Phase-Shifting Interferometry by Amplitude Modulation

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1. Introduction

In optics, the superposition of two or more light beams at any point over space can produce the apparition of interference fringes. When these fringes are applied to resolve a problem in industry or they are related with some property of an investigation matter of interest in some area of physics, chemistry, biology, etc., the evaluation of them is a very necessary task. One of the most used methods for phase extraction, as a result of fringes evaluation, is based on a phase change between the interference beams by a known value, while their amplitudes are keeping constant. It is called phase-shifting interferometry, phase-sampling interferometry, or phase-stepping interferometry, which are abbreviated by "PSI" (Schwider, 1990). In this technique a set of N interferograms changed in phase are created, which are represented by a set of N equations, where each equation has three unknowns called as background light, modulation light and the object phase. These spatial unknowns are considered constant during the application of the PSI technique. Then a $N \times 3$ system is formed and therefore it can be resolved when $N \geq 3$. Many methods to introduce a constant phase have been proposed as for example by changing the optical frequency, wavelength, index of refraction, distance, optical path, for instance; but also with some properties or effect of the light such as the polarization, diffraction, Zeeman effect, Doppler effect, for instance (Schwider, 1990; Malacara, 2007). In this chapter, our major interest aims to propose a new method to generate a phase change in the interferogram based on the amplitude modulation of the electric field (Meneses-Fabian and Rivera-Ortega, 2011).

Interferometry uses the superposition principle of electromagnetic waves when certain conditions of coherence are achieved to extract information about them. If light from a source is divided into two to be superposed again at any point in space, the intensity in the superposition area varies from maxima (when two waves crests reach the same point simultaneously) to minima (when a wave trough and a crest reach the same point) ; having by this what is known as an interference pattern. Interferometry uses interferometers which are the instruments that use the interference of light to make precise measurements of surfaces, thicknesses, surface roughness, optical power, material homogeneity, distances and so on based on wavefront deformations with a high accuracy of the order of a fraction of the wavelength through interference patterns. In a two wave interferometer one wave is typically a flat wavefront known as the reference beam and the other is a distorter wavefront whose shape is to be measured, this beam is known as the probe beam.

There are several well studied and know methods to generate phase-shifts which will be briefly discussed, however the present method of PSI based on the amplitude modulation into two beams named reference beams in a scheme of a three beam interferometry will be amply discussed; this discussion will be done for a particular case where the phase difference between the reference beam is conditioned to be $\pi/2$; and for a general case where the phase difference between the reference beam should be within the range of $(0, 2\pi)$ but $\neq \pi$.

We can represent n optical perturbations in a complex form with elliptical polarization and traveling on z direction as follows:

$$\mathbf{E}_k = (\mathbf{i}E_{kx} + \mathbf{j}E_{ky}e^{i\delta_k})e^{i\phi_k} \tag{1}$$

where k goes from $1 \ldots n$, ϕ_k is the phase of each wave and δ_k is the relative phase difference between the component of each wave (by simplicity the temporal and spatial dependencies have been omitted). In the particular case that $\delta_k = m\pi$ where m is an integer number, the wave will be linearly polarized; on the other hand, if $E_{kx} = E_{ky}$ and $\delta_k = (2m+1)\pi/2$ the wave will be circularly polarized.

The phenomenon of interference can occur when two or more waves overlap in space. Mathematically the resulting wave is the vector addition,

$$\mathbf{E}_T = \sum_{k=1}^{n} \mathbf{E}_k \; . \tag{2}$$

When the field is observed by a detector, the result is the average of the field energy by area unit during the integration time of the detector, that is, the irradiance, which can be demonstrated that is proportional to the squared module of the amplitude. However, it is usually accepted the approximation

$$I = |\mathbf{E}_T|^2 = \left| \sum_{k=1}^{n} E_k \right|^2 . \tag{3}$$

An interferometer is an instrument used to generate wave light interference to measure with high accuracy small deformations of the wave front. The general scheme of a two wave interferometer can be observed in Figure 1, where the electromagnetic wave $\mathbf{E}$ is typically divided in two coherent parts that is, in a wave $\mathbf{E}_1$ and $\mathbf{E}_2$, where $\mathbf{E}_1$ is the reference wave and $\mathbf{E}_2$ is the probe wave.
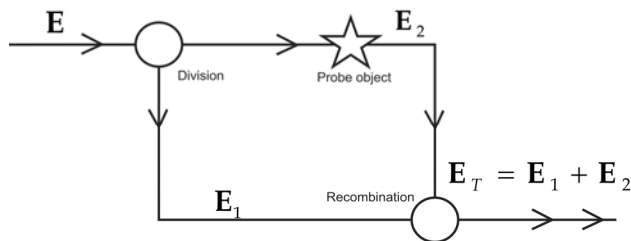

Fig. 1. Scheme of a two wave interferometer.

After the waves have travelled along two separated arms and they have accumulated phase delays, they recombine again by means of a beam splitter giving as a result a field $\mathbf{E}_T$.

For the particular case of two waves ($n = 2$), and using the Eq. (3) for a vector treatment, we can find that the corresponding irradiance is

$$I = |\mathbf{E}_T|^2 = |\mathbf{E}_1 + \mathbf{E}_2|^2 = |\mathbf{E}_1|^2 + |\mathbf{E}_2|^2 + 2\,\mathrm{Re}\{\mathbf{E}_1^* \cdot \mathbf{E}_2\}; \tag{4}$$

It can be seen in Eq. (4) that there are three terms, by doing the math they can be expressed as follows

$$|\mathbf{E}_k|^2 = \mathbf{E}_k \cdot \mathbf{E}_k^* = \left[(\mathbf{i}E_{kx} + \mathbf{j}E_{ky}e^{-i\delta_k})e^{i\varphi_k}\right] \cdot \left[(\mathbf{i}E_{kx} + \mathbf{j}E_{ky}e^{i\delta_k})e^{-i\varphi_k}\right] = E_{kx}{}^2 + E_{ky}{}^2, \tag{5}$$

where the values for $k = 1,2$; by doing this the first two terms of Eq. (4) will be obtained, the third term of this equation is obtained by

$$\mathbf{E}_1^* \cdot \mathbf{E}_2 = \left[(\mathbf{i}E_{1x} + \mathbf{j}E_{1y}e^{-i\delta_1})e^{-\phi_1}\right] \cdot \left[(\mathbf{i}E_{2x} + \mathbf{j}E_{2y}e^{i\delta_2})e^{\phi_2}\right], \tag{6}$$

therefore the resulting interference term is

$$2\,\mathrm{Re}\{\mathbf{E}_1^* \cdot \mathbf{E}_2\} = 2E_{1x}E_{2x}\cos\phi + 2E_{1y}E_{2y}\cos(\phi + \delta), \tag{7}$$

where $\delta = \delta_2 - \delta_1$ y $\phi = \phi_2 - \phi_1$. By taking Eqs. (5-7) a general expression for the interference of two waves is obtained

$$I = a_x + b_x\cos\phi + a_y + b_y\cos(\phi + \delta), \tag{8}$$

where $a_x = E_{1x}{}^2 + E_{2x}{}^2$, $a_y = E_{1y}{}^2 + E_{2y}{}^2$, $b_x = 2E_{1x}E_{2x}$ y $b_y = 2E_{1y}E_{2y}$. It can be observed in Eq. (8) that the interference of two elliptically polarized waves can also be generated by the addition of two interference patterns, one of them with components in x direction ($a_x + b_x\cos\phi$) and the other with components in y ($a_y + b_y\cos(\phi + \delta)$).

On the other hand, it is known that $A\cos\phi + B\sin\phi = C\cos(\phi + \psi)$ if $C = \sqrt{A^2 + B^2}$ and $\tan\psi = B/A$; by applying this identity to Eq. (8) is obtained

$$I = a + b\cos(\phi + \psi), \tag{9}$$

where a is known as the background light, b as the modulation light and ψ indicates an additional phase shifting, which can be expressed by

$$a = a_x + a_y ; \quad b^2 = b_x^2 + 2b_x b_y \cos\delta + b_y^2 ; \quad \tan\psi = \frac{b_y \sin\delta}{b_x + b_y \cos\delta}. \tag{10}$$

It is worth noting from Eq. (10) that both the phase shifting ψ as the background a and the modulation light b depend on the components a_x, a_y, b_x, b_y and also on the phase difference between the amplitude of the waves δ. Note that a phase shifting ψ by varying b_x, b_y and δ will also generate a change in a and b.

2. Phase-shifting interferometry

The first studies in the phase shifting techniques can be found in the work of Carré 1966, but it really started with Crane 1969, Moore 1973 , Brunning *et al*. 1974 and some others. These techniques have also been applied in speckle patter interferometry (Nakadate and Saito 1985; Creath 1985; Robinson and William 1986) and also to holographic interferometry (Nakadate *et al*. 1986; Stetson and Brohinski 1988).

In phase shifting interferometry, a reference wave front is moved along its propagation direction respecting to the probe wave front changing with this the phase difference between them. It is possible to determine the phase of the probe wave by measuring the irradiance changes corresponding to each phase shifting.

2.1 Some phase-shifting methods

There are many experimental ways to generate phase shifting, the most common are:

2.1.1 Moving a mirror

This method is based on the change in the optical path of a beam by means of moving a mirror that is in the beam trajectory. This movement can be made by using a piezoelectric transducer (Soobitsky, 1988; Hayes, 1989). The phase-shifting is given by $\psi = (2\pi/\lambda)(dco)$, where dco is the optical path difference. As an illustrative example if the mirror is translated a distance of $\psi = \lambda/8$ and due to the beam travels two times the same trajectory, the value of the phase-shifting is $\psi = \pi/2$. Examples of interferometers with phase-shifting generated by a piezoelectric are: Twyman-Green, Mach-Zehnder, Fizeau, which are represented in Figure 2. The first two make a phase-shifting by means of moving a mirror that is placed in the reference beam. In the Fizeau interferometer the phase-shifting is made by the translations of the reference or probe object.

2.1.2 Rotating a phase plate

A phase-shifting can also be generated by means of rotating a retarder phase plate (Crane 1969; Okoomian 1969; Bryngdahl 1972; Sommargren 1975; Shagam y Wyant 1978; Hu 1983, Zhi 1983; Kothiyal and Delisle 1984, 1985; Salbut y Patorski 1990). As a particular case, if a circularly polarized wave passes through a half wave retarder plate rotated 45° the direction of rotation will be inverted, thus a phase-shifting of $\psi = \pi/2$ will be present.

2.1.3 Displacing a diffraction grating

This method is based on the perpendicular displacement respecting to the light beam of a diffraction grating (Suzuki y Hioki 1967; Stevenson 1970; Bryngdahl 1976; Srinivasan *et al*.1985, Meneses *et al*. 2009). If the diffraction grating is moved a small distance Δy , the changes in the phase are given by $y = (2\pi m/d)\Delta y$ where d is the period of the grating and m is the diffraction order. As an example, if the grating of Figure 3 is perpendicularly moved respecting to the optical axis, a phase-shifting will be generated.
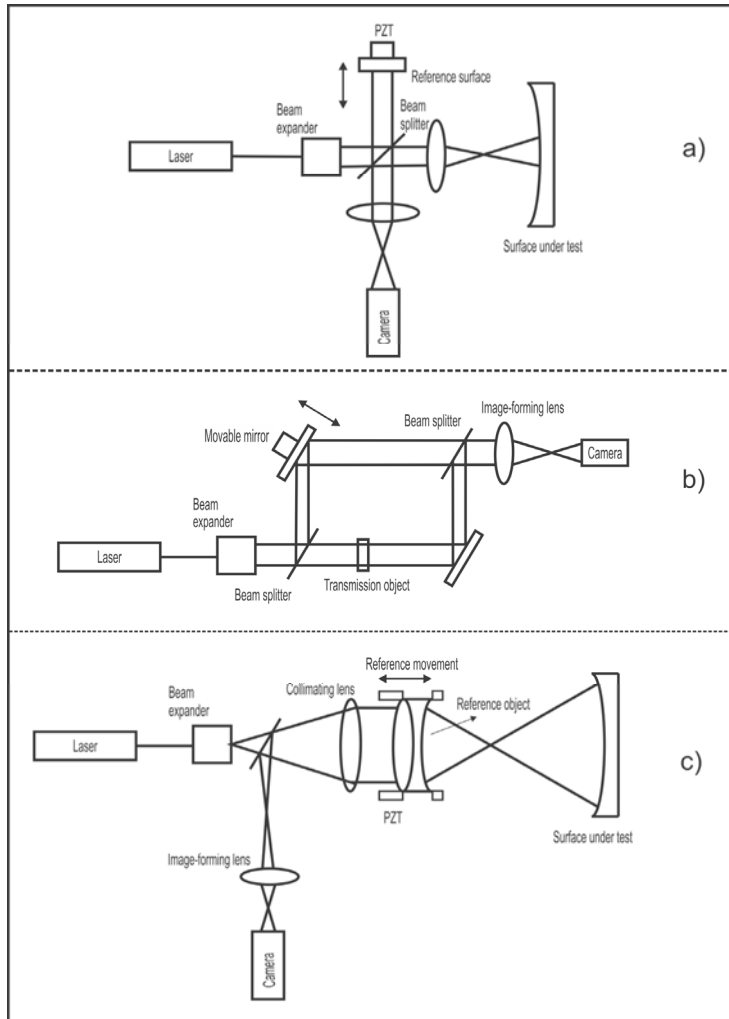
Fig. 2. Scheme of the most commonly used interferometers: a) Twyman-Green, b) Mach-Zehnder, c) Fizeau.

2.1.4 Tilting a glass plate

Another method to generate phase-shifting is by means of inserting a glass plate in the light beam (Wyant y Shagam 1978). The phase-shift ψ_0 is generated when the plate is tilted an angle ϑ respecting to the optical axis hence $\psi = (t/n)(n\cos\vartheta' - \cos\vartheta)$, where t is the thickness of the plate, n is the refraction index and $k = 2\pi/\lambda$. The angles ϑ and ϑ' are the angles formed by the normal and the light beams outside and inside the plate respectively. An special requirement is that the plate must be placed in a collimated light beam to avoid aberrations.

2.1.5 Rotating a polarizer

If in Eq. (1) $k = 1,2$, $E_{1x} = E_{1y} = E_{2x} = E_{2y} = E_0$, $\delta_1 = \pi/2$, $\delta_2 = -\pi/2$ then we have two circularly polarized waves with the same amplitude with opposite rotations, that is

$$\mathbf{E}_1 = (\mathbf{i} + e^{i\frac{\pi}{2}}\mathbf{j})E_0 e^{i\phi_1} , \tag{11}$$

$$\mathbf{E}_2 = (\mathbf{i} + e^{-i\frac{\pi}{2}}\mathbf{j})E_0 e^{i\phi_2} . \tag{12}$$

Expressing the Eqs. (11-12) with the notation for the Jones vector

$$E_1 = E_0 \begin{pmatrix} 1 \\ i \end{pmatrix} e^{i\phi_1} , \tag{13}$$

$$E_2 = E_0 \begin{pmatrix} 1 \\ -i \end{pmatrix} e^{i\phi_2} ; \tag{14}$$

if we interfere those waves, then there will not be an interference term so no fringes will be observed, however if those waves are passed through a polarizer at an angle α and they interfere then

$$E_1' = \begin{bmatrix} \cos^2\alpha & \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha & \sin^2\alpha \end{bmatrix} \begin{bmatrix} 1 \\ i \end{bmatrix} E_0 e^{i\phi_1} = E_0 \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix} e^{i(\alpha+\phi_1)} , \tag{15}$$

$$E_2' = \begin{bmatrix} \cos^2\alpha & \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha & \sin^2\alpha \end{bmatrix} \begin{bmatrix} 1 \\ -i \end{bmatrix} E_0 e^{i\phi_2} = E_0 \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix} e^{-i(\alpha-\phi_2)} , \tag{16}$$

$$I = 2E_0^2 [1 + \cos(\phi - 2\alpha)], \tag{17}$$

where

$$\begin{bmatrix} \cos^2\alpha & \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha & \sin^2\alpha \end{bmatrix}, \tag{18}$$

is the Jones matrix that represents a linear polarizer at an angle α. It is observed in Eq. (17) that there is a phase–shift in the interference pattern which is twice the angle at which the polarizer is placed.

An application of phase-shifting generated by polarization is by using the scheme of Figure 3, which is based in a common path interferometer consisting of two windows in the input plane. In this interferometer is possible to have four interference patters in one shot, each pattern shifted by 90°. To do this a binary grating is used to generate the interference between the diffraction orders of two circularly polarized beams with opposite rotations; a linear polarizer at an angle α is placed in front of each diffraction order thus, the phase-shifting is obtained (Rodriguez-Zurita et al. 2008).
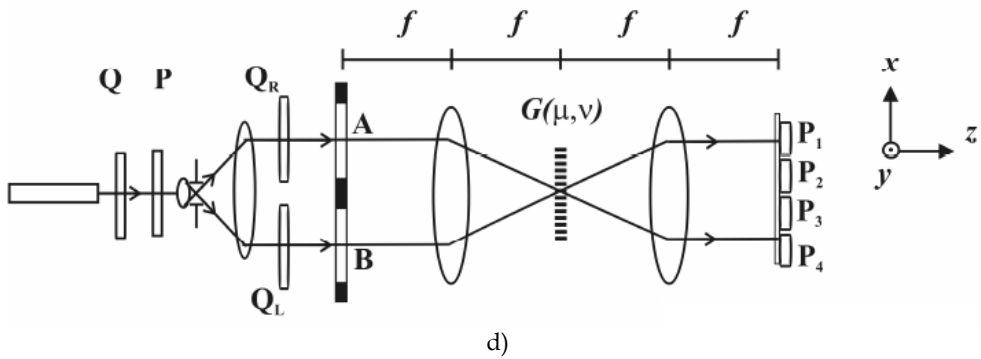
d)

Fig. 3. Scheme of interferometers where the method of phase-shifting by polarizations is used.

2.1.6 Changing the laser source

Other method to generate phase-shift is by means of changing the frequency of the laser source, there are two ways to do this; one of them is by illuminating the interferometer with a Zeeman laser. The laser frequency is divided in two orthogonally polarized output frequencies by means of a magnetic field (Burgwald and Kruger 1970). The other method is by using an unbalanced interferometer, that is an interferometer with a very long optical path difference and using a laser diode with its frequency controlled by electrical current as proposed by Ishii *et al*. (1991) and subsequently studied by Onodera e Ishii (1996). This method is based on the fact that the phase difference in an interferometer is proportional to the product of the optical path difference and the temporal frequency.

In summary, the known techniques for phase-shifting that have been mentioned are applied in two beams interferometers; however none of these techniques use a variation on the amplitude fields, that is why this option will be discussed in this chapter.

2.2 Phase extraction methods

The phase-shifting interferometry is based on the reconstruction of the phase ϕ by sampling a certain number of interference pattern which differ from each other due to different values of ψ_0. If a shift of ψ_0 is made for N steps, then N intensity values I_n will be measured, (where $n = 1,...,N$)

$$I_n = a + b\cos(\phi + \psi_{0n}) . \tag{19}$$

where $\psi_{0n} = 2\pi n / N$.

Eq. (19) can be rewritten as follows

$$I_n = A + B\cos\psi_{0n} + C\sin\psi_{on} , \tag{20}$$

where

$$A = a ; \quad B = b\cos\phi ; \quad C = -b\sin\phi . \tag{21}$$

It can be shown that based on a least-squares fit that B and C meet the next equations in an analytical form

$$B = \frac{2}{N}\sum_{n=1}^{N} I_n \cos\psi_{0n} \; ; \;\; C = \frac{2}{N}\sum_{n=1}^{N} I_n \sin\psi_{0n} \;.$$

(22)

A combination of Eq. (21) and Eqs. (22) can give us the basic equation of the Phase Sampling Interferometry (PSI)

$$\phi = \tan^{-1}\frac{-C}{B} = \tan^{-1}\frac{\sum I_n \sin\psi_{0n}}{\sum I_n \cos\psi_{0n}} \;.$$

(23)

In general, a minimum of three samples are needed to know the phase ϕ because there are three unknowns in the general interference equation Eq. (9): a, b and ϕ. However a better accuracy can be guaranteed with more than three shifts.

2.2.1 Three steps technique

Since we need a minimum of three interferograms to reconstruct the wavefront, the phase can be calculated with a phase-shift of $\pi/2$ per exposition. The three intensity measurements can be expressed as (Wyant, Koliopoulus, Bhushan and George 1984)

$$I_1 = a + b\cos\left(\phi + \frac{1}{4}\pi\right),$$

(24)

$$I_2 = a + b\cos\left(\phi + \frac{3}{4}\pi\right),$$

(25)

$$I_3 = a + b\cos\left(\phi + \frac{5}{4}\pi\right),$$

(26)

the phase at each point is

$$\phi = \tan^{-1}\left(\frac{I_3 - I_2}{I_1 - I_2}\right).$$

(27)

2.2.2 Four steps technique

A common algorithm used to calculate the phase is the four steps method (Wyant 1982). In this case the four intensity measurements can be expressed as

$$I_1 = a + b\cos\phi,$$

(28)

$$I_2 = a + b\cos\left(\phi + \frac{1}{2}\pi\right) = a - b\sin\phi,$$

(29)

$$I_3 = a + b\cos(\phi + \pi) = a - b\cos\phi , \tag{30}$$

$$I_4 = a + b\cos\left(\phi + \frac{3}{2}\pi\right) = a + b\sin\phi , \tag{31}$$

and the phase at each point is

$$\phi = \tan^{-1}\left(\frac{I_4 - I_2}{I_1 - I_3}\right) . \tag{32}$$

2.2.3 The Fourier transform method

The deformations of the wavefront in an interferogram can also be calculated by a method that uses the Fourier transform. This method was originally proposed Takeda *et al.* (1982) using the Fourier transform in one dimension along an scanned line. Later Macy (1983) extended the Takeda's method in two dimensions by adding the information of multiple scanned lines and obtaining slices of the phase in two dimensions. Bone *et al* (1986) extended the Macy's work by applying the Fourier transform in two dimensions.

Once the interferogram is obtained, its Fourier transform is calculated, by doing this an image in the Fourier space as in Figure 4a) is obtained, then one lateral spectrum is taken and filtered by means of a layer so that all the irradiance values outside the layer will be multiplied by cero, after that this spectrum is translated to the origin and its Fourier transform is obtained giving by this the resulting wavefront under test.

To describe mathematically this process, the general equation for the irradiance is taken (Eq. (9)) expressing the cosine as a complex exponential

$$I(x,y) = a(x,y) + c(x,y)e^{-i\psi} + c^*(x,y)e^{i\psi} , \tag{33}$$

being $c(x,y) = (1/2)be^{i\phi(x,y)}$ and the phase of the form $\psi = 2\pi\mu_0 x$, where μ_0 is known as the spatial-carrier frequency

Applying the Fourier transform to Eq. (33)

$$\tilde{I}(\mu,\upsilon) = \tilde{a}(\mu,\upsilon) + \tilde{c}(\mu + \mu_0,\upsilon) + \tilde{c}^*(\mu - \mu_0,\upsilon) , \tag{34}$$

where $\tilde{a}$, $\tilde{c}$ y $\tilde{c}^*$ are complex Fourier amplitudes. By means of a digital filtering one lateral spectrum is isolated by using a filtering window and then translated to the origin by doing $\mu_0 = 0$, as shown in Figure 4b).

Obtaining the inverse Fourier transform of $\tilde{c}(\mu,\upsilon)$ we have $c(x,y) = (1/2)be^{i\phi(x,y)}$, therefore the resulting phase is $\phi(x,y) = \tan^{-1}\dfrac{\operatorname{Im}c(x,y)}{\operatorname{Re}c(x,y)}$.

2.4 Unwrapping the phase

The calculated phase will present discontinuities because it is obtained by using the inverse tangent function Eq. (23). Because the inverse tangent is a multivalued function, the solution
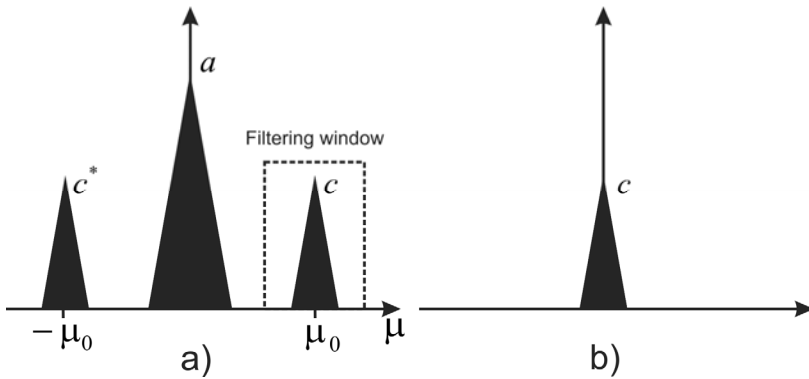
Fig. 4. a) Intensity distribution obtained with the FT of the interference pattern, b) spectrum translated to the origin.

for ϕ is a saw tooth function (Figure 5a), where the discontinuities occur every time ϕ changes by 2π. If ϕ increases, the slope of the function is positive and vice versa if the phase decreases. The final step in the process of measuring the fringe pattern is to unwrap the phase along a line or path counting the discontinuities at 2π and adding 2π each time the angle of the phase jumps from 2π to cero and subtracting 2π if the angle changes from cero to 2π. Figure 5b) shows the dates from Figure 5a) after the unwrapping. The key of a trustable unwrapping algorithm is its capacity of detecting the discontinuities with high accuracy
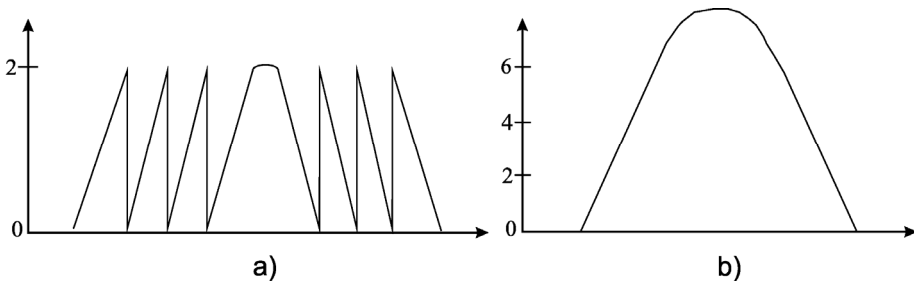

Fig. 5. a) Discontinuities of a wrapped phase, b) unwrapped phase. The basic principle of the phase unwrapping is to "integrate" the wrapped phase (in units of 2π) along a data line. The phase gradient is calculated for each pixel

$$\Delta\phi = \phi_n - \phi_{n-1} , \qquad (35)$$

where n is the pixel number. If $|\Delta\phi|$ exceeds a certain threshold like π, then a discontinuity is assumed. This phase discontinuity is fixed by adding or subtracting 2π depending on the sign of $|\Delta\phi|$, Itoh (1982). The most common principle used to fix those phase discontinuities is based on the fact that the phase difference among any pair of points measured by integrating the phase along a path between these points is independent from the chosen path, provided they don't pass through a phase discontinuity.

3. PSI by amplitude modulation

As it was described in the previous section, many proposed techniques for phase-shifting interferometry (PSI) are based on the interference of two waves where just one interference term is present and the phase shift is done with a constant phase difference between them.

In the present chapter a new method for phase-shifting based on the amplitude variation of the field in a scheme of a three beam interferometer is widely discussed. In that interferometer, two beams will be considered as the reference beam and the other beam as the probe beam. The expression for the irradiance due to the interference of those beams will have three interference terms, however due to a constant phase difference of $\pi/2$ is introduced between the reference beams, one interference term will be canceled and the two remaining will be put in quadrature. Because of this and applying some trigonometric identities to the resulting pattern it is possible to model it mathematically as a two wave interferometer where the interference term will contain an additional phase that depends on the amplitude variations of the reference beam. Since the phase-shift depends on the variation of the amplitude of the fields, the visibility may not remain constant, however it will be shown that if the amplitudes are seen as an ordered pair over an arc segment in the first quadrant of a circle whose radius is the square root of the addition of the squared amplitudes it is possible to keep a constant visibility.

But it could be difficult to get experimentally a phase difference of $\pi/2$ between the reference beams, to overcome this difficulty it is necessary an analysis for a general case where the phase difference between the reference beams is arbitrary; it will be shown that despite that the conditions for the particular case are not obtained it is still possible to generate phase-shifting by means of the amplitude variations of the fields and to keep a constant visibility, those amplitudes must be seen as ordered pairs over ellipses.

3.1 Ideal case

Let's have three waves interfering at any point in space, which are linearly polarized at the same plane and traveling in the z direction

$$E_n = A_n \exp(i\phi_n), \tag{36}$$

with $n = 1,2,3$, being A_n the amplitude considered as nonnegative real, ϕ_n is the phase that contains the wavefront. According to Eq. (2) the interference of these three waves at any point in space is the addition of the three fields, being the irradiance according to Eq. (3)

$$I = |E_1 + E_2 + E_2|^2, \tag{37}$$

doing the math in Eq. (37)

$$I = A_1{}^2 + A_2{}^2 + A_3{}^2 + 2A_1 A_2 \cos(\phi_1 - \phi_2) + 2A_1 A_3 \cos(\phi_1 - \phi_3) + 2A_2 A_3 \cos(\phi_2 - \phi_3), \tag{38}$$

in which three interference terms, the background and the modulation light that is given by the addition of the intensities of each wave are present. Let's consider the first and third wave as the reference wave, and the second wave as the probe wave. By simplicity the next conditions will be chosen

$$\phi_1 = 0 \; ; \; \phi_2 = \phi \; \text{ and } \; \phi_3 = \pi/2 \, . \tag{39}$$

The waves E_1 and E_3 will be chosen as homogeneous no tilted plane waves with a phase difference between them of $\pi/2$, ϕ is the phase of the object contained in the second wave. Substituting Eq. (39) into Eq. (38) we have

$$I = A_1{}^2 + A_2{}^2 + A_3{}^2 + 2A_1 A_2 \cos\phi + 2A_2 A_3 \sin\phi \, . \tag{40}$$

In this equation it can be observed that one of the three interference terms has been cancelled and the two remaining are now in quadrature. Regrouping the above equation

$$I = A_1{}^2 + A_2{}^2 + A_3{}^2 + 2A_2 \left(A_1 \cos\phi + A_3 \sin\phi \right) , \tag{41}$$

that can be rewritten as

$$I = A_1{}^2 + A_2{}^2 + A_3{}^2 + 2A_2 \sqrt{A_1{}^2 + A_3{}^2} \left(\frac{A_1}{\sqrt{A_1{}^2 + A_3{}^2}} \cos\phi + \frac{A_3}{\sqrt{A_1{}^2 + A_3{}^2}} \sin\phi \right) , \tag{42}$$

$$I = A_r{}^2 + A_2{}^2 + 2A_r A_2 \left(\cos\phi \cos\psi + \sin\phi \sin\psi \right) , \tag{43}$$

where $\cos\psi = A_1 / \sqrt{A_1{}^2 + A_3{}^2}$ and $\sin\psi = A_3 / \sqrt{A_1{}^2 + A_3{}^2}$. Eq. (43) can be expressed as

$$I = A_r{}^2 + A_2{}^2 + 2A_r A_2 \cos(\phi - \psi) \, , \tag{44}$$

which as it was indicated in Eq. (9) is the expression for a fringe pattern due to the interference of two waves, being

$$A_r{}^2 = A_1{}^2 + A_3{}^2 \; ; \; \tan\psi = \frac{A_3}{A_1} \, , \tag{45}$$

where A_r is the reference amplitude and ψ is an additional phase, both of them depending on the amplitude variations of the first wave A_1 and the third wave A_3. It can be observed from this relationship that it is possible to generate a phase-shifting with the variations of those amplitudes; however because this also affects A_r, there may be a change in the visibility of the fringes, so it can be thought that it would not be possible to apply the PSI techniques for the phase extraction (because one important condition to apply the PSI phase extraction techniques is that the visibility remains constant). One way to keep A_r constant is to consider the amplitudes of waves one and three as an ordered pair (A_1, A_3) which must be contained over an arc segment of radius A_r in the first quadrant (because the amplitudes are considered positive), hence A_r will be within the range $[0, \pi/2]$, as can be seen in Figure 6. However it is possible to generate a negative amplitude modulation if the phase difference in the reference waves is π, having negative real amplitudes, by this the range of ψ will be $[0, 2\pi]$ and the phase extraction techniques in PSI could be applied without any modification.
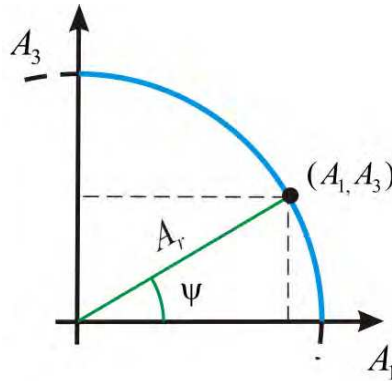
Fig. 6. Amplitudes of first and third wave in a reference system. Any point in the arc keeps A_r constant, and ψ is varied as required for PSI.

In this case the amplitudes are in quadrature, A_1 and A_3 are given by

$$A_1 = A_r \cos\psi \; ; A_3 = A_r \sin\psi \; . \tag{46}$$

In a possible experimental setup the phase difference of $\pi/2$ between the first and third wave could be achieved by means of a retarder plate of a quarter wavelenght. The amplitude variations can be done by using neutral density filters or by using the diffraction orders generated by a grating (for example a Ronchi ruling), where each order is attenuated in accord with the sinc function.

To prove the viability of the present proposal, we have carried out a numerical simulation in which for simplicity, the following considerations have been assumed

$$A_2 = A_r = 1 \; ; \; A_1, A_3 \in [0,1] \; ; \; \phi = x^2 + y^2 \; , \tag{47}$$

such a form guarantees that the interference pattern will have a maximum contrast. Therefore, the three fields could take the form

$$E_1 = A_1 \; ; E_2 = \exp(i\phi) \; E_3 = iA_3 \; , \tag{48}$$

being the interference patter

$$I = 2 + 2\cos(\phi - \psi) \; . \tag{49}$$

Figure 7a) shows the values of the amplitudes needed for a phase-shifting of three steps $N = 3$. The left graphic indicates three points over the arc $(1,0)$, $(\sqrt{2}/2, \sqrt{2}/2)$, $(0,1)$ which are the amplitude variations to obtain the phase steps $\psi = 0, \pi/4, \pi/2$ respectively, while A_r remains constant. Figure 7b) shows the interferograms modeled by Eq. (49) for the phase steps indicated in Figure 7a). A bar diagram above each interferogram indicates the amplitude levels of the waves needed for each phase-shift. In a very similar way Figure 8 show the phase-shifting for the case of four steps $N = 4$.
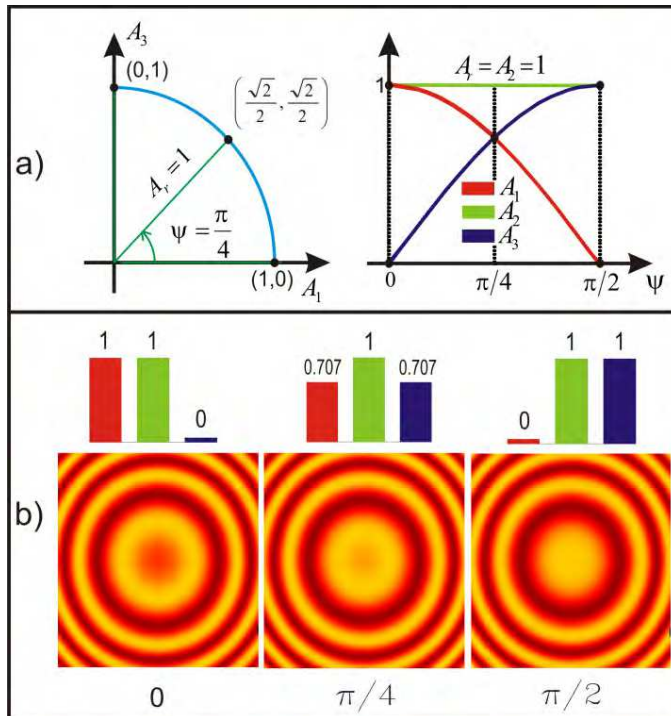
Fig. 7. Amplitude variations for a PSI of three steps: a) a point on the arc yields a phase-shift while the reference amplitude is kept constant; b) interferogram shifted in phase by the amplitude variations indicated in a).

The columns of Figure 9a) show simulated interference pattern for different phase-shifts, which are obtained when the amplitude values A_1 and A_3 are over straight lines that form an angle ψ respecting to the axis A_1, Figure 10b); by doing this, the visibility of the patterns remains constant, but to get a maximum visibility the value of the amplitude A_2 must be equal to the value of A_r, which can be observed in the first row of Figure 9a), where the values of A_1 and A_3 are over an arc segment of radius $A_r = 1$, Figure 9b). If we have different values of ψ but its corresponding amplitudes are not over the same arc segment, the interference patterns will not have a constant visibility, what can be seen in Figure 9a) if for each value of ψ we take a different row (indicated by different symbols).
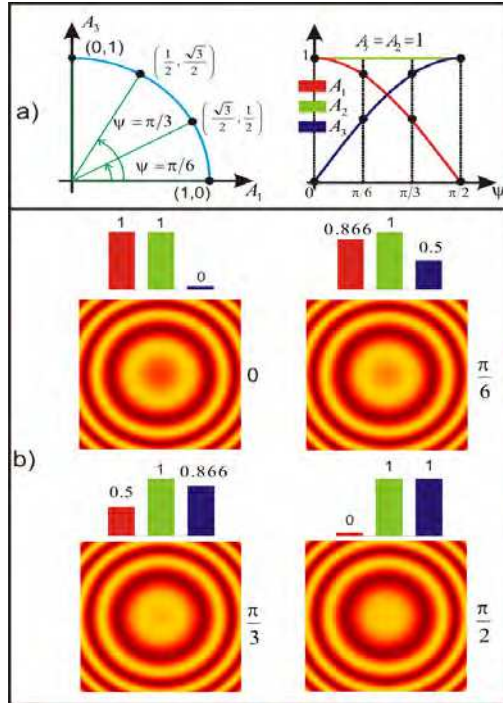
Fig. 8. Amplitude variations for a PSI of four steps: a) a point over the arc yields a phase-shift while the reference amplitude is kept constant; b) interferograms shifted in phase by the amplitude variations indicated in a).
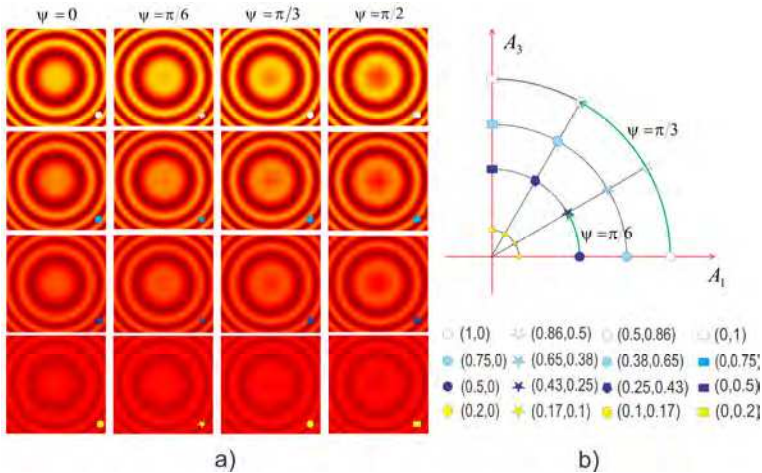

Fig. 9. Interference patterns with different visibility a) Phase-shifted interferograms due to the amplitudes shown in b); b) points over arcs and straight lines that give phase-shift keeping in some cases a constant visibility.

3.2 General case

As demonstrated in the previous section, it is possible to have a new method of PSI by means of the field amplitude variations based on the scheme of a three beam interferometer modeled as a two beam interferometer, where the reference beam and a constant phase term (used to generate the phase-shift) were given in function of the two reference beams. Due to a phase difference of $\pi/2$ between the reference beams one of the interference terms was canceled, leaving the two remaining in quadrature.

However under experimental conditions it is not always possible to obtain a phase difference of $\pi/2$ between the reference beams. Despite of this it will be shown that it is still possible to generate phase-shift by means of the amplitude variation of the fields, where now to keep a constant visibility the amplitudes must be seen as ordered pairs over an ellipse instead of a circle, extending with this the range of the phase-shifting until $[0, \pi]$ instead of $[0, \pi/2]$.

Let's consider again three linearly polarized waves at the same plane traveling on z direction as shown in Eq. (36), whose irradiance can be expressed as in Eq. (37)

$$I = A_1^2 + A_2^2 + A_3^2 + 2A_1 A_2 \cos(\phi_1 - \phi_2) + 2A_1 A_3 \cos(\phi_1 - \phi_3) + 2A_2 A_3 \cos(\phi_2 - \phi_3), \quad (50)$$

where the intensities of the three waves are present in the background light and also in the three interference terms. For the general case, the phases that will be considered are

$$\phi_1 \neq 0 \; ; \; \phi_2 = \phi_1 + \Delta\phi_1 + \phi \; ; \text{ and } \; \phi_3 = \phi_1 + \Delta\phi_2 , \quad (51)$$

$\Delta\phi_1$ is a constant phase difference between the first and second wave, $\Delta\phi_2$ is a constant phase difference between the first and third wave; therefore between the second and third wave will exist a phase difference of $\Delta\phi_2 - \Delta\phi_1$. In summary, in the absence of a phase object it will be considered a phase difference between each pair of waves. It is important to notice that when $\Delta\phi_1 = 0$ and $\Delta\phi_2 = \pi/2$ the general case studied in section 3.1 will be obtained. Substituting Eq. (51) into Eq. (50) we have

$$I = A_1^2 + A_2^2 + A_3^2 + 2A_1 A_2 \cos(\phi + \Delta\phi_1) + 2A_1 A_3 \cos(\Delta\phi_2) + 2A_2 A_3 \cos(\phi + \Delta\phi_1 - \Delta\phi_2) , \quad (52)$$

$$I = A_1^2 + A_2^2 + A_3^2 + 2A_1 A_3 \cos\Delta\phi_2 + 2A_2 [A_1 \cos(\phi + \Delta\phi_1) + A_3 \cos(\phi + \Delta\phi_1 - \Delta\phi_2)], \quad (53)$$

$$I = A_1^2 + A_2^2 + A_3^2 + 2A_1 A_3 \cos\Delta\phi_2 + 2A_2 [A_1 \cos(\phi + \Delta\phi_1) +$$

$$+ A_3 \cos\Delta\phi_2 \cos(\phi + \Delta\phi_1) + A_3 \sin\Delta\phi_2 \sin(\phi + \Delta\phi_1)], \quad (54)$$

$$I = A_1^2 + A_2^2 + A_3^2 + 2A_1 A_3 \cos\Delta\phi_2 + 2A_2 [(A_1 + A_3 \cos\Delta\phi_2)$$

$$\cos(\phi + \Delta\phi_1) + A_3 \sin\Delta\phi_2 \sin(\phi + \Delta\phi_1)], \quad (55)$$

$$I = A_2^2 + A_r^2 + 2A_2 A_r [\cos\psi \cos(\phi + \Delta\phi_1) + \sin\psi \sin(\phi + \Delta\phi_1)], \quad (56)$$

where

$$\cos\psi = \frac{A_1 + A_3 \cos\Delta\phi_2}{A_r} \; ; \; \sin\psi = \frac{A_3 \sin\Delta\phi_2}{A_r} , \tag{57}$$

$$I = A_2^{\;2} + A_r^{\;2} + 2A_2 A_r \cos(\phi + \Delta\phi_1 - \psi) . \tag{58}$$

It has been deduced the general expression of a fringe pattern of two waves where A_r is equivalent to the reference amplitude and ψ is an additional phase, both given by

$$A_r^{\;2} = A_1^{\;2} + A_3^{\;2} + 2A_1 A_3 \cos\Delta\phi_2 \; ; \tag{59}$$

$$\tan\psi = \frac{A_3 \sin\Delta\phi_2}{A_1 + A_3 \cos\Delta\phi_2} , \tag{60}$$

it can be seen that A_r and ψ depend on the amplitude variations of A_1 and A_3 as well as the phase difference between the first and third wave $\Delta\phi_2$.

To apply the PSI techniques, the visibility in Eq. (58) must remain constant for each phase-shift; this can be achieved if the amplitude A_r in la Eq. (59) remains constant while ψ varies with the changes of A_1 and A_3. To define the behavior of the amplitudes which satisfy the condition to keep A_r constant, Eq. (59) will be rewritten as

$$\sin^2\Delta\phi_2 = \frac{A_1^{\;2}}{A_r^{\;2}}\sin^2\Delta\phi_2 + \frac{A_3^{\;2}}{A_r^{\;2}}\sin^2\Delta\phi_2 + 2\frac{A_1 A_3}{A_r^{\;2}}\sin^2\Delta\phi_2 \cos\Delta\phi_2 , \tag{61}$$

which can take the form of the equation of an ellipse

$$\sin^2\alpha = \frac{x^2}{a^2} + \frac{y^2}{b^2} - 2\frac{xy}{ab}\cos\alpha , \tag{62}$$

such that

$$\tan 2\upsilon = \frac{2ab}{a^2 - b^2}\cos\alpha , \tag{63}$$

where υ is the inclination angle of the ellipse, a is the maximum value of the ellipse over the x axis and b is the maximum value over the y axis.

Being in our case $a = -b$, so Eq. (61) can be written as in Eq.(62), where

$$a = \frac{A_r}{\sin\Delta\phi_2} ; \tag{64}$$

substituting the values of a, b and $\alpha = \Delta\phi_2$ in Eq. (60) a family of ellipses inclined to an angle υ will be obtained

$$\tan 2\upsilon = -\frac{\cos\Delta\phi_2}{0} = \begin{cases} \text{indet} & \Delta\phi_2 = \pi/2 + n\pi \\ -\infty & \Delta\phi_2 \in [0,\pi/2) \cup (3\pi/2,2\pi] \\ +\infty & \Delta\phi_2 \in (\pi/2,3\pi/2) \end{cases}, \tag{65}$$

therefore υ can take the next values

$$\upsilon = \begin{cases} \text{indet}, & \Delta\phi_2 = \pi/2 + m\pi \\ -\pi/4, & \Delta\phi_2 \in [0,\pi/2) \cup (3\pi/2,2\pi] \\ \pi/4, & \Delta\phi_2 \in (\pi/2,3\pi/2) \end{cases}. \tag{66}$$

The amplitudes in Eq. (61) can take the next parametric form

$$A_1 = \frac{A_r}{\sin\Delta\phi_2}\sin(\Delta\phi_2 - \psi)\;;\; A_3 = \frac{A_r}{\sin\Delta\phi_2}\sin\psi\;, \tag{67}$$

whose parameter is the phase-shift ψ within a valid range of

$$\psi \in \begin{cases} [0,\Delta\phi_2], & \Delta\phi_2 \in (0,\pi) \\ [\Delta\phi_2 - \pi,\pi], & \Delta\phi_2 \in (\pi,2\pi) \end{cases}, \tag{68}$$

that is obtained considering that the amplitudes A_1 and A_3 are positive, therefore they must be in the first quadrant and that $\Delta\phi_2 \neq m\pi$ being m an integer number. However it is possible to have a negative modulation in the amplitude if a phase difference of π in the reference waves is properly implemented, thus it is possible to have real negative amplitudes, hence ψ will be within the range of $[0,2\pi]$, and the known PSI techniques could be applied without modifications.
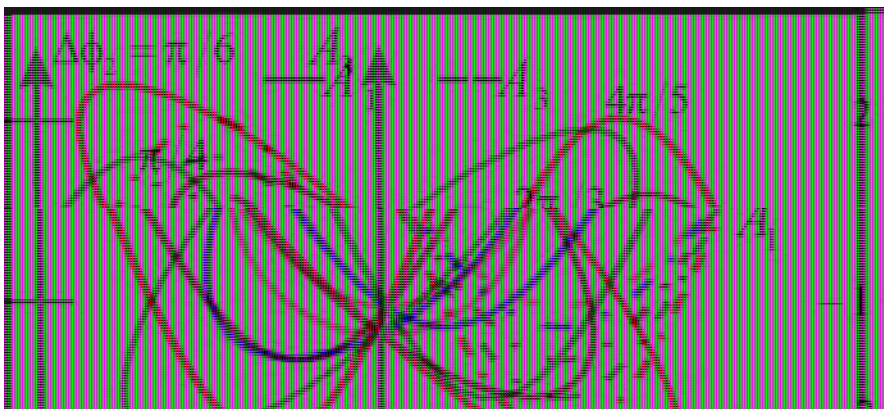

Fig. 10. Amplitudes of the wave one and three put in a reference system for several values of $\Delta\phi_2$. a) family of ellipses; any point on any ellipse keeps A_r constant while ψ is varied as it is required for PSI, b) amplitudes in a parametric form respecting to ψ corresponding to the ellipses in a).

It can be observed form Eq. (68) that $\Delta\phi_2$ determines directly the range of the phase-shifting. When a certain phase-shift is needed, the amplitudes A_1 and A_3 will be given by Eq. (67) which comply the conditions for a constant visibility and they could be seen as points over the arc of an ellipse at the first quadrant as it is shown for several values of $\Delta\phi_2$ in Figure 10.

A numerical simulation will be shown in order to prove the viability of the proposal. For simplicity, the next considerations have been taken

$$A_2 = A_r = 1 \; ; \; \Delta\phi_1 = \phi_1 = 0 \; ; \; \Delta\phi_2 = 2\pi/3 \; ; \; \phi = x^2 + y^2 \; , \tag{69}$$

therefore the three fields can be expressed as

$$E_1 = A_1 \; ; \; E_2 = \exp(i\phi) \; ; \; E_3 = \exp(i\,2\pi/3) \; , \tag{70}$$

the interference pattern can be written as

$$I = 2 + 2\cos(\phi - \psi) \; , \tag{71}$$

such a form that the interference pattern will have a maximum contrast, where the amplitudes A_1, A_3 and the additional phase ψ are given by

$$1 = A_1^{\;2} + A_3^{\;2} - A_1 A_3 \; , \tag{72}$$

$$\tan\psi = \frac{\sqrt{3}A_3}{2A_1 - A_3} \; . \tag{73}$$

Substituting Eq. (69) in Eq. (67) is gotten

$$A_1 = \cos\psi + \frac{1}{\sqrt{3}}\sin\psi \; ; \; A_3 = \frac{2}{\sqrt{3}}\sin\psi \; . \tag{74}$$

Figure 12a) shows the ellipse at the first quadrant for this particular case, which is inclined to $\pi/4$ with an ellipticity of $e = \sqrt{3}$. Figure 12b) shows the amplitudes corresponding to the phase shift within the range $\psi \in \left[0, 2\pi/3\right]$.

Figure 12 shows the interferograms for $N = 9$, being the phase step of $\Delta\psi = \pi/12$; the phase-steps $\psi_k = k\pi/12$ with $k = 0,1,...,8$ were generated by the amplitudes shown in Figure 12b), and these are indicated with points over the arc corresponding to the ellipse in Figure 12a).

This analysis has the advantage that even if there is a phase difference of $\Delta\phi_2$ between the reference waves it is still possible to generate phase-shifting with the proposed method, besides that it will not be necessary to use an optical device (as phase retarders) to generate $\Delta\phi_2$. The variations of the amplitudes can be done by using neutral filters or also by using the diffraction orders produced by a grating as for example a Ronchi ruling, where each diffraction order is attenuated according to the sinc function.
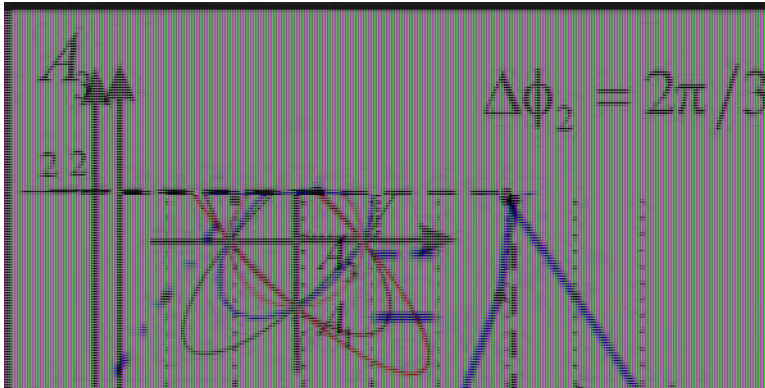
Fig. 11. Amplitude variations for a PSI of 9 steps when $\Delta\phi_2 = 2\pi/3$: a) a point over the arc yields a phase-shifting while the reference amplitude is kept constant; b) amplitudes (A_1, A_3) for the phase steps indicated in a).
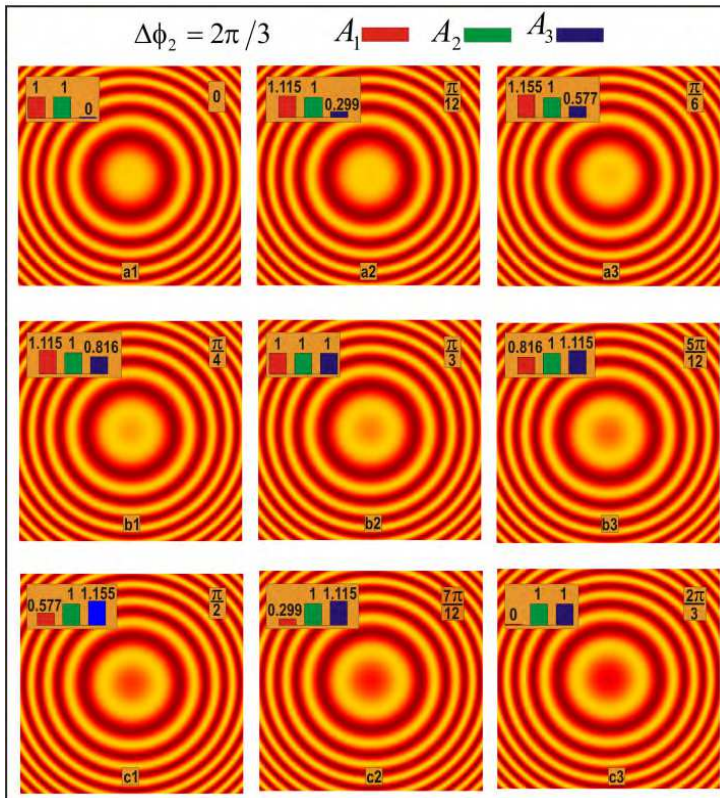

Fig. 12. Interferograms shifted in phase by the amplitude variations indicated in Figure 11. The phase steps are shown from left to right and from top to bottom.

4. Conclusions

In this chapter have been discussed the phenomenon of optical interference for two waves elliptically polarized, the phase shifting interferometry and the commonly used methods to generate that shifting, as well as the methods for phase extraction. It has been demonstrated with a numerical analysis and a computer simulation the viability of a new method of phase-shifting based on the amplitude variation of two fields considered as the reference beams in a scheme of a three beam interferometer, for which two cases were analyzed:

A particular case was considered when the phase difference between the reference waves is $\pi/2$, hence one of the three interference terms is cancelled while the two remaining are put in quadrature. To get a constant visibility, the amplitude of the reference waves must be over an arc segment in the first quadrant of a circle whose radius is A_r. Due to the amplitudes are considered to be real positive, the phase-shifting will be within the range of $[0,\pi/2]$. But, theoretically this range could be extended until $[0,2\pi]$ if the amplitudes are modulated moreover in their negative part, what can be done by an appropriate phase change in the reference waves by π radians.

In a more general case, the phase differences $\Delta\phi_2$ between the reference waves was considered to be arbitrary and within the range $\Delta\phi_2 \in [0,2\pi]$. In this study it was shown that despite of this the phase-shifting by amplitude modulation is also possible, and it includes the particular case given when $\Delta\phi_2 = \pi/2$. Besides in order to keep a constant visibility during the PSI application, the amplitudes must be over an ellipse instead of a circumference, which is inclined at $\pm\pi/4$ if $\Delta\phi_2 \in (\pi/2, 3\pi/2)$ or $\Delta\phi_2 \in [0,\pi/2) \cup (3\pi/2, 2\pi]$, respectively, also it was found that $\Delta\phi_2$ directly influences in both the ellipticity and the phase-stepping range when the amplitudes are modulated no-negative and can also reach a range until of $\psi \in [0,\pi]$, however when the negative part is taken in account, the range can reach until $\psi \in [0,2\pi]$.
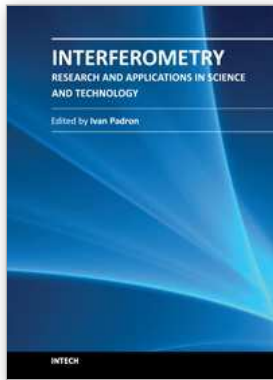
5. Acknowledgment

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