

# Multi-Criteria Optimal Path Planning of Flexible Robots

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## 1. Introduction

Determining the trajectory from the initial to the final end-effector positioning represents one of the most common problems in the path-planning design of serial robot manipulators. The movement is established through the specification of a set of intermediate points. In this way, the manipulator is guided along the trajectory without any concern regarding the intermediate configurations along the path. However, there are applications in which the intermediate points have to be taken into account both for path-planning and control purposes. An example of such an application is the case of robot manipulators that are used in welding operations.

In the context of industrial applications, a previous planning is justified, the so-called *off-line programming*, aiming at establishing a precise control for the movement. This planning includes the analysis of the kinematics and dynamics behavior of the system. The reduction of costs and increase of productivity are some of the most important objectives in industrial automation. Therefore, to make possible the use of robotic systems, it is important that one considers the path planning optimization for a specific task.

The improvement of industrial productivity can be achieved by reducing the weight of the robots and/or increasing their speed of operation. The first choice may lead to power consumption reduction while the second results in a faster work cycle. To successfully achieve these purposes it is very desirable to build flexible robotic manipulators. In some situations it is even necessary to consider the flexibility effects due to the joints and gear components of the manipulators for obtaining an accurate and reliable control.

Compared to conventional heavy robots, flexible link manipulators have the potential advantage of lower cost, larger work volume, higher operational speed, greater payload-to-manipulator-weight ratio, smaller actuators and lower energy consumption.

The study of the control of flexible manipulators started in the field of space robots research. Aiming at space applications, the manipulator should be as light as possible in order to reduce the launching costs (Book, 1984). Uchiyama *et al.* (1990), Alberts *et al.* (1992), Dubowsky (1994), to mention only a few, have also studied flexible manipulators for space

applications. Shi *et al.* (1998) discussed some key issues in the dynamic control of lightweight robots for several applications.

As a consequence of the interest in using flexible structures in robotics, several papers regarding the design of controllers for the manipulation task of flexible manipulators are found in the literature (Latornell *et al.*, 1998), (Choi and Krishnamurthy, 1994) and (Chang and Chen, 1998).

In Tsujita *et al.* (2004), the trajectory and force controller of a flexible manipulator is proposed. From the point of view of structural dynamics, the trajectory control for a flexible manipulator is dedicated to the control of the global elastic deformation of the system, and the force control is dedicated to the control of the local deformation at the tip of the end-effector. Thus, preferably trajectory and force controls are separated in the control strategy.

Static and dynamic hybrid position/force control algorithms have been developed for flexible-macro/rigid-micro manipulator systems (Yoshikawa *et al.*, 1996). The robust cooperative control scheme of two flexible manipulators in the horizontal workspace is presented in Matsuno and Hatayama (1999). A passive controller has been developed for the payload manipulation with two planar flexible arms (Damaren 2000).

In Miyabe *et al.* (2004), the automated object capture with a two-arm flexible manipulator is addressed, which is a basic technology for a number of services in space. This object capturing strategy includes symmetric cooperative control, visual servoing, the resolution of the inverse kinematics problem, and the optimization of the configuration of a two-arm redundant flexible manipulator.

The effective use of flexible robotic manipulators in industrial environment is still a challenge for modern engineering. Usually there are several possible trajectories to perform a given task. A question that arises when programming robots is which is the best trajectory. There is no definitive solution, since the answer to this question depends on the selected performance index. Focused on industrial applications, the optimal path planning of a flexible manipulator is addressed in the present chapter. The manipulator is requested to perform a task in a vertical plane. Under this condition the gravitational effects are taken into account. Energy consumption is minimized when the movement is conducted through a suitable path. Energy is calculated by means of the evaluation of the joint torque along the path. End-effector accuracy is improved by reducing the vibration effect and increasing manipulability. The determination of the position takes into account the influence of structural flexibility. Weighting parameters are used to set the importance of each objective. The optimization scenario is composed by an optimal control formulation, solved by means of a nonlinear programming algorithm. The improvement obtained through a global optimization procedure is discussed. Numerical results demonstrate the viability of the proposed methodology.

A control formulation to determine the optimal torque profile is proposed. The optimal manipulability is also taken into account. The effect of using end-effector positioning error as performance index is discussed. As a result, the contribution of the present work is the proposition of a methodology to evaluate the influence of different performance indexes in a multi-criteria optimization environment.

The paper is organized as follows. In Section 2, model of deflection, torque and manipulability are presented. Section 3 recalls the general optimal control formulation and the performance indexes are defined. Geometrical insight about the design variables is given. Multi-criteria programming aspects such as Pareto-optimality and objective weighting are presented in section 4. The global optimization strategy is outlined in section

5 while section 6 shows numerical results. The conclusions and perspectives for future work are given in section 7.

## 2. Manipulator model

### 2.1 Deflection

Different schemes for modeling of the manipulators have been studied by a number of researchers. The mathematical model of the manipulator is generally derived from energy principles and, for a simple rigid manipulator, the rigid arm stores kinetic energy due to its moving inertia, and stores potential energy due its position in the gravitational field.

A flexible link also stores deformation energy by virtue of its deflection, joint and drive flexibility. Joints have concentrated compliance that may often be modeled as a pure spring storing only strain energy. Drive components such as shafts and belts may appear distributed. They store kinetic energy due to their low inertia, and a lumped parameter spring model often succeeds well to consider such an effect.

The most important modeling techniques for single flexible link manipulators can be grouped under the following categories: assumed modes method, finite element method and lumped parameters technique.

In the assumed modes approach, the link flexibility is usually represented by a truncated finite modal series, in terms of spatial mode eigenfunctions and time-varying mode amplitudes. Although this method has been widely used, there are several ways to choose link boundary conditions and mode eigenfunctions. Some contributions in this field were presented by Cannon and Schmitz (1984), Sakawa *et al.* (1985), Bayo (1986), Tomei and Tornambe (1988), among others. Nagaraj *et al.* (2001), Martins *et al.* (2002) and Tso *et al.* (2003) studied single-link flexible manipulators by using Lagrange's equation and the assumed modes method.

Regarding the finite element formulation, Nagarajan and Turcic (1990) derived elemental and system equations for systems with both elastic and rigid links. Bricout *et al.* (1990) studied elastic manipulators. Moulin and Bayo (1991) also used finite element discretization to study the end-point trajectory tracking for flexible arms and showed that a non-causal solution for the actuating torque enables tracking of an arbitrary tip displacement with any desired accuracy.

By using a lumped parameter model, Zhu *et al.* (1999) simulated the tip position tracking of a single-link flexible manipulator. Khalil and Gautier (2000) used a lumped elasticity model for flexible mechanical systems. Megahed and Hamza (2004) used a variation of the finite segment multi-body dynamics approach to model and simulate planar flexible link manipulator with rigid tip connections to revolute joints.

Santos *et al.* (2007) proposed the computation of flexibility by means of a spring-mass-damper system. According to this analogy, the first spring and damper constants are related to the joint behavior, and the following sets of spring and damper represent link flexibility. The variables and the parameters of the model are interpreted as angular quantities.

In this work the description of the deflection related to a rigid link is proposed. It is achieved by means of an Euler-Bernoulli beam formulation and covers the case for small deflections of a beam subject to lateral loads.

The bending moment  $M$ , shear forces  $Q$  and deflections  $w$  for a cantilever beam subjected to a point load  $P$  at the free end are given by

$$M(x) = P(x - L) \quad (1)$$

$$Q(x) = P \quad (2)$$

$$w(x) = \frac{Px^2(3L - x)}{6EI} \quad (3)$$

where  $L$  is the length of the link,  $E$  is the Young's modulus of the material and  $I$  is the moment of inertia. The variable  $x$  is the distance between the base of the link and the point where the force is applied. In this work, the error of positioning is measured at the end of each link, which yields

$$w(L) = \frac{PL^3}{3EI} \quad (4)$$

The linear displacement at the end of each link is then converted into angular displacement through the expression

$$\Delta\theta = \arcsin\left(\frac{w}{L}\right) \quad (5)$$

Considering that the kinematics position of each link is given by a rigid body transformation  $T_{i-1}^i(\theta)$  the positioning error at the end-effector can be estimated through

$$\delta = \sum_{j=1}^2 T_j(\theta) - T_j(\theta + \Delta\theta) \quad (6)$$

If there is no load at the end of each link, Eq. (4), it follows that  $P=0$ ,  $\Delta\theta = 0$  and  $\delta = 0$ , i.e., the link behaves as a rigid body.

## 2.2 Robot dynamics

Dynamics encompasses relations between kinematics and statics so that the motion of the system is taken into account. To move a robot through a given trajectory, motors provide force or torque to the robotic joints. There are several techniques to model the dynamics of industrial robots. The knowledge about the dynamic behavior of the system is important both to the computational simulation of the movement and to the design of the controller itself (Fu *et al.* 1987).

Characteristics of a two-link planar manipulator follow. The Denavit-Hartenberg parameters are presented in Table 1.

Link	$a$ (m)	$\alpha$ (rad)	$d$ (m)	$\theta$ (rad)
1	$a_1$	0	0	$\theta_1^*$
2	$a_2$	0	0	$\theta_2^*$

Table 1. Denavit-Hartenberg parameters, (\*) joint variable.

The Lagrangian  $L = K - P$  is defined by the difference between the kinetic energy  $K$  and the potential energy  $P$  of the system. The dynamics of the system can be described by the Lagrange's equations as given by:

$$\tau_j = \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\theta}_j} \right) - \frac{\partial L}{\partial \theta_j} \quad (7)$$

where  $\theta_j$  are the generalized coordinates (angle of rotational joints in the present case);  $\dot{\theta}_j$  are the generalized velocities (angular velocity in the present case) and  $\tau_j$  are the generalized forces (torques in the present case). By developing the Euler-Lagrange formalism, the equations that represent the system dynamics are explicitly obtained (Fu *et al.* 1987), and are expressed through

$$Q(q)\ddot{q} + C(q, \dot{q}) + G(q) = \tau \quad (8)$$

where  $q = (\theta_1, \theta_2, \dots, \theta_n)^T$ ,  $\dot{q} = (\dot{\theta}_1, \dot{\theta}_2, \dots, \dot{\theta}_n)^T$  and  $\ddot{q} = (\ddot{\theta}_1, \ddot{\theta}_2, \dots, \ddot{\theta}_n)^T$  are joint vectors of dimension  $n \times 1$  representing position, velocity and acceleration, respectively.  $Q(q)$  is a symmetric  $n \times n$  matrix representing inertia,  $C(q, \dot{q})$  is a  $n \times 1$  vector representing the Coriolis effect,  $G(q)$  is a  $n \times 1$  vector representing the effects of gravitational acceleration, and  $F = (F_1, F_2, \dots, F_n)^T$  is a  $n \times 1$  vector of generalized forces applied at the joints.

The  $Q$ ,  $C$  and  $G$  matrices that represent the dynamics of a two-link planar manipulator are described by the Eqs. (9) to (16)

$$Q_{11} = \left(\frac{1}{3}\right) m_1 a_1^2 + \left(\frac{1}{3}\right) m_2 a_2^2 + m_2 a_1^2 + m_2 a_1 a_2 \cos(\theta_2) \quad (9)$$

$$Q_{12} = \left(\frac{1}{3}\right) m_2 a_2^2 + 0.5 m_2 a_1 a_2 \cos(\theta_2) \quad (10)$$

$$Q_{21} = \left(\frac{1}{3}\right) m_2 a_2^2 + 0.5 m_2 a_1 a_2 \cos(\theta_2) \quad (11)$$

$$Q_{22} = \left(\frac{1}{3}\right) m_2 a_2^2 \quad (12)$$

$$C_{11} = -m_2 a_1 a_2 \sin(\theta_2) \dot{\theta}_1 \dot{\theta}_2 - 0.5 m_2 a_1 a_2 \sin(\theta_2) \dot{\theta}_2^2 \quad (13)$$

$$C_{21} = 0.5 m_2 a_1 a_2 \sin(\theta_2) \dot{\theta}_2^2 \quad (14)$$

$$G_{11} = -0.5 m_1 g a_1 \cos(\theta_1) - 0.5 m_2 g a_2 \cos(\theta_1 + \theta_2) - m_2 g a_1 \cos(\theta_1) \quad (15)$$

$$G_{21} = -0.5 m_2 g a_2 \cos(\theta_1 + \theta_2) \quad (16)$$

where  $g = 9.81 \text{ m/s}^2$  is the gravitational constant.

The energy required to move the robot is an important design issue because in real applications energy supply is limited and any energy reduction leads to smaller operational costs. This point could lead to eco-robots design, in which energy supply is one of the key aspects. Due to the close relationship that exists between energy and force, the minimal energy can be estimated from the generalized force  $\tau_j(t)$  that is associated to each joint  $j$  at time instant  $t$ .

Furthermore, it was observed that the resulting trajectory corresponds to robot configurations that are associated with the minimum mechanical energy and small

variations of the torque along the path. This means that an explicit constraint over the maximum torque is not required since it will be implicitly achieved by the present formulation.

### 2.3 Manipulability

As an approach for evaluating quantitatively the ability of manipulators from the kinematics viewpoint the concepts of manipulability ellipsoid and manipulability measure are used.

The set of all end-effector velocities  $v$  which are realizable by joint velocities such that the Euclidean norm satisfies  $|\dot{\theta}| \leq 1$  are taken into account. This set is an ellipsoid in the  $m$ -dimensional Euclidean space. In the direction of the major axis of the ellipsoid, the end-effector can move at high speed. On the other hand, in the direction of the ellipse minor axis the end-effector can move only at low speed. Also, the larger the ellipsoid the faster the end-effector can move. Since this ellipsoid represents an ability of manipulation it is called as the manipulability ellipsoid.

The principal axes of the manipulability ellipsoid can be found by making use of the singular-value decomposition of the Jacobian matrix  $J(\theta)$ . The singular values of  $J$ , i.e.,  $\sigma_1, \sigma_2, \dots, \sigma_m$ , are the  $m$  larger values taken from the  $n$  roots ( $\sqrt{\lambda_i}, i = 1, \dots, n$ ) of the eigenvalues. Then  $\lambda_i$  are the eigenvalues of the matrix  $J(\theta)^T J(\theta)$ .

One of the representative measures for the ability of manipulation derived from the manipulability ellipsoid is the volume of the ellipsoid. This is given by  $c_m w$ , where

$$w = \sigma_1 \cdot \sigma_2 \dots \sigma_m \quad (17)$$

$$c_m = \begin{cases} (2\pi)^{\frac{m}{2}} / [2.4.6 \dots (m-2).m] & \text{if } m \text{ is even} \\ 2(2\pi)^{\frac{(m-1)}{2}} / [1.3.5 \dots (m-2).m] & \text{if } m \text{ is odd.} \end{cases} \quad (18)$$

Since the coefficient  $c_m$  is a constant value when the dimension  $m$  is fixed, the volume is proportional to  $w$ . Hence,  $w$  can be seen as a representative measure, which is called the manipulability measure, associated to the manipulator joint angle  $\theta$ .

Due to the direct relation between singular configuration and manipulability (through the Jacobian), the larger the manipulability measure the larger the singular configuration avoidance.

The distance from singular points can be obtained from the solution of an optimization problem in which the following objective function is minimized:

$$f(\theta) = \frac{1}{w} \quad (19)$$

Consequently, the minimization of Equation (19) means to increase the manipulability measure.

## 3. Optimal control formulation

### 3.1 Design variables

The design variables are the key parameters that are updated during the workflow, aiming at increasing (or decreasing) the performance index. In this study, the design variables are

reference nodes, presented by circles in Figure 1. The abscissa presents the time instants ( $t_i = 0, t_f = 1$ ) while the ordinate is the joint angle. The value of intermediate joint angles between the references nodes are evaluated by means of a cubic spline interpolation.

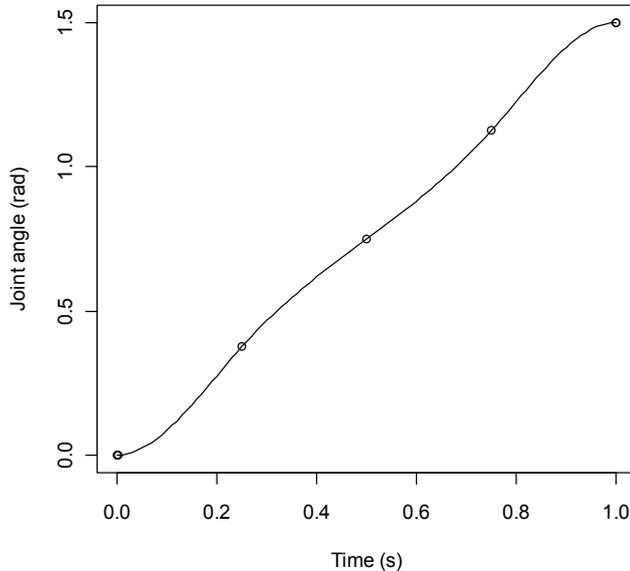


Fig. 1. Reference nodes (o) and joint angle (—).

This approach ensures a smooth transition on the joint angles, velocities and accelerations, which are positive aspects from the mechanical perspective. A smooth movement preserves the mechanism from fatigue effects, for example.

Optimal programming problems for continuous systems are included in the field of the calculus of variations. A continuous-step dynamic system is described by an  $n$ -dimensional state vector  $\mathbf{x}(t)$  at time  $t$ . The choice of an  $m$ -dimensional control vector  $\mathbf{u}(t)$  determines the time rate of change of the state vector through the dynamics given by the equation below:

$$\dot{\mathbf{x}}(t) = f(\mathbf{x}(t), \mathbf{u}(t), t) \quad (20)$$

A general optimization problem for such a system is to find the time history of the control vector  $\mathbf{u}(t)$  for  $t_i \leq t \leq t_f$  to minimize a performance index given by

$$J = \varphi[\mathbf{x}(t_f)] + \int_{t_0}^{t_f} L[\mathbf{x}(t), \mathbf{u}(t), t] dt \quad (21)$$

subject to Equation (20) with  $t_0$ ,  $t_f$  and  $\mathbf{x}(t_0)$  specified.

In the present context, the interest is focused on the joint angles and the reference nodes. Performance indexes such as positioning error, manipulability and mechanical power are derived from this information. The variable  $x_{i,j} = \theta_j(t_i)$  is an element of the state vector  $\mathbf{x}$ , which represents the joint angle at each time  $t_i$ , related to the joint  $j$ . The control vector  $u_{k,j}$

represents the position of the reference node  $k$  with respect to the joint  $j$ . In the present study, the dimension of the state vector is  $n = 202$  (the total traveling time is divided into  $i=1, \dots, 101$  steps for each joint) and the control vector has dimension  $m = 1, \dots, 10$  (five reference nodes for each joint).

### 3.2 Performance indexes

Initially, the specification of a feasible project is modeled as an optimization problem. To achieve this purpose, let  $\mathbf{p}_t = [x_{\text{target}}, y_{\text{target}}]^T$  be the Cartesian target position where the end-effector may intersect when performing a given task. Then, a feasible project is the one whose torque profile is able to conduct each end-effector to a given cartesian position in which the prescribed task will be performed. This profile is obtained by the optimum value of the objective function

$$\varphi[\mathbf{x}(t_f)] = \left[ \mathbf{e}_1(\mathbf{x}(t_f)) - \mathbf{p}_t \right]^2 \quad (22)$$

where  $\mathbf{e}_1(\mathbf{x}(t))$  is the Cartesian position of the robot end-effector, respectively. By using Eq. (22) as the only performance index of the general formulation, Eq. (21), the objective function is

$$J = \left[ \mathbf{e}_1(\mathbf{x}(t_f)) - \mathbf{p}_t \right]^2 \quad (23)$$

The torque required by the actuators to achieve the final position is approximated by a quadratic expression

$$L_1(\mathbf{x}(t), \mathbf{u}(t), t) = \sum_{j=1}^2 \tau_{i,j}^2 \quad (24)$$

The manipulability index is evaluated by the expression

$$L_2(\mathbf{x}(t), \mathbf{u}(t), t) = \sum_{j=1}^2 \left( \frac{1}{w_{i,j}^2} \right) \quad (25)$$

The end-effector positioning error along the path is evaluated by using the equation

$$L_3(\mathbf{x}(t), \mathbf{u}(t), t) = \sum_{j=1}^2 \delta_{i,j}^2 \quad (26)$$

A number of methods are found in the literature to deal with optimal control (OC) problems (Bryson, 1999), (Bertsekas, 1995). In the present contribution, the results are computed through a classical nonlinear programming (NLP) procedure (Betts, 2001). In this case, there is no need of extra parameter computations and the derivatives are numerically evaluated. This choice characterizes a strong point of the proposed methodology since it provides an efficient formulation to solve the optimal control problem addressed.

Since the NLP procedure requires a finite number of points as design variables, the continuity of the OC variables along the time interval is obtained by interpolating the discrete set along the time. Those objectives can be evaluated individually or combined according to the importance of each objective.

#### 4. Multi-criteria programming problem

In multiple-criteria optimization problems one deals with a design variable vector  $\mathbf{u}$ , which satisfies all constraints and makes as small as possible the scalar performance index that is calculated by taking into account the  $m$  components of an objective function vector  $J(\mathbf{u})$ . This goal can be achieved by the vector optimization problem

$$\min_{\mathbf{u} \in \Omega} \{J(\mathbf{u}) | h(\mathbf{u}) = 0, g(\mathbf{u}) \leq 0\} \quad (27)$$

where  $\Omega \subset R^n$  is the domain of the objective function (the design space).

An important feature of such multiple-criteria optimization problems is that the optimizer has to deal with objective conflicts (Eschenauer *et al.*, 1990). Other authors discuss the so-called compromise programming, since there is no unique solution to the problem (Vanderplaats, 1999). In this context, the optimality concept used in this work is presented below.

##### 4.1 Pareto-optimality

A vector  $\mathbf{u}^* \in \Omega$  is Pareto optimal for the problem (27) if and only if there are no vector  $\mathbf{u} \in \Omega$  with the characteristics:

- i.  $J_k(\mathbf{u}) \leq J_k(\mathbf{u}^*)$  for all  $k \in \{1, \dots, m\}$ .
- ii.  $J_k(\mathbf{u}) < J_k(\mathbf{u}^*)$  for at least one  $k \in \{1, \dots, m\}$ .

For all non-Pareto-optimal vectors, the value of at least one objective function  $J_k$  can be reduced without increasing the functional values of the other vector components.

Solutions to multi-criteria optimization problems can be found in different ways by defining the so-called substitute problems. Substitute problems represent different form of obtaining the corresponding scalar objective function (Eschenauer *et al.*, 1990).

##### 4.2 Method of objective weighting

Weighting Objectives is one of the most usual (and simple) substitute models for multi-objective optimization problems (Oliveira and Saramago, 2010). It permits a preference formulation that is independent from the individual minimum for positive weights. The preference function or utility function is here determined by the linear combination of the criteria  $J_1, \dots, J_m$  together with the corresponding weighting factors  $\alpha_1, \dots, \alpha_m$  so that

$$p[J(\mathbf{u})] = \sum_{k=1}^m \alpha_k J_k(\mathbf{u}), \mathbf{u} \in \Omega. \quad (28)$$

It is usually assumed that  $0 \leq \alpha_k \leq 1$  and  $\sum \alpha_k = 1$ . It is possible to generate Pareto-optima for the original problem (Equation 27) by varying the weights  $\alpha_k$  in the preference function.

##### 4.3 The proposed formulation

Multiple objectives usually have different magnitude values. Numerical comparison among them have no real meaning since they have distinct measurement units and one objective may have a much higher value than another, since they represent different physical quantities. In this context, an initial scaling procedure is justified. In the case of two objective functions considered simultaneously, the following steps are adopted:

1. Select initial parameters  $\mathbf{x}_0(t), \mathbf{u}_0(t), t \in [t_0, t_f]$  and  $0 \leq \alpha_1 \leq 1$ .
2. Set  $\alpha_2 = 1 - \alpha_1$ .

3. Set  $\bar{J}_k = J_k(\mathbf{x}_0, \mathbf{u}_0, t)$ .
4. Update  $\alpha_k = \frac{\alpha_k}{\bar{J}_k}, k = 1, 2$ .
5. Set  $p[J(\mathbf{u})] = \alpha_1 J_1 + \alpha_2 J_2$ .

It follows that  $J = 1$  at the beginning of the optimization process. At the end,  $0 < J < 1$ . By using this scaling procedure, the final objective function value provides a non-dimensional index that describes the percentage of improvement. As an example, the final value  $J=0.3$  means that the overall objective was reduced to 30% of this initial value.

## 5. Global optimization

Along the investigation of the optimization problem, there are two kinds of solution points (Luenberger, 1984): *local minimum points*, and *global minimum points*. A point  $\mathbf{u}^* \in \Omega$  is said to be a *global minimum point* of  $f$  over  $\Omega$  if  $f(\mathbf{u}) \geq f(\mathbf{u}^*)$  for all  $\mathbf{u} \in \Omega$ . If  $f(\mathbf{u}) > f(\mathbf{u}^*)$  for all  $\mathbf{u} \in \Omega, \mathbf{u} \neq \mathbf{u}^*$ , then  $\mathbf{u}^*$  is said to be a *strict global minimum point* of  $f$  over  $\Omega$ .

There are several search methods devoted to find the global minimum of a nonlinear objective function, since it is not an easy task. Well known methods such as genetic algorithms (Vose, 1999), differential evolution algorithm (Price *et al.*, 2005) and simulated annealing (Kirkpatrick, 1983) could be used in this case. The main characteristic of these methods is that the global (or near global) optimum is obtained through a high number of functional evaluations.

As proposed by Santos *et al.* (2005), to use the best feature of local optimization method (low computational cost) and global optimization method (global minimum), it is considered using the so-called tunneling strategy (Levy and Gomez, 1985) (Levy and Montalvo, 1985), a methodology designed to find the global minimum of a function. It is composed of a sequence of cycles, each cycle consisting of two phases: a minimization phase having the purpose of lowering the current function value, and a tunneling phase that is devoted to find a new initial point (other than the last minimum found) for the next minimization phase. This algorithm was first introduced in Levy and Gomez (1985), and the name derives from its graphic interpretation.

The first phase of the tunneling algorithm (minimization phase) is focused on finding a local minimum  $\mathbf{u}^*$  of Equation (21), while the second phase (tunneling phase) generates a new initial point  $\mathbf{u}^0 \neq \mathbf{u}^*$  where  $f(\mathbf{u}^0) \leq f(\mathbf{u}^*)$ . In summary, the computation evolves through the following phases:

- a. Minimization phase: Given an initial point  $\mathbf{u}^0$ , the optimization procedure computes a local minimum  $\mathbf{u}^*$  of  $f(\mathbf{u})$ . At the end, it is considered that a local minimum is found.
- b. Tunneling phase: Given  $\mathbf{u}^*$  found above, since it is a local minimum, there exists at least one  $\mathbf{u}^0 \in \Omega$ , so that  $f(\mathbf{u}^0) \leq f(\mathbf{u}^*), \mathbf{u}^0 \neq \mathbf{u}^*$ .

In other words, there exists  $\mathbf{u}^0 \in Z = \{\mathbf{u} \in \Omega - \{\mathbf{u}^*\} \mid f(\mathbf{u}) \leq f(\mathbf{u}^*)\}$ . To move from  $\mathbf{u}^*$  to  $\mathbf{u}^0$  along the tunneling phase a new initial point  $\mathbf{u} = \mathbf{u}^* + \delta, \mathbf{u} \in \Omega$  is defined and used in the auxiliary function

$$F(\mathbf{u}) = \frac{f(\mathbf{u}) - f(\mathbf{u}^*)}{[(\mathbf{u} - \mathbf{u}^*)^T(\mathbf{u} - \mathbf{u}^*)]^\gamma} \quad (29)$$

that has a pole in  $\mathbf{u}^*$  for a sufficient large value of  $\gamma$ . By computing both phases iteratively, the sequence of local minima leads to the global minimum. Different values for  $\gamma$  are

suggested (Levy and Gomez, 1985) (Levy and Montalvo, 1985) for being used in Equation (29) to avoid undesirable points and prevent the search algorithm to fail.

## 6. Numerical results

Computational evaluation was performed aiming at evaluating the effectiveness of the proposed methodology, as outlined in Figure 2.

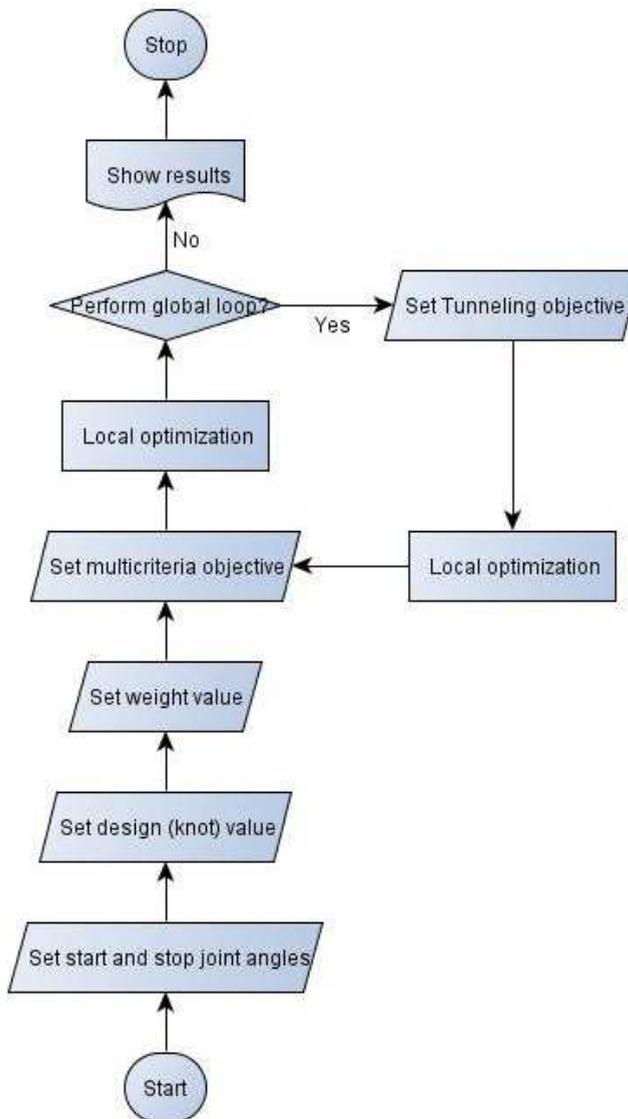


Fig. 2. Computational workflow.

An important point when using multi-objective formulation is the choice of performance indexes. A discussion about the influence of torque, manipulability and end-effector positioning error is presented in the following.

Figure 3 presents the Pareto frontier for the case in which torque and manipulability are considered in the same objective function.

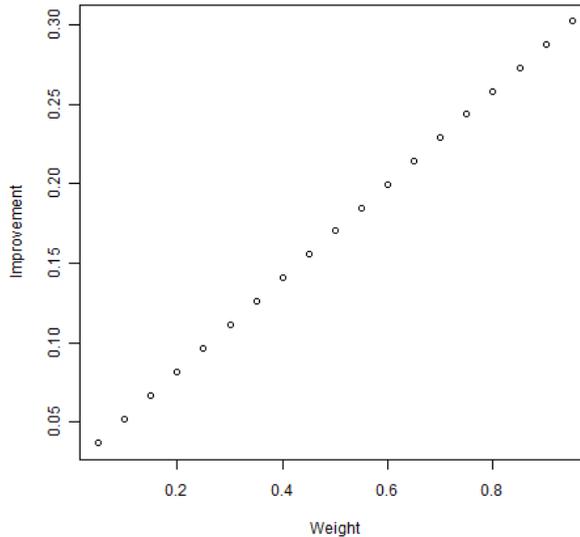


Fig. 3. Minimum value of the performance index.

The abscissa contains several values of the weight factor  $\alpha \in (0,1)$ . If the factor is zero, only manipulability is taken into account. If the factor is one, only the torque is considered. The factor 0.5 sets the same importance to both objectives. The ordinate is the percentage related to the initial value of the objective function. At the beginning, the objective function value is one. The value 0.10 in the improvement scale means that the final value is 10% of the initial value.

Results presented in the figure confirm that the improvement of manipulability is easier to be achieved by the numerical procedure than the reduction of torque. Those are the final results of the global optimization.

Another important point is the contribution of the global optimization strategy. Figure 4 shows the corresponding improvement achieved.

The abscissa contains several values of the weight factor  $\alpha \in (0,1)$ . The ordinate represents a range of values where 1.0 means no improvement, that is, no further improvement was obtained with respect to the local minimum. The value 0.75 means that the new (possibly global) minimum has a value corresponding to 75% of the local minimum.

A general view about the contribution of the tunneling strategy to the local minimum is presented in Figure 5. The reduction varies between 0.7 and 1.0, that is, there was a reduction to 70% of the value of the local minimum in the best scenario and no reduction (the corresponding value is 1.0) in the worst case.

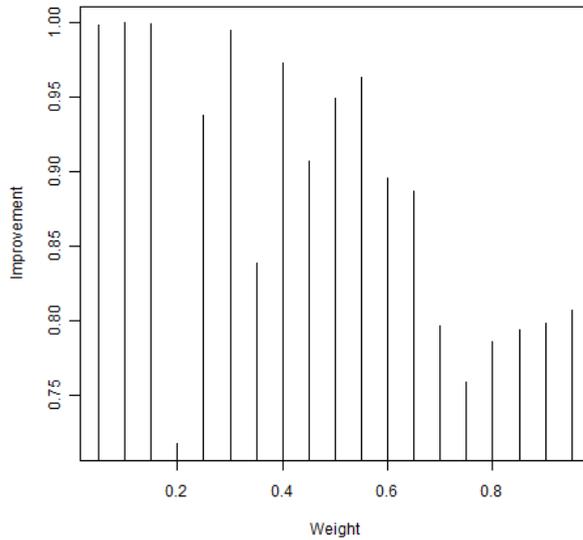


Fig. 4. Contribution of the global strategy.

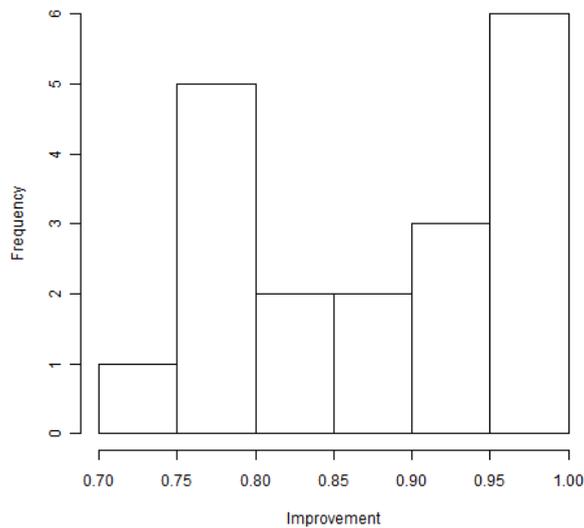


Fig. 5. Range of improvement obtained by the global strategy.

Performance indexes are affected differently by the presence of other objectives and the corresponding priority. The influence of the weight factor on individual objectives is discussed in the following.

The sum of torque is a performance index to be minimized. The higher the importance of the torque in the objective formulation better the improvement achieved by the optimization process. This effect is graphically presented by Figure 6.

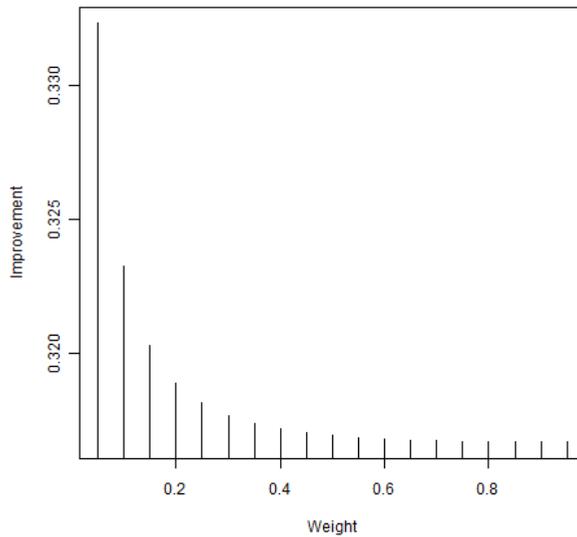


Fig. 6. Influence of weight on the torque.

Manipulability is a performance index to be maximized. It follows that larger the performance index value, better the performance. The influence of the weight coefficient over the manipulability is presented in Figure 7.

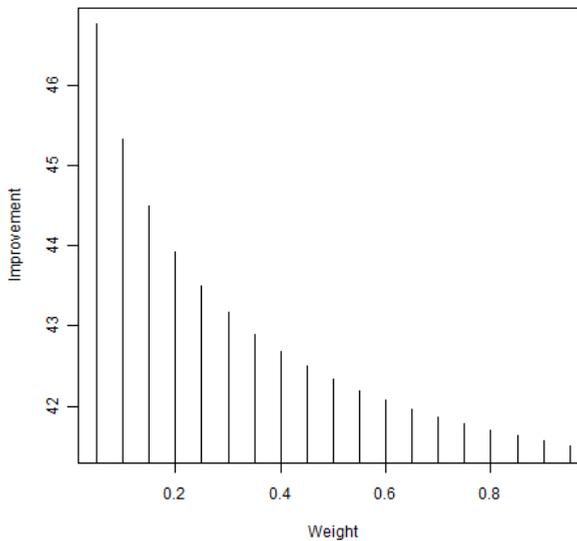


Fig. 7. Influence of weight coefficients on the manipulability.

According to Figure 7, the lower the value of the weight coefficient, better the manipulability. Note that manipulability and torque are conflicting objectives that are addressed by the present formulation.

Results comparing the effects of the end-effector positioning error and torque are presented next. The Pareto frontier for torque and end-effector displacement is shown in Figure 8.

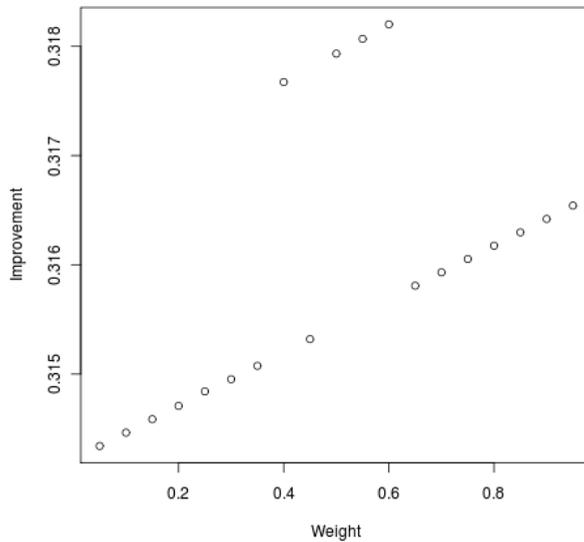


Fig. 8. Minimum value of the performance index.

Despite the small difference in the results, for all cases studied the optimum is between 31% and 32% of the initial value of the objective function. The contribution of the global optimization strategy is also similar for all cases and the global minimum was found between 80.4% and 81.4% of the value of the local minimum.

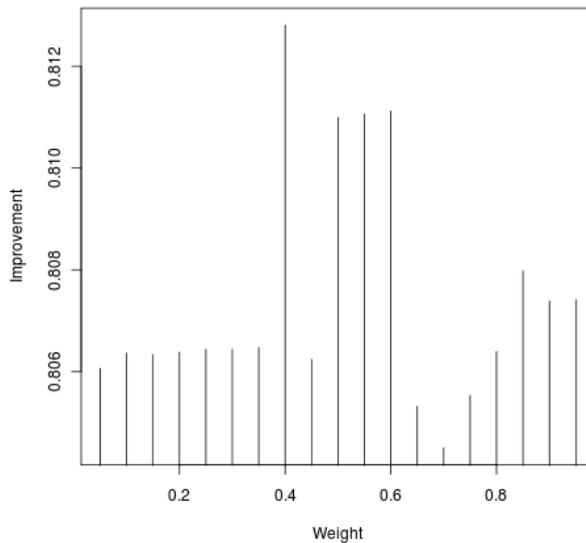


Fig. 9. Contribution of the global strategy.

The average contribution of the tunneling process to the global minimum is shown in Figure 10.

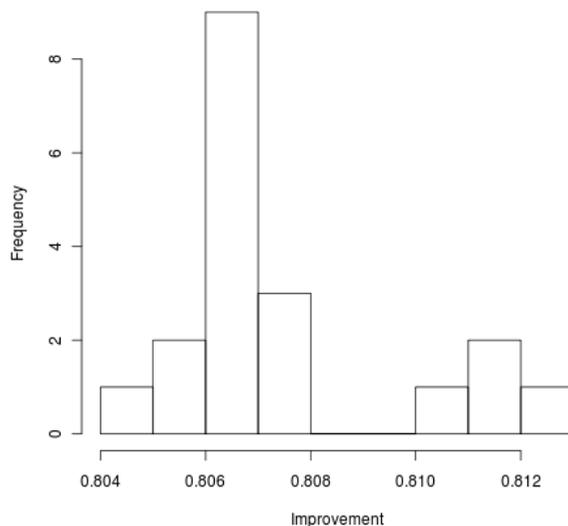


Fig. 10. Range of improvement obtained by the global strategy.

The optimal value of torque index is between 31.6% and 31.9% of the initial value, as presented in Figure 11.

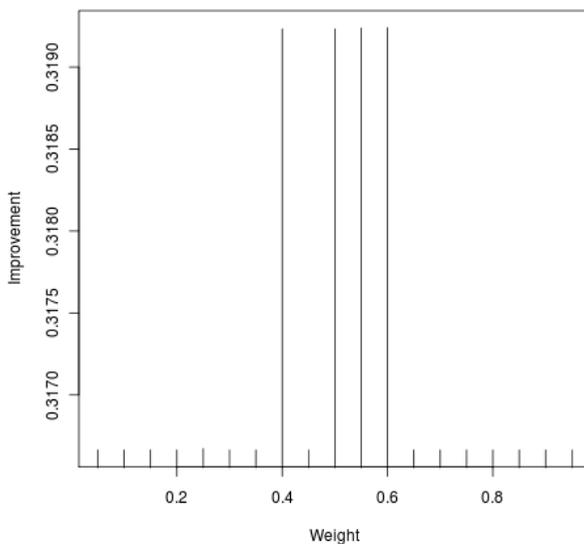


Fig. 11. Influence of weight on the torque.

The end-effector positioning error index was increased about 36 times, as presented in Figure 12.

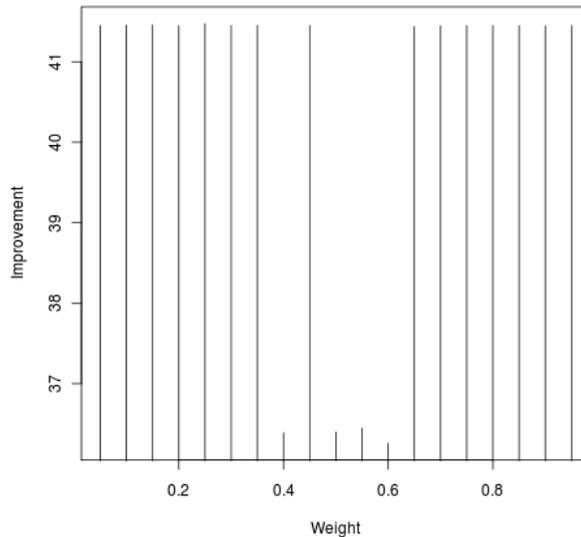


Fig. 12. Influence of weight on the end-effector positioning error.

Results presented in Figures 6 and 7 appear frequently when conflicting objectives are taken into account in the optimization procedure. On the other hand, correlated indexes usually have small deviation in the values obtained, as presented in Figures 11 and 12.

This analysis suggest that torque and manipulability are better choices to compose a multi-criteria analysis as compared with results given by torque and end-effector positioning error. This is justified by the fact that end-effector disturbance is affected by the torque profile. As a result, if the torque is reduced, then the effect of flexibility at the end-effector is also reduced.

## 7. Conclusion

This work was dedicated to multi-criteria optimization problems applied to flexible robot manipulators. Initially, the model of deflection, torque and manipulability were presented for the system analyzed. Next, optimal control formalism and multi-criteria strategy were outlined. It was concluded that the effectiveness of the numerical procedure depends on the choice of the objective functions and weighting factors.

The first numerical evaluation considered torque and manipulability as performance indexes. The effect of changing the weighting coefficients was presented in Figure 3. This is a typical trend of concurrent objectives, i.e., when one performance index is improved, the other degenerates. In this context it is important to identify the contribution of each index, as shown in Figures 6 and 7.

The effect of the global optimization procedure was also discussed. This point was shown to be effective to obtain the global minimum.

The second numerical evaluation considered the end-effector error positioning and the torque as performance indexes. It was shown that there are no significant changes in the performance index while the weighting factor values are changed. It is explained by the correlation between the objectives, i.e., both indexes are directly affected by the joint torque.

As a result, no further improvement is achieved by changing the relative importance of the objectives.

The numerical experiments conducted during the present research work lead to the following conclusions. The interdependence of the performance indexes is an important aspect that impacts the performance of the numerical method. The evaluation of different objectives that evaluates correlated information increases the computational cost but have no contribution to the optimal design. The use of several weighting factors is recommended. The nonlinear nature of the objective indexes requires a heavy exploration of the design space, aiming at finding a significant improvement for a particular combination of weighting factors. Finally, the use of a global optimization methodology is the most important contribution of this work that will distinguish the proposed methodology as a fast and accurate solver. Few iterations of the tunneling process provided an effective evolution to the global minimum.

From the author's perspective, the effective combination of different techniques is the key to obtain a high performance engineering result. Since this study deals with off-line planning, even a small improvement in the path performed by the robot may lead to expressive economic benefits. This is justified by the fact that the same movement of the robot is repeated several times during the working cycle.

The next step of this research is the evaluation of the effect of uncertainty in the design, through a stochastic approach.

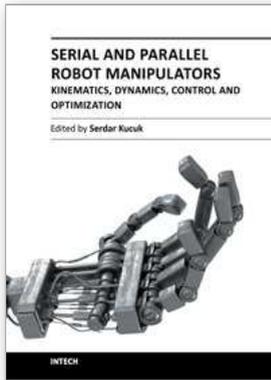
As the proposed methodology is efficient to obtain an improved manipulator trajectory while dealing with the flexibility effect, joint torque and manipulability, the authors believe that the present contribution is a useful tool for robotic path planning design.

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## **Serial and Parallel Robot Manipulators - Kinematics, Dynamics, Control and Optimization**

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The robotics is an important part of modern engineering and is related to a group of branches such as electric & electronics, computer, mathematics and mechanism design. The interest in robotics has been steadily increasing during the last decades. This concern has directly impacted the development of the novel theoretical research areas and products. This new book provides information about fundamental topics of serial and parallel manipulators such as kinematics & dynamics modeling, optimization, control algorithms and design strategies. I would like to thank all authors who have contributed the book chapters with their valuable novel ideas and current developments.

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