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Energy Generation Mechanisms in Stellar Interiors

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1. Introduction

It is well known that in the calculation of thermonuclear reaction rates in dense and cool plasma, the screening effects must be taken into account. The internal structure and evolution of stars can be obtained with the calculations of equation of state of gas structure, opacity and thermonuclear reaction rates. For high density it is necessary to calculate the equation of state by inserting deviations from ideal gas under high density and temprature. For low mass high density stars the effect of degenerate non-ideal interactions must be taken into account in equation of state. The rate of fusion nuclear reactions in stellar matter is important for the evolution of the star. In dense ionized matter, the rate of nuclear reactions is enhanced by screening effects (Alastuey and Jancovici 1978). One measure of nonideality in plasmas is the so-called coupling parameter Γ. In a plasma where particles have average distance $<r>$ from each other, we can define Γ as the ratio of average potential binding energy over mean kinetic energy kT,

$$\Gamma = \left(\frac{e^2 / <r>}{kT} \right) \tag{1}$$

Plasmas with $\Gamma >> 1$ are strongly coupled, and those with $\Gamma << 1$ are weakly coupled (Basu et al. 1999).

2. Thermonuclear reaction rates

The reaction rate, r_{12}, between two nuclei,1 and 2, is given by,

$$r_{12} = \frac{N_1 N_2 \langle \sigma \upsilon \rangle}{(1+\delta_{12})} cm^{-3} / sn \tag{2}$$

where N_1 and N_2 are the number densities of 1 and 2. For a gas of mass density, ρ, the number density, N_i, of the nucleide, *i*, is often expressed in terms of its mass fraction, X_i, by the relation

$$N_i = \rho N_A \frac{X_i}{A_i}\ cm^{-3} \tag{3}$$

where N_A is the Avogadro's number and A_i is the atomic mass of i in atomic mass units. In a stellar environment, the reaction rate per particle pair is calculated as

$$\langle \sigma \upsilon \rangle = \frac{(8/\pi)^{1/2}}{M^{1/2}(kT)^{3/2}} \int \sigma\, E \exp(-E/kT)\, dE \tag{4}$$

where σ is the energy-dependent reaction cross section.

At low energies, nonresonant charged particle interactions are dominated by the coulomb-barrier penetration factors. It is therefore convenient to factor out this energy dependence and express the cross section, $\sigma(E)$,by

$$\sigma = \frac{S(E)}{E} \exp[-(E_G/E)^{1/2}] \tag{5}$$

where the kinetic energy, E, of the center-of-mass system, the cross section factor S(E), the Gamow energy, E_G , is given by

$$E_G = (2\pi\alpha\ \ Z_0 Z_1)^2 \left(Mc^2/2\right) = \left[\ 9.8948 Z_1 Z_2 A^{1/2}\ \right]^2 keV \tag{6}$$

(Lang 1999). Far from a nuclear resonance, the cross section factor, S(E), is a slowly varying function of E, and can be conveniently expressed as the first three terms of a Maclaurin Series in the center-of-momentum energy E. Thus,

$$S(E) = S(0)\left[1 + \frac{S'(0)}{S(0)} E + \frac{1}{2}\frac{S''(0)}{S(0)} E^2\right] \tag{7}$$

where the prime indicates differentiation with respect to E. The values of S and associated derivates are quoted at zero energy. Substitution of Equations 5 and 7 into Equation 4 yields,

$$\langle \sigma \upsilon \rangle = \frac{(8/\pi)^{1/2}}{M^{1/2}(kT)^{3/2}} \int S(E) \exp\left(-E_G{}^{1/2}/E^{1/2} - E/kT\right) dE\ = \left(\frac{2}{M}\right)^{1/2} \frac{\Delta E_0}{(kT)^{3/2}} S_{eff} \exp(-\tau) \tag{8}$$

(Fowler, Caughlan, and Zimmerman 1967) where,

$$\Delta E_0 = 4(E_0 kT/3)^{1/2} \tag{9}$$

$$E_0 = \left[\ \pi\alpha\ Z_0 Z_1 kT \left(Mc^2/2\right)^{1/2}\ \right]^{2/3} = 1.2204\left[Z_1^2 Z_2^2 A T_6^2\right]^{1/3} keV \tag{10}$$

$$\tau = 3E_0/kT\ = 3\left[\ \pi\alpha\ Z_0 Z_1 \left(Mc^2/2kT\right)^{1/2}\ \right]^{2/3} = 42.487\left[Z_1^2 Z_2^2 A\right]^{1/3} T_6^{-1/3} \tag{11}$$

$$S_{eff} = S(0)\left[1+\frac{5}{12\tau}+\frac{S'(0)}{S(0)}\left(E_0+\frac{35}{36}kT\right)+\frac{1}{2}\frac{S''(0)}{S(0)}\left(E_0{}^2+\frac{89}{36}E_0kT\right)\right] \text{ keV-barn} \quad (12)$$

Putting these equations into Equation 8 yields,

$$\langle\sigma\upsilon\rangle = \left\{1.3006\times10^{-15}\left(Z_0Z_1/A\right)^{1/3}S_{eff}\right\}T_6{}^{-2/3}\exp(-\tau)\ cm^3s^{-1} \quad (13)$$

This is the obtained relation of cross section ($T_6 = T/10^6$). The mean lifetime, $\tau_2(1)$, of nucleus 1 for destruction by nucleus 2 is given by the relation,

$$\lambda_2(1) = \frac{1}{\tau_2(1)} = N_2\langle\sigma\upsilon\rangle = \rho\, N_A\frac{X_2}{A_2}\langle\sigma\upsilon\rangle \quad sn^{-1} \quad (14)$$

where $\lambda_2(1)$, is the decay rate of 1 for interaction with 2 (Fowler, Caughlan, and Zimmerman 1967). Putting equations (12), (13) and (5) into equation (2) yields for unscreened reaction rate

$$r_{unscreened} = KX_1X_2g\rho\exp(-\tau)\,T_6^{-2/3} \quad (15)$$

where g is:

$$g = \left[1+\frac{5}{12\tau}+\frac{S'(0)}{S(0)}\left(E_0+\frac{35}{kT}\right)+\frac{1}{2}\frac{S''(0)}{S(0)}\left(E_0+\frac{89}{36}E_0kT\right)\right] \quad (16)$$

and K is given as,

$$K = 7.83x10^8\left(\frac{N_A}{A_1A_2}\right)\left(\frac{Z_1Z_2}{A}\right)^{1/3}S(0). \quad (17)$$

3. The electron screening effect and enhancement factors

At the high temperatures in stellar interiors all the atoms are ionized and the gas density ρ is high. The average distance between nucleus and neighbouring electrons is small. Each nucleus is then completely screened by a spherically symmetric negative charge cloud. The radius of this charge cloud is of the same order as the interparticle distance or larger, depending on the ratio of coulomb repulsion between neighbouring charges to the mean thermal energy. Hence, when two nuclei approach each other in a collision each of them carries its screening affects the interaction energy between the nuclei. The nucleus is surrounded in its immediate vicinity by electrons only the nuclei staying outside a sphere containing nearly electrons which effectively screen the nucleus (Salpeter 1954; hereafter S54). As explained by S54, the rate of a fusion of two nuclei charges Z_1 and Z_2 is increased by factor

$$f = \exp\left[-\frac{U(0)}{kT}\right] \quad (18)$$

$$= \exp \Lambda$$

where $\Lambda = U(0)/kT$ is the natural strength screening parameter.

$$\left(-\frac{U(0)}{kT}\right)_{ws} = 0.188 Z_1 Z_2 \frac{\rho^{\frac{1}{2}}}{T_6^{\frac{3}{2}}} \zeta \tag{19}$$

This is the so-called standard assumption or weak screening which was orginally presented by S54 (Dzitko et al. 1995). In Table 1 the results of screening strength parameter and enhancement factors calculated by using Küçük, Kızıloğlu and Civelek 1998 stellar evolutionary program are listed. The input physics is as follows: OPAL opacity has been used and the ratio of mixing length to the scale height is taken to be α=1.50. The accepted chemical composition is X=0.699, and Z=0.019 (Küçük, Kızıloğlu and Civelek 1998).

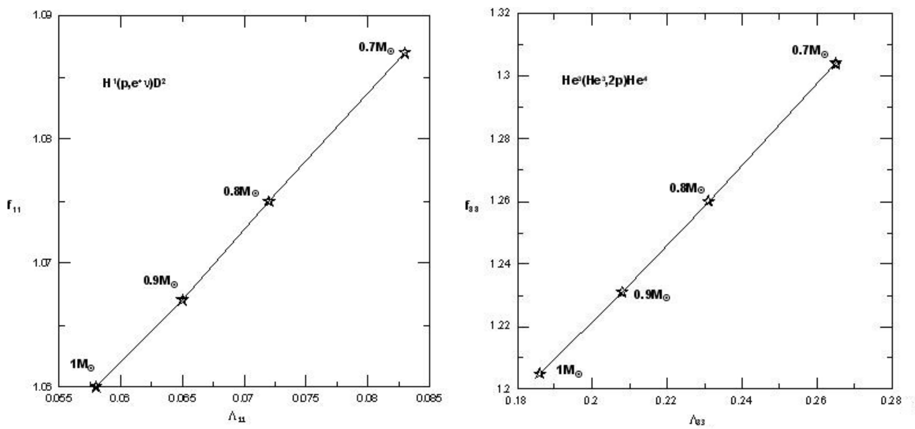

Fig. 1. Variation of the screening factors (S54) for $H^1\left(p,e^+\upsilon\right)D^2$ reaction with the screening strength parameter Λ (left). Variation of the screening factors (S54) for $He^3\left(He^3,2p\right)He^4$ reaction with the screening strength parameter (right)

Mass	$T_c \times 10^7$ (K)	ρ_c $\left(gr/cm^3\right)$	$\Lambda_{1,1}$	$\Lambda_{3,3}$	$\Lambda_{3,4}$	$f_{1,1}$	$f_{3,3}$	$f_{3,4}$
0.7M$_\odot$	10.26	76.99	0.083	0.265	0.265	1.087	1.304	1.304
0.8M$_\odot$	10.85	68.59	0.072	0.231	0.231	1.075	1.260	1.260
0.9M$_\odot$	12.17	78.03	0.065	0.208	0.208	1.067	1.231	1.231
1.0M$_\odot$	13.02	76.79	0.058	0.186	0.186	1.060	1.205	1.205

Table 1. Calculated screening strength parameter and enhancement factors.

Table 2 gives enhancement factors calculated with the various prescriptions for the main reactions involved in hydrogen burning: Küçük and Çalışkan 2010 (hereafter KÇ2010), S54, Salpeter and Van Horn 1969 (hereafter SVH), Graboske et al. 1973 (hereafter GDGC), Mitler 1977 hereafterML) and Tsytovich 2000 (hereafter TS).

	$f_{1,1}$	$f_{3,3}$	$f_{3,4}$
KÇ2010	1.046	1.166	1.166
S54	1.050	1.215	1.215
SVH	1.045	1.186	1.186
GDGC	1.050	1.115	1.115
ML	1.045	1.176	1.176
TS	0.950	0.830	0.827

Table 2. Enhancement factors.(T_c~15.54$x10^6$ K, ρ_c~160.8 gr/cm^3, X_c=0.35 and Y_c=0.62) (Dzitko, H., et al. 1995).

$$r_{3,3} = 2.671 \times 10^5 \,, E_{3,3} = 5.503 \,.$$

Thermonuclear reaction rates in stars, are calculated by multiplying the screened reaction rate enhancement factor, $f_{i,j}$ (i,j=1,2,3,…), with unscreened reaction.

$$r_{screened} = f_{i,j} \times r_{unscreened} \tag{20}$$

Then energy generation rate in stars given by,

$$\varepsilon_{nuc} = Q \times r_{screened} \tag{21}$$

In this study, various data (Weiss et al., 2001, Morel et.al. 1999, Bahcall 1989) are used for the calculation of thermonuclear reaction rates. The results for some reaction rates are as follows:

For $H^1\left(p,e^+\upsilon\right)D^2$ reaction,

$$r_{1,1} = 1.15 \times 10^{11} X_1^2 f_{1,1} g_{1,1} \rho \exp\left(-\frac{33.81}{T_6^{1/3}}\right) T_6^{-2/3}$$

$$g_{1,1} = 1 + 0.0123 T_6^{1/3} + 0.00114 T_6^{2/3} + 9.8 \times 10^{-4} T_6 \tag{22}$$

$$Q_{1,1} = 1.442 \quad MeV \,,$$

for $D^2\left(p,\gamma\right)He^3$ reaction,

$$r_{2,1} = 5.305 \times 10^{28} X_1 X_2 f_{2,1} g_{2,1} \rho \exp\left(-\frac{37.21}{T_6^{1/3}}\right) T_6^{-2/3}$$

$$g_{2,1} = 1 + 0.0112 T_6^{1/3} + 0.299 T_6^{2/3} + 0.00234 T_6$$

$$Q_{2,1} = 5.494 \quad MeV \tag{23}$$

for $D^2(d,n)He^3$ reaction,

$$r_{2,2} = 3.154 \times 10^{33} X_2^2 f_{2,2} g_{2,2} \rho \ \exp\left(-\frac{42.58}{T_6^{1/3}}\right) T_6^{-2/3}$$

$$g_{2,2} = 1$$

$$Q_{2,2} = 3.269 \quad MeV \tag{24}$$

for $He^3(He^3, 2p)He^4$ reaction,

$$r_{3,3} = 1.859 \times 10^{35} X_3^2 f_{3,3} g_{3,3} \rho \exp\left(-\frac{122.76}{T_6^{1/3}}\right) T_6^{-2/3}$$

$$g_{3,3} = 1 + 3.39 \times 10^{-3} T_6^{1/3}$$

$$Q_{3,3} = 12.860 \quad MeV \tag{25}$$

for $He^3(\alpha,\gamma)Be^7$ reaction

$$r_{3,4} = 2.795 \times 10^{31} X_3 X_4 f_{3,4} g_{3,4} \rho \exp\left(-\frac{128.26}{T_6^{1/3}}\right) T_6^{-2/3}$$

$$g_{3,4} = 1 + 3.25 \times 10^{-3} T_6^{1/3} - 3.547 \times 10^{-3} T_6^{2/3} - 8.07 \times 10^{-5} T_6$$

$$Q_{3,4} = 1.588 \quad MeV \tag{26}$$

for $Be^7(p,\gamma)B^8$ reaction

$$r_{7,1} = 2.32 \times 10^{30} X_7 X_1 f_{7,1} g_{7,1} \rho \exp\left(-\frac{102.62}{T_6^{1/3}}\right) T_6^{-2/3}$$

$$g_{7,1} = 1 + 4.06 \times 10^{-3} T_6^{1/3}$$

$$Q_{7,1} = 0.137 \quad MeV \tag{27}$$

for $C^{12}(p,\gamma)N^{13}$ reaction

$$r_{1,12} = 1.089 \times 10^{33} X_1 X_{12} f_{1,12} g_{1,12} \rho \exp\left(-\frac{136.9}{T_6^{1/3}}\right) T_6^{-2/3}$$

$$g_{1,12} = 1 + 3.04 \times 10^{-3} T_6^{1/3} + 6.41 \times 10^{-3} T_6^{2/3} + 1.36 \times 10^{-4} T_6$$

$$Q_{1,12} = 1.944 \quad MeV \tag{28}$$

for $N^{14}(p,\gamma)O^{15}$ reaction

$$r_{1,14} = 2.088 \times 10^{32} X_1 X_{14} f_{1,14} g_{1,14} \rho \exp\left(-\frac{152.28}{T_6^{1/3}}\right) T_6^{-2/3}$$

$$g_{1,14} = 1 + 2.74 \times 10^{-3} T_6^{1/3} - 8.08 \times 10^{-3} T_6^{2/3} - 1.547 \times 10^{-4} T_6$$

$$Q_{1,14} = 7.297 \quad MeV \tag{29}$$

for $O^{16}(p,\gamma)F^{17}$ reaction

$$r_{1,16} = 5.54 \times 10^{33} X_1 X_{16} f_{1,16} g_{1,16} \rho \exp\left(-\frac{166.92}{T_6^{1/3}}\right) T_6^{-2/3}$$

$$g_{1,16} = 1 + 2.491 \times 10^{-3} T_6^{1/3} - 1.18 \times 10^{-2} T_6^{2/3} - 2.07 \times 10^{-4} T_6$$

$$Q_{1,16} = 0.60 \quad MeV \tag{30}$$

The star's thermonuclear reaction rate and energy generation can be obtained from these relations. For example; $H^1\left(p,e^+\upsilon\right)D^2$ and $He^3\left(He^3,2p\right)He^4$ reactions occured at the center, when 0.7M$_\odot$ model reaches its ZAMS. The reaction rates and energy generations are found to be $r_{1,1} = 2.154 \times 10^5$, $E_{1,1} = 2.305$ and $r_{3,3} = 2.013 \times 10^5$, $E_{3,3} = 4.147$, respectively. Similiarly, for 0.8M$_\odot$ the results are $r_{1,1} = 3.262 \times 10^5$, $E_{1,1} = 3.490$ and $r_{3,3} = 2.671 \times 10^5$, $E_{3,3} = 5.503$.

4. Conclusion

During the lifetime of a star its composition, the relative abundances of different elements and isotopes, change basically because of nuclear reactions in the deep interior. In the evolution of 0.7, 0.8, 0.9 and 1M$_\odot$ stars, the hydrogen burning begins and gets equlibrium with increasing central pressure and temperature. Equlibrium time increases when going lower masses. These are ~2×10^{10} and ~5×10^9 years for 0.7M$_\odot$, and 1M$_\odot$, respectively. It is well known that in the calculation of nuclear reaction rates in dense, relatively cool stellar plasma, the screening of the coulomb interaction between the reacting nuclei by the surrounding ions and electrons must be taken into account. When screening factors calculated with different methods are considered one can see the differences. When the energies released due to nuclear reaction rates are calculated for ^{1}H - ^{1}H and ^{3}He - ^{3}He reactions and for each mass, respectively, it is seen that, the released energy increases with increasing mass. As a conclusion reaction rates and electron screening factors which are added to the evolutionary programs in details, causes some shifts.

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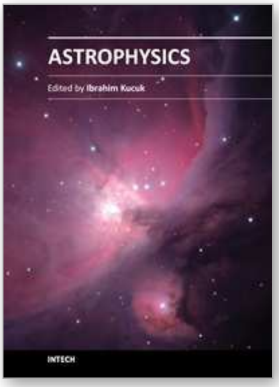

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