

Using the General Gamma Distribution to Represent the Droplet Size Distribution in a Spray Model

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1. Introduction

The production of small sized droplets is the main purpose of using sprays. Sprays are produced when bulk liquid or large liquid sheets or jets are shattered in a process called atomization. In the atomization process, the shattering of the bulk liquid can occur if it is subjected to a high-velocity gas, or due to the kinetic energy of the liquid. An external device can also be used to induce these processes that lead to atomization.

Sprays occur naturally, in waterfall sprays and drizzle, for instance. Thus, they are of interest in meteorology. The industrial applications of sprays, however, are numerous. Drug applications in medical inhalers, chemical and irrigation applications in agriculture, dispersion of liquid fuels for combustion in engines, food preservation and processing applications, fire suppression, paint spraying and spray drying are just a few examples.

The proliferation of spray producing devices (atomizers) for use in the various applications of sprays has led to a keen interest in the hydrodynamics of sprays. Consequently, a lot of progress has been made with regards to spray measuring devices and numerical spray models in order to better match atomizers for particular applications, to predict, and improve the atomization process.

The process of atomization involves the production of new droplets that are of different sizes to the initial parent droplets. Thus atomizers, in practice, do not produce droplets of uniform size. To aid in evaluating sprays, some knowledge of the distribution of the droplet sizes is required. This is especially crucial in spray models as some representation of the droplet size distribution is needed in the prediction of the multi-phase flow hydrodynamics.

This chapter presents the application of a general Gamma distribution to represent the droplet size distribution in a spray model. This is a recently developed spray model in which the hydrodynamics characteristics of multi-phase flows are evaluated by calculating three moments of the droplet size distribution from transport equations and one moment from a general Gamma distribution. Three areas in this model require a particular distribution function for the droplet sizes: at the inlet conditions to describe the drag model, and at the droplet break up and collision processes. This is a novel concept as spray models normally track parcels of droplets and the Rosin-Rammler expression is commonly used for

representing droplet size distributions. The next section in this chapter presents the concept of droplet size distribution functions, including frequency curves and cumulative droplet distribution curves. The third section outlines the mathematical representation of the droplet size distribution functions, and, the various droplet size distribution functions that have been applied to the spray model. The fourth section presents the representation of droplet sizes in terms of mean diameters. In the fifth section, the application of the Gamma distribution to represent the droplet size distribution in a spray model is presented. Finally, in section six, the conclusions of the present chapter are summarised.

2. Representation of droplet size distributions

2.1 Introduction

A spray can be characterised as a grouping of droplets immersed in a gaseous environment. As these groupings contain droplets that are of different sizes in practical atomizing conditions, atomizers do not produce droplets of uniform size. Therefore, to help in evaluating sprays, some knowledge and definition of the droplet size distribution is needed.

The spectrum of droplet sizes that can be found in sprays can vary depending on the particular atomizing process. This spectrum of droplets can be represented graphically. Droplet size histograms can be used to represent the droplet size distributions. Classifying droplet sizes and specifying the number of droplets that fall within the prescribed class limits, a number droplet size histogram can be plotted. A histogram of the volume droplet size distribution can be plotted by using the volume of the droplets within the prescribed droplet size classes. If the droplet classes in the histograms are prescribed with very small ranges, then the histograms can be developed into frequency curves. These frequency distribution curves can also be used to provide cumulative droplet distribution curves that give the percentage of the total number of droplets in the spray below the given droplet size or the percentage of the total volume of the liquid droplets contained in the spray below the given percentage of the total volume of the spray.

Thus the number distribution, $n(D)$, can be defined such that

$$\int_0^d n(D)dD \quad (1)$$

is the fraction of droplets with diameter less than d , where D is the droplet diameter.

The volume size distribution, $v(D)$, can also be defined such that

$$\int_0^d v(D)dD \quad (2)$$

is the fraction of the total droplet volume contained in droplets with diameter less than d , where D is the droplet diameter .

Figure 1 presents a volume frequency distribution and a cumulative volume distribution for a water spray (Emekwuru, 2007).

These droplet size distributions can be used to show how changes in the atomization conditions can affect the droplet size distribution in a spray. Thus, for instance, figure 2 shows the effect of the change in atomizing air pressure on the droplet size distribution, while figure 3 shows the effect of the distance from the spray nozzle on the droplet size distribution.

From figure 2(b), with the increase in atomizing air pressure value, there is a shift to more smaller sized droplets. The volume frequency of the larger sized droplets fall with increase in atomizing air pressure, and this is also reflected in the cumulative volume curve being skewed more to the smaller sized droplets compared to the figure 2(a). At higher atomizing air pressure values, increased droplet momentum results in increased droplet breakup and, thus, smaller droplet sizes. Figures 3(a) and 3(b) show the opposite trend to figures 2(a) and 2(b); a trend towards larger droplet sizes at locations further from the nozzle. The effects of droplet break up diminish at distances further from the nozzle, droplet coalescence becomes more important, and larger sized droplets travel further than smaller droplets as they experience less drag forces.

These droplet size distributions give an indication of not only how heterogeneous the spray droplets are, but also of the relative atomization performance of various nozzle designs and how these nozzles perform with changing operating conditions. Thus, droplet size distributions are a very good way of describing sprays.

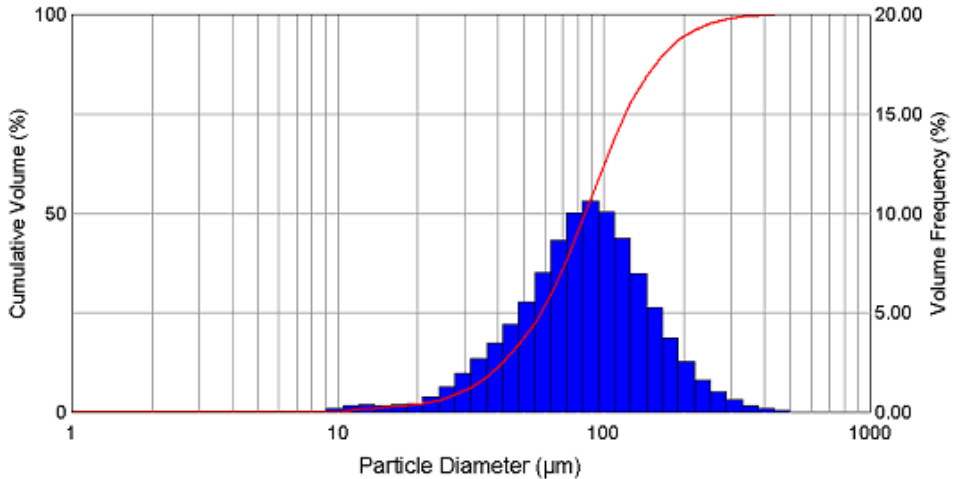
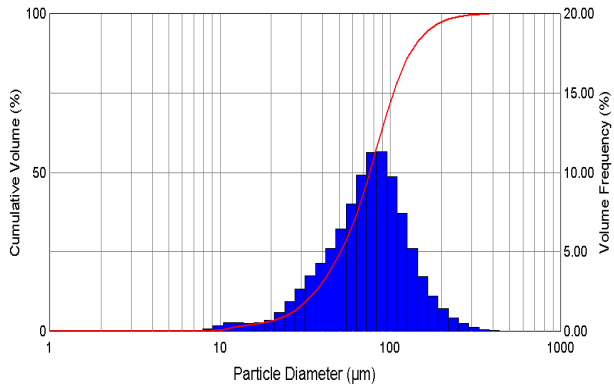
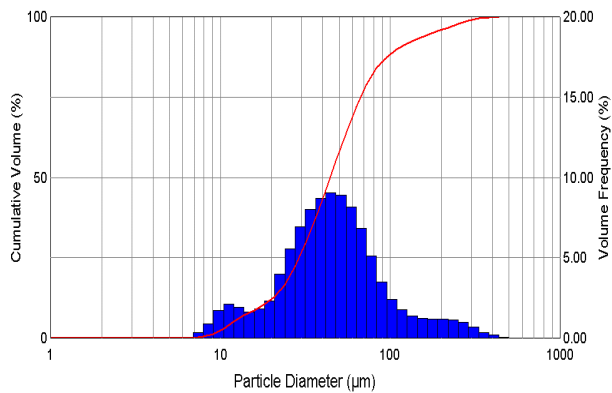


Fig. 1. Droplet size distribution showing both the cumulative volume distribution and the volume frequency distribution. These have been obtained from a water spray at 103 kPa atomizing pressure from a 3.52 mm diameter nozzle at an axial distance 275 mm from the nozzle using a laser-diffraction based droplet analyzer (Emekwuru, 2007).



(a) 103 kPa



(b) 138 kPa

Fig. 2. Droplet size distribution showing both the cumulative volume distribution and the volume frequency distribution. These show the effects of the atomizing air pressure values on the droplet size distribution. (Emekwuru & Watkins, 2010a).

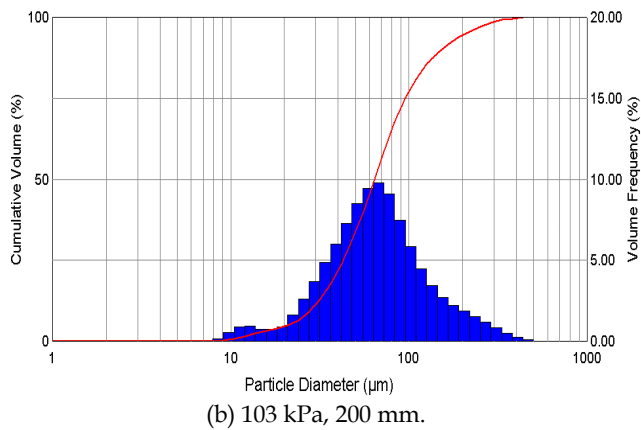
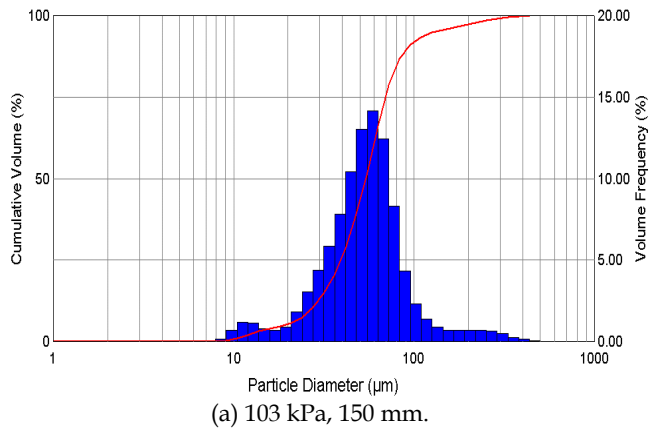


Fig. 3. Droplet size distribution showing both the cumulative volume distribution and the volume frequency distribution. These show the effects of the axial distance from the nozzle on the droplet size distribution. (Emekwuru & Watkins, 2010a).

3. Droplet size distribution functions

As outlined in section 2, presenting the droplet size distribution graphically greatly aids in evaluating the characteristics of sprays. However, the construction of such graphs can require specialist spray measurement equipment and expertise that is not readily available to everyone with interest in spray droplet distribution characteristics. Besides, the droplet size distribution changes with both time and position, as shown in section 2, thus formulating the graphs for droplet size distributions can be involving. Therefore, many scholars have presented empirical droplet size distribution functions that attempt to give a good representation of the droplet sizes in sprays with parameters obtained from a limited knowledge of the droplet measurements. According to Miesse & Putnam (1957) if any proposed mathematical distribution function is to be able to adequately represent the droplet size distribution in a spray, it should have these qualities:

- Give a good fit to the droplet size data.
- Give a means for bringing large quantities of data together.
- Present a way of calculating mean and representative droplet diameters.
- Permit the extrapolation of the known droplet sizes to droplet sizes beyond the range of these known values.
- Ideally, beyond linking the mathematical expressions to droplet size data, provide further understanding of the basic atomization process.

Many droplet size distribution functions are in use. The *normal distribution* is based on the principle that a given droplet size occurs randomly. This distribution function is relatively easy to use but is constrained to situations where droplets occur randomly. The number distribution function $f(D)$ of the normal distribution, gives the number of droplets with a given diameter D . The expression is:

$$\frac{dN}{dD} = f(D) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left[-\frac{(D - \bar{D})^2}{2\sigma^2} \right] \quad (3)$$

In equation (3), σ is the standard deviation of the values of D from a given mean value \bar{D} and σ^2 is the variance.

In the *log-normal distribution*, the logarithm of the droplet diameter is used as a variable, and this is based on the normal distribution law that many droplet size distributions have been found to fit. Thus, with the logarithm of the droplet diameter as the variable, equation (3) becomes

$$\frac{dN}{dD} = f(D) = \frac{1}{\sqrt{2\pi\sigma_g^2}} \exp \left[-\frac{(\ln D - \ln \bar{D}_g)^2}{2\sigma_g^2} \right] \quad (4)$$

In equation (4), σ_g is the geometric standard deviation of the values of D from a given number geometric mean droplet value \bar{D}_g and σ_g^2 is the geometric variance. These have the same significance as σ , and \bar{D} in the normal distribution.

Mugele & Evans (1951) found the log-normal distribution to produce reasonable approximations to both the number and volume droplet size distribution functions. The *Nukiyama & Tanasawa distribution* (Nukiyama & Tanasawa, 1939) is also commonly used, but the expression was found to give a good fit to experimental droplet number size distributions but a poor fit to the volume size distributions (Mugele & Evans, 1951). Conversely, the same study found the *Rosin & Rammler distribution* (Rosin & Rammler, 1933), which is also widely used by scholars, a good fit to experimental droplet volume size distributions but a poor fit to number size distributions. Boulderston et al. (1981) also found that the Rosin-Rammler distribution gave a good fit to their experimental droplet volume size distribution. Other droplet distributions have been mentioned in literature, for example, the *chi-square distribution* (Hirohasu & Kadota, 1974; Bai & Gosman, 1999), a *modified log-normal distribution* (Mugele & Evans, 1951), and a *modified Rosin-Rammler function* (Rizk & Lefebvre, 1985).

There are many references on droplet size distribution functions for the interested reader, for example, Lefebvre (1989) and Mugele & Evans (1951). The rest of the section outlines the droplet size distribution functions used in the moments spray models.

3.1 Droplet size distribution functions used in the moments spray models

The moments spray model presents a novel way of characterising the complete polydisperse nature of a spray flow by evaluating the moments of the droplet size distribution. To date three types of the moments spray model have been developed based on the number of moments of the droplet size distribution that are calculated from transport equations. Different droplet size distribution functions have been used in the models. In the initial moments spray model (Beck & Watkins, 2003a), the first four moments of the of the droplet size distribution are used to describe the distribution of the of the droplet sizes. Transport equations are written for the third and fourth moments that represent the surface area and liquid volume, respectively. Thus, this model is termed the two-moments spray model. The first two moments, representing the droplet number and total radius respectively, are approximated by using a presumed distribution function. This presumed function was required to be an analytically integrable number distribution such that the volume distribution produced from the function was an approximation to the Rosin-Rammler distribution (Beck & Watkins, 2003a). The Rosin-Rammler volume distribution is defined as

$$v(r) = \left(\frac{\alpha_R}{r_R^{\alpha_R}} \right) r^{\alpha_R - 1} \exp \left(- \left(\frac{r}{r_R} \right)^{\alpha_R} \right) \quad (5)$$

where r_R is called the Rosin-Rammler mean radius, and α_R the Rosin-Rammler exponent. In this case r_R is the droplet radius for which 63% of the liquid mass is made up of droplets with smaller radii. The shape of the distribution is determined by the exponent, and Beck &

Watkins (2003a) chose an exponent such as to match a Rosin-Rammler distribution of exponent 2, because most sprays have distributions with exponents close to this value. The number distribution they found is

$$n(r) = \frac{16r}{\bar{r}_{32}^2} \exp\left(-\frac{4r}{\bar{r}_{32}}\right) \tag{6}$$

where the Sauter Mean Radius \bar{r}_{32} is used because all the droplet moments are defined with respect to the droplet radii, and is not equal to the Rosin-Rammler mean radius. The volume distribution produced by this approximation is compared with the Rosin-Rammler distribution in Figure 4.

This distribution provided an adequate representation of the spray droplet size distribution in the two-moments spray model (Beck & Watkins, 2002, 2003a, 2003b).

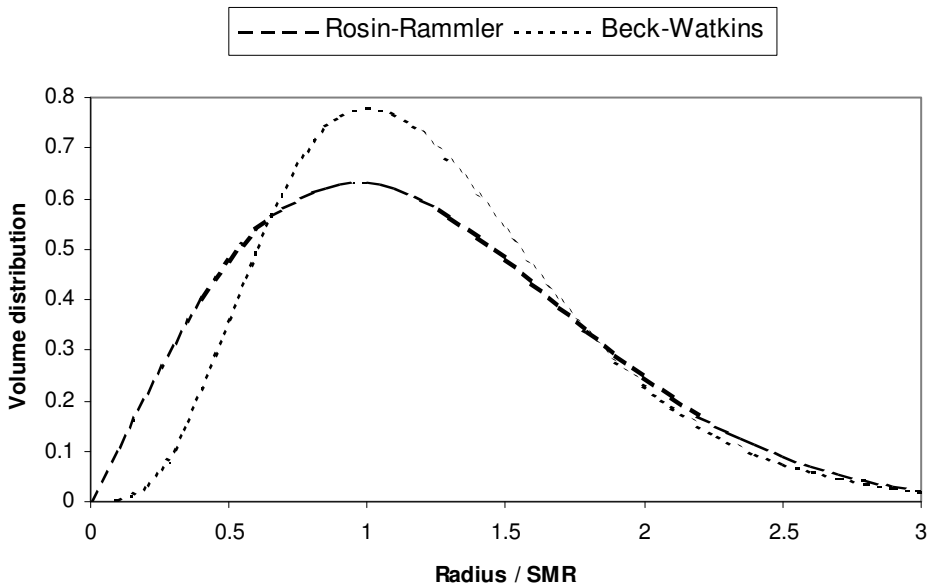


Fig. 4. Comparison of Beck-Watkins distribution used in the two moments model (Beck & Watkins, 2003a) with the Rosin-Rammler distribution of exponent 2.

The second moments model is the four-moments model of Yue & Watkins (2004). In this spray model the four moments of the droplet size distribution used in the Beck & Watkins (2002) model and their respective moment-averaged velocities are calculated from transport equations. Three areas in the spray model still require a distribution; the inlet conditions, the droplet drag model, and the droplet breakup and collision models. A Gamma distribution function is used in these sub-models. The three-moments model of Emekwuru & Watkins (2010b) is the third moments spray model, and the last three moments of the droplet size distribution are calculated from transport equations while the first moment is calculated from a Gamma distribution function which is also used for the three other areas in the spray model that require a distribution as in the four-moments model. The general Gamma distribution used in the three moments model is given by

$$n(r) = \frac{\alpha^k}{\Gamma(k)r_{32}^k} r^{k-1} e^{-\alpha(\frac{r}{r_{32}})} \tag{7}$$

where r_{32} is the Sauter mean radius and $\Gamma(k)$ is the Gamma function defined by the integral

$$\Gamma(k) = \int_0^\infty e^{-x} x^{k-1} dx \tag{8}$$

The two parameters defining the functional form are r_{32} and k .

The functional forms of equation (7) for various values of k are shown in figure (5).

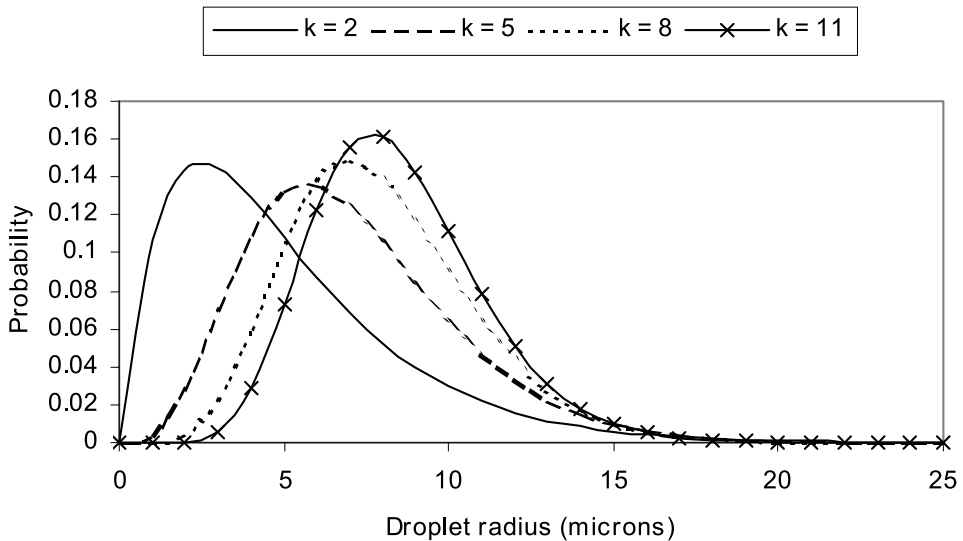


Fig. 5. Gamma distributions with $r_{32} = 10\mu\text{m}$

The results of the application of this distribution function to the three-moments spray model will be further highlighted in section 5.

4. The mean diameters concept

In practical calculations involving sprays many researchers use the mean diameters obtained from the size distributions presented in the previous sections, rather than the whole size distributions, in order to define an average droplet size in the given spray. Generally, these mean diameters can be defined thus:

$$D_{pq}^{p-q} = \frac{\int_0^{\infty} n(D)D^p dr}{\int_0^{\infty} n(D)D^q dr} \quad (9)$$

Mean diameters (from equation 9)			
<i>p</i>	<i>q</i>	<i>Symbol</i>	<i>Name</i>
1	0	D_{10}	Number
2	0	D_{20}	Surface area
3	0	D_{30}	Volume
2	1	D_{21}	Surface area-length
3	1	D_{31}	Volume-length
3	2	D_{32}	Sauter

Table 1. Some mean diameters

Some of the more commonly used mean diameters and their relation to equation (9) are presented in Table 1. For example, the *number mean diameter*, D_{10} , is the average value of all the droplets in the spray; the *volume mean diameter*, D_{30} , is the diameter of a droplet whose volume, when multiplied by the number of droplets in the spray, equals the total volume of the spray; and the *Sauter mean diameter*, D_{32} is the diameter of a droplet with a ratio of volume to surface area which is similar to that of the whole spray. Mugele & Evans (1951) provide a good representation of mean diameters. Some other researchers such as Yule et al. (2000) prefer to use the volume median diameter, $D_{v,0.5}$, defined as

$$\int_0^{D_{v,0.5}} v(D)dD = 0.5 \quad (10)$$

This is akin to the concept of *representative diameters* of which some of the definitions have been presented in Table 2.

REPRESENTATIVE DIAMETERS	
$D_{0.1}$	10% of total liquid volume is in droplets of smaller diameter than this diameter
$D_{0.5}$	50% of total liquid volume is in droplets of smaller diameter than this diameter
$D_{0.9}$	90% of total liquid volume is in droplets of smaller diameter than this diameter

Table 2. Some representative diameters

Thus, the droplet size distribution function (see equations (1) & (2) for instance), can be described with the knowledge of a representative diameter and a mean diameter.

5. Application of the gamma distribution to the moments spray model

Equation (1) can be represented in terms of a droplet number probability distribution, denoted by $N(r)$ where the fraction of droplets having radii between limits r_1 and r_0 , is given by

$$\int_{r_0}^{r_1} N(r) dr \quad (11)$$

As equation (11) is a probability density function, the integral over all the droplet radii within the limits is equal to one. By multiplying by the total number of droplets per unit total volume, a multiple of this distribution, $n(r)$, can be presented, and the moments of the droplet number probability distribution is

$$Q_i = \int_0^{\infty} n(r) r^i dr \quad (12)$$

As was mentioned in section 3.1, the three moments spray model uses the first four moments, Q_0 to Q_3 , as was first used by Beck & Watkins (2003):

- Q_0 is the total number of droplets, per unit total volume.
- Q_1 is the total sum of radii of the droplets, per unit total volume.
- $4\pi Q_2$ is the total surface area of the droplets, per unit total volume.
- $\frac{4\pi Q_3}{3}$ is the total volume of the droplets, per unit total volume.

These four moments can be used to represent all the mean droplet diameters in Table 1, from D_{10} to D_{32} , and a relationship similar to equation (9) for the droplet distribution moments is

$$D_{pq}^{p-q} = \frac{2^{p-q} Q_p}{Q_q} \quad (13)$$

Sowa (1992) discusses this relationship in detail.

In the three moments spray model, the last three moments of the droplet size distribution, Q_1 to Q_3 , are obtained from transport equations. These transport equations have been described by Emekwuru & Watkins (2010b). The first moment of the droplet size distribution, Q_0 , is obtained from the Gamma size distribution function. Thus from section 3.1, the moments of equation (7) can be given as

$$Q_i = Q_0 \int_0^{\infty} \frac{\alpha^k}{\Gamma(k) r_{32}^k} r^{k+i-1} e^{-\alpha(\frac{r}{r_{32}})} dr \quad (14)$$

By partial integration,

$$Q_3 = \frac{(k+2)}{\alpha} r_{32} Q_2 \quad (15)$$

By definition, $Q_3 = r_{32} Q_2$ thus $\alpha = k + 2$.

Therefore,

$$Q_2 = \frac{(k+1)}{(k+2)} r_{32} Q_1, \quad Q_1 = \frac{k}{(k+2)} r_{32} Q_0 \quad (16)$$

And finally Q_0 can be evaluated as

$$Q_0 = \frac{(k+2)}{kr_{32}} Q_1 \quad (17)$$

5.1 Discussion of application to water sprays

A water spray study, using a twin-fluid sprayer typically used in agricultural spraying processes, was presented by Emekwuru & Watkins (2010a). In the study, the characteristics of the water spray are defined by the droplet size data obtained using a laser-diffraction-based measuring instrument. The droplet size data measurements were taken at different atomizing air pressure values, at various axial distances from the spray nozzle, and radial distances from the spray centreline. The test results indicate that the performance of the air-atomizer depends largely on the air atomizing pressure. The nozzle diameter is 3.52 mm and the results at the 300 mm axial distance from the nozzle for different atomising air pressure values are presented in figures 6(a) to (c). Some of the conditions are presented in table 3, the complete results can be found in Emekwuru & Watkins (2010a, 2010b). The three moments model provides the capability to represent both the local droplet sizes and the local droplet size distributions. Of interest here is the comparison of the local droplet size distribution from the experimental data with the predictions from the three moments spray model. As the air atomizing pressure value is increased, the droplet sizes decrease (figures 6 (a) to (c)) as expected since droplet break up increases due to more unstable droplets resulting from higher droplet momentum and increased shear in the atomization area due to higher atomizing pressures. The volume frequency of the larger sized droplets reduces and that of the smaller sized droplets increases with these increasing air atomizing pressure values. The numerical data capture these trends. The model also captures the relatively poor atomization noticed at low air atomizing pressure cases characterised by larger droplet sizes. As the atomizing air pressure values increase, the droplet size distributions become increasingly bi-modal. The model captures this as well, though there is a difference between the observed and calculated bi-modal droplet distribution peaks.

The data from the numerical predictions suggest that, for the moments spray model, the Gamma number size distribution function can be used to model the droplet size distribution data obtained from experimental data.

The moments spray model using the general Gamma distribution to represent the droplet size distribution has also been applied to diesel spray cases (Emekwuru & Watkins, 2011) and work continues on applying this distribution to other spray cases, including combining the distribution with other droplet size distribution functions for different parts of the spray model (Emekwuru et al., 2012).

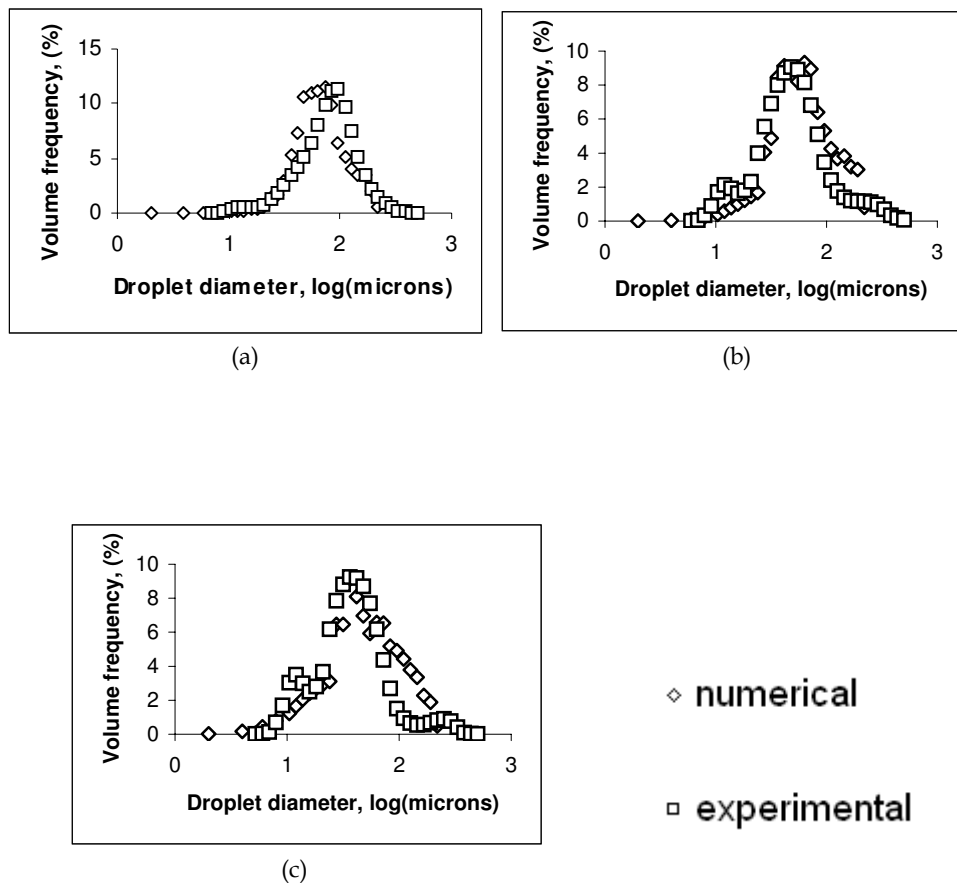


Fig. 6. Comparison of the local droplet size distribution from experimental data (Emekwuru & Watkins, 2010a) with the predictions from the three moments spray model (Emekwuru & Watkins, 2010b). The measurement location is 300 mm axial distance from the nozzle at air atomizing pressure values of (a) 103 kPa, (b) 138 kPa, and (c) 172 kPa.

Air atomizing pressure, (kPa)	103	138	172
Ambient pressure, (kPa)	100		
Injector nozzle diameter, (mm)	3.52		
Spray cone angle, (degrees)	15.94	16.25	16.46

Table 3. Some of the conditions used for the experimental (Emekwuru & Watkins, 2010a) and computational work (Emekwuru & Watkins, 2010b) presented in figure 6.

6. Conclusions

A spray can be defined as a grouping of droplets immersed in a gaseous environment. In practical conditions these groupings do not contain droplets of the same size. Thus, to help in evaluating sprays, some knowledge and definition of the droplet size distributions is necessary.

Droplet size distributions can be represented graphically as histograms, and cumulative distribution curves, but constructing such graphs can be tedious. Empirical droplet size distribution functions get round this issue by attempting to give a good representation of the droplet sizes with parameters obtained from a limited knowledge of the droplet data measurements. A number of droplet size functions exist but none can yet be judged to be a universal function for characterizing droplet size distribution in sprays. The use of these functions is useful in spray models, as some knowledge of the distribution of the droplet sizes is needed in the modelling process. The moments spray model evaluates the hydrodynamics characteristics of sprays without using droplet size classes. Instead moments of the droplet size distribution, which are calculated from transport equations and a size distribution function, are used. The application of a general Gamma distribution function to represent the droplet size distribution in a moments spray model indicate that the predictions from the model agree with experimental data, and thus is a viable distribution function to use to model the droplet size distributions in sprays. The results not only capture the changes in the local droplet size distribution with changes in the atomizing air pressure values, but also predict the shift from mono to bi-modal droplet distributions. More accurate representations of droplet size distribution functions in spray models can help in the better prediction, and hence, analysis of the hydrodynamic characteristics of sprays and spray making devices.

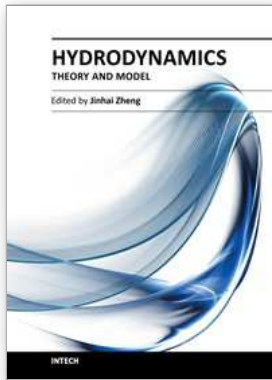
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8. References

- Bai, C., & Gosman, A.D., (1999). Prediction of spray wall impingement in reciprocating engines. *15th Annual Conf. Liquid Atomisation and Spray Systems.*, Toulouse, France, July 1999.
- Beck, J.C., & Watkins, A.P. (2002). On the development of spray sub-models based on the droplet size moments. *J. Comp. Phys.*, Vol. 182, Issue 2., pp. 586-621, ISSN: 0021-9991.
- Beck, J.C., & Watkins, A.P. (2003a). On the development of a spray model based on drop-size moments. *Proc. R. Soc. Lond. A*, Vol. 459, No. 2034, pp. 1365-1394, ISSN: 1471-2946.
- Beck, J.C., & Watkins, A.P. (2003b). Simulation of water and other non-fuel sprays using a new spray model. *Atomization and Sprays*, Vol. 13, Issue. 1, pp. 1-26, ISSN: 1044-5110.
- Boulderston, C., Ahseng, C., Yule, A.J., Felton, P.G., & Chigier, N.A., (1981). The structure of diesel injector sprays. CARL IM 81-01.
- Emekwuru, N.G. (2007). Statistical Experimental Analysis of a Two-Fluid Sprayer and Its Use to Develop the Number Size Distribution Moments Model of Sprays. PhD., University of Manchester, UK.
- Emekwuru, N.G., & Watkins, A.P. (2010a). Analysis of a two-fluid sprayer and its use to develop the number size distribution moments spray model, Part I: Experimental Analysis. *Atomization and Sprays*, Vol. 20, Issue 6, pp. 467-484, ISSN: 1044-5110.
- Emekwuru, N.G., & Watkins, A.P. (2010b). Analysis of a two-fluid sprayer and its use to develop the number size distribution moments spray model, Part II: Computational Analysis. *Atomization and Sprays*, Vol. 20, Issue 8, pp. 653-672, ISSN: 1044-5110.
- Emekwuru, N.G., & Watkins, A.P. (2011). Application of a moments spray model to solid cone diesel sprays. *JSAE Paper 20119080, SAE Paper 2011-01-1843*.
- Emekwuru, N.G. (2012). A number size distribution moments based solid cone diesel spray model: Assessment of droplet breakup models based on different distribution functions. *SAE Paper 2012-01-1260*
- Hiroyasu, H., & Kadota, T., (1974). Fuel droplet size distribution in diesel combustion chamber. *SAE Paper 740715*.
- Lefebvre, A.H. (1989). *Atomization and Sprays. Hemisphere*. ISBN: 0-89116-603-3, New York, USA.
- Miesse, C.C., & Putnam, A.A., (1957) Mathematical Expressions for Drop-Size Distributions, *Injection and Combustion of Liquid Fuels*, Section II, WADC Technical Report 56-344, Mattelle Memorial Institute, March.
- Mugele, R.A., & Evans, H.D., (1951). Droplet Size Distributions in Sprays. *Ind. Eng. Chem.*, Vol. 43, No. 6, pp. 1317-1324.
- Nukiyama, S., & Tanasawa, Y., (1939). Experiments on the Atomization of Liquids in an Air Stream, Report 3, On the Droplet-Size Distribution in an Atomized Jet, Defense Research Board, Department of National Defense, Ottawa, Canada; translated from *Trans. Soc. Mech. Eng. Jpn.*, Vol. 5, No. 18, pp. 62-67.
- Rizk, N.N., & Lefebvre, A.H., (1985) Drop-Size Distribution Characteristics of Spill-Return Atomizers. *AIAA J. Propul. Power*, Vol. 1, No. 3, pp. 16-22, ISSN: 0748-4658.
- Rosin, P., & Rammler, E. (1933). The Laws Governing the Fineness of Powdered Coal. *Journal of the Institute of Fuel*, Vol. 7, pp. 29-36.
- Sowa, W.A. (1992). Interpreting mean drop diameters using distribution moments. *Atomization and Sprays*, Vol. 2, Issue 1, pp. 1-15, ISSN: 1044-5110.

- Yue, B., & Watkins, A.P. (2004). Mathematical Development and Numerical Analysis of Further Transport Equations for the Droplet Size Moment Theory. *19th Annual Meeting of the Institute for Liquid Atomization and Spraying Systems (Europe)*. Nottingham, UK, September 2004.
- Yule, A.J., Sharief, R.A., Jeong, J.R., Nasr, G.G., & James, D.D. (2000). The performance characteristics of solid cone spray pressure swirl atomizers. *Atomization and Sprays*, Vol. 10, Issue. 6, pp. 627-646, ISSN: 1044-5110.



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