t)}\right)$	Product A		Product B		Product C		Profit
	1	2	1	2	1	2	
$2p_j^{(t)}$	149	124.5	176	79.5	125	142.5	11606
$\left(p_j^{(t)}\right)^2$	149.4	122.1	176.4	77.1	125.4	140.1	12496
$\exp\left(p_j^{(t)}\right)$	149.1	122.3	176.1	77.3	125.1	140.3	12493
$\sqrt{p_j^{(t)}}$	154.5	129.8	181.5	84.8	130.5	147.8	9106

Table 4. Optimal solution of production volume to each function $g_j^{(t)}\left(p_j^{(t)}\right)$

From Table 4., in the case of concave function, whole production volume tends to be larger than the linear and convex-based proposed models. On the contrary, the total profit in the case of concave function is much smaller than linear and convex functions. This means that the cost derived from the CSR activity is not used as promotion activity to increase the customers' demands, but used as the recycling activity to collect old-products.

5. Conclusion

In this paper, we have considered the optimal production decision model with CSR and recycling of old-products in the closed-loop supply chain. We have integrated the evaluation of environmental conservation activities and resource recycling as CSR activities into the closed-loop supply chain management, and proposed the new optimal production policy based on CSR. In mathematical programming, the proposed model including CSR activities have been formulated as stochastic and fuzzy programming problem due to linguistic information and subjectivities as well as random demands, and we have transformed the initial proposed model into the deterministic equivalent problem using CVaR as the advanced risk measure and fuzzy goals for the multi-criteria model. Consequently, we have developed the analytical and effective solution algorithm based on linear programming. Furthermore, in order to represent useful features and advantages of the proposed model, we have provided a numerical example, and obtained that the whole production volume for the CSR-based model was larger than the standard model and that the cost derived from the CSR activity is used as promotion activity or the recycling activity according to the continuous function for the recycling volume.

As the future works, we will apply this proposed model to practical systems of actual production companies, and evaluate the performance or advantage of the proposed model compared with the previous standard and useful models using real numerical and linguistic data.

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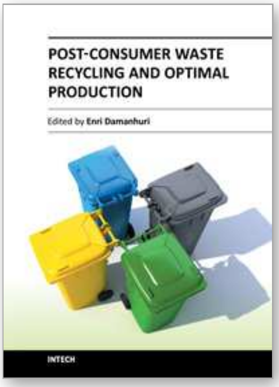

Post-Consumer Waste Recycling and Optimal Production

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This book deals with several aspects of waste material recycling. It is divided into three sections. The first section explains the roles of stakeholders, both informal and formal sectors, in post-consumer waste activities. It also discusses waste collection programs for recycling. The second section discusses the analysis tools for recycling system. The third section focuses on the recycling process and optimal production. I hope that this book will convey both the need and means for recycling and resource conservation activities to a wide readership, at both academician and professional level, and contribute to the creation of a sound material-cycle society.