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Modification of the Charnock Wind Stress Formula to Include the Effects of Free Convection and Swell

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1. Introduction

The estimate of the surface fluxes of momentum, sensible heat, and water vapor over the ocean is important in numerical modeling for weather forecasting, air quality modeling, environmental impact assessment, and climate modeling, which, in particular, requires more accurate flux calculations. The accuracy of flux calculations depends on an important parameter: the surface roughness length. More than five decades ago Charnock (1955) formulated the surface roughness length over the ocean, relating the sea-surface roughness to wind stress on the basis of dimensional argument. Even now the Charnock wind stress formula is widely used. However, in the free convection limit, the horizontal mean wind speed and the wind stress approach zero, therefore the friction velocity also approaches zero; in this condition, the Charnock wind stress formula is no longer applicable and the Monin-Obukhov similarity theory (MOST) has a singularity; MOST relies on the dimensional analysis of relevant key parameters that characterize the flow behavior and the structure of turbulence in the atmospheric surface layer. Thus, in the conditions of free convection, the traditional MOST is no longer valid, because the surface roughness length also approaches zero. In numerical modeling, the surface fluxes, wind, and temperature in the surface layer are important, because they are used as lower boundary conditions.

In the Charnock formulation of surface roughness, two important physical processes were not considered: one is the free convection and the other is the swell. Swell is the long gravity waves generated from distant storms, which modulate the short gravity waves. To date the formulation of the effect of swell on the aerodynamic roughness length has not been theoretically developed. Thus, the purpose of this study is to modify the Charnock wind stress relation to include the effects of the free convection and the swell on the surface roughness length. A formula of the surface roughness length that relates to the convective velocity is proposed by Abdella and D'Alessio (2003). However, in this study, the alternative approaches, which are much simpler than the Abdella and D'Alessio derivation, are used to derive the surface roughness length, which takes into account the effects of the free convection. The convective process is important in affecting the air-sea transfer of momentum, sensible heat, and water vapor, especially at very light winds. Businger (1973) suggested that under the conditions of free convection and in the absence of the horizontal mean wind, averaged over the horizontal area, a local wind profile still exists. This wind profile is generated by the convective circulations in the atmospheric boundary layer and, hence, we can observe a wind stress and a shear production of turbulent energy near the surface. As a consequence of the convective circulations, it will induce the “virtual” horizontal stress, the so-called convection-induced stress, which contributes to the increase of the surface roughness. Figure 1 illustrates the concept of convective circulations over the ocean.

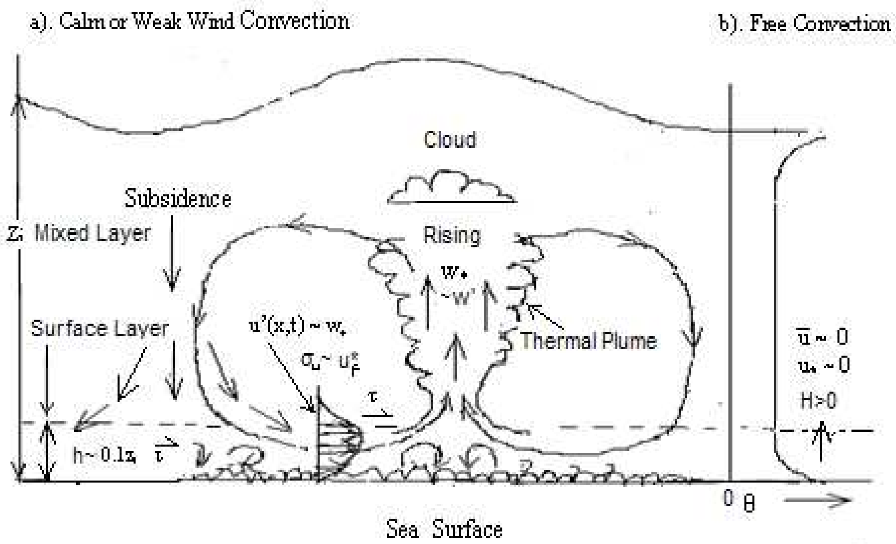

Fig. 1. The figure shows the large scale circulations associated with the convective activities. a. Near Calm or weak wind convection, b. Free Convection. Where z_i is the mixed-layer height, h the surface layer (inner layer), $\bar{u}$ the mean wind speed, u_* the friction velocity, w_* the convective velocity, τ the wind stress, θ the potential temperature, H the sensible heat flux, σ_u the standard deviation of the horizontal mean wind speed, u_{*f} the induced friction velocity, and $u'(x, t)$ the horizontal velocity fluctuations induced by large eddies in connection with the large scale convective circulations.

In free convection, the velocity should be scaled by the convective velocity as proposed by Deardorff (1970) rather than by the friction velocity. The convective velocity is an important scaling parameter in the study of the atmospheric convection. Therefore, in this paper, we introduce a new velocity scale that is a combination of the friction velocity and the convective velocity. This velocity scale is equal to the total friction velocity. With the use of this velocity scale to obtain the surface roughness length, the singularity in the traditional MOST is avoided. In general, the surface roughness length also depends on the sea state and can be expressed in terms of friction velocity, convective velocity, air viscosity, and wave age. The effect of the wave-induced stress or swell on the surface roughness is also investigated in this paper.

In the free convection limit, Abdella and D'Alessio (2003) found that the Charnock relation substantially underestimates the value of the momentum roughness. To remedy this, they proposed a sea surface roughness formula for z_o by including a convective velocity in the Charnock relation as

$$z_o = \frac{\alpha}{g}\left(u_*^2 + \gamma_a w_*^2\right), \tag{1}$$

where the parameter a is the Charnock constant, which also depends on the wave age, g is the acceleration of gravity, u_* is the friction velocity (The squared value of friction velocity is related to wind stress, $\tau/\rho_a = u_*^2$, where ρ_a is the density of air), w_* is the convective velocity (as defined below in Eq. (27); the convection velocity is also used in the literature), and γ_a is an empirical constant. Eq. (1) reduces to the Charnock relationship if the convective velocity is neglected.

In this paper, we propose an alternative approach for the parameterization of roughness length, z_o, for flow over the sea in forced and free convection (forced convection is without buoyancy forces), which is expressed as

$$z_o = \frac{\alpha}{g}\left(u_*^3 + \gamma w_*^3\right)^{\frac{2}{3}}, \tag{2}$$

where γ is an empirical constant. Equation (2) is similar to that of the roughness length proposed by Abdella and D'Alessio (2003) as shown in Eq. (1). However, in the limiting conditions Eq. (1) and Eq. (2) are identical.

In this paper, first we provide necessary background information about MOST in Section 2 and about the bulk formulations for the surface fluxes of momentum, heat, and water vapor in Section 3. Then, two alternative approaches are used to derive the surface roughness length as given in Eqs. (1) and (2) in Section 4. The first approach is based on the Prandtl mixing length theory and the standard deviation of the vertical velocity component to derive a new velocity scale that is applicable to the conditions of free convection. The second approach is to use the standard deviations of the horizontal velocity fluctuations induced by large eddies in connection with the large scale convective circulations to derive the relationship between surface roughness and the convective velocity, which is consistent with the concept of gustiness proposed by Businger (1973) (also see Schumann, 1988; Sykes et al., 1993).

The swell may also alter the surface roughness. Thus, in this paper, the Charnock wind stress relation including the effect of swell is derived theoretically for the first time.

The background information about MOST is provided in the following section.

2. The Monin-Obukhov similarity theory

The Monin-Obukhov similarity theory is based on dimensional analysis, which basically states that the flow is in quasi steady state over the horizontally homogeneous surface. And the vertical profiles of the horizontal mean wind and temperature, and the characteristics of turbulence in the atmospheric surface layer can be described as a universal function of relevant parameters, including the height above the surface, the surface wind stress, the buoyancy parameter, and the surface heat flux. In the atmospheric surface layer, the vertical variations of surface fluxes are within 10%. This surface layer is also called the constant stress layer or constant flux layer; the wind stress is aligned with the direction of the horizontal mean wind.

According to the well-known Monin-Obukhov similarity theory (MOST) (Monin-Obukhov, 1954), the non-dimensional profiles of wind shear, ϕ_m, temperature gradient, ϕ_h, and moisture gradient, ϕ_q, are expressed as:

$$\frac{kz}{u_*}\frac{\partial \overline{u}}{\partial z} = \phi_m\left(\frac{z}{L}\right), \tag{3}$$

$$\frac{kz}{\theta_*}\frac{\partial \overline{\theta}}{\partial z} = \phi_h\left(\frac{z}{L}\right), \tag{4}$$

$$\frac{kz}{q_*}\frac{\partial \overline{q}}{\partial z} = \phi_q\left(\frac{z}{L}\right). \tag{5}$$

In the above equations, it states that in the atmospheric surface layer, the non-dimensional wind shear, and temperature and moisture gradients can be expressed as a universal function of the atmospheric stability parameter, *z/L*, where *L* is the Monin-Obukhov stability length (as defined in Eq. (8-1)). And where θ_* and q_* are the temperature and moisture scales, respectively, defined as

$$\theta_* = -\frac{\overline{w'\theta'}}{u_*} \tag{6}$$

$$q_* = -\frac{\overline{w'q'}}{u_*} \tag{7}$$

Where $\overline{u}$, is the horizontal mean wind speed; $\overline{\theta}$, is the mean potential temperature; $\overline{q}$, is the mean specific humidity; k is the von Kármán constant (k=0.4); $\overline{w'\theta'}$ is the surface kinematic heat flux; $\overline{w'q'}$ is the moisture flux, and L is the Monin-Obukhov stability length (also called Obukhov length (Obukhov,1946)) defined as

$$L = -\frac{\theta_v u_*^3}{kg\left(\overline{w'\theta_v'}\right)}, \tag{8-1}$$

or

$$L = -\frac{T(1+0.61q)u_*^2}{kg[\theta_* + 0.61Tq_*)}, \tag{8-2}$$

where T is the temperature, and θ_v is the virtual temperature. Eq. (8-1) and Eq.(8-2) are equal to each other; Eq.(8-2) is expressed in terms of the scaling parameters, , u_*, θ_*, and q_*.

For practical applications, we assume the following flux-profile relations (see Huang, 1979):

$$\phi_h = \phi_q = \phi_m^2,\ \text{z/L} < 0 \text{ for unstable conditions} \tag{9}$$

where $\phi_h(z/L)$ is the non-dimensional function of temperature gradient.
Based on various meteorological experiments, different forms of the universal function for $\phi_m(z/L)$ have been proposed. For example, the universal function for the non-dimensional wind shear $\phi_m(z/L)$ empirically obtained from the observational data (Dyer, 1974; also see Huang, 1979) can be expressed as:

$$\phi_m\left(\frac{z}{L}\right) = 1 + \alpha_1 \frac{z}{L} \qquad z/L \geq 0 \text{ for stable conditions} \tag{10-1}$$

$$\phi_m\left(\frac{z}{L}\right) = \left(1 - \alpha_2\left(\frac{z}{L}\right)\right)^{-\beta_1}, \quad z/L < 0 \text{ for unstable conditions} \tag{10-2}$$

where the values of a_1 = 5, a_2 = 16 and β_1 = 1/4 are recommended by Dyer (1974) for unstable conditions. Other values of a_2 = 7 and β_1 = 1/3 for unstable conditions are given by Troen and Mahrt (1986) and others.
In general, the function of $\phi_h(z/L)$ can be expressed as (Hogstrom, 1998; Wilson, 2001):

$$\phi_h = \mathrm{Pr}_t\left(1 + \beta_h \frac{z}{L}\right) \quad z/L \geq 0 \text{ for stable conditions} \tag{11-1}$$

$$= \mathrm{Pr}_t \quad z/L = 0 \text{ for neutral conditions} \tag{11-2}$$

$$= \mathrm{Pr}_t\left(1 - \gamma_h \frac{z}{L}\right)^{-\frac{1}{2}} \quad z/L < 0 \text{ for unstable conditions} \tag{11-3}$$

Where β_h and γ_h are empirical constants and the values of β_h and γ_h are equal to 5 and 16, respectively. Pr_t is the turbulent Prandtl number and is defined as

$$\mathrm{Pr}_t = \frac{K_m}{K_h}. \tag{11-4}$$

Where K_m and K_h are exchange coefficients (or eddy diffusivities) for momentum and heat, respectively. The turbulent Prandtl number for neutral stability, Pr_t, is equal to 0.95 .

3. Fluxes of momentum, heat, and moisture

The standard bulk formulations for the surface fluxes of momentum (τ), heat (H), and moisture (E) can be expressed as

$$\tau = \rho_a C_d (u_s - u_r)^2 = \rho_a u_*^2, \tag{12}$$

$$H = \rho_a c_{pa} C_h u_r (T_s - \theta) = -\rho_a c_{pa} u_* \theta_* \tag{13}$$

$$E = \rho_a L_e C_e u_r (q_s - q) = -\rho_a u_* q_*. \tag{14}$$

Where ρ_a is the air density, c_{pa} the specific heat of air at constant pressure, u_r the mean wind speed at the reference height z_r above the sea surface, u_s the speed of surface current, T_s the skin temperature, q_s the saturation specific humidity at the sea surface, and L_e the latent heat of water vapor. The C_d, C_h, and C_e are the bulk transfer coefficients for wind stress or momentum, sensible heat, and latent heat, respectively. The typical values of the bulk transfer coefficients are about 0.001 at the reference height of about 10 m. For the ease of computation, these transfer coefficients can be further partitioned into individual components and written as analytical functions that depend on the atmospheric stability parameter (z/L) as

$$C_d = C_d^{1/2} C_d^{1/2} = \frac{C_{dn}}{\left[1 - \frac{C_{dn}^{1/2}}{k}\psi_m\left(\frac{z}{L}\right)\right]^2}, \tag{15}$$

$$C_h = C_d^{1/2} C_T^{1/2} = \frac{C_{dn}^{1/2} C_{hn}^{1/2}}{\left[1 - \frac{C_{dn}^{1/2}}{k}\psi_m\left(\frac{z}{L}\right)\right]\left[1 - \frac{C_{hn}^{1/2}}{k}\psi_h\left(\frac{z}{L}\right)\right]}, \tag{16}$$

$$C_e = C_d^{1/2} C_q^{1/2} = \frac{C_{dn}^{1/2} C_{hn}^{1/2}}{\left[1 - \frac{C_{dn}^{1/2}}{k}\psi_m\left(\frac{z}{L}\right)\right]\left[1 - \frac{C_{qn}^{1/2}}{k}\psi_q\left(\frac{z}{L}\right)\right]}. \tag{17}$$

These components can be expressed on the basis of the Monin-Obukhov similarity theory as functions of reference height, z, surface roughness length (z_o for momentum, z_{oT}, for heat, and z_{oq} for water vapor), and the Monin-Obukhov length, L, defined in Eq. (8). For more detailed information on the functional expressions of the bulk transfer coefficient, refer to texts such as Garratt (1992) or Stull (1997). Where $\psi_m(z/L)$, $\psi_h(z/L)$, and $\psi_q(z/L)$ are the similarity functions, which are given in Eq. (19) to (21) below. It is usually assumed that there is similarity between water vapor and heat transfer, which implies that $\psi_h(z/L) = \psi_q(z/L)$.

For example, the bulk transfer coefficient of momentum or the wind profile can be expressed as (see Paulson, 1970; Huang 1979)

$$C_d \equiv \left(\frac{u_*}{u}\right)^2 = \frac{k^2}{\left(\ln\left(\frac{z}{z_o}\right) - \psi_m\right)^2}. \tag{18}$$

Where the similarity function $\psi_m(z/L)$ in Eq. (15) or (18) is given as

$$\psi_m = \psi_h = -5\frac{z}{L} \quad \text{for } z/L \geq 0 \tag{19}$$

$$\psi_m = 2\ln\left(\frac{1+x}{2}\right) + \ln\left(\frac{1+x^2}{2}\right) - 2\tan^{-1}(x) + \frac{\pi}{2} \quad \text{for } z/L<0 \tag{20}$$

and the function $\psi_h(z/L)$ in Eq. (16) is given as

$$\psi_h = 2\ln\left(\frac{1+x^2}{2}\right) \quad \text{for } z/L<0 \tag{21}$$

where

$$x = \left(1-16\frac{z}{L}\right)^{1/4} \quad \text{for } z/L < 0$$

Under neutral conditions, that is when z/L approaches 0, the transfer coefficients in Eqs. (15) – (17) become

$$C_{dn}^{1/2} = \frac{k}{\ln\left(\frac{z_r}{z_o}\right)}, \tag{22}$$

$$C_{hn}^{1/2} = \frac{k}{\ln\left(\frac{z_r}{z_{oT}}\right)}, \tag{23}$$

$$C_{qn}^{1/2} = \frac{k}{\ln\left(\frac{z_r}{z_{oq}}\right)}. \tag{24}$$

Where z_{oT} and z_{oq} are roughness lengths for heat and water vapor, respectively.

4. Relationship between the friction and convective velocity

Two approaches are used here to derive the relationship between the induced friction velocity and the convective velocity. These approaches are based on the Prandtl mixing length theory and the turbulent velocity fluctuations, which are described in the following sections.

4.1 Prandtl mixing length theory and velocity scale

In this section, we use the Prandtl mixing length theory and the standard deviation of the vertical turbulent velocity to derive the surface roughness length that includes the effect of free convection on the surface roughness length. The Prandtl mixing length theory has also been used to derive the Abdella and D'Alessio formula in Eq. (2) (also see Huang, 2009).

According to MOST, the standard deviation of the vertical turbulent velocity component (w'), $\sigma_w = \sqrt{(w')^2}$, in the surface layer can be described by:

$$\frac{\sigma_w}{u_*} = \phi_w\left(\frac{z}{L}\right), \tag{25}$$

In Eq. (25), ϕ_w is a non-dimensional function, which depends only on the height, z, and the Monin-Obukhov stability length, L.
The similarity function for the vertical turbulent velocity postulated by Wyngaard and Cote (1974) and suggested by Panofsky et al. (1977) can be expressed as

$$\frac{\sigma_w}{u_*} = 1.25\left(1 - 3\frac{z}{L}\right)^{\frac{1}{3}}. \tag{26}$$

Equation (26) is based on the similarity theory and observational data. Under the conditions of free convection, buoyancy is the driving force, which, through the free convection in connection with the large-scale convective circulation, induces the horizontal motions (see Fig. 1).
The following, through the Prandtl mixing length theory, shows that the standard deviation of the vertical turbulent velocity is related to the total horizontal wind stress.
The convective velocity scale, w_*, is expressed by Deardorff (1970) as

$$w_* \equiv \left(\frac{g}{T}H_o z_i\right)^{\frac{1}{3}} = \left(-\frac{z_i}{kL}\right)^{\frac{1}{3}}.u_* . \tag{27}$$

Where T is the temperature of air, H_o is the surface heat flux, and z_i is the depth of the mixed layer. The appropriate scaling parameters for the mixed-layer similarity are the convective velocity, w_*, and the mixing height, z_i.
By using the definition of the convective velocity, w_*, in Eq. (27), Eq. (26) becomes

$$\sigma_w = 1.25\left(u_*^3 + \gamma w_*^3\right)^{\frac{1}{3}}, \tag{28}$$

where γ is an empirical constant. It also can be specified as $\gamma = 3k\varepsilon$ and $\varepsilon = z/z_i$, the ratio of the surface boundary layer to the mixed-layer height, where k is the von Kármán constant and ε is commonly specified to be 0.1 (Troen and Mahrt, 1986).
In the limit of free convection, the friction velocity approaches zero and the component of vertical turbulent velocity, σ_w, in Eq. (28), becomes

$$\sigma_w = \sqrt{w'^2} = 1.25\gamma^{\frac{1}{3}} w_* \approx 0.6 w_* . \tag{29}$$

From Eq. (29), we obtain the value of 0.11 for γ. If we set the value of $\varepsilon = 0.1$, then we obtain the value of 0.12 for γ in Eq. (28).
According to the Prandtl mixing length theory, the eddy diffusivity for momentum can be written as

$$K_m = Ckz\sigma_w , \tag{30}$$

where C is a constant to be determined and K_m is related to the wind stress by definition.
Substituting Eq. (28) into Eq. (30), we obtain

$$K_m = C'kz\left(u_*^3 + \gamma w_*^3\right)^{\frac{1}{3}}, \tag{31}$$

where C' is a constant. Under the neutral condition, C' is equal to 1. Therefore, Eq. (31) becomes

$$K_m = kz\left(u_*^3 + \gamma w_*^3\right)^{\frac{1}{3}} \quad . \tag{32}$$

From the above equation, now for convenience, we define a new velocity scale, w_s, as

$$w_s = \left(u_*^3 + \gamma w_*^3\right)^{1/3} . \tag{33}$$

Then Eq. (32) becomes

$$K_m = kz\tilde{u}_* = kzw_s , \tag{34}$$

where $\tilde{u}_*$ is the total friction velocity.
Therefore, from Eqs. (33) and (34), the total stress or the square of friction velocity, $\tilde{u}_*$, can be written as

$$\tilde{u}_*^2 = \left(u_*^3 + \gamma w_*^3\right)^{\frac{2}{3}} . \tag{35}$$

Eq. (35) shows that the free convection represented by the convective velocity, w_*, will induce the surface stress, $\tau = \rho \tilde{u}_*^2$, as proposed by Businger (1973). Eq. (35) can be used to calculate the surface roughness length. The Abdella and D'Alessio derivation of the roughness length formula is much more complicated and involves more assumptions (Abdella and D'Alessio, 2003). The present derivation confirms the Abdella and D'Alessio formulation (see Eq. (1)) and is much simpler than their derivation.
The above approach is consistent with the concept of gustiness produced by large eddies in connection with the convective circulation (see Fig. 1, Schuman, 1988, and Sykes et al. 1993 for details). In the following section, we derive the relationship between the convection-induced wind stress and the convective velocity using this concept.

4.2 Horizontal velocity variance and gustiness

In this section, the horizontal turbulent velocity representing the gustiness is used to obtain the relationship between the convection-induced stress and the convective velocity, which in turn can be used to estimate the surface roughness length.
In convective conditions, it has been found that the gustiness, the large-scale turbulence, will enhance the wave growth and the energy transfer from wind to wave (see Janssen, 1989).
In free convection conditions, the fluctuations of the horizontal wind velocity are controlled primarily by the large-scale convective circulations in the convective boundary layer (CBL) (see Fig. 1). Therefore, the relationship between the standard deviations of the horizontal turbulence components and the convective velocity can be expressed as (e.g., see Fairall et al., 1996)

$$\sigma_u^2 + \sigma_v^2 = \beta^2 w_*^2 , \tag{36}$$

where the gustiness parameter β is an empirical constant, σ_u and σ_v are the standard deviations of horizontal turbulent velocity components, respectively, and w_* the convective velocity. A value of β (=1.25) was determined from the Moana Wave measurements of the horizontal velocity variances (Fairall et al. 1996). However, according to Businger (1973), in the limit of free convection, as the mean "friction velocity", u_* (or the mean wind stress, $\rho_a u_*^2$), and the horizontal mean wind speed, averaged over an area, approach zero, close to the ground, the fluctuations of horizontal velocity induced by the large-scale convective circulations locally promote a convection-induced stress. The standard deviations of horizontal turbulent velocity components that represent the fluctuations of horizontal wind speed will produce the horizontal-induced wind stress. Under convective conditions, the standard deviation of horizontal turbulent velocity is related to the surface friction velocity (see e.g., Monin-Yaglom, 1971; Businger, 1973); therefore, we express the relation between the velocity variances and the convection-induced friction velocity u_{*f} or the velocity scale ($w_s = u_{*f}$) as

$$\sigma_u^2 + \sigma_v^2 = \beta_2 u_{*f}^2 , \tag{37}$$

where β_2 is an empirical constant. For example, under convective conditions, the velocity variances are estimated to be $\sigma_u/u_{*f} \approx 2.1$ and $\sigma_v/u_{*f} \approx 2.0$ for the Monin-Obukhov stability parameter of $z/L = -0.6$ (see Monin and Yaglom , 1971); therefore, in this case, we obtain a value of $\beta_2 = 8.41$. This value is used in this study.
From Eqs. (36) and (37), we have the convection-induced stress ($\tau_f = \rho u_{*f}^2$) as

$$u_{*f}^2 = \frac{\beta^2}{\beta_2} w_*^2 = \gamma' w_*^2 , \tag{38}$$

where γ' is an empirical constant and is estimated to be $\gamma' = \beta^2/\beta_2 = 0.19$ from Eq. (38), the value of γ' ($=\gamma^{2/3} = 0.23$) has also been estimated above in Section 4.1. Eq. (38) is consistent with the concept of gustiness proposed by Businger (1973) and others (Schumann, 1988; Sykes et al., 1993). Under strong convection conditions, the production of turbulent energy due to buoyancy force is more effective than the shear production of turbulent energy. That is, under free convection, the convective velocity is an important scaling parameter that has been recognized by Deardorff (1970).

5. Effective total stress

Over ocean waves, the effective total stress, τ_{eff}, above the sea surface can be partitioned into four parts: turbulent shear stress, τ_t; wave-induced stress, τ_w; convection-induced stress, τ_f; and viscous stress, τ_v. Thus the effective total stress is defined here as:

$$\tau_{eff}(z, z_i) = \tau_t(z) + \tau_w(z) + \tau_f(z_i) + \tau_v(z) . \tag{39}$$

In the above equation, the effective total stress is a function of height, z, above the sea surface and the mixed layer height, z_i, in the atmospheric boundary layer. The third term on the right-hand side of Eq. (39) has not hitherto been considered. That is under the free

convection and in the absence of swell, τ_{eff} is equal to τ_f (i.e., $\tau_{eff} = \tau_f$). The value of the convection induced stress is equal to $\tau_f/\rho_a = \gamma' w_*^2 = 0.2$ if we set $\gamma' = 0.2$ and $w_* = 1$.
The momentum flux corresponding to each term in Eq. (39) can be written in the form:

$$u_{*eff}^2 = u_{*t}^2 + u_{*w}^2 + u_{*f}^2 + u_{*v}^2 . \tag{40}$$

Omitting the convection-induced stress, τ_f, in Eq. (39), the effective total stress reduces to (see Phillips, 1966, p. 93):

$$\tau(z) = \tau_t(z) + \tau_w(z) + \tau_v(z) . \tag{41}$$

On the basis of the conservation of momentum flux in the marine atmospheric surface layer, the total stress is independent of height, i.e., one assumes that the flow is in a steady state, with horizontal homogeneity in the surface boundary layer (see e.g., Phillips, (1966); Janssen (1991); the WAM model). It is commonly assumed that the total stress (both turbulent and wave momentum and energy) is constant in the atmospheric surface layer, which is usually considered as about the lowest 10 – 30 m above the sea surface.

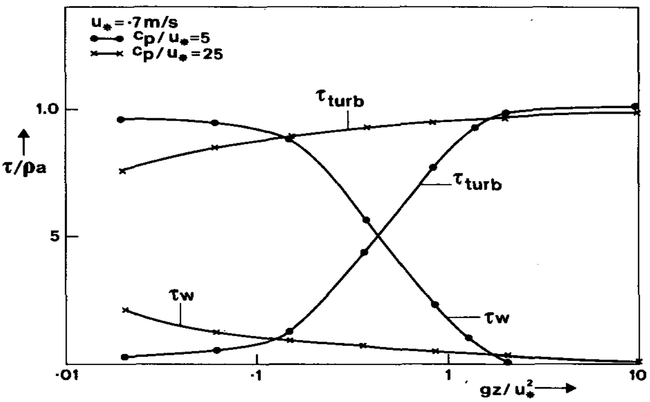

Fig. 2. Distribution of turbulent stress, viscous stress and wave-induced stress as a function of height. The contribution of viscous stress is not shown because it is smaller than 5%. (From Janssen, 1989).

Figure 2 shows the distribution of turbulent stress, viscous stress and wave-induced stress as a function of height, the change of each term in Eq. (41) with height. According to Phillips (1966), the viscous stress can be neglected, except possibly very near the water surface. The wave-induced stress is important only in the wave boundary layer (WBL; see Janssen, 1989). Near the water surface, the wave-induced stress would be significant; however, it decays rapidly with height in the surface layer. Well above the water surface, the wave-induced disturbance vanishes (see Phillips, 1966; Makin and Kudryavtsev, 1999; and Makin, 2008).

6. Modification of the charnock wind stress formula

The surface roughness length, z_o, will be shown to relate to the friction velocity, wave-induced friction velocity, and convective velocity. The well-known Charnock wind stress formula is a relationship between the surface roughness length over the sea and the wind stress or momentum flux; it is expressed as (Charnock, 1955)

$$z_o = \alpha \frac{u_*^2}{g}, \tag{42}$$

where the parameter $a = a_{ch}$ is the Charnock constant with the value ranging from 0.011 (Smith, 1980, 1988) to 0.035 (e.g., see Garratt, 1977; 1992, Table 4.1). Charnock (1955) postulated the roughness-wind stress formulation in Eq. (42) on the basis of a dimensional argument. In the Charnock wind stress relation, Eq. (42), the roughness length depends only on the friction velocity and the gravitational acceleration; the buoyancy effect due to thermal convection is ignored. In this paper, we have shown that under strong convection, the roughness length also depends on the convective velocity (see Section 4).

The dependencies of the surface roughness length on the convective velocity, the wave age, and the effect of the wave-induced stress or swell are investigated in the following sections.

6.1 General Charnock relation and free convection

A general formula for the Charnock relation and the free convection is discussed in this section.

6.1.1 General Charnock relation

In this section, we focus on the effect of free convection on the surface roughness. The general equation for the Charnock relation is obtained by replacing the friction velocity scale (u_*) in the Charnock relation, Eq. (42), with the effective total friction velocity, $u*_{\text{eff}}$, in Eq. (40) or the vertical velocity scale, w_s, which is a combination of the friction velocity and convective velocity. Therefore, the Charnock formula is also applicable under the conditions of free convection.

Substituting Eq. (40) into Eq. (42) and neglecting the viscous stress for the moment, we have

$$\begin{aligned} z_o &= \frac{\alpha}{\rho_a g}\left(\tau_{eff}\right) \\ &= \frac{\alpha}{\rho_a g}\left(\tau_t + \tau_w + \tau_f\right),. \end{aligned} \tag{43}$$

where ρ_a is the density of air. Taking the flow over the aerodynamically smooth surface into consideration by including the viscous term, the equation for the surface roughness, Eq. (43), becomes

$$\begin{aligned} z_o &= \frac{\alpha}{\rho_a g}\left(\tau_{eff}\right) + 0.11\frac{\nu}{u_*} \\ &= \frac{\alpha}{\rho_a g}\left(\tau_t + \tau_w + \tau_f\right) + 0.11\frac{\nu}{u_*}. \end{aligned} \tag{44}$$

The above equation includes the effect of the wave-induced stress or swell on the surface roughness (see Section 6.3 and 6.4 for more information), and ν is the kinematic viscosity (molecular) of air.

The velocity scale as shown in Eq. (33) also includes the effect of buoyancy that will enhance the surface roughness length. By substituting the vertical velocity scale, w_s, in Eq. (33) or the total friction velocity in Eq. (35) into the Charnock formula (42) for the friction velocity, u_*, we obtain:

$$z_o = \frac{\alpha}{g}\left(u_*^3 + \gamma w_*^3\right)^{2/3}, \tag{45}$$

where the value of γ is equal to 0.11. Eq. (45) reduces to the Charnock relation (42) if the free convection term is neglected.

In comparing Eq. (45) with Eq. (43), if we identify the friction velocity in Eq. (45) with the total stress, $\tau/\rho_a = u_*^2$ (the friction velocity is related to the wind stress) in Eq. (43) and incorporate the viscous effect, then Eq. (45) or Eq. (44) becomes

$$\begin{aligned} z_o &= \frac{\alpha}{g}\left(u_*^3 + \gamma w_*^3\right)^{2/3} + 0.11\frac{\nu}{u_*} \\ &= \frac{\alpha}{g}\left((u_{*t}^2 \pm u_{*w}^2)^{\frac{3}{2}} + \gamma w_*^3\right)^{2/3} + 0.11\frac{\nu}{u_*}. \end{aligned} \tag{46}$$

The surface roughness length is dominated by short gravity waves for relatively high wind speed. For light winds, the term 0.11 ν/u_* should be added to Eq. (44) or Eq. (46), as suggested by Smith (1988), where $\nu = 1.5 \times 10^{-5}$ m^2/s is the kinematic viscosity of air. This term is for the air flow over an aerodynamically smooth surface. The wave-induced stress usually decreases rapidly with height (see Fig. 2.); therefore, for the height far above the sea surface (above the WBL), $u_* \approx u_{*t}$ (see Phillips, 1966; Makin and Kudryavtsev, 1999; and Makin, 2008). The negative sign in Eq. (46) indicates that the upward transfer of momentum flux from the sea surface to the atmosphere is due to the wave effect. The addition of the viscous term in Eq. (46) is to show that Eq. (46) reduces to Smith's formulation (see Eq. (50) below).

In light wind conditions, swell may influence the roughness length and the momentum transfer. The Charnock parameter in reality is not a constant; it depends on the wave age and may also be influenced by the presence of swell. The effects of the wave age and the wave-induced stress or swell on the Charnock parameter are discussed in Sections 6.2, 6.3, and 6.4, respectively.

6.1.2 Free convection

Equation (46) is a general equation for determining the surface roughness length for the flow over the sea. Thus, we have extended the Charnock wind stress formula to include the conditions of free convection and the wave effect (see Sections 6.3 and 6.4).

In the free convection limit, the horizontal mean wind speed and the shear stress vanish; therefore, the surface roughness length in Eq. (45) becomes

$$z_o = \frac{\alpha\gamma'}{g} w_*^2, \tag{47}$$

Where $\acute{\gamma} = \gamma_a$ is a constant, which is equal to 0.19 to 0.23 (see Section 4). Eq. (47) shows that the surface roughness length is generated under free convection. Eq. (47) has also been proposed by Abdella and D'Alessio (2003); they give a value of 0.15 for $\acute{\gamma}$.

Here we give two examples to show the effect of free convection on the sea-surface roughness. If we set $\gamma' = 0.23$ and $a = 0.015$, and, in addition, if we assume that $w_* = 1$ m/s, from Eq. (47) we have the surface roughness length $z_o = 3.5\text{x}\ 10^{-4}$ m. If we set $w_* = 0.5$ m/s, we have $z_o = 9 \times 10^{-5}$ m .

For illustration, the variation of the surface roughness length with the wind speed under the forced or free convection is plotted in Fig. 3, which shows that the surface roughness length is obtained from the Charnock wind stress formula, omitting the convective term in Eq. (45), and, in the free convection limit as the mean wind speed and shear stress approach zero, the surface roughness is obtained from Eq. (47).

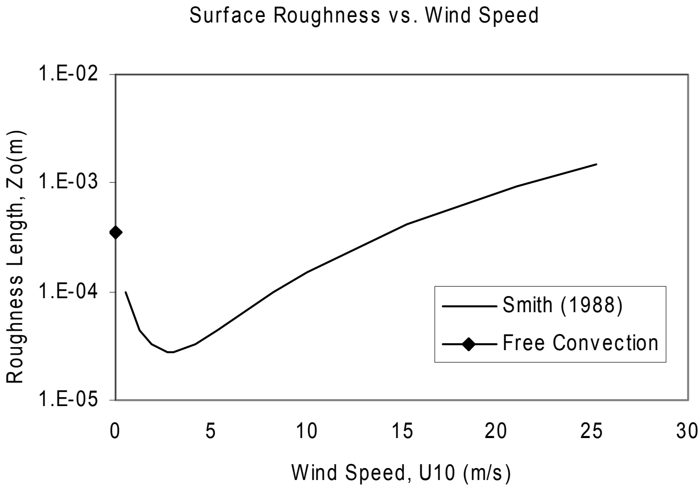

Fig. 3. Variations of the surface roughness over the sea with wind speed at 10 m. The solid line (-) indicates the roughness length obtained from the Charnock relation (see Eq. 50 below). The diamond symbol (♦) shows the roughness length for the free convection; the convective velocity here is set at 1 m/s.

6.2 Effect of wave age

The sea state controls the direction of momentum transfer and also influences the surface roughness. The wave age is often used to characterize the sea state, which is a measure of the sea state for wind wave development. The wave age parameter is defined as c_p/U_{10}, or c_p/u_* (In general the friction velocity, u_* , increases with increasing the wind speed, $U_{10.}$), where c_p is the phase speed, the speed of wave, and U_{10} is the wind speed at 10 m height above the sea surface, is used as criteria to distinguish the wind sea from the swell. For pure wind sea or for a young wave, the wind speed is larger than the phase speed, i.e., $c_p/U_{10} <$ 1.2, i.e., the wind is moving faster than the wave, while under swell conditions, the waves generated from distant storms travel faster than the wind speed, i.e., $c_p/U_{10} >1.2$.

In reality, the Charnock constant, a, is not a constant; it also depends on the sea state. The Charnock constant, a, in the roughness length equation, Eq. (45) or Eq. (46), can be modified to better reflect the sea state. The effect of the sea state on the roughness can be modeled by expressing the Charnock constant as a function of the wave age (e.g., Smith et al., 1992) or the wave-induced stress (Janssen, 1989; 1991). A number of authors have suggested that the Charnock parameter a depends on the characteristics of the sea state in the developing stage of wind sea (e.g., Smith et al., 1992), specifically, the wave age, which may be expressed, for example, as

$$\alpha = f\left(\frac{u_*}{c_p}\right) = \alpha_3\left(\frac{u_*}{c_p}\right)^{\beta_3}, \tag{48}$$

where c_p is the phase speed of the dominant waves and c_p/u_* is the wave age. Young wave has small value of c_p/u_*, while old wave has large value of c_p/u_*. Several authors give different formulations for the function of wave age, $f\ (c_p/u_*)$. Eq. (48) indicates that the young wave has the larger surface roughness, while the older, more mature wave has the smaller surface roughness. Smith et al. (1992) also suggested including the effect of wave age into the Charnock parameter, a. For example, the values of $a_3 = 0.48$ and $\beta_3 = 1$ are given by Smith et al. (1992), while Johnson et al. (1998) give the values of $a_3 = 1.89$ and $\beta_3 = 1.59$.
Eq. (46), with the Charnock parameter a replaced by Eq. (48), becomes

$$z_o = \frac{\alpha_3}{g}\left(\frac{u_*}{c_p}\right)^{\beta_3}\left(u_*^3 + \gamma w_*^3\right)^{2/3} + 0.11\frac{\nu}{u_*}. \tag{49}$$

Eq. (49) shows that the sea surface roughness length, z_o, is related to the wind stress ($u_* = u_{*t}$), convective velocity, wave age, and air viscosity. Thus, in the free convection limit, the singularity in the Charnock relation is avoided, neglecting the viscous term in Eq. (49). If the effects of free convection and wave age on the surface roughness are neglected, then Eq. (49) becomes

$$z_o = \frac{\alpha}{g}u_*^2 + 0.11\frac{\nu}{u_*}. \tag{50}$$

This equation, suggested by Smith (1988), is commonly used to calculate the sea roughness length, z_o, and hence to compute the surface fluxes over the ocean (e.g., Smith, 1988; also see Fairall et al., 1996). For an aerodynamically smooth surface, Eq. (50) reduces to

$$z_o = 0.11\frac{\nu}{u_*}. \tag{51}$$

Eq. (51) corresponds to light wind conditions for flow over aerodynamically smooth surface (see Nikuradse, 1933).

6.3 Effect of wave-induced stress

Now, let us take a look at the contribution of the wave-induced stress, τ_w, in Eq. (39) or Eq. (40) to the total stress, τ_{eff}.

For old sea, the magnitude of the wave-induced stress is approximately 10% of the total turbulent stress (see Phillips, 1966; Janssen, 1989). In the atmospheric surface layer (or the WBL) the wave-induced stress usually decays rapidly with height (Janssen, 1989; (see Phillips, 1966; Makin and Kudryavtsev , 1999; and Makin , 2008) (see Section 6.3.1. below for further discussion).

6.3.1 Wind sea and wave-induced stress

The wave-induced stress may influence the surface roughness. To take into consideration the effect of wave-induced stress on the roughness length, Janssen (1989; 1991) suggested modifying the Charnock constant, a by including the wave-induced stress. This approach has been implemented in the wave model (WAM; WAMDI, 1988). The WAM model is widely used in predicting the characteristics of the surface wave.

In the ocean wave modeling, the effect of waves on the surface roughness is usually accommodated through the parameterization of the Charnock "constant" (see Eq. (52) below). The Charnock parameter has been used successfully in the WAM model in the prediction of ocean waves and the effect of wind waves on the transfer of fluxes.

Now, let us consider the effect of the wave-induced stress on the Charnock parameter. Janssen (1991) suggested modifying the Charnock parameter by including the effect of the wave-induced stress on the surface roughness. Thus, Eq. (44) can be written as

$$z_o = \frac{\alpha}{\rho_a g}\left(\tau_t + \tau_f\right) + 0.11\frac{\nu}{u_*}, \tag{52}$$

where

$$\alpha = \frac{\alpha_{ch}}{\sqrt{1-\eta}} \tag{53}$$

and

$$\eta = \frac{\tau_w}{\tau}$$

where a_{ch} is the Charnock constant. The value of a in Eq. (52) is the modified Charnock constant; it includes the effect of the wave-induced stress that depends on the wave age. In the Charnock relation, a is equal to a_{ch} = 0.0144 (see Janssen, 1991, p.1634), while in a coupled ocean-wave-atmospheric model, the value of a_{ch} = 0.01 is used (WAM; WAMDI, 1988). The WAM model is widely used to forecast the characteristics of ocean wave at meteorological centers around the world.

Substituting Eq. (53) into Eq. (45) or Eq. (46), we obtain

$$z_o = \frac{\alpha_{ch}}{g\sqrt{1-\eta_{eff}}}\left(u_{*t}^3 + \gamma w_{:*}^3\right)^{2/3} + 0.11\frac{\nu}{u_*}, \tag{54}$$

where

$$\eta_{eff} = \frac{\tau_w}{\tau_{eff}}.$$

Under strong convection, the second term on the right-hand side of Eq. (54), the convection-induced stress, is the dominant term. If we neglect the convection-induced stress and the viscous stress, then Eq. (54) becomes

$$z_o = \frac{\alpha_{ch} u_{*t}^2}{g\sqrt{1-\eta_{eff}}}, \tag{55}$$

where the wave parameter, η, is defined as $\eta_{eff} = \tau_w/\tau_{eff;}$ and the effective wave parameter is the ratio of the wave-induced stress, τ_w, to the effective total stress. This wave parameter, η, depends on the wave age and decreases with increasing wave age. The typical value of η is about 10% to 20% (see Phillips, 1966; Phillips, 1977; Janssen, 1989). Phillips (1966) and Janssen (1989) suggested that the total stress for old sea is, in general, $|\tau_w/\tau| \le 0.2$ and that, with the wind speed less than 2.5 m/s at the height of 10 m, the value of η for old sea is about 10% of the total stress (Phillips, 1966; Janssen, 1989). These values indicate that the effect of wave-induced stress on the value of the Charnock parameter is about 10%; that is, if we assume that, $|\tau_w/\tau| \le 0.2$, a change of 20% in the wave-induced stress will result in the departure of about 10% from the Charnock constant, a_{ch}.

The direct impact of swell-induced momentum and energy fluxes are confined in the WBL, a thin layer near the water surface for light winds (see Janssen, 1991; Makin and Kudryavtsev, 1999; Makin, 2008). In their theory, the wave boundary layer is in the order of O (1 m), a value less than 10 m. They suggested the height of the WBL is 10 m (Makin and Kudryavtsev, 1999, p. 7615) which is a good estimate (in their Fig. 2, the WBL is much lower and about 1-2 m). The wave-induced stress is exponential decay with height in the WBL, e.g., $f(z) \to 0$ at $z \to \delta_w$ (WBL) (see Makin, 2008, p.472; also see the critical layer theory by Miles, 1957).

Thus, the Charnock relation can be considered as a good approximation for moderate wind speeds, that is, $a \approx a_{ch}$, in view of the variability and uncertainty of the data.

In the following section, we consider the effect of swell on the surface roughness.

6.4 Swell effect

Swell is the long gravity waves generated from distant storms. It is common practice to assume that for the air flow over the ocean, under neutral conditions and in the absence of swell, the vertical distribution of the mean wind follows the well-known logarithmic wind profile. Davidson's data were obtained under the influence of heavy swell (Davidson, 1974). Davidson's formulation of the drag coefficient under swell conditions (Davidson, 1974) can be used to transform the wave-dependent formula of the drag coefficient into the following form:

$$Cd = \frac{Cd_o}{\left(1+\beta'\left(\frac{c_p}{u_*}-\alpha_o\right)\right)^2}, \tag{56-1}$$

where

$$Cd_o = \left[\frac{k}{\ln(\frac{z}{z_{och}})}\right]^2, \tag{56-2}$$

$$\beta' = 0.13Cd_o^{1/2} / k \,. \tag{56-3}$$

where Cd_o is the drag coefficient in the absence of wave effect, β' an empirical constant, c_p the phase speed at the peak frequency, c_p/u_* the wave age, and a_o another empirical constant. Eq. (56-1) shows the effect of wave age or swell on the drag coefficient. In Eq. (56-1), the drag coefficient, Cd, is expressed in terms of the drag coefficient Cd_o and the relative wave age, c_p/u_* - a_o, that is the wave age relative to the value of an empirical constant a_o. The drag coefficient Cd_o. in Eq. (56-2) is a result of the logarithmic wind profile under the neutral condition and z_{och} is the corresponding surface roughness. According to Large and Pond (1981), the expected average value of Cd_o = 1.3x10^{-3} and thus from Eq. (56-3), we obtain the value of β' = 0.012. A value of wave age, c_p/u_*, near 25 is associated with minimal wind-wave coupling influence (see Davidson, 1974). Therefore, we set the value of a_o = 25, which implies that when the wave age is equal to 25, $c_p/u_* = a_o$, the drag coefficient Cd is equal to Cd_o which is in the absence of swell effect. Theoretical justification of Eq. (56-1) is given by Brutsaert (1973). Eq. (56-1) indicates that when the wave age is greater than 25, i.e., $c_p/u_* > a_o$, the drag coefficient C_d decreases with increasing wave age; this condition corresponds to the older sea. When the wave age is less than 25, i.e., $c_p/u_* < a_o$, the drag coefficient C_d increases with decreasing wave age; this condition corresponds to the younger sea or developing waves.

By using Eq. (52) and from Eq. (56-1), we obtain the following equation for the surface roughness in a more general form:

$$z_o = \frac{\alpha}{g}(u_*^2 + \gamma' w_*^2) + 0.11\frac{\nu}{u_*} \tag{57-1}$$

where

$$\alpha = \frac{\alpha_{ch}}{e^{\beta^*(\frac{c_p}{u_*} - \alpha_o)}}\,, \tag{57-2}$$

$$\approx \frac{\alpha_{ch}}{1 + \beta^*\left(\frac{c_p}{u_*} - \alpha_o\right)}, \text{ for } \beta^*\left(\frac{c_p}{u_*} - \alpha_o\right) < 1 \tag{57-3}$$

Where β^* is an empirical constant and is equal to $\beta^* = k\beta'/cd_o^{1/2} = 0.13$. Eq. (57-2) implies that the modified Charnock parameter, a, or the non-dimensional roughness, and hence the surface roughness length, decrease with increasing wave age for $c_p/u_* > a_o$. Therefore, the singularity of the surface roughness is removed when the turbulent shear stress approaches zero, as indicated in Eq. (46) or Eq. (57-1) (if the viscous term is neglected). The non-dimensional roughness in Eq. (57-3) is for the value of $|\beta(c_p/u_* - a)| < 1$.

For young or developing waves, Toba et al. (1990) proposed the following formula for the non-dimensional surface roughness

$$\frac{gz_o}{u_*^2} = \gamma^* \frac{c_p}{u_*}, \tag{58}$$

where γ^* is an empirical constant and is equal to 0.025. For young and growing waves, the non-dimensional surface roughness is also dependent on the wave age, c_p/u_*, in the form as indicated in Eq. (58), and furthermore, the non-dimensional roughness length is also related to the Charnock parameter, i.e., $gz_o/u_*^2 = \alpha$; therefore, from Eq. (57-2) and Eq. (58), the non-dimensional roughness length can be expressed in the following functional form

$$\frac{gz_o}{u_*^2} = \frac{q' . \left(\frac{c_p}{u_*} \right)}{e^{\beta^* \left(\frac{c_p}{u_*} - \alpha_o \right)}} . \tag{59}$$

Where q' is an empirical constant that can be obtained from the experimental data. We obtain the value of $q' = 0.001$ (= $\gamma^* e^{-24\beta^*}$) by setting the wave age to $c_p/u_* = 1$ and fitting the curve through the cluster of the observational data (see Fig. 4.).

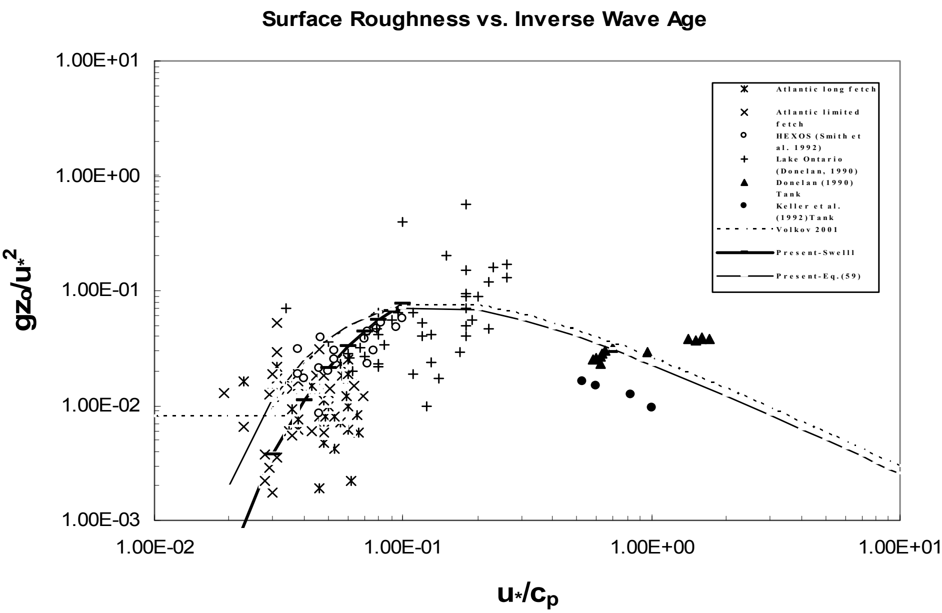

Fig. 4. Dependency of the non-dimensional roughness gz_o/u_*^2 on the inverse wave age u_*/c_p. Data are the collection of Donelan et al. (1993). Dotted line is from Volkov (2001), thin line from Eq. (59), and solid line in the presence of swell Eq. (57-2).

The semi-empirical formulas of the surface roughness suggested by Volkov (2001) are expressed as follows

$$\frac{gz_o}{u_*^2} = 0.03\left(\frac{c_p}{u_*}\right)e^{-0.14\left(\frac{c_p}{u_*}\right)}, \text{ for } 0.35 < \frac{c_p}{u_*} < 35 \tag{60-1}$$

$$= 0.008\,. \qquad \text{for } \frac{c_p}{u_*} > 35 \tag{60-2}$$

In Eq. (60-2), it is assumed that the non-dimensional roughness is a constant value of gz_0/u_*^2 = 0.008 for the wave age c_p/u_* > 35, which implies that even under the conditions of swell, the non-dimensional roughness is a constant value of 0.008. Eq. (60-1) is the non-dimensional roughness in the absence of swell. The comparison of Eq. (59) and Eq. (60-1) with experimental data is given in Fig. 4; the experimental data were collected by Donelan (1993). The two datasets by Davidson (1974) and by Donelan et al. (1993) were obtained under the influence of heavy swell. The Atlantic Ocean data are dominated by swell (see Fig. 4 and Donelan et at., 1993). Eq. (59) was derived from the wave-dependent formulation of Davidson. The theoretical justification was also provided by Brusaert (1973).

As can be seen from Fig. 4, the result obtained from Eq. (59) is remarkably close to the roughness formula (Eq. 60-1) proposed by Volkov (2001), considering the different approaches used to derive the surface roughness lengths; the two curves are almost identical to each other. However, under the conditions of strong swell, the two curves diverge significanly. It appears that Eq. (57-2) has the better fit to the experimental data. The data in Fig. 4 show that under the influence of swell, the surface roughness is much less than the value of 0.008 proposed by Volkov (2001). That is, under the influence of heavy swell, Eq. (57-2) shows that the sea surface is very smooth, much smoother than suggested by Volkov (2001), i.e., there exits a "super" smooth sea surface.

Equations (35), (38), (39), (46), (49), (54), (56), (57), and (59) are the main results of this paper. In theory, the total stress is controlled mainly by the short to moderate gravity waves, the large eddies associated with the convective activity, the wave-induced stress, or the swell. For moderate wind speeds, the Charnock relation is a good approximation for the estimate of the surface roughness length. Under the conditions of swell, part of momentum flux may be transferred from the ocean to the atmosphere. The swell also has influence on the surface roughness and the total stress. Eq. (57) and Eq. (59) imply that the wave-induced stress is to modulate the Charnock parameter. For young and developing waves, the surface roughness increases, while for old sea with large wave age, the wave damping occurs. Abdella and D'Alessio (2003) conducted numerical experiments using the second-order turbulence closure scheme and the convection–induced stress in the Charnock relation; their results of sea surface temperature and heat content obtained from the model simulations are in good agreement with the observations. The equations (57-2) and (59), the non-dimensional surface roughness under the influence of swell, are for the first time theoretically derived in this study.

Concerning the practical applications of the proposed formulation for the surface roughness, it has been demonstrated that the approaches similar to the proposed approaches have already been used for practical applications, for examples, Janssen's formulation used in WAM model (the European Centre for Medium-Range Weather Forecasts (ECMWF); Hasselmann et al., 1988) and numerical simulations conducted by Abdella and D'Alessio (2003) under the light wind conditions. These results are in good agreement with the observational data.

7. Summary and conclusions

The Charnock wind stress formula is well known and has been widely used for the study of the air-sea interaction. In his formula, Charnock (1955) considered only forced convection and did not consider two other important physical processes that affect the surface roughness: free convection and swell. A parameterization of surface roughness length for the air-sea interaction in free convection has been suggested by Abdella and D'Alessio (2003). In the free convection limit, as the horizontal mean wind speed and the friction velocity vanish, the well-known Charnock wind stress formulation is inadequate and the traditional Monin-Obukhov similarity theory breaks down. According to Deardorff (1970), under free convection conditions, the convective velocity is an important scaling parameter; in the parameterization of the surface roughness, the velocity or the relevant similarity variables should be scaled by the convective velocity rather than the friction velocity. In free convection, the large-scale convective eddies in connection with the convective circulations, which are expected to reach near the surface, would induce the "virtual" surface wind stress, the so-called convection-induced stress, that, in turn, contributes to the increase of surface roughness.

In this study, two alternative approaches were used to derive the surface roughness length. In the first approach, we derived a new parameterization scheme of the sea surface roughness for the conditions of the forced and free convection based on the Prandtl mixing length theory and the standard deviation of the vertical turbulent velocity. In the derivation of this new parameterization scheme, first we introduced a new velocity scale, which squared value is equal to the "total stress" and is a combination of the friction velocity and the convective velocity. We then showed that, by replacing the friction velocity in the Charnock formula with the new velocity scale or the effective friction velocity, the Charnock formula can be extended to include the conditions of free convection, that is, to show that the surface roughness length depends on a new velocity scale, which is a combination of the forced convection and free convection. The second approach is based on the standard deviations of the horizontal velocity components to derive the relationship between the friction velocity and the convective velocity, which can be used to estimate the surface roughness length. This approach is consistent with the concept of gustiness proposed by Businger (1973) and others (see Schumann, 1988 and Sykes et al. , 1993). Friction velocity and convective velocity are also used as scaling parameters. Wind stress is in the horizontal direction that represents friction velocity; therefore, in a sense friction velocity (stress) and convective velocity are perpendicular to each other.

The dependence of the roughness length on the sea state and the effect of the wave-induced stress or the swell on the roughness length are also investigated in this paper. The influence of the wave-induced stress or the swell on the surface roughness is taken into account by modifying the Charnock parameter that depends on the wave age. For moderate wind speeds, the Charnock relation is a good approximation for the estimate of the surface roughness length. Abdella and D'Alessio (2003) performed the numerical simulations including the convective velocity; they showed that their results of sea surface temperature and heat content are in good agreement with the observations. For young and developing waves, the wave-induced stress enhances the surface roughness; the surface roughness in the absence of swell derived in this paper is remarkably close, almost identical to the wind stress formula proposed by Volkov (2001). However, in this paper, the proposed formula for the aerodynamic roughness in the presence of heavy swell, deviated significantly from

Volkov's formula, is for the first time theoretically derived and substantiated by experimental data; it shows that the surface roughness depends on the relative wave age. The results are in better agreement with experimental data. The results show that under the influence of heavy swell, there exists a very smooth sea surface, a "super" smooth sea surface, much smoother than that, a constant value, as suggested by Volkov (2001). Therefore, in this study, we have extended the Charnock wind stress relation to the forced and free convection in the presence of swell. An advanced wind-wave and atmospheric boundary layer measurement system has been deployed to an offshore oil platform in the Gulf of Mexico (Huang et al., 2011). The data collected from this wind-wave measurement system may shed additional light on this subject and further elucidate the behavior of air flow over ocean waves and the physical processes of the air-sea interaction.

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9. References

Abdella, K. and S. J. D. D'Alessio. (2003). A parameterization of the roughness length for the air-sea interaction in free convection. *Envir. Fluid Mechanics*, *3,* 55-77.

Brusaert, W. (1973). Similarity functions for turbulence in neutral air above swell. *J. Phys. Oceanogr*, 4, 479-482.

Businger, J. A. (1973). A note on free convection. *Boundary-Layer Meteorol.*, 4, 323–326.

Charnock, H. (1955). Wind stress on a water surface. *Quart. J. Roy. Meteorol.* Soc., 81, 639-640.

Davidson, K. L. (1974). Observational results on the influence of stability and wind-wave coupling on momentum transfer and turbulent fluctuations over ocean waves. *Boundary- Layer Meteorol.*, *6,* 305–331.

Deardorff, J. W. (1970). Convective velocity and temperature scales for the unstable boundary layer and for Rayleigh convection. *J. Atmos. Sci.*, 27, 1211-1213.

Donelan, M. A., F. W. Dobson, S. D. Smith and R. J. Anderson. (1993). On the dependence of sea surface roughness on wave development. *J. Phys. Oceanogr., 23*, 2143-2149.

Dyer, A. J. (1974). A review of flux-profile relationships. *Boundary-Layer Meteorol.*, 7, 363-372.

Fairall, C, W., E. F. Bradley, D. P. Rogers, J. B. Edson and G. S. Young. (1996). Bulk parameterization of air-sea fluxes for Tropical Ocean-Global Atmosphere Coupled-Ocean Atmosphere Response Experiment. *J. Geophys. Res.,* 101(C2), 3747-3764.

Garratt, J. R. (1977). Review of drag coefficients over oceans and continents. *Mon Wea. Rev.*, *105*, 915-929.

Garratt, J. R. (1992). *The atmospheric Boundary Layer*. Cambridge University Press, Cambridge, 316 pp.

Hogstrom, U. (1988). Non-dimensional wind and temperature profiles in the atmospheric surface layer: A re-evaluation. *Boundary-Layer Meteorol.*, 42, 55-78.

Huang, C.H. (1979). A theory of dispersion in turbulence shear flow. *Atmospheric Envir.*, 13, 453-463.

Huang, C. H. (2009). Parameterization of the roughness length over the sea in forced and free convection. *Environmental Fluid Mech.*, 2009, 9:359-366.

Huang, C. H., C. MacDonald, Randolph "Stick" Ware, A. Ray, C. A. Knoderer, J. Hare, W. Gibson, L. Bariteau, P. Roberts, C.W. Fairall, S. Pezoa. (2011). Thermodynamic Profiling and Temporal Evaluation of Mixing Height during the Cold Air Outbreak in the Gulf of Mexico. *Thermodynamic Profiling Technologies Workshop (poster session)*. April 12-14, 2011 held at Boulder, Colorado.

Janssen, P. A. E. M. (1989). Wave-induced stress and the drag of air flow over sea waves. *J. Phys. Oceanogr.*, 19, 745-754.

Janssen, P. A. E. M. (1991). Quasi-linear theory of wind-wave generation applied to wave forecasting. *J. Phys. Oceanogr.*, 21, 1631-1642.

Johnson, H. K., J. Hojstrup, H. J. Vested and S. E. Larsen. (1998). On the dependence of sea surface roughness on wind waves. *J. Phys. Oceanogr.*, 28, 1702-1716.

Large, W. G. and S. Pond. (1981). Open ocean momentum flux measurements in moderate to strong winds. *J. Phys. Oceanogr.*, 11, 324-336.

Makin V. K. (2008). On the possible impact of a following-swell on the atmospheric boundary layer. *Boundary-Layer Meteorol*. 129:469–78.

Makin V. K. & Kudryavtsev, V. N. (1999). Coupled sea surface-atmosphere model. Part 1. Wind over waves coupling. *J. Geophys. Res.*, 104(C4), 7613--7623.

Miles, J. W. (1957). On the generation of surface waves by shear flow. *J. Fluid Mech.*, 3, 185-204.

Monin, A. S. and A. M. Obukhov. (1954). Basic turbulence mixing laws in the atmospheric surface layer. *Trudy Geofiz. Inst. AN SSSR.*, No. 24 (151), 163-187.

Monin, A .S. and A. M. Yaglom. (1971). *Statistical fluid mechanics; mechanics of turbulence,* MIT Press, Cambridge, MA. 769 pp.

Nikuradse, J. (1933). Stromungs gesetze in rauhen Rohren. *Forschungsheft*, No. 361.

Panofsky, H. A., H. Tennekes, D. H. Lenschow and J. C. Wyngaard. (1977). The characteristics of turbulent velocity components in the surface layer under convective conditions. *Boundary-Layer Meteorol., 11*, 355-361.

Paulson, C.A. (1970): The mathematical representation of wind speed and temperature profiles in the unstable atmospheric surface layer, *J. Appl. Meteorol.*, 9, 857-861.

Obukhov, A. M. (1946). Turbulence in an atmosphere with a non-homogenous temperature. *Trudy Inst. Theor. Geophy.*, USSR, 1:95-115.

Phillips, O. M. (1966). *The Dynamics of the Upper Ocean*. Cambridge Univ. Press, Cambridge, U.K., 261 pp.

Phillips, O. M. (1977). *The Dynamics of the Upper Ocean*, 2nd ed. Cambridge Univ. Press, Cambridge, U.K., 336 pp.

Schumann, U. (1988). Minimum friction velocity and heat transfer in the rough surface layer of a convective boundary layer. *Boundary-Layer Meteorol.*, 44, 311-326.

Smith, S. D. (1980). Wind stress and heat flux over the ocean in gale force winds. *J. Phys. Oceanogr.*, 10, 709-726.

Smith, S. D. (1988). Coefficients for sea surface wind stress, heat flux, and wind profiles as a function of wind speed and temperature. *J. Geophys. Res.*, 93, 15467-15472.

Smith, S. D., R. J. Anderson, W. A. Oost, C. Kraan, N. Maat, J. DeCosmo, K. B. Katsaros, K. L. Davison, K. Bumke, L. Hasse and H. M. Chadwick. (1992). Sea surface wind stress and drag coefficients: the HEXOS results. *Boundary-Layer Meteorol.*, 60, 109-142.

Stull, B. R. (1988). *An Introduction to Boundary Layer Meteorology*. Kluwer Academic Publishers. 666 p.

Sykes, R. I., D. S. Henn and W. S. Lewellen. (1993). Surface-layer description under free convection conditions. *Quart. J. Roy. Meteorol.* Soc., 119, 409–421.

Troen, I. and L. Mahrt. (1986). A simple model of the atmospheric boundary layer: sensitivity to surface evaporation. *Boundary-Layer Meteorol.*, 37, 129-148.

Toba, Y., N. Iida, H. Kawamura, N. Ebuchi and I. S. F. Jones. (1990). Wave dependence of sea-surface stress. *J. Phy. Oceanogr.*, 20, 705-721.

Volkov, Y. (2001). The dependence on wave age. p. 206-217. *In Wind Stress over the Ocean*, ed. by I. S. F. Jones and Y. Toba. Cambridge Univ. Press, New York, 307 pp.

Wyngaard, J. C., and O. R. Coté. (1974). The evolution of a convective planetary boundary layer-A higher-order-closure model study. *Boundary.-Layer Meteorol.,* 7, 289–308.

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Wilson D. K. (2001). An alternative function for the wind and temperature gradients in unstable surface layers. *Boundary-Layer Meteorol.*, 99, 151-158.

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Whereas the field of Fluid Mechanics can be described as complicated, mathematically challenging, and esoteric, it is also imminently practical. It is central to a wide variety of issues that are important not only technologically, but also sociologically. This book highlights a cross-section of methods in Fluid Mechanics, each of which illustrates novel ideas of the researchers and relates to one or more issues of high interest during the early 21st century. The challenges include multiphase flows, compressibility, nonlinear dynamics, flow instability, changing solid-fluid boundaries, and fluids with solid-like properties. The applications relate problems such as weather and climate prediction, air quality, fuel efficiency, wind or wave energy harvesting, landslides, erosion, noise abatement, and health care.