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Graviton Emission in the Bulk and Nucleosynthesis in a Model with Extra Dimension

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1. Introduction

Brane-world scenarios with a 3-brane identified with the observable Universe which is embedded in a higher-dimensional space-time provide an alternative to the standard 4D cosmology (Binetruy, Deffayet & Langlois 2000; Binetruy et al., 2000; Collins & Holdom, 2000; Kraus, 1999; Shiromizu; Maeda & Sasaki, 2000), reviews (Durrer, 2005; Maartens, 2004; Rubakov, 2001). A necessary requirement on these models is that they should reproduce the main observational cosmological data, the age of the Universe, abundances of elements produced in primordial nucleosynthesis, etc.

A general property of the models with extra dimensions is that gravity propagates in the extra dimensions independent of whether the ordinary matter is confined to the brane or not. This entails a peculiar property of the models with extra dimensions which is absent in the standard cosmology: gravitons which are produced in reactions of particles of matter on the brane can escape from the brane and propagate in the bulk (Hebecker & March-Russel, 2001; Langlois; Sorbo & Rodriguez-Martinez, 2002; Langlois & Sorbo, 2003; Maartens, 2004; Tanaka & Himemoto, 2003). As a consequence the Einstein equations contain terms accounting for the graviton emission. Cosmological evolution of matter on the brane is also affected by this process.

Roughly the energy loss due to the process $a + b \to G + X$ can be estimated (Gorbunov & Rubakov, 2008) as

$$\frac{d\hat{\rho}}{dt} = - < n_a n_b \sigma_{a+b\to G+X} v E_G >,$$

where in the radiation-dominated period of the evolution of the Universe $n_a, n_b \sim T^3$ and $E_G \sim T$. This yields

$$\frac{d\hat{\rho}}{dt} \sim -\kappa^2 T^8.$$

Here T is temperature of the Universe and $\kappa^2 = 8\pi/M^3$, is the 5D gravitational constant, where M is the 5D Planck mass,

Below we consider the problem of graviton emission to the bulk in a model with one 3-brane embedded in the bulk with one infinite extra dimension (Randall-Sundrum type II model

(Randall & Sundrum, 1999) with matter). In a class of metrics defined below, the 5D model can be treated in two alternative approaches. In the first approach the brane is moving in the static 5D space-time, constructed by attaching two AdS spaces to the brane (Birmingham, 1999; Chamblin & Reall, 1999; Collins & Holdom, 2000; Kraus, 1999). In the second approach the brane is located at a fixed position in the extra dimension, and the metric is time-dependent (Binetruy, Deffayet & Langlois 2000; Binetruy et al., 2000). We review a method to connect both approaches.

The Einstein equations including the terms due to graviton production are solved in the perturbative approach. In the leading order we neglect the graviton production, and include it in the next order. We perform our calculations in a picture in which the metric is time-dependent and the brane is located at a fixed position in the extra dimension. In the leading order solution of the system of 5D Einstein equations is the warped extension of the metric on the brane to the bulk. Restriction of the 5D metric to the brane has the form of the FRW metric $ds^2 = -dt^2 + a^2(t)\eta_{\mu\nu}dx^\mu dx^\nu$ where the scale factor $a(t)$ is determined from the generalized Friedmann equation (Binetruy, Deffayet & Langlois 2000; Binetruy et al., 2000; Collins & Holdom, 2000; Kraus, 1999; Maartens, 2004; Mukohyama, Shiromizu & Maeda). The generalized Friedmann equation is obtained by solving the system of the 5D Einstein equations. In the 5D models the generalized Friedmann equation contains the terms linear and quadratic in energy density on the brane. In the leading order the Friedmann equation does not contain terms due to graviton production which are included in the next order.

Evolution of the energy density on the brane is determined by the Boltzmann equation. The collision term in the Boltzmann equation accounts for the graviton emission from the brane resulting from annihilation of the Standard model particles to gravitons. To calculate the collision term, solving the field equations for fluctuations over the background metric, we find the spectrum of the tower of Kaluza-Klein gravitons.

Explicit form of the bulk energy-momentum tensor is obtained by identifying the collision term in the Boltzmann equation which accounts for the loss of energy density on the brane with the component of the energy-momentum tensor in the 5D conservation equation which also represents the energy flow from the brane to the bulk (Langlois & Sorbo, 2003).

Graviton emission changes cosmological evolution of matter on the brane. Time (temperature) dependence of of the Hubble function determined from the Friedmann equation which includs the components of the bulk graviton energy-momentum tensor is different from that in the standard cosmological model. This, in turn, results in a change of abundances of light elements produced in primordial nucleosynthesis (Steigman; Walker & Zentner, 2001). Perturbatively solving the system of the Friedmann and 5D conservation equations, we find the difference of abundances of ^4He produced in primordial nucleosynthesis calculated in the models with and without graviton production. Calculations are performed in the period of late cosmology, in which in the Friedmann equation the term linear in matter energy density is dominant. We find that the difference of abundances of ^4He calculated in both models is a small number, much smaller than that estimated in (Hebecker & March-Russel, 2001; Langlois; Sorbo & Rodriguez-Martinez, 2002; Langlois & Sorbo, 2003). We make an estimate of production of ^4He in the period of early cosmology, in which in the Friedmann equation the quadratic term in matter energy density is dominant. In this period it is important to account for multiple bounces of gravitons back to the brane (Hebecker & March-Russel, 2001; Langlois & Sorbo, 2003). Under the assumption that the component of the graviton energy-momentum tensor representing the bouncing gravitons cancels the large terms contained in the energy-momentum tensor of the emitted gravitons, we find that production of ^4He is consistent with the result of the standard cosmological model.

The plan of the paper is as follows. After briefly reviewing in Sect. 2 two approaches to the 5D model, in Sects. 3 and 4 we establish explicit connection between the two pictures. In Sect. 5, starting from the system of the Einstein equations containing the terms due to graviton emission to the bulk, we obtain the generalized Friedmann equation. In Sect. 8, using the results of Sect. 7, we calculate the collision integral in the Boltzmann equation and find the components of the energy-momentum tensor of the gravitons emitted to the bulk. In Sects. 9 and 10 we calculate the effect of the graviton emission on abundance of ^4He produced in primordial nucleosynthesis. In Sect. 11 we find condition for bouncing of the emitted gravitons back to the brane.

2. Two pictures of 3-brane in the 5D space-time

We consider the 5D model with one 3D brane embedded in the bulk. Matter is confined to the brane, gravity extends to the bulk. In the leading approximation we neglect gravitation emission from the brane to the bulk. The action is taken in the form

$$S_5 = \frac{1}{2\kappa^2}\left[\int_{\Sigma} d^5x\sqrt{-g^{(5)}}(R^{(5)} - 2\Lambda) + 2\int_{\partial\Sigma} K\right] - \int_{\partial\Sigma} d^4x\sqrt{-g^{(4)}}\,\hat{\sigma} - \int_{\partial\Sigma} d^4x\sqrt{-g^{(4)}}L_m, \quad (1)$$

where $x_4 \equiv y$ is coordinate of the infinite extra dimension, $\kappa^2 = 8\pi/M^3$.

The 5D model can be treated in two alternative approaches. Each approach proves to be useful for certain problems discussed below. In the first approach metric is non-static, and the brane is located at a fixed position in the extra dimension (Binetruy, Deffayet & Langlois 2000; Binetruy et al., 2000). We consider the class of metrics of the form [1]

$$ds_5^2 = g_{ij}^{(5)}dx^i dx^j = -n^2(y,t)dt^2 + a^2(y,t)\eta_{ab}dx^a dx^b + dy^2 \equiv dy^2 + g_{\mu\nu}dx^\mu dx^\nu. \quad (2)$$

The brane is spatially flat and located at $y = 0$. Making use of the freedom in parametrization of t, we can make $n(0,t) = 0$. Reduction of the metric (2) to the brane is

$$ds^2 = dt^2 + a^2(0,t)\eta_{ab}dx^a dx^b. \quad (3)$$

The energy-momentum tensor of matter on the brane is taken in the form

$$\hat{T}_\mu^\nu = diag\,\delta(y)\{-\hat{\rho},\hat{p},\hat{p},\hat{p}\}. \quad (4)$$

For the following it is convenient to introduce the normalized expressions for energy density, pressure and cosmological constant on the brane which all have the same dimensionality $[GeV]$

$$\mu = \sqrt{-\frac{\Lambda}{6}}, \quad \sigma = \frac{\kappa^2\hat{\sigma}}{6}, \quad \rho = \frac{\kappa^2\hat{\rho}}{6}, \quad p = \frac{\kappa^2\hat{p}}{6}. \quad (5)$$

[1] The indices i, j run over 0,...,4, the Greek indices are 0,...,3, and $a, b = 1, 2, 3$

In the leading approximation the system of 5D Einstein equations and junction conditions admits a solution (Binetruy et al., 2000)

$$a^2(y,t) = \frac{a^2(0,t)}{4}\left[e^{2\mu|y|}\left(\left(\frac{\rho+\sigma}{\mu}-1\right)^2+\frac{\rho_w}{\mu}\right)+e^{-2\mu|y|}\left(\left(\frac{\rho+\sigma}{\mu}+1\right)^2+\frac{\rho_w}{\mu}\right)\right. \quad (6)$$
$$\left.-2\left(\left(\frac{\rho+\sigma}{\mu}\right)^2-1+\frac{\rho_w}{\mu}\right)\right],$$
$$n(y,t) = \frac{\dot{a}(y,t)}{\dot{a}(0,t)}. \quad (7)$$

The function $a(t) = a(0,t)$ is solution of the generalized Friedmann equation (Binetruy, Deffayet & Langlois 2000; Binetruy et al., 2000)

$$H^2(t) = -\mu^2 + (\rho+\sigma)^2 + \mu\rho_w(t). \quad (8)$$

Here we introduced the Hubble function $H(t)$ and the so-called Weyl radiation term $\rho_w(t)$ (Maartens, 2004; Shiromizu; Maeda & Sasaki, 2000)

$$H(t) = \frac{\dot{a}(0,t)}{a(0,t)}, \qquad \rho_w(t) = \rho_{w0}\left(\frac{a(0,t_0)}{a(0,t)}\right)^4. \quad (9)$$

In the second approach the brane separates two static 5D AdS spaces attached to both sides of the brane (Birmingham, 1999; Chamblin & Reall, 1999; Collins & Holdom, 2000; Kraus, 1999). The bulk actions are as the bulk part of the action (1). The metrics, which are solutions of the Einstein equations, are

$$ds^2 = -f_i(R)dT^2 + \frac{dR^2}{f_i(R)} + \mu_i^2 R^2 dx^a dx_a, \quad (10)$$

where

$$f_i(R) = \mu_i^2 R^2 - \frac{P_i}{R^2}.$$

Below we consider the case $\mu_1 = \mu_2$ and $P_1 = P_2$. Reduction of the 5D metric to the brane is

$$ds^2 = -dt^2 + R_b^2(t)dx^a dx_a. \quad (11)$$

Trajectory of the moving brane is defined through the proper time t on the brane as

$$R = R_b(t),\; T = T_b(t), \text{where} - f(R_b)\dot{T}_b^2 + f^{-1}(R_b)\dot{R}_b^2 = -1.$$

The junction conditions on the brane are (Binetruy, Deffayet & Langlois 2000; Binetruy et al., 2000; Chamblin & Reall, 1999; Collins & Holdom, 2000; Kraus, 1999; Shiromizu; Maeda & Sasaki, 2000)

$$[h_i^k \nabla_k n_j] = \tau_{ij} - \frac{1}{3}\tau h_{ij}. \quad (12)$$

Here $h_{ij} = g_{ij} - n_i n_j$ is the induced metric on the brane, $v^i = \pm(\dot{T}_b, \dot{R}_b, 0)$ and $n_i = \pm(-\dot{R}_b, \dot{T}_b, 0)$ are velocity and normal vector to the brane, $\tau_{ij} = (\rho+p)v_i v_j + p h_{ij}$. [2] [X]

[2] Here and below prime and dot denote differentiation over y and t.

denotes the difference of expressions calculated at the opposite sides of the brane. From the spatial components $(a, b = 1, 2, 3)$ of the junction conditions follows the generalized Friedmann equation

$$\left(\frac{\dot{R}_b}{R_b}\right)^2 = -\mu^2 + (\rho + \sigma)^2 + \frac{P}{R_b^4}. \tag{13}$$

Equation (13) is of the same form as (8). $a^2(0,t)$ can be identified with $\mu^2 R_b^2(t)$, and the term $\rho_w(t) = \rho_{w0}/a^4(t)$ can be identified with the term $P/\mu R_b^4(t)$. Below we set $\sigma = \mu R_b^2(t)$ and $a^2(0,t)$ are interpreted as scale factors of the Universe. [3] .

3. Geodesic equations in the picture with static metric

To prepare necessary relations for establishing connection between the two pictures, we consider the geodesic equations in the picture with the static metric and moving brane. Let y be a parameter along a geodesic. Geodesic equations are

$$\frac{d^2T}{dy^2} + 2\Gamma^T_{TR}\frac{dT}{dy}\frac{dR}{dy} = 0 \tag{14}$$

$$\frac{d^2x^a}{dy^2} + 2\Gamma^a_{bR}\frac{dx^b}{dy}\frac{dR}{dy} = 0 \tag{15}$$

$$\frac{d^2R}{dy^2} + \Gamma^R_{RR}\left(\frac{dR}{dy}\right)^2 + \Gamma^R_{TT}\left(\frac{dT}{dy}\right)^2 + \Gamma^R_{ab}\frac{dx^a}{dy}\frac{dx^b}{dy} = 0, \tag{16}$$

where the Christoffel symbols calculated with the metric (10) are

$$\Gamma^T_{TR} = \frac{f'}{2f}, \quad \Gamma^R_{RR} = -\frac{f'}{2f}, \quad \Gamma^R_{TT} = \frac{1}{2}ff', \quad \Gamma^R_{ab} = -\eta_{ab} f\mu^2 R, \quad \Gamma^a_{Rb} = \frac{\delta^a_b}{R}.$$

Here $(T, R) \equiv (T^\pm, R^\pm)$ are coordinates in the AdS spaces at the opposite sides of the brane. Integrating the geodesic equations, we obtain

$$\frac{dT^\pm}{dy} = \frac{E^\pm}{f(R)}, \qquad \frac{dx^a}{dy} = \frac{C^a}{\mu^2R^2}, \qquad \left(\frac{dR^\pm}{dy}\right)^2 = f(R)(C^{R\,\pm})^2 + E^{\pm 2} - \frac{C^{a2}f}{\mu^2R^2}, \tag{17}$$

where $(E^\pm, C^a, C^{R\,\pm})$ are integration parameters. $(dT/dy, dR/dy, dx^a/dy)$ are the components of the tangent vector to the geodesic which we normalize to unity. Imposing normalization condition

$$\frac{dx^i}{dy}\frac{dx^j}{dy}g_{ij} = 1, \qquad i = T, R, a$$

we obtain that $(C^{R\,\pm})^2 = 1$.

Let us consider the foliation of the hypersurface $(T,\, R,\, x^a = 0)$ by geodesics $(T(y,t),\, R(y,t))$ that intersect the trajectory of the brane $(T_b(t),\, R_b(t))$ and at the intersection point are orthogonal to it (cf. (Mukohyama, Shiromizu & Maeda)). The geodesics are subject to the

[3] From the fit of the cosmological data in the leading approximation of the present model it follows that $\sigma^2 = \mu^2(1 + O(H_0^2/\mu^2))$, where H_0 is the present-time Hubble parameter (Iofa, 2009a;b). For $\mu \sim 10^{-12} GeV$ correction is $\sim 10^{-60}$.

initial conditions at $y = 0$: $R^{\pm}(0,t) = R_b(t)$, $T^{\pm}(0,t) = T_b(t)$. We consider solutions of the geodesic equations even in y: $T^{+}(y) = T^{-}(-y)$, $R^{+}(y) = R^{-}(-y)$

$$\frac{dT^{\pm}(y)}{dy} = \frac{E\varepsilon(y)}{f(R)}, \qquad \frac{dR^{\pm}(y)}{dy} = \alpha\varepsilon(y)\left(f(R) + E^2\right)^{1/2}, \tag{18}$$

where $\alpha = \pm 1$, and $E^{+}(t) = E^{-}(t) = E(t)$.

The normalized velocity of the brane and the normal vector to the brane are

$$v_b^i = (\dot{T}_b, \dot{R}_b), \qquad n_b^{i\pm} = \eta\varepsilon(y)\left(\frac{\dot{R}_b}{f(R_b)}, f(R_b)\dot{T}_b\right), \tag{19}$$

where $\eta = \pm 1$. From (17), setting $C^a = 0$, we obtain the tangent vector to the geodesic

$$u^{i\pm} = \left(\frac{E\varepsilon(y)}{f(R)}, \alpha\varepsilon(y)\sqrt{f(R) + E^2}\right), \tag{20}$$

By construction, at the intersection point with the trajectory of the brane the tangent vector to a geodesic is parallel to the normal to the trajectory of the brane,

$$u^i\,|_{y=0}||n_b^i. \tag{21}$$

Substituting $\dot{T}_b = \xi\sqrt{f(R_b) + \dot{R}_b^2}/f(R_b)$, where $\xi = \pm 1$, we have

$$E = \eta\dot{R}_b, \qquad \alpha = \xi\eta. \tag{22}$$

Integrating Eq.(18) for R, we express coordinate R in the hyperplane $C^a = 0$ through coordinates (y,t)

$$R^2(y,t) = R_b^2(t)\left[\cosh(2\mu y) + \frac{H^2}{2\mu^2}(\cosh(2\mu y) - 1) \pm \sqrt{1 + \frac{H^2}{\mu^2} - \frac{P}{R_b^4\mu^2}}\sinh(2\mu|y|)\right]. \tag{23}$$

Here $H^2 = (\dot{R}_b(t)/R_b(t))^2$. Substituting H^2 from the Friedmann equation (13) with $\sigma = \mu$, we transform (23) to the form (6) with $\sigma = \mu$. Thus, we can identify

$$\mu^2 R^2(y,t) = a^2(y,t). \tag{24}$$

4. Integration of geodesic equations and connection between two forms of the metric

In the following we consider the metric (10) with $P = 0$, i.e. $f(R) = \mu^2 R^2$. In the hyperplane $x^a = const$ we set $C^a = 0$. Integrating Eq. (18)

$$\frac{dR^{\pm}(y,t)}{dy} = \alpha\varepsilon(y)\sqrt{\mu^2 R^2(y,t) + E^2} \tag{25}$$

we obtain

$$R^{\pm}(y,t) = R_b(t)\cosh\mu y + \alpha\sqrt{\frac{\dot{R}_b^2}{\mu^2} + R_b^2}\,\sinh\mu|y|. \tag{26}$$

Using the Friedmann equation

$$H^2 = \dot{R}_b^2/R_b^2 = \rho^2 + 2\mu\rho \tag{27}$$

omitting $(\pm)$ we rewrite (26) as

$$R(y,t) = R_b(t)\left(\cosh \mu y + \alpha\beta\left(1+\frac{\rho}{\mu}\right)\sinh \mu|y|\right), \tag{28}$$

where $\beta \equiv sign(R_b(t))$. To identify $\mu^2 R^2(y,t)$ with

$$a^2(y,t) = \frac{a^2(0,t)}{4}\left[e^{2\mu|y|}\left(\frac{\rho}{\mu}\right)^2 + e^{-2\mu|y|}\left(\frac{\rho}{\mu}+2\right) - 2\frac{\rho}{\mu}\left(\frac{\rho}{\mu}+2\right)\right], \tag{29}$$

(cf. (6)) we set

$$\alpha\beta = -1.$$

and obtain

$$R(y,t) = \frac{R_b(t)}{2}\left[e^{-\mu|y|}\left(\frac{\rho}{\mu}+2\right) - e^{\mu|y|}\frac{\rho}{\mu}\right]. \tag{30}$$

Also we have

$$\frac{dR(y,t)}{dy} = -\varepsilon(y)\frac{\mu R_b}{2}\left[e^{-\mu|y|}\left(\frac{\rho}{\mu}+2\right) + e^{\mu|y|}\frac{\rho}{\mu}\right]. \tag{31}$$

Introducing y_0, such that

$$e^{\mu y_0} = \left(\frac{\rho}{\rho+2\mu}\right)^{1/2} \tag{32}$$

we express $R(y,t)$ and $R'(y,t)$ as

$$R(y,t) = -\frac{HR_b(t)}{\mu}\sinh(\mu|y|+\mu y_0), \qquad R'(y,t) = -\varepsilon(y)HR_b(t)\cosh(\mu|y|+\mu y_0). \tag{33}$$

Substituting in Eq. (18) for $T(y,t)$ expression (33) for $R(y,t)$, and integrating the equation, we have

$$T^{\pm}(y,t) = -\frac{1}{\mu E}\frac{\cosh(\mu|y|+\mu y_0)}{\sinh(\mu|y|+\mu y_0)} + C^{\pm}(t). \tag{34}$$

Taking $C^+(t) = C^-(t) = C(t)$, we obtain that the limits $y = 0$ of $T^{\pm}(y,t)$ from both sides of the brane are the same .

To determine $C(t)$, first, we consider transformation of the metric (10) from coordinates $R(y,t)$, $T(y,t)$ to coordinates y,t. We have

$$ds^2 = dy^2\left(-\mu^2R^2T'^2 + \frac{R'^2}{\mu^2R^2}\right) + 2dtdt\left(-\mu^2R^2\dot{T}T' + \frac{\dot{R}R'}{\mu^2R^2}\right) \tag{35}$$
$$+dt^2\left(-\mu^2R^2\dot{T}^2 + \frac{\dot{R}^2}{\mu^2R^2}\right) + \mu^2R^2dx^{a2}.$$

On solutions of the geodesic equations (18) the coefficient at dy^2 is $\varepsilon^2(y)$. The coefficient at $dydt$ is zero, if

$$\dot{T} = \frac{\dot{R}\,R'}{\mu^4R^4\,T'} = \frac{\dot{R}\,R'}{\mu^2R^2(\varepsilon(y)E)}, \tag{36}$$

where in the second equality we have used (18). Substituting (36) in the coefficient at dt^2 and using (18) and (33), we obtain $\dot{R}^2/\dot{R_b}^2 = n^2(y,t)$ (cf. (6)).

Writing (34) as

$$T(y,t) = -\frac{R'(y,t)/\varepsilon(y)}{\mu^2 R(y,t)E} + C(t) \tag{37}$$

and taking the time derivative, we obtain

$$\dot{T} = \frac{\dot{R}R'}{\mu^2 R^2(\varepsilon(y)E)} - \frac{1}{\mu^2 R}\frac{d}{dt}\left(\frac{R'/\varepsilon(y)}{E}\right) + \dot{C} \tag{38}$$

The first term in the rhs of (38) is the same as (36). Remarkably, substituting explicit expressions (33) for R and R', we find that the second term in the right-hand side of (38) is independent of y

$$\frac{1}{\mu^2 R}\frac{d}{dt}\left(\frac{R'\varepsilon(y)}{E}\right) = \frac{\eta\dot{y}_0(t)}{\dot{R}_b(t)}. \tag{39}$$

Choosing

$$\dot{C} = \eta\frac{\dot{y}_0}{\dot{R}_b},$$

we obtain $\dot{T}$ in the form (36). To conclude, we have transformed the metric (10) to the form (2).

5. Generalized Friedmann equation with the graviton emission terms included

In this section, keeping in mind application of the results of this section to calculation of primordial nucleosynthesis, we derive the generalized Friedmann equation containing the bulk energy-momentum tensor due to the graviton emission from the brane to the bulk. We use formulation based on the metric (2). Dynamics of the model is contained in the system of 5D Einstein equations

$$G_{ij} \equiv R_{ij} - \frac{1}{2}g_{ij}R = \kappa^2 T_{ij}^{(5)} - g_{ij}\Lambda - \delta_{ij}^{\mu\nu}\frac{\sqrt{-g^{(4)}}}{\sqrt{-g^{(5)}}}\delta(y)g_{\mu\nu}\kappa^2\hat{\sigma}. \tag{40}$$

Here $T_{ij}^{(5)}$ is the sum of the energy-momentum tensor of matter confined to the brane $\hat{T}_{ij}$ (11) and the bulk energy-momentum tensor $\check{T}_{ij}$.

The components of the Einstein tensor $G_{ij}(y,t)$ are

$$G_{00} = 3\left[\frac{\dot{a}^2}{a^2} - n^2\left(\frac{a''}{a} + \frac{a'^2}{a^2}\right)\right] \tag{41}$$

$$G_{44} = 3\left[\left(\frac{a'^2}{a^2} + \frac{a'n'}{a\,n}\right) - \frac{1}{n^2}\left(\frac{\dot{a}^2}{a^2} - \frac{\dot{a}\dot{n}}{a\,n} + \frac{\ddot{a}}{a}\right)\right] \tag{42}$$

$$G_{04} = 3\left(\frac{n'}{n}\frac{\dot{a}}{a} - \frac{\dot{a}'}{a}\right). \tag{43}$$

The components $G_{ij}(y,t)$ satisfy the relations (cf. (Binetruy et al., 2000))

$$G_0^0 - G_4^0 \frac{\dot{a}}{a'} = \frac{3}{2a'a^3} F' \tag{44}$$

$$G_{44} - G_{04} \frac{a'}{\dot{a}} = \frac{3}{2\dot{a}a^3} \dot{F}, \tag{45}$$

where

$$F = (a'a)^2 - \frac{(\dot{a}a)^2}{n^2} \tag{46}$$

The functions $a(y,t)$ and $n(y,t)$ satisfy junction conditions on the brane [4]

$$\frac{a'(0,t)}{a(0,t)} = -\sigma - \rho(t), \qquad \frac{n'(0,t)}{n(0,t)} = 2\rho(t) + 3p(t) - \sigma \tag{47}$$

Reparametrization of t allows to set $n(0,t) = 1$, i.e. t is the proper time on the brane.

Eq. (45) can be rewritten as

$$\dot{F} = -\mu^2 (\dot{a^4}) - \frac{\kappa^2}{6} (a^4)' \check{T}_{04} + \frac{\kappa^2}{6} (\dot{a^4}) \check{T}_{44}. \tag{48}$$

On the brane, at $y = 0$, using junction conditions and setting $\sigma \simeq \mu$, we have

$$\dot{F} = \mu^2 (\dot{a^4}) + \frac{2\kappa^2 a^4}{3} (\rho + \mu) \check{T}_{04} + \frac{2\kappa^2 a^3 \dot{a}}{3} \check{T}_{44}. \tag{49}$$

Integrating (48) in the interval $(t,\ t_l)$, where the initial time t_l is defined below, we obtain

$$F(0,t) = \mu^2 a^4(0,t) + \frac{2\kappa^2}{3} \int_{t_l}^{t} dt' \check{T}_{04}(t') (\rho(t') + \mu) a^4(0,t') + \frac{2\kappa^2}{3} \int_{t_l}^{t} dt' \check{T}_{44}(t') \dot{a}(0,t') a^3(0,t') - C, \tag{50}$$

where C is an integration constant. Substituting expression (46) for F and using the junction conditions, we rewrite (50) in a form of the generalized Friedmann equation (cf. (Binetruy, Deffayet & Langlois 2000; Binetruy et al., 2000))

$$H^2(t) = \rho^2(t) + 2\mu\rho(t) + \mu\rho_w(t) - \frac{2\kappa^2}{3a^4(0,t)} \int_{t_l}^{t} dt' \left[\check{T}_{04}(t') (\rho(t') + \mu) + \check{T}_{44}(t') H(t') \right] a^4(0,t'). \tag{51}$$

On the brane, at $y = 0$, substituting the expressions for n'/n and a'/a from the junction conditions (47), we transform the (04) component of the Einstein equations (43) to the form

$$\dot{\rho} + 3H(\rho + p) = \frac{\kappa^2 \check{T}_{04}}{3}. \tag{52}$$

On the other hand, the same equation, which is the generalization of the conservation equation for the energy-momentum tensor of the matter confined to the brane to the case with the energy-momentum flow in the bulk, is obtained by integration of the 5D conservation law $\nabla_i T_0^i = 0$ across the brane (Tanaka & Himemoto, 2003).

[4] We assume invariance $y \leftrightarrow -y$.

6. Period of late cosmology

The generalized Friedmann equation contains the terms linear and quadratic in the energy density ρ. The period, in which the term linear in energy density is dominant, i.e. $\rho/\mu < 1$ is called the period of late cosmology. Graviton production by hot matter is sufficiently intensive in the radiation-dominated period of cosmology. In the radiation-dominated phase of the Universe condition of late cosmology is

$$\frac{\rho_r(T)}{\mu} = \frac{\kappa^2\hat{\rho}_r(T)}{6\mu} = \frac{4\pi^3 g_*(T)T^4}{90\mu M^3} \simeq \frac{4\pi^3 g_*(T)T^4}{90(\mu M_{pl})^2} < 1, \tag{53}$$

where we used that $\mu M_{pl}^2/M^3 \simeq 1$ Iofa (2009a;b). Taking $\mu \sim 10^{-12}GeV$, we find that the approximation of late cosmology is valid up to the temperatures of order $5 \cdot 10^2 GeV$.

Without the Weyl radiation term, the function $a^2(y,t)$ (29) has the minimum equal to zero at the point $|\bar{y}|$

$$e^{2\mu|\bar{y}|} = 1 + \frac{2\mu}{\rho}. \tag{54}$$

In the region $0 < |y| < |\bar{y}|$ and for $\rho/\mu \ll 1$ the functions $a(y,t)$ and $n(y,t) = \dot{a}(y,t)/\dot{a}(0,t)$ can be approximated as

$$\begin{aligned} a(y,t) &\simeq a(0,t)e^{-\mu|y|} \\ n(y,t) &\simeq e^{-\mu|y|}, \end{aligned} \tag{55}$$

and the approximate 5D metric is

$$ds^2 \simeq dy^2 + e^{-2\mu|y|}(-dt^2 + a^2(0,t)\eta_{ab}dx^a dx^b). \tag{56}$$

7. Fluctuations of the background metric

in the models with extra dimensions, in interactions of particles of the hot plasma on the brane, are produced not only massless gravitons, but the whole Kaluza-Klein tower of gravitons. Gravitons are fluctuations over the background metric. To calculate the energy loss from the brane due to graviton emission we need the spectrum of gravitons. The part of the action quadratic in fluctuations is

$$\begin{aligned} I = \frac{1}{2}\int d^5x\sqrt{-g^{(5)}}\Big[&(R-\Lambda_{(5)})\left(-\frac{1}{2}h_i^j h_j^i + \frac{1}{4}h^2\right) - R_i^j h_j^i h + 2R_i^j h_j^k h_k^i \\ &+\frac{1}{2}\left(2h_{qi;k}h^{ik;q} - h_{ik;q}h^{ik;q} + h_{,q}h^{,q} - 2h_{,i}h^{ik}{}_{;k}\right)\Big]. \end{aligned} \tag{57}$$

The action I is invariant under the gauge transformations

$$\tilde{h}_{kl} = h_{kl} - (\nabla_k\xi_l + \nabla_l\xi_k), \tag{58}$$

where ∇ is defined with respect to the background metric. The gauge freedom allows to set the components h_{4i} to zero. There remain residual gauge transformations, allowing for subsequent simplifications.

With the full metric $g_{\mu\nu}(y,t)$ (2) it is complicated to solve the equations for $h_{\mu\nu}$ analytically. To solve the equations, instead of the exact metric, in the regions $0 < y < \bar{y}$ and $y > \bar{y}$ we use the approximate metrics with the separated dependence on y and t. In the region $0 < y < \bar{y}$, we use the approximate metric (56). In the region $y > \bar{y}$ the approximate metric is

$$ds^2 = dy^2 + e^{2\mu y}\left(-dt^2 + \frac{a^2(0,t)}{4}\left(\frac{\rho}{\mu}\right)^2 \eta_{ab}dx^a dx^b\right). \tag{59}$$

In the metrics (56) and (59), using the residual gauge transformations and field equations for h_{ij}, it is possible to transform h_{ij} to the traceless, transverse form $h_\mu{}^\mu = 0,\ D^\mu h_{\mu\nu} = 0$ (Iofa, 2011).

Equations for the eigenmodes are considered in the next subsection. We show that the norm of the function $h_m^>$ is smaller than that of $h_m^<$. Effectively, in the period of late cosmology, this allows to consider the contribution from the region $0 < y < \bar{y}$ only.

We obtain the spectrum

$$m_n \simeq \mu e^{-\mu\bar{y}}\left(n\pi + \frac{\pi}{2}\right) \tag{60}$$

and the normalized eigenmode $h_m(0)$

$$h_m(0) \simeq (\mu e^{-\mu\bar{y}})^{1/2}. \tag{61}$$

For the following we need the sum $\sum_n h^2_{m_n}(0)$, where m_n is determined by (60). Because of a narrow spacing between the levels, we change summation to integration and obtain

$$\sum_n h^2_{m_n}(0) \simeq \int \frac{dm\, e^{\mu\bar{y}}}{\mu\pi}\mu e^{-\mu\bar{y}} = \int \frac{dm}{\pi}. \tag{62}$$

The integral (62) is independent of $\bar{y}$. The same measure of integration was obtained in Langlois; Sorbo & Rodriguez-Martinez (2002), where the authors used the graviton modes of the Randal-Sundrum II model (Randall & Sundrum, 1999) without matter, in which case the integration over y extends to infinity and the spectrum is continuous. Similarity of the results can be traced to the fact that we performed calculations in the period of late cosmology neglecting the terms of order $O(\rho/\mu)$ as compared to unity.

7.1 Equations for eigenmodes

In the background of the approximate metric (56), in the region $0 < y < \bar{y}$, in the gauge $D^\mu h_{\mu\nu}(y,x) = 0,\ h^\mu_\mu(y,x) = 0$, the $(\mu\nu)$ components of the field equations for fluctuations are

$$h''_{\mu\nu} - 4\mu^2 h_{\mu\nu} + b^{-1}(y)D_\rho D^\rho h_{\mu\nu} + \delta(y)4\mu h_{\mu\nu} = 0. \tag{63}$$

We expand the functions $h^\mu_\nu(x,y)$ as

$$h^\mu_\nu(x,y) = \sum_m \phi^\mu_{(m)\nu}(x)h_m(y),$$

where the functions $h^<_m(y)$ satisfy the equation [5]

$$h''_m(y) - 4\mu^2 h_m(y) + e^{2\mu|y|}m^2_< h_m(y) + \delta(y)4\mu h_m(y) = 0. \tag{64}$$

[5] Wherever it does not lead to ambiguity, we omit the (sub)superscripts $<$ and $>$.

Solution of the equation

$$h''_m(y) - 4\mu^2 h_m(y) + e^{2\mu|y|} m_<^2 h_m(y) = 0 \tag{65}$$

is

$$h_m^<(y) = C_1 J_2(\tilde{m} e^{\mu|y|}) + C_2 N_2(\tilde{m} e^{\mu|y|}), \tag{66}$$

where

$$\tilde{m} = \frac{m_<}{\mu}.$$

The terms with $\delta(y)$ are taken into account by the boundary condition

$$\left[\frac{dh_m(y)}{dy} + \frac{2}{\tilde{m}} h_m(y)\right]_{y=0_+} = 0$$

which yields the relation $C_1 J_1(\tilde{m}) + C_2 N_1(\tilde{m}) = 0$. The eigenfunctions (66) take the form

$$h_{m_<}^<(y) = C\left[(N_1(\tilde{m}) J_2(\tilde{m} e^{\mu|y|}) - J_1(\tilde{m}) N_2(\tilde{m} e^{\mu|y|})\right]. \tag{67}$$

Introducing the functions

$$f_k(y) = N_1(\tilde{m}) J_k(\tilde{m} e^{\mu|y|}) - J_1(\tilde{m}) N_k(\tilde{m} e^{\mu|y|})$$

we obtain the norm of h_m

$$||h_{m_<}^<||^2 = 2C^2 \int_0^{\bar{y}} e^{2\mu y} f_2^2(y) dy = \frac{C^2}{\mu}\left[e^{2\mu\bar{y}}\left(f_2^2(\bar{y}) - f_1(\bar{y}) f_3(\bar{y})\right) - \left(f_2^2(0) - f_1(0) f_3(0)\right)\right]. \tag{68}$$

Typical masses (energies) of the emitted Kaluza-Klein gravitons are of order of temperature of the Universe T. In the case of small $\mu \sim 10^{-12}$ GeV we have $m/\mu \gg 1$. Substituting the asymptotics of the Bessel functions, we obtain

$$||h_{m_<}^<||^2 = \frac{C^2}{\mu}\left(\frac{2}{\pi\tilde{m}}\right)^2 (e^{\mu\bar{y}} - 1). \tag{69}$$

In the region $y > \bar{y}$ we use the metric in (59) with the increasing exponent. The equations for the eigenmodes $h^>$ is

$$h''_m(y) - 4\mu^2 h_m(y) + e^{-2\mu|y|} m_>^2 h_m(y) = 0. \tag{70}$$

The eigenfunctions are

$$h_{m_>}^>(y) = \tilde{C}_1 J_2(\tilde{m} e^{-\mu y}) - \tilde{C}_2 N_2(\tilde{m} e^{-\mu y}), \tag{71}$$

where $\tilde{m} = m_>/\mu$. For large y, such that $\tilde{m} e^{-\mu y} \ll 1$, the function $N_2(\tilde{m} e^{-\mu y}) \sim (\tilde{m} e^{-\mu y})^{-2}$ rapidly increases, and to have normalizable eigenfunctions we set $\tilde{C}_2 = 0$. The norm of the function $h_m^>(y)$ is

$$||h_{m_>}^>||^2 = \tilde{C}_1^2 \int_{\bar{y}}^{\infty} e^{-2\mu y} J_2^2(\tilde{m} e^{-\mu y}) dy = \tilde{C}_1{}^2 \frac{e^{-2\mu\bar{y}}}{2\mu}[J_2^2(\tilde{m} e^{-\mu\bar{y}}) - J_1(\tilde{m} e^{-\mu\bar{y}}) J_3(\tilde{m} e^{-\mu\bar{y}})]. \tag{72}$$

At temperatures $T \gg \mu$ at which production of gravitons is sufficiently intensive, the argument of the Bessel function

$$\tilde{m}e^{-\mu\bar{y}} \sim \frac{T}{\mu}\left(\frac{\rho}{\mu}\right)^{1/2} \sim \frac{T}{\mu}g_*^{1/2}(T)\frac{T^2}{\mu M_{pl}}$$

is large and we can substitute the asymptotics of the Bessel functions

$$h^{>}_{m_>}(y) = \tilde{C}_1\sqrt{\frac{2}{\pi\tilde{m}}}e^{\mu\bar{y}/2}\cos(\tilde{m}(e^{\mu\bar{y}}-1)). \tag{73}$$

Instead of sewing the oscillating functions $h^{<}_{m}(y)$ and $h^{>}_{m}(y)$, we sew the envelopes of their asymptotics

$$\frac{2C}{\pi\tilde{m}_<}e^{-\mu\bar{y}/2} = \tilde{C}_1\left(\frac{2}{\pi\tilde{m}_>}\right)^{1/2}e^{\mu\bar{y}/2} \tag{74}$$

giving

$$\tilde{C}_1 = Ce^{-\mu\bar{y}}\sqrt{\frac{2}{\pi}}\frac{\sqrt{\tilde{m}_>}}{\tilde{m}_<}$$

The norm (72) is smaller than (69). Effectively, we neglect the contribution from the region $y > \bar{y}$ and impose the condition $h_m(\bar{y}) = 0$, which yields us the spectrum (60). Using the norm (69), we obtain the normalized eigenmode $h_m(0)$ (61).

8. Production of Kaluza-Klein gravitons

In this section we calculate the rate of production of Kaluza-Klein gravitons in interactions of particles of the hot matter on the brane in the radiation-dominated period. The leading contribution to this process is given by the annihilation reactions $\psi^i + \bar{\psi}^i \to G$, where ψ and $\bar{\psi}$ are the standard model particles on the brane (vector, spinor, scalar) and G is a state of mass m_n from the graviton Kaluza-Klein tower. Production of Kaluza-Klein gravitons is calculated with the interaction Lagrangian

$$I = \kappa\int d^4x\,\sqrt{-\bar{g}}\,h_{\mu\nu}(0,x)T^{\mu\nu}(x),$$

where $T^{\mu\nu}$ is the energy-momentum of particles on the brane. Evolution of energy density of matter on the brane is determined from the Boltzmann equation (Gorbunov & Rubakov, 2008; Kolb & Turner, 1990)

$$\frac{d\hat{\rho}}{dt} + 4H\hat{\rho} = -\sum_n\sum_i\int\frac{d^3p}{(2\pi)^3}\int\frac{d^3k_1}{(2\pi)^3 2E_1}\frac{d^3k_2}{(2\pi)^3 2E_2}f_1^i(E_1)f_2^i(E_2)|M_n^i|^2(2\pi)^4\delta^4(k_1+k_2-p). \tag{75}$$

Here f^i are the Bose/Fermi distributions of colliding particles and M_n^i is the amplitude of the annihilation reaction. The square of the annihilation amplitude is (Langlois & Sorbo, 2003)

$$|M_n^i|^2 = A_i\frac{\kappa^2}{8}h_n^2(0)s^4, \tag{76}$$

where $A_i = A_s, A_v, A_f = 2/3, 4, 1$ for scalars, vectors and fermions and $s^2 = (k_1+k_2)^2$. The sum over the graviton states is transformed to the integral as in (62). Integrating over the angular variables of the momenta of interacting particles, we obtain Boltzmann equation in the form

$$\frac{d\hat{\rho}}{dt} + 4H\hat{\rho} = -\frac{\kappa^2 \sum_i A_i}{8(2\pi)^4} \int \frac{dme^{\mu\bar{y}}}{\mu} h_m^2(0)m^4 \int dk_1 dk_2 f(k_1) f(k_2)(k_1+k_2)\theta\left(1 - \left|1 - \frac{m^2}{2k_1k_2}\right|\right). \tag{77}$$

Integrating over m, we obtain (cf. (Hebecker & March-Russel, 2001; Langlois & Sorbo, 2003))

$$\frac{d\hat{\rho}}{dt} + 4H\hat{\rho} = -\frac{\kappa^2 \sum_i A_i}{8(2\pi)^4} \int_0^{(2k_1k_2)^{1/2}} dmm^4 \int dk_1 dk_2 f(k_1) f(k_2)(k_1+k_2) \tag{78}$$

$$= -\frac{\kappa^2 A 315 \zeta(9/2)\zeta(7/2) T^8}{\pi^3 2^8},$$

where the sum extends over relativistic degrees of freedom

$$A = \sum_i A_i = \frac{2g_s}{3} + g_f(1 - 2^{-7/2})(1 - 2^{-9/2}) + 4g_v. \tag{79}$$

In the high-energy period, when all the standard model degrees of freedom are relativistic, $A = 166,2$.

Eq.(78) has the same form as Eq.(52) and the right hand side of (78) can be identified with the component $\check{T}_{04}$ of the bulk energy-momentum tensor (Langlois & Sorbo, 2003).

The energy-momentum tensor of the emitted gravitons is taken in the form of free radiation of massless particles (Langlois & Sorbo, 2003)

$$\check{T}^{ij}(x) = \int d^5p \sqrt{-g}\delta(p_i p^i) \tilde{f}(x,p) p^i p^j, \tag{80}$$

where $\tilde{f}(x,p)$ is the phase space density of the distribution function of gravitons.

Expanding the graviton momentum p^i in the orthonormal basis v_b^i, n_b^i (19), we have

$$p^i = Ev_b^i + mn_b^i + p^a e_a^i,$$

where $p_i p^i = -E^2 + m^2 + p_a p^a$. Substituting these expressions in (80), we obtain

$$\check{T}_{nu} = u^i n^j T_{ij} = -\frac{1}{2} \int d^3p dm \, m f(E,m,p^a)\Big|_{E=\sqrt{m^2+p^2}} \tag{81}$$

The interaction tern in (75) can be rewritten as

$$\frac{1}{(2\pi)^5} \int d^3p \frac{dm}{\pi} \int \frac{d^3k_1}{2E_1} \frac{d^3k_2}{2E_2} f_1^i(E_1) f_2^i(E_2) \sum_i |M_n^i|^2 \delta^4(k_1 + k_2 - p) \tag{82}$$

Comparing (81) and (82), one determines $f(E,m,p^a)$ and calculates the other components of $\check{T}_{ij}$

$$\check{T}_{nn}(0,t) = \frac{3A\zeta(9/2)\zeta(7/2)}{2\pi^4}\kappa^2 T^8, \qquad \check{T}_{uu}(0,t) = \frac{21\, A\zeta(9/2)\zeta(7/2)}{8\pi^4}\kappa^2 T^8. \tag{83}$$

In the basis (19), the components $\check{T}_{nu}$ and $\check{T}_{nn}$ are equal to $\check{T}_{04}$ and $\check{T}_{44}$.

9. Graviton emission in the bulk in the period of late cosmology and nucleosynthesis

To estimate the effect of the graviton emission on nucleosynthesis we solve perturbatively the system of the generalized Friedmann equation (51) and the 5D conservation equation (52). In the period of late cosmology the leading approximation of the Friedmann equation coincides with that in the standard cosmological model. Supposing that the terms with the bulk energy-momentum tensor do not alter the leading-order results significantly, we treat these terms perturbatively. The Weyl radiation term and the term quadratic in radiation energy density can be treated perturbatively also.

The Friedmann equation can be written as

$$H^2 \simeq 2\mu\rho - I_{04} - I_{44}. \tag{84}$$

Using the expressions (83) for the components $\check{T}_{ij}$, we obtain

$$I_{04} = \frac{2\kappa^2}{3a^4(0,t)} \int_{t_l}^{t} dt' \check{T}_{04}(0,t')(\mu + \rho(t'))a^{-4}(0,t') = \tag{85}$$
$$\mu\rho(t)A_{04}\left(\frac{1}{12}\left(\frac{1}{\mu t_l} - \frac{1}{\mu t}\right) + \frac{1}{288}\left(\frac{1}{(\mu t_l)^3} - \frac{1}{(\mu t)^3}\right)\right),$$

and

$$I_{44} = \frac{2\kappa^2}{3a^4(0,t)} \int_{t_l}^{t} dt' \check{T}_{44}(t')(2\mu\rho(t'))^{1/2}a^{-4}(0,t') = \frac{\mu\rho(t)A_{44}}{48}\left(\frac{1}{(\mu t_l)^2} - \frac{1}{(\mu t)^2}\right), \tag{86}$$

Here we substituted

$$\kappa^2\check{T}_{04} = A_{04}\rho^2 = -\frac{315\, A\, \zeta(9/2)\zeta(7/2)}{2^9\pi^3}\left(\frac{180}{g_*\pi^2}\right)^2 \rho^2(t) \tag{87}$$

$$\kappa^2 T_{44} = A_{44}\rho^2 = \frac{3\, A\, \zeta(9/2)\zeta(7/2)}{4\pi^4}\left(\frac{180}{g_*\pi^2}\right)^2 \rho^2(t) \tag{88}$$

The integrals have a strong dependence on the value of the lower limit. Therefore, in the integrand the slowly varying functions can be taken at the times when all the Standard model degrees of freedom are relativistic. In this period $A = 166.2$, $g_*(T) = 106.7$ and $A_{04} \simeq -0.126$. In the leading approximation $\rho(t) \simeq 1/8\mu t^2$ and $1/\mu t_l \simeq (8\rho(t_l)/\mu)^{1/2}$ Taking $\rho(t_l)/\mu \sim 0.1 \div 0.001$ and $\mu \sim 10^{-12} GeV$, we have $1/\mu t_l \simeq 0.9 \div 0.09$ and $T_l \sim (5.1 \div 1.6) \cdot 10^2 GeV$. For $\rho(t_l)/\mu \sim 0.1$ we obtain

$$I_{04} \simeq -2\mu\rho(t) \cdot 0.0048 \tag{89}$$
$$I_{44} \simeq 2\mu\rho(t) \cdot 0.00094. \tag{90}$$

In the radiation-dominated period,the 5D conservation equation is

$$\dot{\rho} + 4H\rho = -\frac{A_{04}\rho^2}{3} \tag{91}$$

Let $\bar{\rho}$ and $\bar{H}$ be the energy density and the Hubble function in the leading order

$$\bar{H}^2 = 2\mu\bar{\rho}.$$

Defining

$$\rho = \bar{\rho} + \rho_1, \qquad H = \bar{H} + H_1,$$

and separating in (84) and (91) the leading-order terms, we obtain

$$2\bar{H}H_1 = 2\mu\rho_1 - \frac{2\bar{\rho}}{3}\int dt' \left[A_{04}(\mu+\bar{\rho}) + A_{44}\bar{H}\right]\bar{\rho}(t') \tag{92}$$

$$\dot{\rho}_1 + 4\bar{H}\rho_1 + 4\bar{\rho}H_1 = \frac{A_{04}\bar{\rho}^2}{3}. \tag{93}$$

Substituting in the above system $\bar{\rho}(t) \simeq 1/8\mu t^2$ and performing integration, we have

$$H_1(t) \simeq \frac{\mu}{\bar{H}}\rho_1 - \frac{\bar{H}A_{04}}{48\mu}\left[\frac{1}{t_l} - \frac{1}{t} + \frac{1}{24\mu^2}\left(\frac{1}{t_l^3} - \frac{1}{t^3}\right)\right] - \frac{\bar{H}A_{44}}{192\mu^2}\left(\frac{1}{t_l^2} - \frac{1}{t^2}\right). \tag{94}$$

Substituting in (92) expression (94) for H_1 and noting that $\bar{H} = 1/2t$, we obtain

$$\dot{\rho}_1 + \frac{3}{t}\rho_1 = \frac{A_{04}}{192\mu^2 t^3}\left[\frac{1}{t_l} + \frac{1}{24\mu^2}\left(\frac{1}{t_l^3} - \frac{1}{t^3}\right)\right] + \frac{A_{44}}{768\mu^3 t^3}\left(\frac{1}{t_l^2} - \frac{1}{t^2}\right).$$

Solving this equation, we find

$$\rho_1(t) = \frac{C_1}{t^3} + \frac{A_{04}}{192\mu^2 t^3}\left[\frac{t-t_l}{t_l} + \frac{1}{24\mu^2}\left(\frac{t-t_l}{t_l^3} - \frac{1}{2}\left(\frac{1}{t_l^2} - \frac{1}{t^2}\right)\right)\right] + \frac{A_{44}}{768\mu^3 t^3}\left(\frac{t}{t_l^2} - \frac{2}{t_l} + \frac{1}{t}\right) \tag{95}$$

The constant C_1 can be determined by sewing solutions of Friedmann equation in the periods of early and late cosmologies. For a moment we set $C_1 = 0$, i.e. we look for a contribution from the period of late cosmology. For H_1 we obtain

$$H_1(t) = -\frac{A_{04}}{1536\mu^3 t^2}\left(\frac{1}{t_l^2} - \frac{1}{t^2}\right) + \frac{A_{44}}{192\mu^2 t^2}\left(-\frac{1}{t_l} + \frac{1}{t}\right) \simeq -\bar{\rho}(t)\left(\frac{A_{04}}{192\mu^2 t_l^2} + \frac{A_{44}}{24\mu t_l}\right). \tag{96}$$

The expressions (95) and (96) show that corrections to the leading terms are small, i.e. the perturbative approach is justified. Note that the leading term proportional to $A_{04}/\mu t_l$ was canceled in H_1.

The mass fraction of ^4He produced in primordial nucleosynthesis is (Gorbunov & Rubakov, 2008; Kolb & Turner, 1990)

$$X_4 = \frac{2(n/p)}{(n/p)+1},$$

where the ratio n/p is taken at the end of nucleosunthesis. Characteristic temperature at the onset of the period of nucleosynthesis (freezing temperature T_n of the reaction $n \leftrightarrow p$), estimated as the temperature at which the reaction rate $\sim G_F T^5$ is approximately equal to the Hubble parameter $G_F T_n^5 \sim H$ (Gorbunov & Rubakov, 2008; Kolb & Turner, 1990), is $T_n \sim 10^{-3} GeV$. The difference of the freezing temperatures in the models with and without the account of the graviton emission is

$$\frac{\delta T_n}{T_n} \simeq \frac{H_1}{5\bar{H}} \simeq \frac{1}{5}\sqrt{\frac{\bar{\rho}(t_n)}{2\mu}}\left(\frac{A_{04}}{192\mu^2 t_l^2} + \frac{A_{44}}{24\mu t_l}\right), \tag{97}$$

where

$$\frac{\bar{\rho}(t_n)}{\mu} = \frac{4\pi^3 g_*(T_n)}{90} \frac{T_n^4}{\mu M^3}. \tag{98}$$

Substituting $T_n \sim 10^{-3} GeV$ and $\mu M^3 \sim (\mu M_{pl})^2 \sim 10^{14} GeV^4$, we find that the ratio $\delta T_n/T_n$ is very small. The equilibrium value of the $n - p$ ratio at the freezing temperature is

$$\left(\frac{n}{p}\right)_n = \exp\left[-\frac{(m_n - m_p)}{T_n}\right].$$

Substituting $\delta(n/p)_n = (n/p)\ln(n/p)_n \delta T_n/T_n$, we obtain variation of X_4 under variation of the freezing temperature

$$\delta X_4 \simeq \frac{2}{(n/p+1)^2} \ln\left(\frac{n}{p}\right)\left(\frac{n}{p}\right)_n \frac{\delta T_n}{T_n} \tag{99}$$

which is also a very small number (Iofa, 2011).

10. Estimate of the graviton emission in the bulk in the period of early cosmology

In the period of early cosmology in the Friedmann equation the ρ^2 term is dominant, i.e. $\rho(t)/\mu > 1$. For the value of $\mu \sim 10^{-12} GeV$ assumed in the present study the characteristic temperatures of the period of early cosmology are above $5 \cdot 10^2 GeV$. Not much is known about physics at such temperatures. To make a crude estimate of the effect of the graviton emission on the nucleosynthesis we adopt a conservative point of view assuming that the collision integral in the Boltzmann equation and the expressions for $\check{T}_{ij}$ calculated in the period of late cosmology remain qualitatively valid in the early cosmological period.

The new phenomenon in the early cosmological period is that some of the emitted gravitons can return to the brane and be again reflected in the bulk with a different momentum. These gravitons do not contribute to the component $\check{T}_{04}$, because they are not produced, but reflected, but contribute to the component $\check{T}_{44}$. The new ingredient in the Friedmann equation is the term $\check{T}_{44}^{(b)}$ representing the energy-momentum tensor of gravitons bouncing back to the brane (Hebecker & March-Russel, 2001; Langlois & Sorbo, 2003)

$$\begin{aligned} H^2(t) &= \rho^2(t) + 2\mu\rho(t) + \mu\rho_w(t) \\ &- \frac{2\kappa^2}{3a^4(0,t)} \int_{t_c}^{t} dt' \left[\check{T}_{04}(t')(\rho(t') + \mu) + \check{T}_{44}(t')H(t') - \check{T}_{44}^{(b)}(t')H(t')\right] a^4(0,t'). \end{aligned} \tag{100}$$

Here the initial time t_c is the time of reheating. Numerical estimates and considerations from the Vaidya model (Langlois; Sorbo & Rodriguez-Martinez, 2002; Langlois & Sorbo, 2003) suggest that at the period of early cosmology the dominant contributions from T_{04}, T_{44} and $\check{T}_{44}^{(b)}$ mutually cancel. Below we solve the system of the Friedmann and 5D conservation equations assuming that this cancellation takes place.

Let $\bar{\rho}$ and $\bar{H}$ be the energy density of matter on the brane and the Hubble function calculated in the model without the graviton emission in the period of early cosmology $\bar{H}^2 \simeq \bar{\rho}^2(t)$. For the equation of state of the hot plasma $\bar{\rho} = \bar{p}/3$, we have $\bar{\rho}(t) \simeq 1/4t$. Defining

$$\rho = \bar{\rho} + \rho_2, \qquad H = \bar{H} + H_2,$$

we obtain

$$2\bar{H}H_2 \simeq 2\bar{\rho}\rho_2 - \frac{2\bar{\rho}}{3}\int_{t_c}^{t} dt' \left[(A_{04}(\mu+\bar{\rho}) + A_{44}\bar{H})\,\bar{\rho}(t') - \kappa^2 \check{T}_{44}^{(b)}(t') \right] \tag{101}$$

$$\dot{\rho}_2 + 4\bar{H}\rho_2 + 4\bar{\rho}H_2 = \frac{A_{04}\bar{\rho}^2}{3}. \tag{102}$$

The equation for ρ_2 is

$$\dot{\rho}_2 + \frac{2}{t}\rho_2 - \frac{1}{t}\left[\frac{A_{04}}{12}\left(\mu \ln\frac{t}{t_c} + \frac{1}{4}\left(\frac{1}{t_c} - \frac{1}{t}\right)\right) + \frac{A_{44}}{48}\left(\frac{1}{t_c} - \frac{1}{t}\right)\right] + \frac{1}{12t}\int_{t_c}^{t} \kappa^2 \check{T}_{44}^{(b)}(t')dt' = \frac{A_{04}}{48t^2}, \tag{103}$$

Integrating (102) with the initial condition $\rho_2(t_c) = 0$, we obtain

$$\rho_2(t) = \frac{A_{04}\mu}{24}\left(\ln\frac{t}{t_c} - \frac{1}{2} + \frac{t_c^2}{2t^2}\right) + \frac{A_{04}+A_{44}}{96t_c}\left(1 - \frac{t_c^2}{t^2}\right) - \frac{A_{44}}{48}\left(\frac{1}{t} - \frac{t_c}{t^2}\right) \tag{104}$$
$$-\frac{1}{12t^2}\int_{t_c}^{t} dy y \int_{t_c}^{y} dx \kappa^2 \check{T}_{44}^{(b)}(x)$$

The time t_c of reheating is estimated for the reheating temperature $T_R \sim 5 \cdot 10^6 GeV$ (Mielczarek, 2009). Using the relation (53) and substituting $\bar{\rho}(t_c)/\mu \simeq 1/4\mu t_c$, we have

$$\frac{1}{4\mu t_c} \simeq \frac{4\pi^3 g_*(T_R) T_R^4}{90\mu M^3} \sim 9 \cdot 10^{14}.$$

It follows that $t_l/t_c \sim 1/\mu t_c \simeq 3.5 \cdot 10^{15}$.

In (104) there is a large term $(A_{04} + A_{44})/t_c$. Omitting the small terms, we have

$$\rho_2(t) \simeq \frac{A_{04}\mu}{24}\left(\ln\frac{t}{t_c} - \frac{1}{2}\right) + \frac{A_{04}+A_{44}}{96t_c} - \frac{A_{44}}{48}\frac{1}{t} - \frac{1}{12t^2}\int_{t_c}^{t} dy y \int_{t_c}^{y} dx \kappa^2 \check{T}_{44}^{(b)}(x) \tag{105}$$

On dimensional grounds at small t the term $\kappa^2 \check{T}_{44}^{(b)}(t)$ has the following structure

$$\kappa^2 \check{T}_{44}^{(b)}(t) = \frac{b_2}{t^2} + \frac{b_1\mu}{t} + b_0\mu^2 + \cdots \tag{106}$$

Performing integration of the last term in (105) and taking $t \sim t_l$, we obtain

$$\rho_2(t_l) \simeq \frac{A_{04}\mu}{24}\left(\ln\frac{t_l}{t_c} - \frac{1}{2}\right) - \frac{A_{44}}{48t_l} + \frac{A_{04}+A_{44}}{96t_c} - \frac{b_2}{24t_c} + \frac{b_2}{12t_l} + \frac{b_1\mu}{24}\ln\frac{t_l}{t_c}. \tag{107}$$

Next, we equate $\rho_2(t_l)$ and C_1/t_l^3 in (95). At the times $t \sim t_l$, where $\mu t_l \sim 1$, we have

$$C_1\mu^3 \simeq \mu\left(\frac{A_{04}}{24}\left(\ln\frac{1}{\mu t_c} - \frac{1}{2}\right) - \frac{A_{44}}{48}\right) + \frac{A_{04}+A_{44}}{96t_c} - \frac{b_2}{24t_c} + \frac{b_2\mu}{12} + \frac{b_1\mu}{24}\ln\frac{1}{\mu t_c}. \tag{108}$$

The term C_1/t^3 is

$$\frac{C_1}{t^3} \simeq \frac{1}{\mu^2 t^3}\left[\mu\left(\frac{A_{04}}{24}\left(\ln\frac{1}{\mu t_c} - \frac{1}{2}\right) - \frac{A_{44}}{48}\right) + \frac{A_{04}+A_{44}}{96t_c} - \frac{b_2}{24t_c} + \frac{b_2\mu}{12} + \frac{b_1\mu}{24}\ln\frac{1}{\mu t_c}\right]. \tag{109}$$

In the period of late cosmology the term C_1/t^3 generates in Eq.(94) for H_1 the contribution $\Delta H_1 = \mu C_1/\bar{H}t^3$

$$\Delta H_1 = 16\bar{\rho}\left[\mu\left(\frac{A_{04}}{24}\left(\ln\frac{1}{\mu t_c} - \frac{1}{2}\right) - \frac{A_{44}}{48}\right) + \frac{A_{04}+A_{44}}{96t_c} - \frac{b_2}{24t_c} + \frac{b_2\mu}{12} + \frac{b_1\mu}{24}\ln\frac{1}{\mu t_c}\right]. \quad (110)$$

Here $\bar{\rho}(t) \simeq 1/8\mu t^2$ is the matter energy density in the period of late cosmology.

If the term $(A_{04}+A_{44})/96t_c \sim 3\cdot 10^{-4}/t_c$ was not canceled, it would produce in ρ_1 the contribution

$$\Delta\rho_1 = \frac{A_{04}+A_{44}}{96t_c}\frac{1}{(\mu t)^3} \simeq \frac{3\cdot 10^{-4}}{(\mu t)^3 t_c}.$$

From (94) we would have

$$\frac{\Delta H_1}{\bar{H}} = \frac{\mu}{\bar{H}^2}\Delta\rho_1 \simeq \frac{1.2\cdot 10^{-3}}{(\mu t_c)(\mu t)}. \quad (111)$$

At time of the nucleosynthesis

$$\frac{1}{8(\mu t_n)^2} \simeq \frac{4\pi^3 g_*(T_n)}{90}\frac{T_n^4}{(\mu M_{pl})^2}.$$

For $T_n \sim 10^{-3}GeV$, we have $\mu t_n \sim 10^{12}$. From (111) we obtain $\Delta H_1(t_n)/\bar{H}(t_n) \simeq 4$, which is too large a value, and would contradict the experimental data.

Assuming that the large terms in (110) cancel, we have

$$\Delta H_1 = \frac{2\bar{\rho}(t_n)}{3}\left[A_{04}\left(\ln\frac{1}{\mu t_c} - \frac{1}{2}\right) + \frac{A_{04}}{2} + b_1\ln\frac{1}{\mu t_c}\right]. \quad (112)$$

Because in the period of nucleosynthesis $\Delta H_1/\bar{H} \sim (\bar{\rho}(t_n)/\mu)^{1/2}$ is a small number, contribution from the early cosmology would result in a small variation of $\delta X_4/X_4$.

11. Gravitons bouncing to the brane

In this section, in the framework of the brane moving in the static space-time we present arguments that in the period of early cosmology gravitons can bounce back to the brane.

Using Friedmann equation, $\left(\dot{R}_b/R_b\right)^2 = \rho^2 + 2\mu\rho$ the equation for the brane trajectory can be written as (Langlois & Sorbo, 2003)

$$\frac{dR_b}{dT_b} = \frac{\mu^2 R_b^2 \dot{R}_b}{\xi\sqrt{\mu^2 R_b^2 + \dot{R}_b^2}} = \xi\beta\mu^2 R_b^2\frac{H}{\sqrt{\mu^2+H^2}}, \quad (113)$$

where $\beta = sign(R_b(t))$. Expanding the right hand side of (113) in powers of $\mu/\rho < 1$, we obtain

$$\frac{dR_b}{dT_b} \simeq \xi\beta\mu^2 R_b^2\left(1 - \frac{\mu^2}{2\rho^2}\right). \quad (114)$$

Using that $\rho(t) = \rho(t_1)(R_b(t_1)/R_b(t))^4$, we have

$$\left(1 - \frac{\mu^2}{2\rho^2}\right)^{-1} \simeq 1 + \frac{\mu^2 R_b^8(t)}{2\rho^2(t_1) R_b^8(t_1)}.$$

Integrating Eq. (113) with the initial conditions $R_b = R_b(t_1),\ T_b = T_b(t_1)$, we obtain

$$\xi\beta\mu^2(T_b(t) - T_b(t_1)) = -\frac{1}{R_b(t)} + \frac{1}{R_b(t_1)} + \frac{\mu^2}{14(\rho(t_1)R_b^4(t_1))^2}\left(R_b^7(t) - R_b^7(t_1)\right). \tag{115}$$

Gravitons propagate along the null geodesics which are found from the geodesic equations (14)-(16). The tangent vectors to a null geodesic satisfy the relation

$$g_{ij}\frac{dx^i}{d\lambda}\frac{dx^j}{d\lambda} = 0, \tag{116}$$

where, to distinguish the case of the null geodesics, we relabeled the affine parameter from y to λ. Substituting in (116) solution (17), we obtain that $C^R = 0$. The tangent vectors to a null geodesic are

$$\frac{d\tilde{T}}{d\lambda} = \frac{C^T}{f(\tilde{R})}, \qquad \frac{d\tilde{x}^a}{d\lambda} = \frac{C^a}{\mu^2\tilde{R}^2}, \qquad \frac{d\tilde{R}}{d\lambda} = \nu|C^T|\,(1-\gamma)^{1/2}, \tag{117}$$

where *tilde* indicates that the point is at the null geodesic, $\nu = \pm 1$. From the equations (117) we obtain

$$\frac{d\tilde{R}}{d\tilde{T}} = \nu\kappa(1-\gamma)^{1/2}\mu^2\tilde{R}^2, \tag{118}$$

where $\kappa \equiv sign(C^T)$. Integrating this equation with the initial conditions $\tilde{R} = R_b(t_1),\ \tilde{T} = T_b(t_1)$, we have

$$\frac{1}{R_b(t_1)} - \frac{1}{\tilde{R}} = \nu\kappa(1-\gamma)^{1/2}\mu^2(\tilde{T} - T_b(t_1)). \tag{119}$$

To find, if at a time t the graviton trajectory returns to the brane world sheet, i.e. $\tilde{R} = R_b(t)$ and $\tilde{T} = T_b(t)$, we combine equations (115) and (119). In the case $\xi\beta = \nu\kappa$ we obtain

$$[(1-\gamma)^{-1/2} - 1]\left(\frac{1}{R_b(t_1)} - \frac{1}{R_b(t)}\right) = \frac{\mu^2}{14\rho^2(t_1)}\frac{1}{R_b(t_1)}\left(\left(\frac{R_b(t)}{R_b(t_1)}\right)^7 - 1\right). \tag{120}$$

Eq. (120) means that graviton emitted from the brane at the time t_1 has bounced back on the brane at the time t. Setting $z \equiv R_b(t_1)/R_b(t)$, and $\hat{\gamma}/2 \equiv [(1-\gamma)^{-1/2} - 1]$, and noting that in the period of early cosmology $\mu/\rho(t) \simeq 4\mu t \ll 1$, we rewrite (120) as

$$\frac{(1-z^7)}{z^7(1-z)} = \frac{14\hat{\gamma}\rho^2(t_1)}{\mu^2} \simeq \frac{14\hat{\gamma}}{(4\mu t_1)^2} \tag{121}$$

The function $(1-z^7)z^{-7}(1-z)^{-1}$ is monotone decreasing with the minimum at $z = 1$, and Eq. (121) has a unique solution provided $2\hat{\gamma}/(4\mu t_1)^2 > 1$.

From these relations, using that in the period of early cosmology $z = R_b(t_1)/R_b(t) \simeq (t_1/t)^{1/4}$, is calculated the time of the bounce. The time of the bounce is small, if $z \sim 1$, or $\gamma/(4\mu t)^2 \sim 1$, that is if gravitons are emitted at small angles to the brane.

12. Conclusion

In a model of 3-brane embedded in 5D space-time we calculated graviton emission of interacting hot matter on the brane in the bulk. Reliable calculations can be performed in the period of late cosmology, when $\rho(T)/\mu < 1$, where $\rho(T)$ is the normalized radiation energy density of matter on the brane, $\mu = \sqrt{-\Lambda/6}$ is the scale of the warping factor in the metric, Λ is the 5D cosmological constant. For $\mu \sim 10^{-12} GeV$, which we adopted in this paper, the limiting temperatures of the Universe at which the approximation of late cosmology is valid are of order $T_l \sim 5 \cdot 10^2 GeV$. In the period of late cosmology it was possible to make a number of approximations, which enabled us to obtain the analytic expression for the energy loss from the brane to the bulk.

The 5D model of the present paper can be treated in two alternative approaches - with moving brane in the static 5D bulk and with static brane in 5D bulk with time-dependent metric. Each picture proves to be useful for particular problems. We establish explicit connection between two pictures.

From the system of the Einstein equations containing the terms due to graviton emission we obtained the generalized Friedmann equation. The Einstein equations and Friedmann equation were solved perturbatively, the leading-order solution being that without the graviton emission. We have shown that in the period of late cosmology corrections to the leading-order approximation are small.

Graviton emission changes cosmological evolution of matter on the brane and thus production of light elements in primordial nucleosynthesis. Solving the system of the generalized Friedmann and the 5D energy conservation equations, which included the graviton emission terms, we found the difference of the mass fractions of ^4He produced in primordial nucleosynthesis calculated in the models with and without the graviton emission

$$\frac{\delta X_4}{X_4} \sim C\sqrt{\frac{\rho(T_n)}{\mu}}.$$

Here $\rho(T)$ is is the radiation energy density of matter on the brane, $T_n \sim 10^{-3} GeV$ is the freezing temperature of the reaction $p \leftrightarrow n$, and $C \ll 1$ is a small number. The ratio $\rho(T_n)/\mu \sim 10^{-26}$ is a small number, and thus correction to abundance of ^4He due to the graviton emission is small.

To estimate the effect of the graviton emission in the period of early cosmology on Helium production, we assumed that the collision integral in the Boltzmann equation obtained for the period of late cosmology qualitatively retains its form in the period of early cosmology. As the upper limit of temperatures of matter on the brane at which we estimate the graviton production, we take the reheating temperature $\sim 10^6 GeV$. It appears that straightforward calculation of abundance of ^4He produced in the early cosmology is too large to comply with the data. In the period of early cosmology a new effect becomes important (Hebecker & March-Russel, 2001; Langlois & Sorbo, 2003): gravitons emitted from the brane can bounce back to the brane. Assuming that the large terms in the components of the energy-momentum tensors from the emitted and bouncing gravitons mutually cancel, we find that the graviton emission in the period of early cosmology yields a small correction to abundance of ^4He calculated in the standard cosmological model.

13. References

Binetruy, P., Deffayet, C. & Langlois, D. (2000). Non-conventional cosmology from a brane-universe, *Nucl. Phys.* B565, 269-287. [hep-th/9905012]

Binetruy, P. et al. (2000). Brane cosmological evolution in a bulk with cosmological constant, *Phys. Lett.* B477, 285-291. [hep-th/9910219]

Birmingham, D. (1999). Topological Black Holes in Anti-de Sitter Space. *Class. Quant. Grav.* 16, 1197-1205. [hep-th/9808032],

Chamblin, H.A. & Reall, H.S. (1999) Dynamic Dilatonic Domain Walls. *Nucl. Phys.* B562, 133-157. [hep-th/9903225],

Collins, H. & Holdom, B. (2000). Brane Cosmologies without Orbifolds, *Phys. Rev.* D62, 105009. [hep-ph/0003173]

Durrer, R. (2005). Braneworlds, *XI Brazilian School of Cosmology and Gravitation, AIP Conf. Proc.* 782. [hep-th/0507006]

Gorbunov, D. & Rubakov, V. (2008). *Introduction to the theory of early Universe*, ISBN 978-5-382-00657-4, Moscow.

Hebecker, A. & March-Russel, J. (2001) Randall-Sundrum II Cosmology, AdS/CFT, and the Bulk Black Hole, *Nucl. Phys.* B608, 375-393. [hep-ph/0103214]

Iofa, M. (2009)a. Constraints on parameters of models with extra dimension from primordial nucleosynthesis. *Mod. Phys. Lett.* A24, 2307-2318. [0907.0547]

Iofa, M. (2009)b. Cosmological constraints on parameters of one-brane models with extra dimension. *JCAP*, 11, 023. [0907.4039]

Iofa, M. (2011). Graviton emission from the brane in the bulk in a model with extra dimension *JCAP*, 06, 025. [1012.2445]

Kolb, E. & Turner, M. (1990). *The Early Universe*. Addison-Wesley, Frontiers in Physics, Redwood City.

Kraus, P. (1999) Dynamics of Anti-de Sitter Domain Walls. *JHEP* 9912 (1999) 011, [hep-th/9910149]

Langlois, D.; Sorbo, L. & Rodriguez-Martinez, M. (2002). Cosmology of a brane radiating gravitons into the extra dimension. *Phys. Rev. Lett.* 89, 171301. [hep-th/0206146]

Langlois, D. & Sorbo, L. (2003) Bulk gravitons from a cosmological brane. *Phys. Rev.* D68, 084006. [hep-th/0306281]

Maartens, R. (2004). Brane-world gravity. *Liv. Rev. Rel.* 7, 7. [gr-qc/0312059]

Mielczarek, J. (2009) Reheating temperature from the CMB. *Phys. Rev.* D83, 023502. [astro-ph/1009.2359]

Mukohyama, S., Shiromizu,T. & Maeda, K-i (2000). Global structure of exact cosmological solutions in the brane world. *Phys. Rev.* D62 (2000) 024028. [arXiv:hep-th/9912287]

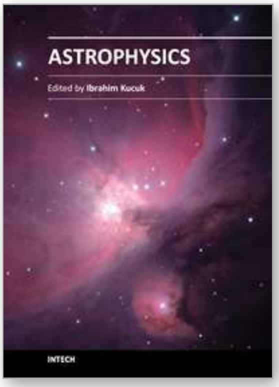
Randall, L. & Sundrum, R. (1999) An Alternative to Compactification. *Phys. Rev. Lett.* B83, 4690-4693. [arXiv:hep-th/9906064]

Rubakov, V. (2001). Large and infinite extra dimensions. *Phys. Usp.* 44, 871-893. [hep-ph/0104152]

Shiromizu, T; Maeda, K-i. & Sasaki, M. (2000) The Einstein equations on the 3-brane world. *Phys. Rev.* D62, 024012. [gr-qc/9910076]

Steigman, G.; Walker, T. & Zentner, A. (2001) Global Constraints On Key Cosmological Parameters. [astro-ph/0012149]

Tanaka, T. & Himemoto, Y. (2003) Generation of dark radiation in the bulk inflaton model, *Phys. Rev.* D67 (2003) 104007 [gr-qc/0301010]



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