13

Asteroseismology of Vibration Powered Neutron Stars

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1. Introduction

There is a general recognition today that basic features of asteroseismology of non-convective final stage (FS) stars, such as white dwarfs, neutron stars and strange quark star, can be properly understood working from the model of vibrating solid star, rather than the liquid star model lying at the base of asteroseismology of convective main-sequence (MS) stars. In accord with this, most of current investigations of an FS-star vibrations rests on principles of solid-mechanical theory of continuous media, contrary to the study of the MS-star vibrations vibrations which are treated in terms of fluid-mechanical theory. This means that super dense matter of FS-stars (whose gravitational pressure is counterbalanced by degeneracy pressure of constituting Fermi-matter), possesses elasticity and viscosity generic to solid state of condensed matter, whereas a fairly dilute matter of the MS-stars (whose internal pressure of self-gravity is opposed by radiative pressure) possesses property of fluidity which is generic to the liquid state of a highly conducting condensed matter. This feature of the MS-star matter plays crucial role in generation of their magnetic fields in the dynamo processes involving macroscopic flows which are supported by energy of nuclear reactions in the central, reactive zone, of these stars. In the meantime, in the finale-stage (FS) stars, like white dwarfs and neutron stars, there are no nuclear energy sources to support convection. The prevailing today view, therefore, is that a highly stable to spontaneous decay dipolar magnetic fields of neutron stars are fossil. The extremely large intensity of magnetic fields of degenerate solid stars is attributed to amplification of fossil magnetic field in the magnetic-flux-conserving process of core-collapse supernova.

Even still before discovery of neutron stars, it has been realized that, for absence of nuclear sources of energy, the radiative activity of these of compact objects should be powered by energy of either rotation or vibrations and that the key role in maintaining the neutron star radiation should play an ultra strong magnetic field. As is commonly know today, the neutron star capability of accommodating such a field is central to understanding pulsating character of magneto-dipole radiation of radio pulsars whose radiative power is provided, as is commonly believed, by the energy of rigid-body rotation. The discovery of soft gamma-ray repeaters and their identification with magnetars (1) – quaking neutron stars endowed with ultra strong magnetic fields experiencing decay – has stimulated enhanced interest in the study of models of quake-induced magneto-mechanical seismic vibrations of neutron star and resultant electromagnetic radiation. Of particular interest in this study are torsional magneto-mechanical vibrations about axis of magnetic dipole moment of the star driven by forces of magnetic-field-dependent stresses in a perfectly conducting matter and in a permanently magnetized non-conducting matter (2). Most, if not all, reported up to now computations of frequency spectra of poloidal and toroidal Alfvén vibration modes in pulsars and magnetars rest on tacitly adopted assumption about constant-in-time magnetic field in which a perfectly conducting neutron star matter undergoes Lorentz-force-driven oscillations (3–12) (see, also, references therein). A special place in the study of the above Alfvén modes of pure shear magneto-mechanical seismic vibrations (a-modes) occupies a homogeneous model of a solid star with the uniform density ρ and frozen-in poloidal static magnetic field of both homogeneous and inhomogeneous internal and dipolar external configuration. The chapter is organized as follows. In section 2 with a brief outline of the solid-star model undergoing node-free torsional Alfvén vibrations in uniform internal (and dipolar external) magnetic field of constant intensity. In section 3, a model is extended to the case of vibrations in magnetic field experiencing decay. Here emphasis is placed on the loss of vibration energy caused by depletion of internal magnetic field pressure and resulting vibration-energy powered magneto-dipole radiation of vibrating neutron star. The decreasing of magnetic field pressure in the star is presumed to be caused by coupling between vibrating star and outgoing material which is expelled by quake, but mechanisms of star-envelope interaction resulting in the decay of magnetic field, during the time of vibrational relaxation, are not considered. It is shown that physically meaningful inferences regarding radiative activity of quaking neutron star can be made even when detailed mechanisms of depletion of magnetic field pressure in the process of vibrations triggered by quakes are not exactly known. This statement is demonstrated by a set of representative examples of magnetic field decay. The basic results of the model of vibration powered neutron star are summarized in section 4 with emphasis of its relevance to astrophysics of magnetars.

2. Lorentz-force-driven torsion vibrations of neutron star

To gain better understanding of the basic physics behind the interconnection between seismic and radiative activity of quaking neutron star, we start with a brief outline a fiducial model of a solid star with frozen-in homogeneous internal magnetic field

$$\mathbf{B}(\mathbf{r}) = B\,\mathbf{b}(\mathbf{r}), \quad B = \text{constant}, \tag{1}$$
$$\mathbf{b}(\mathbf{r}) = [b_r = \cos\theta, b_\theta = -\sin\theta, b_\phi = 0]$$

and dipolar external magnetic field, as is shown in Fig.1. In the last equation, B is the field intensity [in Gauss] and $\mathbf{b}(\mathbf{r})$ stands for the dimensionless vector-function of spatial distribution of the field.

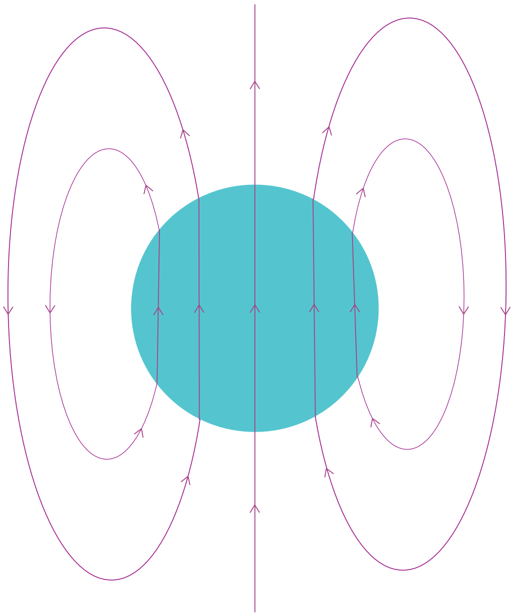

Fig. 1. The lines of magnetic field in fiducial model of neutron star with dipolar external and homogeneous internal magnetic field.

The above form of $\mathbf{B}(\mathbf{r})$ has been utilized in recent works(10–12) in which the discrete spectra of frequencies of node-free torsional Alfvén oscillations has been computed in analytic form on the basis of equations of linear magneto-solid mechanics

$$\rho\ddot{\mathbf{u}} = \frac{1}{c}[\delta\mathbf{j} \times \mathbf{B}], \quad \nabla \cdot \mathbf{u} = 0, \tag{2}$$

$$\delta\mathbf{j} = \frac{c}{4\pi}[\nabla \times \delta\mathbf{B}],\ \delta\mathbf{B} = \nabla \times [\mathbf{u} \times \mathbf{B}],\ \nabla \cdot \mathbf{B} = 0. \tag{3}$$

Here $\mathbf{u} = \mathbf{u}(\mathbf{r}, t)$ is the field of material displacement, the fundamental dynamical variable of solid-mechanical theory of elastically deformable (non-flowing) material continua. These equations describe Lorentz-force-driven non-compressional vibrations of a perfectly conducting elastic matter of a non- convective solid star with Ampére form of fluctuating current density $\delta\mathbf{j}$. Equation for $\delta\mathbf{B}$ describing coupling between fluctuating field of material displacements $\mathbf{u}$ and background magnetic field $\mathbf{B}$ pervading stellar material is the mathematical form of Alfvén theorem about frozen-in lines of magnetic field in perfectly conducting matter. The adopted for Alfvén vibrations of non-convective solid stars terminology [which is not new of course, see for instance(13), namely, equations of magneto-solid mechanics and/or solid-magnetics is used as a solid-mechanical counterpart of well-known terms like equations of magneto-fluid mechanics, magnetohydrodynamics (MHD) and hydromagnetics $\rho\delta\dot{\mathbf{v}} = (1/c)[\delta\mathbf{j} \times \mathbf{B}]$, $\delta\mathbf{j} = (c/4\pi)[\nabla \times \delta\mathbf{B}]$, $\delta\dot{\mathbf{B}} = \nabla \times [\delta\mathbf{v} \times \mathbf{B}]$, describing flowing magento-active plasma in terms of the velocity of fluctuating flow $\delta\mathbf{v} = \dot{\mathbf{u}}$ and fluctuating magnetic field $\delta\mathbf{B}$. The MHD approach is normally utilized in astrophysics of convective main-sequence (MS) liquid stars such, for instance, as rapidly oscillating Ap (roAp) stars, chemically peculiar magnetic stars exhibiting high-frequency oscillations that have been and still remain the subject of extensive investigation(14; 15).

Inserting (2) in (3) the former equation of solid-magnetics takes the form

$$\rho\,\ddot{\mathbf{u}}(\mathbf{r},t) = \frac{1}{4\pi}[\nabla\times[\nabla\times[\mathbf{u}(\mathbf{r},t)\times\mathbf{B}(\mathbf{r})]]]\times\mathbf{B}(\mathbf{r}). \quad (4)$$

The analogy between perfectly conducting medium pervaded by magnetic field (magneto-active plasma) and elastic solid, regarded as a material continuum, is strengthened by the following tensor representation of the last equation $\rho\,\ddot{u}_i = \nabla_k \delta M_{ik}$, where $\delta M_{ik} = (1/4\pi)[B_i\delta B_k + B_k\delta B_i - B_j\delta B_j\delta_{ik}]$ is the Maxwellian tensor of magnetic field stresses with $\delta B_i = \nabla_k[u_i B_k - u_k B_i]$. This form is identical in appearance to canonical equation of solid-mechanics $\rho\,\ddot{u}_i = \nabla_k\sigma_{ik}$, where $\sigma_{ik} = 2\mu\, u_{ik} + [\kappa - (2/3)\mu]\, u_{jj}\delta_{ik}$ is the Hookean tensor of mechanical stresses and $u_{ik} = (1/2)[\nabla_i\, u_k + \nabla_k\, u_i]$ is the tensor of shear deformations in an isotropic elastic continuous matter with shear modulus μ and bulk modulus κ (having physical dimension of pressure). The most prominent manifest of physical similarity between these two material continua is their capability of transmitting non-compressional perturbation by transverse waves. Unlike incompressible liquid, an elastic solid can respond to impulsive non-compressional load by transverse waves of shear material displacements traveling with the speed $c_t = \sqrt{\mu/\rho}$. The unique feature of an incompressible perfectly conducting and magnetized continuous matter (in liquid or solid aggregated state) is the capability of transmitting perturbation by transverse magneto-mechanical, Alfvén, wave in which mechanical displacements of material and fluctuations of magnetic field undergo coupled oscillations traveling with the speed $v_A = B/\sqrt{4\pi\rho} = \sqrt{2P_B/\rho}$ (where $P_B = B^2/8\pi$ is magnetic field pressure) along magnetic axis. It is stated, therefore, that magnetic field pervading perfectly conducting medium imparts to it a supplementary portion of solid-mechanical elasticity (16; 17). This suggests that hydromagnetic Alfvén vibrations of a spherical mass of a perfectly conducting matter with frozen-in magnetic field can be specified in a manner of eigenstates of elastic vibrations of a solid sphere. As for the general asteroseismology of compact objects is concerned, the above equations seems to be appropriate not only for neutron stars but also white dwarfs(18) and quark stars. The superdense material of these latter yet hypothetical compact stars is too expected to be in solid state(19; 20).

Equation (4) serves as a basis of our further analysis. The studied in above works regime of node-free torsion vibrations under the action of Lorentz restoring force is of some interest in that the rate of differentially rotational material displacements

$$\dot{\mathbf{u}}(\mathbf{r},t) = [\boldsymbol{\omega}(\mathbf{r},t)\times\mathbf{r}], \quad \boldsymbol{\omega}(\mathbf{r},t) = [\nabla\chi(\mathbf{r})]\,\dot{\alpha}(t), \quad (5)$$

$$\nabla^2\chi(\mathbf{r}) = 0, \quad \chi(\mathbf{r}) = f_\ell\, P_\ell(\cos\theta), \quad f_\ell(r) = A_\ell\, r^\ell \quad (6)$$

has one and the same form as in torsion elastic mode of node-free vibrations under the action of Hooke's force of mechanical shear stresses. Hereafter $P_\ell(\cos\theta)$ stands for Legendre polynomial of degree ℓ specifying the overtone of toroidal a-mode. Fig.2 shows quadrupole and octupole overtones of such vibrations. The time-dependent amplitude $\alpha(t)$ describes temporal evolution of above vibrations; the governing equation for $\alpha(t)$ is obtained form

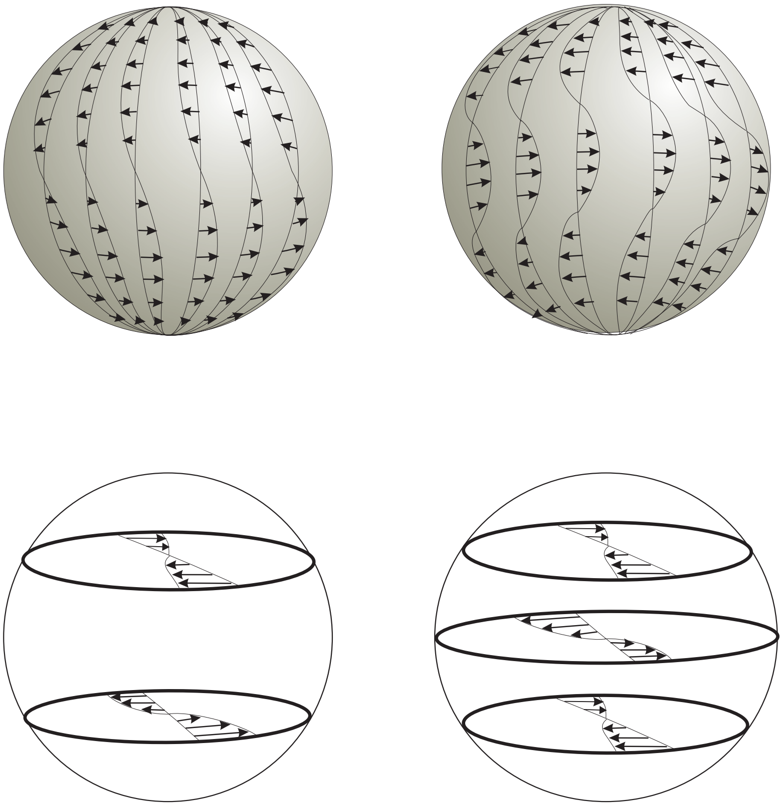

Fig. 2. The fields of material displacements in a neutron star undergoing torsional quadrupole (left) and octupole (right) node-free vibrations about magnetic axis.

equation (4). The prime purpose of above works was to get some insight into difference between spectra of discrete frequencies of toroidal *a*-modes in neutron star models having one and the same mass M and radius R, but different shapes of constant-in-time poloidal magnetic fields. By use of the energy method, it was found that each specific form of spatial configuration of static magnetic field about axis of which the neutron star matter undergoes nodeless torsional oscillations is uniquely reflected in the discrete frequency spectra by form of dependence of frequency upon overtone ℓ of nodeless vibration. It worth noting that first computation of discrete spectra of frequencies of toroidal Alfvén stellar vibrations in the standing wave-regime, has been reported by Chandrasekhar(23). The extensive review of other earlier computations of discrete frequency spectra of *a*-modes, $\omega_\ell = \omega_A s_\ell = B\kappa_\ell$, is given in well-known review of Ledoux and Walraven(14). The assumption about constant in time undisturbed magnetic field means that the internal magnetic field pressure, P_B, the velocity v_A of Alfvén wave in the star bulk

$$P_B = \frac{B^2}{8\pi}, \quad v_A = \sqrt{\frac{2P_B}{\rho}} \tag{7}$$

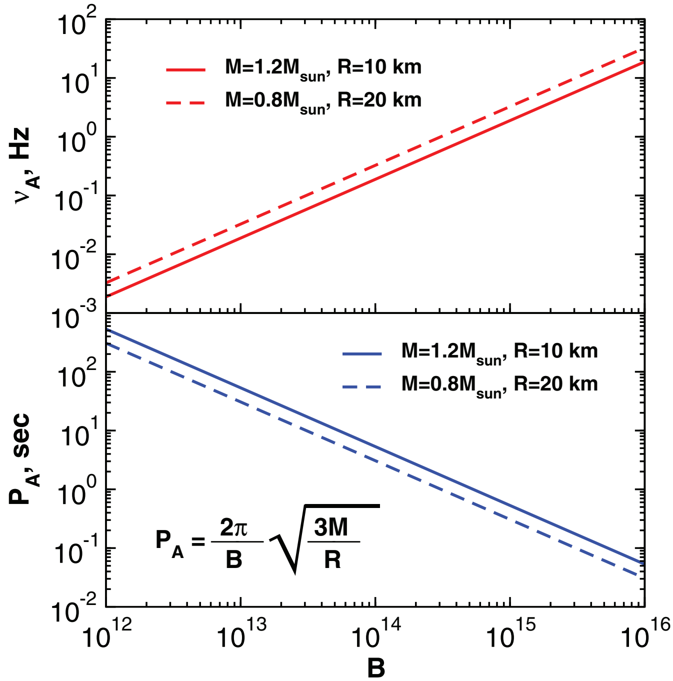

Fig. 3. The basic frequency and period of global Alfvén oscillations, equations (8), as functions of magnetic field intensity in the neutron star models with indicated mass and radius.

and, hence, the frequency $\nu_A = \omega_A/2\pi$ (where $\omega_A = v_A/R$) and the period $P_A = \nu_A^{-1}$ of global Alfvén oscillations

$$\nu_A = \frac{B}{2\pi}\sqrt{\frac{R}{3M}}, \quad P_A = \frac{2\pi}{B}\sqrt{\frac{3M}{R}} \tag{8}$$

remain constant in the process of vibrations whose amplitude $\alpha(t)$ subjects to standard equation of undamped harmonic oscillator(10; 11). The allow for viscosity of stellar material leads to exponential damping of amplitude, but the frequency ν_A and, hence, the period P_A preserve one and the same values as in the case of non-viscous vibrations(12). In Fig.3 these latter quantities are plotted as functions of intensity B of undisturbed poloidal magnetic field in the neutron star models with indicated mass M and radius R. The practical usefulness of chosen logarithmic scale in this figure is that it shows absolute vales of ν_A and P_A for global vibrations of typical in mass and radius neutron stars.

In what follows, we relax the assumption about constant-in-time magnetic field and examine the impact of its decay on the vibration energy and period. A brief analysis of such a case has been given in the context of magnetic white dwarfs(21). In this paper we present a highly extensive consideration of this problem in the context of neutron stars with emphasis on its relevance to the post-quake radiation of magnetars.

3. Vibration powered neutron star

In the following we consider a model of a neutron star with time-dependent intensity of homogeneous poloidal magnetic field which can be conveniently represented in the form

$$\mathbf{B}(\mathbf{r},t) = B(t)\,\mathbf{b}(\mathbf{r}), \quad [b_r = \cos\theta, b_\theta = -\sin\theta, b_\phi = 0]. \tag{9}$$

On account of this the equation of solid-magnetics, (4), takes the form

$$\rho\,\ddot{\mathbf{u}}(\mathbf{r},t) = \frac{B^2(t)}{4\pi}\,[\nabla\times[\nabla\times[\mathbf{u}(\mathbf{r},t)\times\mathbf{b}(\mathbf{r},t)]]]\times\mathbf{b}(\mathbf{r}). \tag{10}$$

Inserting here the following separable form of fluctuating material displacements

$$\mathbf{u}(\mathbf{r},t) = \mathbf{a}(\mathbf{r})\,\alpha(t) \tag{11}$$

we obtain

$$\{\rho\,\mathbf{a}(\mathbf{r})\}\,\ddot{\alpha}(t) = 2P_B(t)\,\times\{[\nabla\times[\nabla\times[\mathbf{a}(\mathbf{r})\times\mathbf{b}(\mathbf{r})]]]\times\mathbf{b}(\mathbf{r})\}\,\alpha(t). \tag{12}$$

Scalar product of (12) with the time-independent field of instantaneous displacements $\mathbf{a}(\mathbf{r})$ followed by integration over the star volume leads to equation for amplitude $\alpha(t)$ having the form of equation of oscillator with depending on time spring constant

$$\mathcal{M}\ddot{\alpha}(t) + \mathcal{K}(t)\alpha(t) = 0, \tag{13}$$

$$\mathcal{M} = \rho\,m_\ell, \quad \mathcal{K}(t) = 2P_B(t)\,k_\ell, \tag{14}$$

$$m_\ell = \int \mathbf{a}(\mathbf{r})\cdot\mathbf{a}(\mathbf{r})\,d\mathcal{V},\ \mathbf{a} = A_t\nabla\times[\mathbf{r}\,r^\ell\,P_\ell(\cos\theta)], \tag{15}$$

$$k_\ell = \int \mathbf{a}(\mathbf{r})\cdot[\mathbf{b}(\mathbf{r})\times[\nabla\times[\nabla\times[\mathbf{a}(\mathbf{r})\times\mathbf{b}(\mathbf{r})]]]]d\mathcal{V}. \tag{16}$$

The solution of equation of non-isochronal (non-uniform in duration) and non-stationary vibrations with time-dependent frequency $[\ddot{\alpha}(t) + \omega^2(t)\alpha(t) = 0$ where $\omega^2(t) = \mathcal{K}(t)/\mathcal{M}]$ is non-trivial and fairly formidable task(22). But solution of such an equation, however, is not a prime purpose of this work. The main subject is the impact of depletion of magnetic-field-pressure on the total energy of Alfvén vibrations $E_A = (1/2)[\mathcal{M}\dot{\alpha}^2 + \mathcal{K}\alpha^2]$ and the discrete spectrum of frequency of the toroidal a-mode

$$\omega_\ell^2(t) = \omega_A^2(t)\,s_\ell^2,\ \omega_A^2(t) = \frac{v_A^2(t)}{R^2},\ s_\ell^2 = \frac{k_\ell}{m_\ell}\,R^2, \tag{17}$$

$$\omega_\ell^2(t) = B^2(t)\kappa_\ell^2, \quad \kappa_\ell^2 = \frac{s_\ell^2}{4\pi\rho R^2}, \quad s_\ell^2 = \left[(\ell^2-1)\frac{2\ell+3}{2\ell-1}\right], \quad \ell \geq 2. \tag{18}$$

It is to be stated clearly from the onset that it is not our goal here to speculate about possible mechanisms of neutron star demagnetization and advocate conceivable laws of magnetic field decay. The main purpose is to gain some insight into the effect of arbitrary law of magnetic field decay in quaking neutron star on period of Lorentz-force-driven torsional seismic vibrations (whose quadrupole and octupole overtones are pictured in Fig.2) and radiative activity of the star brought about by such vibrations. In the reminder of the paper we focus on the case of torsional Alfvén vibrations in quadrupole ($\ell = 2$) overtone. In so doing we omit index ℓ putting $\omega(t) = \omega_{\ell=2}(t)$.

3.1 Magnetic-field-decay induced loss of vibration energy

The total energy stored in quake-induced Alfvén seismic vibrations of the star is given by

$$E_A(t) = \frac{\mathcal{M}\dot{\alpha}^2(t)}{2} + \frac{\mathcal{K}(B(t))\alpha^2(t)}{2}, \tag{19}$$

$$\mathcal{K}(B(t)) = \omega^2(B(t))\mathcal{M}. \tag{20}$$

Perhaps most striking consequence of the magnetic-field-pressure depletion during the post-quake vibrational relaxation of neutron star is that it leads to the loss of vibration energy at a rate proportional to the rate of magnetic field decay

$$\frac{dE_A(t)}{dt} = \dot{\alpha}(t)[\mathcal{M}\ddot{\alpha}(t) + \mathcal{K}(B(t))\alpha(t)] + \frac{\alpha^2(t)}{2}\frac{d\mathcal{K}(B(t))}{dt}$$
$$= \frac{\mathcal{M}\alpha^2(t)}{2}\frac{d\omega^2(B)}{dB}\frac{dB(t)}{dt} = \mathcal{M}\kappa^2\alpha^2(t)B(t)\frac{dB(t)}{dt}. \tag{21}$$

In the model under consideration, a fairly rapid decay of magnetic field during the time of post-quake vibrational relaxation of the star is though of as caused, to a large extent, by coupling of the vibrating star with material expelled by quake. In other words, escaping material removing a part of magnetic flux density from the star is considered to be a most plausible reason of depletion of internal magnetic field pressure in the star. It seems quite likely that, contrary to viscous dissipation, the loss of vibration energy due to decay of magnetic field must be accompanied by coherent (non-thermal) electromagnetic radiation. Adhering to this supposition in the next section special consideration is given to the conversion of the energy of Lorentz-force-driven seismic vibrations into the energy of magneto-dipole emission whose flux oscillates with frequency of torsional Alfvén magneto-mechanical vibrations of the final stage solid stars, like magnetic white dwarf and neutron stars. Fig.4 replicates seismic torsional Alfvén vibrations of the star which are accompanied by oscillations of lines of dipolar magnetic field defining the beam direction of outburst X-ray emission.

3.2 Conversion of vibration energy into power of magneto-dipole radiation

The point of departure in the study of vibration-energy powered magneto-dipole emission of the star (whose radiation power, $\mathcal{P}$, is given by Larmor's formula) is the equation

$$\frac{dE_A(t)}{dt} = -\mathcal{P}(t), \quad \mathcal{P}(t) = \frac{2}{3c^3}\delta\ddot{\boldsymbol{\mu}}^2(t). \tag{22}$$

Consider a model of quaking neutron star whose torsional magneto-mechanical oscillations are accompanied by fluctuations of total magnetic moment preserving its initial (in seismically

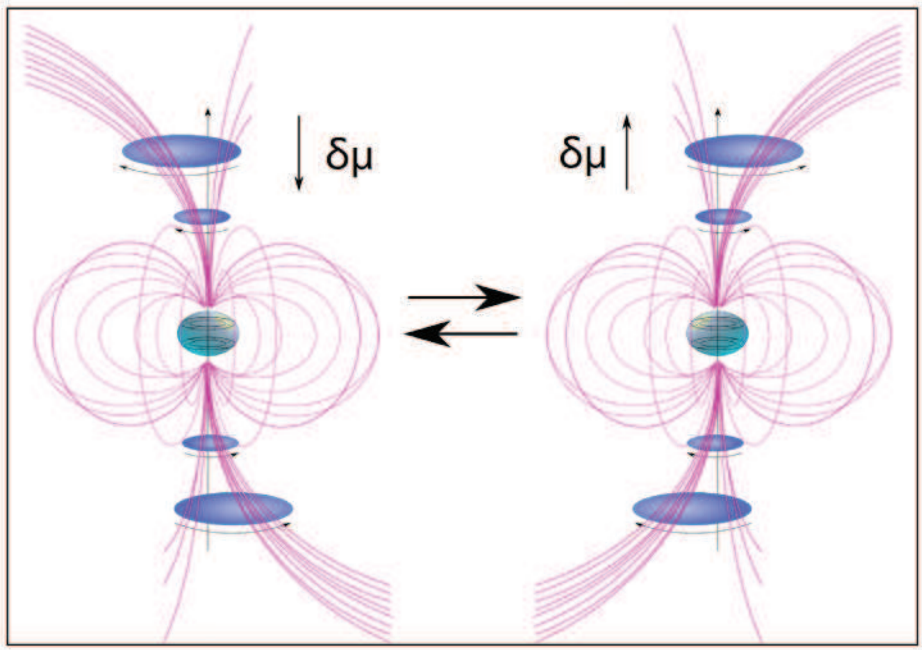

Fig. 4. Schematic view of quadrupole overtone of seismic torsional Alfvén vibrations of neutron star with homogeneous internal and dipolar external field whose lines, defining direction of outburst beam, oscillate with the frequency of this seismic a-mode.

quiescent state) direction: $\boldsymbol{\mu} = \mu\,\mathbf{n}$ = constant. The total magnetic dipole moment should execute oscillations with frequency $\omega(t)$ equal to that for magneto-mechanical vibrations of stellar matter which are described by equation for $\alpha(t)$. This means that $\delta\boldsymbol{\mu}(t)$ and $\alpha(t)$ must obey equations of similar form, namely

$$\delta\ddot{\boldsymbol{\mu}}(t) + \omega^2(t)\delta\boldsymbol{\mu}(t) = 0, \tag{23}$$

$$\ddot{\alpha}(t) + \omega^2(t)\alpha(t) = 0, \quad \omega^2(t) = B^2(t)\kappa^2. \tag{24}$$

It is easy to see that equations (23) and (24) can be reconciled if

$$\delta\boldsymbol{\mu}(t) = \boldsymbol{\mu}\,\alpha(t). \tag{25}$$

Then, from (23), it follows $\delta\ddot{\boldsymbol{\mu}} = -\omega^2\boldsymbol{\mu}\alpha$. Given this and equating

$$\frac{dE_A(t)}{dt} = \mathcal{M}\kappa^2\alpha^2(t)B(t)\frac{dB(t)}{dt} \tag{26}$$

with

$$-\mathcal{P} = -\frac{2}{3c^3}\mu^2\,\kappa^4 B^4(t)\alpha^2(t) \tag{27}$$

we arrive at the equation of time evolution of magnetic field

$$\frac{dB(t)}{dt} = -\gamma B^3(t), \quad \gamma = \frac{2\mu^2\kappa^2}{3\mathcal{M}c^3} = \text{constant} \tag{28}$$

which yields the following law of its decay

$$B(t) = \frac{B(0)}{\sqrt{1+t/\tau}}, \quad \tau^{-1} = 2\gamma B^2(0). \tag{29}$$

The lifetime of magnetic field τ is regarded as a parameter whose value is established from below given relations between the period P and its time derivative $\dot{P}$ which are taken from observations. Knowing from observations $P(t)$ and $\dot{P}(t)$ and estimating τ one can get information about the magnitude of total magnetic moment and the strength of undisturbed magnetic field.

It is worth noting that in the model of vibration-energy powered magneto-dipole emission under consideration, the equation of magnetic field evolution is obtained in similar fashion as equation for the angular velocity Ω does in the standard model of rotation-energy powered emission of neutron star. As is shown in the next section, the substantial physical difference between models of rotation-energy and vibration-energy powered pulsating emission of neutron stars is that in the model of quaking neutron star vibrating in toroidal a-mode, the elongation of period of pulses is attributed to magnetic field decay, whereas in canonical Pacini-Gold model of radio-pulsar the lengthening of period of pulses is ascribed to the slow down of the neutron star rotation(39; 40; 49).

3.3 Lengthening of vibration period

The immediate consequence of above line of argument is the magnetic-field-decay induced lengthening of vibration period

$$B(t) = \frac{B(0)}{\sqrt{1+t/\tau}} \;\rightarrow\; P(t) = \frac{C}{B(t)}, \; C = \frac{2\pi}{\kappa}, \tag{30}$$

$$P(t) = P(0)\,[1 + (t/\tau)]^{1/2}, \; P(0) = \frac{C}{B(0)}, \tag{31}$$

$$\dot{P}(t) = \frac{1}{2\tau}\frac{P(0)}{[1 + (t/\tau)]^{1/2}}. \tag{32}$$

It follows that lifetime τ is determined by

$$P(t)\dot{P}(t) = \frac{P^2(0)}{2\tau} = \text{constant}. \tag{33}$$

This inference of the model under consideration is demonstrated in Fig.5. The difference between periods evaluated at successive moments of time $t_1 = 0$ and $t_2 = t$ is given by

$$\begin{aligned} \Delta P(t) &= P(t) - P(0) \\ &= -P(0)\left[1 - \frac{B(0)}{B(t)}\right] > 0, \quad B(t) < B(0). \end{aligned} \tag{34}$$

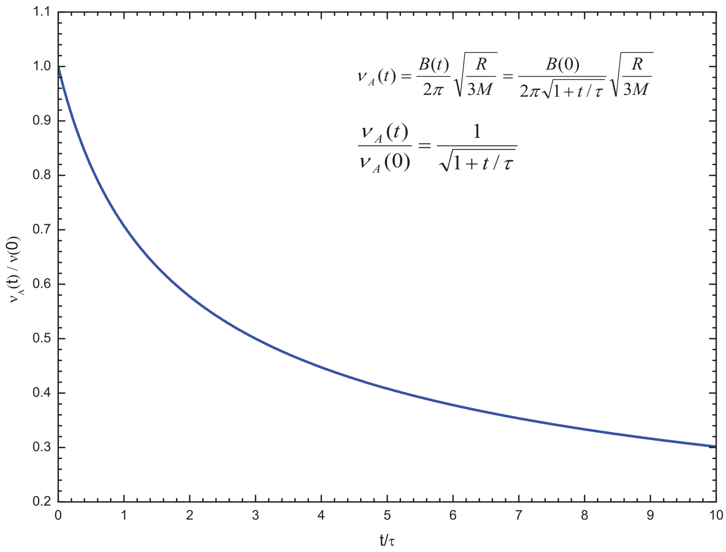

Fig. 5. Time evolution of Alfvén frequency of Lorentz-force-driven vibrations converting vibration energy into power of magneto-dipole radiation.

The practical usefulness of these general relations is that they can be used as a guide in search for fingerprints of Alfvén seismic vibrations in data on oscillating emission from quaking neutron star.

3.4 Time evolution of vibration amplitude

The considered model permits exact analytic solution of equation for vibration amplitude $\alpha(t)$ which is convenient to represent as

$$\ddot{\alpha}(t) + \omega^2(t)\alpha(t) = 0, \tag{35}$$

$$\omega^2(t) = \frac{\omega^2(0)}{1 + t/\tau}, \;\; \omega(0) = \omega_A \kappa. \tag{36}$$

The procedure is as follows. Let us introduce new variable $s = 1 + t/\tau$. In terms of $\alpha(s)$, equation (35) takes the form

$$s\alpha''(s) + \beta^2 \alpha(s) = 0, \quad \beta^2 = \omega^2(0)\tau^2 = \text{const} \tag{37}$$

This equation permits exact analytic solution(24)1.5mm

$$\alpha(s) = s^{1/2}\{C_1 J_1(2\beta s^{1/2}) + C_2 Y_1(2\beta s^{1/2})\} \tag{38}$$

where $J_1(2\beta s^{1/2})$ and $Y_1(2\beta s^{1/2})$ are Bessel functions(25)

$$J_1(z) = \frac{1}{\pi}\int_0^{\pi} \cos(z\sin\theta - \theta)d\theta, \tag{39}$$

$$Y_1(z) = \frac{1}{\pi}\int_0^{\pi} \sin(z\sin\theta - \theta)d\theta, \quad z = 2\beta s^{1/2}. \tag{40}$$

The arbitrary constants C_1 and C_2 can be eliminated from two conditions

$$\alpha(t=0) = \alpha_0, \quad \alpha(t=\tau) = 0. \tag{41}$$

where zero-point amplitude

$$\alpha_0^2 = \frac{2\bar{E}_A(0)}{M\omega^2(0)} = \frac{2\bar{E}_A(0)}{K(0)}, \quad \omega^2(0) = \frac{K(0)}{M} \tag{42}$$

is related to the average energy $\bar{E}_A(0)$ stored in torsional Alfvén vibrations at initial (before magnetic field decay) moment of time $t = 0$. Before magnetic field decay, the star oscillates in the harmonic in time regime, that is, with amplitude $\alpha = \alpha_0\cos(\omega(0)t)$, so that $< \alpha^2 >= (1/2)\alpha_0^2$ and $< \dot{\alpha}^2 >= (1/2)\omega^2(0)\alpha_0^2$. The average energy of such oscillations is given by equation $\bar{E}_A(0) = (1/2)M < \dot{\alpha}^2 > +(1/2)K(0) < \alpha^2 >= (1/2)M\omega^2(0)\alpha_0^2 = (1/2)K(0)\alpha_0^2$ which relates the energy stored in vibrations with vibration amplitude α_0. As a result, the general solution of (35) can be represented in the form

$$\alpha(t) = C\,[1 + (t/\tau)]^{1/2}$$

$$\times\{J_1(2\beta\,[1 + (t/\tau)]^{1/2}) - \eta\, Y_1(2\beta\,[1 + (t/\tau)]^{1/2})\}, \tag{43}$$

$$\eta = \frac{J_1(z(\tau))}{Y_1(z(\tau))}, \quad C = \alpha_0[J_1(z(0)) - \eta\, Y_1(z(0)]^{-1}. \tag{44}$$

The vibration period lengthening in the process of vibrations is illustrated in Fig.6 and Fig.7, where we plot $\alpha(t)$, equation (43), at different values of parameters β and η pointed out in the figures. Fig.7 shows that η is the parameter regulating magnitude of vibration amplitude, the larger η, the higher amplitude. However this parameter does not affect the rate of period lengthening. As it is clearly seen from Fig.6 both the elongation rate of vibration period and magnitude of vibration amplitude are highly sensitive to parameter β. All the above shows that the magnetic-field-decay induced loss of vibration energy is substantially different from the vibration energy dissipation caused by shear viscosity of matter resulting in heating of stellar material(26; 27). As was noted, the characteristic feature of this latter mechanism of vibration energy conversion into the heat (i.e., into the energy of non-coherent electromagnetic emission responsible for the formation of photosphere of the star) is that the frequency and, hence, period of vibrations are the same as in the case of viscous-free vibrations(12). However, it is no longer so in the case under consideration. It follows from above that depletion of magnetic field pressure resulting in the loss of total energy of Alfvén vibrations of the star causes its vibration period to lengthen at a rate proportional to the rate of magnetic field decay.

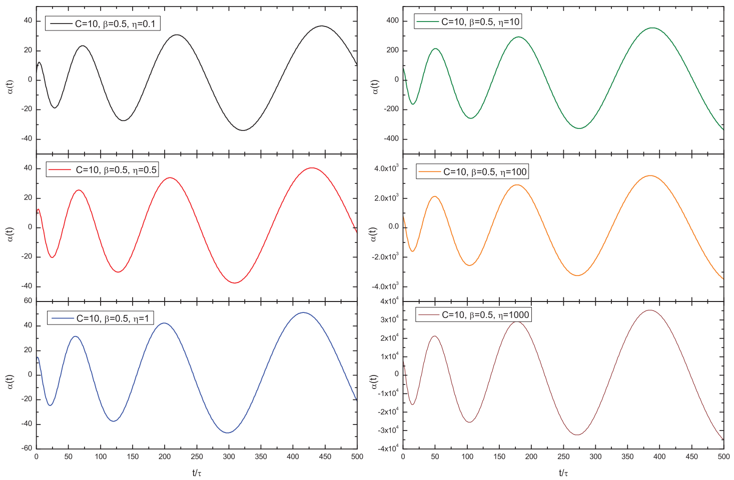

Fig. 6. Vibration amplitude $\alpha(t)$ computed at fixed C and β and different values of η. This shows that variation of this parameter is manifested in change of magnitude of $|\alpha|$, but period elongation is not changed.

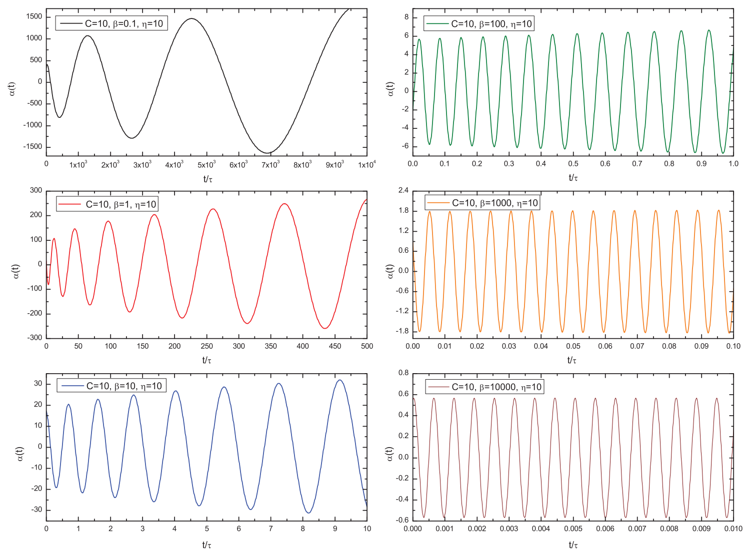

Fig. 7. Vibration amplitude $\alpha(t)$ computed at fixed C and η and different values of β.

	$M(M_\odot)$	R(km)	B(G)	ν_A(Hz)	τ(yr)
Pulsars	0.8	20	10^{12}	3.25×10^{-3}	4.53×10^{10}
	1.0	15	10^{13}	2.52×10^{-2}	2.98×10^{7}
Magnetars	1.1	13	10^{14}	0.22	7.4×10^{3}
	1.2	12	10^{15}	2.06	1.31
	1.3	11	10^{15}	1.89	2.38
	1.4	10	10^{16}	17.4	4.44×10^{-4}

Table 1. The Alfvén frequency of Lorentz-force-driven torsion vibrations, ν_A, and their lifetime equal to decay time of magnetic field, τ, in neutron stars with magnetic fields typical for pulsars and magnetars.

4. Oscillating luminosity of vibration powered neutron star

To get an idea of the magnitude of characteristic parameter of vibrations providing energy supply of magneto-dipole radiation of a neutron star, in Table 1 we present results of numerical computations of the fundamental frequency of neutron star oscillations in quadrupole toroidal a-mode and time of decay of magnetic field τ as functions of increasing magnetic field. As was emphasized, the most striking feature of considered model of vibration powered radiation is the lengthening of periods of pulsating emission caused by decay of internal magnetic field. This suggests that this model is relevant to electromagnetic activity of magnetars - neutron stars endowed with magnetic field of extremely high intensity the radiative activity of which is ultimately related to the magnetic field decay. Such a view is substantiated by estimates of Alfvén frequency presented in the table. For magnetic fields of typical rotation powered radio pulsars, $B \sim 10^{12}$ G, the computed frequency ν_A is much smaller than the detected frequency of pulses whose origin is attributed to lighthouse effect. In the meantime, for neutron stars with magnetic fields $B \sim 10^{14}$ G the estimates of ν_A are in the realm of observed frequencies of high-energy pulsating emission of soft gamma repeaters (SGRs), anomalous X-ray pulsars (AXPs) and sources exhibiting similar features. According to common belief, these are magnetars - highly magnetized neutron stars whose radiative activity is related with magnetic field decay. The amplitude of vibrations is estimated as (28)

$$\alpha_0 = \left[\frac{2\bar{E}_{\rm A}(0)}{\mathcal{M}\omega^2(0)}\right]^{1/2} = 3.423 \times 10^{-3} \bar{E}_{\rm A,40}^{1/2} B_{14}^{-1} R_6^{-3/2}. \quad (45)$$

where $\bar{E}_{\rm A,40} = \bar{E}_{\rm A}/(10^{40}$ erg) is the energy stored in the vibrations. $R_6 = R/(10^6$ cm) and $B_{14} = B/(10^{14}$ G). The presented computations show that the decay time of magnetic field (equal to duration time of vibration powered radiation in question) strongly depends on the intensity of initial magnetic field of the star: the larger magnetic field B the shorter time of radiation τ at the expense of energy of vibration in decay during this time magnetic field. The effect of equation of state of neutron star matter (which is most strongly manifested in different values mass and radius of the star) on frequency ν_A is demonstrated by numerical vales of this quantity for magnetars with one and the same value of magnetic field $B = 10^{15}$ G but different values of mass and radius. The luminosity powered by neutron star vibrations

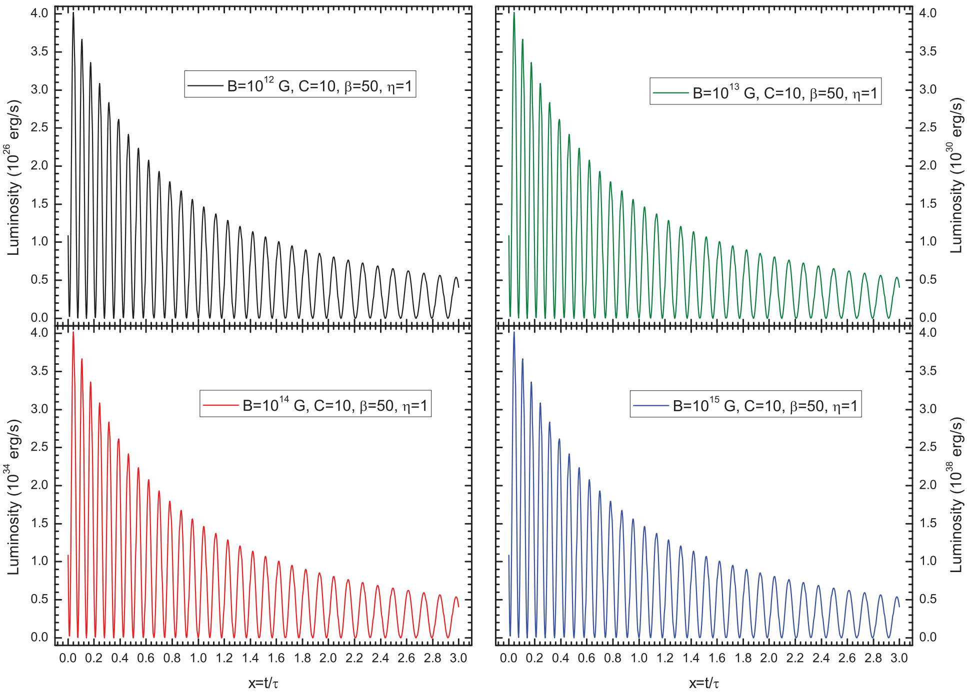

Fig. 8. Time evolution of luminosity of magneto-dipole radiation powered by energy of torsional Alfvén seismic vibrations of neutron star with mass $M = 1.2M_{\odot}$ and radius $R = 15$ km with intensities of magnetic field.

in quadrupole toroidal a-mode is given by

$$\mathcal{P} = \frac{\mu^2}{c^3} B^4(t)\alpha^2(t), \quad \mu = (1/2)B(0)R^3, \quad B(t) = B(0)[1 + t/\tau]^{-1/2}, \tag{46}$$

$$\alpha(t) = C\, s^{1/2}\{J_1(z(t)) - \eta\, Y_1(z(t))\}, \quad z = 2\omega(0)\tau\,(1 + t/\tau) \tag{47}$$

The presented in Fig.8 computations of power of magneto-dipole radiation of a neutron stars (of one and the same mass and radius but different values of magnetic fields) exhibit oscillating character of luminosity. The frequency of these oscillations equal to that of torsional Alfvén seismic vibrations of neutron star. All above suggests that developed theory of vibration-energy powered emission of neutron star is relevant to electromagnetic activity of magnetars - neutron stars endowed with magnetic field of extremely high intensity the radiative activity of which is ultimately related to the magnetic field decay. This subclass of highly magnetized compact objects is commonly associated with soft gamma repeaters and anomalous X-ray pulsars(29–32) – young isolated and seismically active neutron stars(33; 34). The magnetar quakes are exhibited by short-duration thermonuclear gamma-ray flash followed by rapidly oscillating X-ray flare of several-hundred-seconds duration. During this latter stage of quake-induced radiation of magnetar, a long-periodic (2-12 sec) modulation of brightness was observed lasting for about 3-4 minutes. Such long period of pulsed emission might be expected from old rotation-energy powered pulsars (due to slow down of rotation), but not from magnetars which are young neutron stars, as follows from their association with pretty young supernovae. Taking this into account (and also the fact that energy release during X-ray flare is much larger than the energy of rigid-body rotation with so long periods) it has been suggested(2) that detected long-periodic pulsating emission of magnetars is powered by the energy of torsional magneto-elastic vibrations triggered by quake. In view of key role of ultra-strong magnetic field it is quite likely that quasi-periodic oscillations (QPOs) of outburst flux from SGR 1806-20 and SGR 1900+14 detected in(35–37) are produced by torsional seismic vibrations predominately sustained by Lorentz force(10–12) which are accompanied, as was argued above, by monotonic decay of background magnetic field. If so, the predicted elongation of QPOs period of oscillating outburst emission from quaking magnetars should be traced in existing and future observations.

4.1 Rotation powered neutron star

For a sake of comparison, in the considered model of vibration powered radiation, the equation of magnetic field evolution is obtained in similar fashion as that for the angular velocity $\Omega(t)$ does in the standard model of rotation powered neutron star which rests on equations (38)

$$\frac{dE_R}{dt} = -\frac{2}{3c^3}\delta\ddot{\boldsymbol{\mu}}^2(t), \tag{48}$$

$$E_R(t) = \frac{1}{2}I\,\Omega^2(t), \quad I = \frac{2}{5}MR^2 \tag{49}$$

One of the basic postulates of the model of rotation-energy powered emission is that the time evolution of total magnetic moment of the star is governed by the equation

$$\delta\ddot{\boldsymbol{\mu}}(t) = [\boldsymbol{\Omega}(t) \times [\boldsymbol{\Omega}(t) \times \boldsymbol{\mu}]], \quad \boldsymbol{\mu} = \text{constant}. \tag{50}$$

It follows

$$\delta\ddot{\boldsymbol{\mu}}^2(t) = \mu_\perp^2\,\Omega^4(t), \quad \mu_\perp = \mu\,\sin\theta \tag{51}$$

where θ is angle of inclination of $\boldsymbol{\mu}$ to $\boldsymbol{\Omega}(t)$. The total magnetic moment of non-rotating neutron star is parametrized by equation of uniformly magnetized (along the polar axis) sphere (38)

$$\boldsymbol{\mu} = \mu\mathbf{n},\; \mu = \frac{1}{2}BR^3 = \text{constant}. \tag{52}$$

This parametrization presumes that in the rotation powered neutron star, the frozen-in the star magnetic field operates like a passive promoter of magneto- dipole radiation, that is, intensity of the internal magnetic field remain constant in the process of radiation. As a result, the equation of energy conversion from rotation to magnetic dipole radiation is reduced to equation of slow down of rotation

$$\dot{\Omega}(t) = -K\Omega^3(t), \quad K = \frac{2\mu_\perp^2}{3Ic^3}, \tag{53}$$

$$\Omega(t) = \frac{\Omega(0)}{\sqrt{1+t/\tau}}, \quad \tau^{-1} = 2\,K\,\Omega^2(0). \tag{54}$$

where θ is angle of inclination of $\boldsymbol{\mu}$ to $\boldsymbol{\Omega}(t)$. From the last equation it follows

$$P(t) = P(0)\,[1+(t/\tau)]^{1/2}, \; P(0) = \frac{2\pi}{\Omega(0)}, \tag{55}$$

$$\dot{P}(t) = \frac{1}{2\tau}\frac{P(0)}{[1+(t/\tau)]^{1/2}} \tag{56}$$

and, hence, the lifetime τ is related with $P(t)$ and $\dot{P}(t)$ as

$$P(t)\dot{P}(t) = \frac{P^2(0)}{2\tau} = \text{constant}. \tag{57}$$

Equating two independent estimates for τ given by equations (54) and (57) we arrive at widely utilized analytic estimate of magnetic field on the neutron star pole:

$$B = [3Ic^3/(2\pi^2\,R^6)]^{1/2}\sqrt{P(t)\,\dot{P}(t)}.$$

For a neutron star of mass $M = M_\odot$, and radius $R = 13$ km, one has

$$B = 3.2\,10^{19}\sqrt{P(t)\,\dot{P}(t)},\ \text{G}.$$

The outlined treatment of rotation-powered magneto-dipole radiation of a neutron star emphasizes kinematic nature of variation of magnetic moment of pulsar whose magnetic field is regarded as independent of time. Thus, the substantial physical difference between vibration-powered and rotation-powered neutron star models is that in the former the elongation of pulse period is attributed to magnetic field decay, whereas in the latter the period lengthening is ascribed to slow down of rotation (39–41).

4.2 Comment on magnetic field decay in quaking neutron star

The considered law of magnetic field decay cannot be, of course, regarded as universal because it reflects a quite concrete line of argument regarding the fluctuations of magnetic moment of the star. The interrelation between quake-induced oscillations of total magnetic moment of the star, $\delta\boldsymbol{\mu}(t)$, and the amplitude, $\alpha(t)$, of its seismic magneto-mechanical oscillations can be consistently interpreted with the aid of the function of dipole demagnetization $\mathbf{f}(B(t))$ which is defined by the following condition of self-consistency in α of right and left hand sides of equation (22), namely

$$\delta\ddot{\boldsymbol{\mu}}(t) = \mathbf{f}(B(t))\alpha(t). \tag{58}$$

The vector-function of dipole demagnetization $\mathbf{f}(B(t))$ depending on decaying magnetic field reflects temporal changes of electromagnetic properties of neutron star matter as well as evolution of magnetic-field-promoted coupling between neutron star and its environment. This means that specific form of this phenomenological function should be motivated by heuristic arguments taking into account these factors. With this form of $\delta\ddot{\boldsymbol{\mu}}(t)$, equation (22) is transformed to magnetic field decay of the form

$$\frac{\mathrm{d}B(t)}{\mathrm{d}t} = -\eta\,\frac{\mathbf{f}^2(B(t))}{B(t)}, \quad \eta = \frac{2}{3\mathcal{M}\kappa^2 c^3} = \text{const.} \tag{59}$$

In the above considered case, this function is given by

$$\mathbf{f}(B(t)) = \beta\, B(t)\,\mathbf{B}(t), \quad \beta = \kappa^2\mu = \text{constant.} \tag{60}$$

The practical usefulness of the dipole demagnetization function, $\mathbf{f}(B(t))$, consists in that it provides economic way of studying a vast variety of heuristically motivated laws of magnetic field decay, $B = B(t)$, whose inferences can ultimately be tested by observations. In this subsection, with no discussing any specific physical mechanism which could be responsible for magnetic field decay, we consider a set of representative examples of demagnetization function $\mathbf{f}(B(t))$ some of which have been regarded before, though in a somewhat different context [42-49]. Here we stress again that the model under consideration deals with magnetic field decay in the course of vibrations triggered by starquake, not with long-term secular decay which has been the subject of these latter investigations (see also references therein). 1. As a first representative example, a model of quaking neutron star whose function of dipole

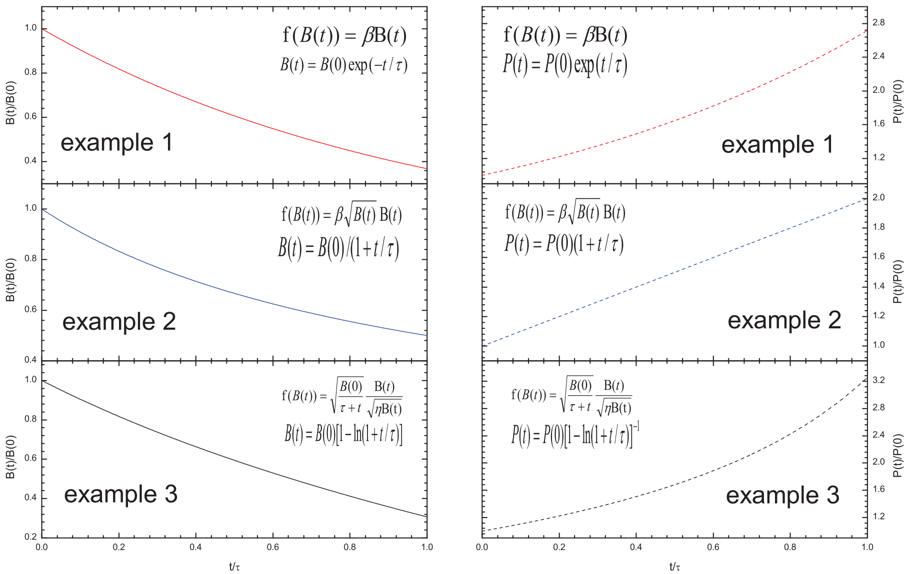

Fig. 9. Three representative examples with different laws of the magnetic field decay resulting in the elongation of the vibration period for each case.

demagnetization has the following form

$$\mathbf{f}(B(t)) = \beta\mathbf{B}(t) \tag{61}$$

is been considered. In such demagnetization model, the temporal evolution of the magnetic field and the lengthening of the vibration period obey the following laws

$$B(t) = B(0)\, e^{-t/\tau},\ P(t) = P(0)\, e^{t/\tau},\ \tau = \frac{P(t)}{\dot{P}(t)}. \tag{62}$$

where $B(0)$ is the intensity of magnetic field before quake. 2. For a quaking neutron star model whose function of demagnetization is given by

$$\mathbf{f}(B(t)) = \beta\sqrt{B(t)}\,\mathbf{B}(t) \tag{63}$$

the resultant equation of magnetic field evolution and the vibration period elongation read

$$B(t) = B(0)\left(1+\frac{t}{\tau}\right)^{-1},$$

$$P(t) = P(0)\left(1+\frac{t}{\tau}\right),\ \tau = \frac{P(0)}{\dot{P}(0)}. \tag{64}$$

Similar analysis can be performed for the demagnetization function of the form

$$\mathbf{f}(B(t)) = \beta\sqrt{B^{m-1}(t)}\,\mathbf{B}(t), \quad m = 3,4,5...$$

Namely,

$$\frac{dB(t)}{dt} = -\gamma\, B^m(t), \quad \gamma = \eta\beta^2,$$

$$B(t) = \frac{B(0)}{[1+t/\tau_m]^{1/(m-1)}}, \quad \tau_m^{-1} = \gamma(m-1)B^{m-1}(0).$$

3. Finally, let's consider a model with quite sophisticated function of dipole demagnetization

$$\mathbf{f}(B(t)) = \sqrt{\frac{B(0)}{\tau+t}}\frac{\mathbf{B}(t)}{\sqrt{\eta B(t)}}. \tag{65}$$

which lead to a fairly non-trivial logarithmic law of magnetic field decay and vibration period lengthening

$$B(t) = B(0)\left[1-\ln\left(1+\frac{t}{\tau}\right)\right], \tag{66}$$

$$P(t) = P(0)\left[1-\ln\left(1+\frac{t}{\tau}\right)\right]^{-1}, \quad \tau = \frac{P(0)}{\dot{P}(0)}. \tag{67}$$

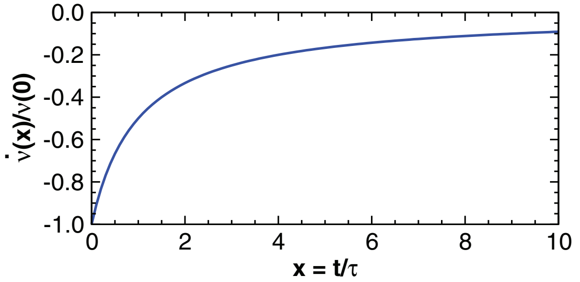

Fig. 10. The rate of frequency $\dot{\nu}(x)$ normalized to $\nu(0)$ as a function of $x = t/\tau$ for logarithmic law of magnetic filed decay.

For each of above examples, the magnetic field and the resultant lengthening of vibration period, computed as functions of fractional time t/τ, are shown in Fig. 9. This third example is interesting in that computed in this model ratio $\dot{\nu}(t)/\nu(0)$, pictured in Fig.10, as a function of t, is similar to that which exhibit data on post-glitch emission of PSR J1846-0258 [50]. These examples show that the period elongation is the common effect of magnetic field decay. The interrelations between periods and its derivatives substantially depend on specific form of the magnetic field decay law although physical processes responsible for magnetic field decay remain uncertain. Practical significance of above heuristic line of argument is that it leads to meaningful conclusion (regarding elongation of periods of oscillating magneto-dipole emission) even when detailed mechanisms of magnetic field decay in the course of vibrations are not exactly known.

5. Summary

It is generally realized today that the standard model of inclined rotator, lying at the base of our understanding of radio pulsars, faces serious difficulties in explaining the long-periodic ($2 < P < 12$ s) pulsed radiation of soft gamma repeaters (SGRs) and anamalous X-ray pulsars (AXPs). Observations show that persistent X-ray luminosity of these sources ($10^{34} < L_X < 10^{36}$ erg s^{-1}) is appreciably (10-100 times) larger than expected from neutron star deriving radiation power from energy of rotation with frequency of detected pulses. It is believed that this discrepancy can be resolved assuming that AXP/SGR-like sources are magnetars – young, isolated and seismically active neutron stars whose energy supply of pulsating high-energy radiation comes not from rotation (as is the case of radio pulsars) but from different process involving decay of ultra strong magnetic field, $10^{14} < B < 10^{16}$ G. Adhering to this attitude we have presented the model of quaking neutron star deriving radiation power from the energy of torsional Lorentz-force-driven oscillations. It is appropriate to remind early works of the infancy of neutron star era (51; 52) in which it has been pointed out for the first time that vibrating neutron star should operate like Hertzian magnetic dipole deriving radiative power of magneto-dipole emission from the energy of magneto-mechanical vibrations(53). What is newly disclosed here is that the main prerequisite of the energy conversion from vibrations into radiation is the decay of magnetic field in the star. Since the magnetic field decay is one of the most conspicuous features distinguishing magnetars from rotation powered pulsars, it seems meaningful to expect that at least some of AXP/SGR - like sources are magnetars deriving power of pulsating magnetic dipole radiation from the energy of quake-induced Alfvén torsion vibrations about axis of dipole magnetic field experiencing decay.

6. Acknowledgment

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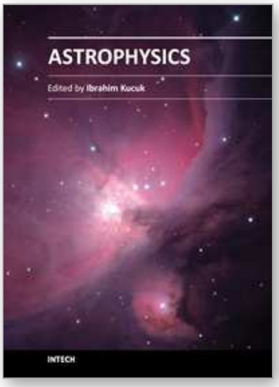

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