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The Equivalent Non-Linear Single Degree of Freedom System of Asymmetric Multi-Storey Buildings in Seismic Static Pushover Analysis

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1. Introduction

In order to estimate seismic demands at low seismic performance levels (such as life safety or collapse prevention), the application of Non-Linear Response History Analysis (NLRHA) is recommended for reasons related to its accuracy. However, for the purposes of simplification, the application of Pushover Analysis is also often recommended. Here, in order to obtain the seismic demands of asymmetric multi-storey reinforced concrete (r/c) buildings, a new seismic non-linear static (pushover) procedure that uses inelastic response acceleration spectra, is presented. The latter makes use of the optimum equivalent Non-Linear Single Degree of Freedom (NLSDF) system, which is used to represent a randomly-selected, asymmetric multi-storey r/c building. As is proven below, for each asymmetric multi-storey building, a total of twelve suitable non-linear static analyses are required according to this procedure, while at least two hundred and twenty-four suitable non-linear dynamic analyses are needed in the case of NLRHAnalysis, respectively, while if accidental ecentricity is ignored (or external floor moments loads around the vertical axis are used) then a total number of fifty six NLRH Analyses are required. The seismic non-linear static procedure that is presented here is a natural extension of the documented seismic equivalent static linear (simplified spectral) method that is recommended by the established contemporary Seismic Codes, with reference to torsional provisions. From the numerical parametric documentation of this proposed seismic non-linear static procedure, we can reliably evaluate the extreme values of floor inelastic displacements, with reference to results provided by the Non-Linear Response History Analysis.

More specifically, it is well-known from past research that, as regards the pushover procedure (i.e. pushover analyses of buildings with inelastic response acceleration spectra), its suitability for use on asymmetric multi-storey r/c buildings is frequently questioned because of the following five points:

1. For a known earthquake, an ideal equivalent NLSDF system, which represents the real asymmetric multi-storey r/c building, must first be defined, in order to calculate the seismic target-displacement. In other words, the above-mentioned r/c NLSDF system has an equivalent viscous damping ratio $\xi = 0.05$ and is combined with "inelastic acceleration spectra with an equivalent viscous damping ratio $\xi = 0.05$"; thus its demand seismic target-displacement is obtained for each earthquake level. The latter

displacement is transformed into a demand extreme seismic displacement of the monitoring point of the real Multi-Degree of Freedom (MDoF) system, namely the real multi-storey building. After that, the remaining demand seismic displacements (and seismic stress on the members) at the other points of the building are easily calculated using the pushover analysis image. Therefore, successfully defining the ideal equivalent NLSDF system is always the most important part of this procedure, and for this reason, it also constitutes a major concern. Many attempts have been made in the past to resolve this problem in the planar frames (Saiidi & Sozen, 1981; Fajfar & Fischinger, 1987a,b; Uang & Bertero, 1990; Qi & Moehle, 1991; Rodriquez, 1994; Fajfar & Gaspersic, 1996; Hart & Wong, 1999; Makarios, 2005;). On the other hand, in the case of asymmetric multi-storey buildings, a mathematically documented optimum equivalent Non-Linear Single Degree of Freedom System can be defined (Makarios, 2009). According to the latter proposal, three coupled degrees of freedom (two horizontal displacements and a rotation around a vertical axis that passes through the floor mass centre) are taken into account for each floor mass centre. The above-mentioned definition of the optimum equivalent NLSDF system was mathematically derived by observing suitable dynamic loadings on the floor mass centres of the multi-storey building, using simplified assumptions. The use of this optimum equivalent NLSDF system, in combination with the inelastic design spectra, provides an acceptable evaluation of the extreme (maximum/minimum) seismic floor displacements required for a known design earthquake, and for this reason constitutes the core issue of the present chapter.

2. It is known that in the case of Response History Analysis, three uncorrelated accelerograms (Penzien & Watabe, 1975;) are used simultaneously along the three axes x, y & z. For this reason, in the case of Response Spectrum Analysis (RSA) or Simplified Spectral Method (SSM) or the equivalent static method, the three response acceleration spectra are "statistically independent" (sect.3.2.2.1(3)P of Eurocode EN-1998.01). This practically means that the response spectra (in the case of RSA), or the lateral floor static forces (in the case of SSM), must be inserted separately for each main direction of the building, and then a suitable superposition (i.e. rule of Square Root of Sum of Squares – SRSS) must be applied to the results of each independent analysis, always in the linear area. However, in the non-linear area, as in the example of static pushover analysis, where superposition is generally forbidden due to its non-linearity, a basic question arises; which is the most suitable way of taking the "spatial seismic action" of the three seismic components into account? In order to answer this question, the rule of Eq.(1) that has resulted from a parametric numerical analysis, provides an approximate evaluation of the spatial seismic action during the static pushover procedure (Makarios, 2011):

$$E_{,\mathrm{ex}} = \pm\left(E_{,\mathrm{I}}^{\kappa} + E_{,\mathrm{II}}^{\kappa} \right)^{1/\kappa} \tag{1}$$

where $\kappa = 0.75$ is a mean value in the case of displacements/deformations, and $E_{,\mathrm{I}}$ is the extreme seismic demand inelastic displacement/deformation due to static pushover analysis using the first seismic component only, along the horizontal real (or fictitious) principal *I*-axis of the building. Similarly, $E_{,\mathrm{II}}$ is the extreme seismic demand inelastic displacement/deformation due to pushover analysis using the second seismic component only, the other horizontal real (or fictitious) principal *II*-axis of the building.

On the contrary, when $E_{,\mathrm{I}}$ and $E_{,\mathrm{II}}$ represent stress, then $\kappa = 2.00$, as is the case in the linear area. In Eq.(1) above, we consider that the vertical seismic component is often ignored (for example, when the vertical ground acceleration is less than 0.25g, according to sect.4.3.3.5.2(1)/EN-1998.01; g is the acceleration of gravity).

3. What is the most suitable "monitoring point" and its suitable characteristic degree of freedom (control displacement) that is related to the degree of freedom of the equivalent NLSDF system, in the case of asymmetric buildings? As has been proven by the mathematical analysis provided below, concerning the definition of the optimum NLSDF system, the centre of mass of the last floor at the top of the building can play the role of the "monitoring point", whilst its "control displacement" must be parallel to the lateral floor static forces.

4. What is the most suitable distribution of lateral floor static forces in elevation? There are various opinions on this matter, since the distribution can be "triangular", "uniform" or in accordance with the pure translational "fundamental mode-shape". Moreover, the distribution can be adapted to the fundamental mode-shape of the building in each step of the pushover analysis, an issue related to the action of higher order mode-shapes in tall buildings. According to sect.4.3.3.4.2.2(1)/EN-1998.01, at least two different distributions ("uniform" and "first mode-shape") in elevation should always be taken into account. On the other hand, fundamental mode-shape distribution is often applied to common asymmetric buildings, while in the case of very tall buildings, Non-Linear Response History Analysis is mainly reccommended.

5. At which point in the plan should the lateral floor static forces be applied during the pushover analysis of asymmetric buildings? According to sect.4.3.3.4.2.2(2)P/EN-1998.01, the lateral static loads must be applied at the location of the mass centres, while simultaneously, accidental eccentricities should also be taken into account. Moreover, it is known that in the case of dynamic methods, such as "Response Spectrum Analysis" or "Response History Analysis", accidental eccentricities are also considered, with a suitable movement of the floor mass centres by $e_{\mathrm{a}i} = \pm 0.05 L_i$ or $\pm 0.10 L_i$, where L_i is the dimension of the building's floor that is perpendicular to the direction of the seismic component (sect.4.3.2(1)P/EN-1998.01). However, in the case of static seismic methods, such as the "equivalent static method", "simplified spectrum method", "lateral force method" and "static pushover analysis", the accidental eccentricities are examined with a suitable movement (by $e_{\mathrm{a}i} = \pm 0.05 L_i$ or $\pm 0.10 L_i$) of the point at which the lateral floor static forces are applied. In the latter case, and for one seismic component, the final two design eccentricities ($\max e_{\mathrm{II},i}$ & $\min e_{\mathrm{II},i}$ for i floor) consist of the following two parts: In order to take into account the dynamic amplification of the torsional effects of the building, two dynamic eccentricities $e_{\mathrm{f},i}$ & $e_{\mathrm{r},i}$ are examined (first part). According to several Seismic Codes (NBCC-95, EAK/2003), dynamic eccentricities are defined as $e_{\mathrm{f},i} = 1.50 \cdot e_{\mathrm{o}}$ & $e_{\mathrm{r},i} = 0.50 \cdot e_{\mathrm{o}}$, where e_{o} is the static eccentricity of the building along the examined horizontal principal axis of the building, perpendicular to the direction of the seismic component. Dynamic eccentricities are measured from the real/fictitious stiffness centre of the building to its mass centre (CM); more accurate closed mathematical relationships of the former have been developed by Anastassiadis *et al* (1998). As we can observe, a calculation of (a) the location of the real/fictitious stiffness

centre of the asymmetric building in the plan, (b) the orientation of its horizontal principal directions *I & II* and (c) the magnitude of its torsional-stiffness radii, must be carried out before the seismic static pushover analysis begins. For this triple purpose, the fictitious elastic Reference Cartesian System $P_o(I, II, III)$ of the asymmetric multi-storey building is the most rational, documented method for use (Makarios & Anastassiadis 1998a,b; Makarios, *et al* 2006; Makarios, 2008;). On the contrary, it is worth noting that, various other floor centres, such as centres of rigidity/twist/shear, are not suitable for seismic design purposes, because they are dependent on external floor lateral static loads (Cheung & Tso, 1986; Hejal & Chopra, 1987;). At a next stage, the static eccentricities $e_{o,I}$ and $e_{o,II}$ of an asymmetric multi-storey building are defined as the distance between the floor mass centre (CM) and the real/fictitious stiffness centre P_o of the building, along the two horizontal principal *I & II* –axes.

In relation to my previous point, and in order to account for uncertainties as regards the real location of floor mass centres, as well as the spatial variation of the seismic motion, accidental eccentricities are also examined (second part). However, it is well-known that according to documented Seismic Codes with reference to torsional provisions, the critical horizontal directions for static lateral loading (in linear equivalent static seismic methods) are the real/fictitious principal *I & II*-axes of a building. Therefore, the documented static pushover procedure must define the same point; the lateral floor static loads that will be used in the static pushover analysis must be oriented along the real/fictitious principal *I & II*-axes of the building. In this case, and since the final design eccentricities ($_{max}e$ & $_{min}e$ along the two principal *I & II*-axes) have also been used, we can observe that this non-linear static pushover analysis is, undoubtedly, a natural extension of the linear equivalent static seismic method.
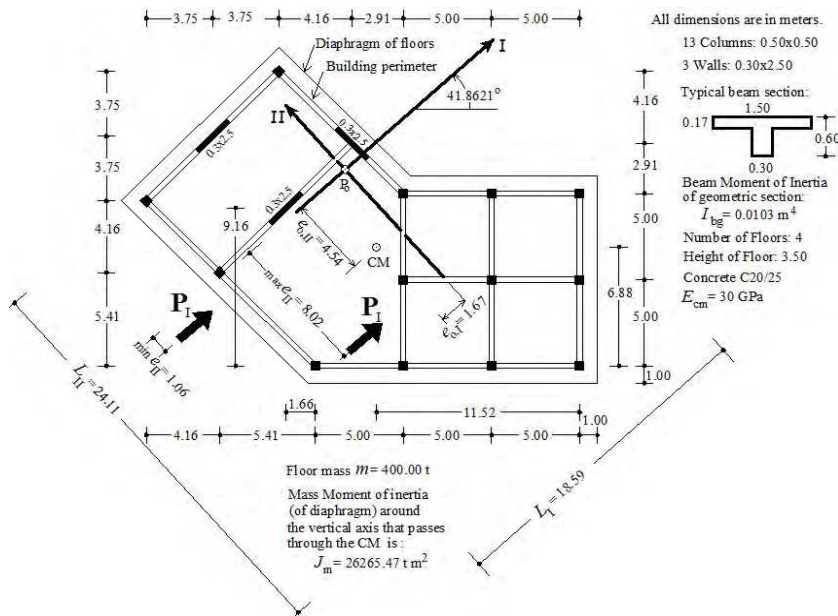

Fig. 1. Typical plan of an asymmetric four-storey r/c building.

For illustration purposes, we can see the numerical example of an asymmetric four-storey r/c building with a typical plan, whose real/fictitious principal axes are initially unknown (Fig.1). The location in the plan and the orientation of the $P_o(I, II, III)$ system have been calculated according to the above-mentioned relative references, using a 50% reduction of the stiffness properties of all structural elements, due to cracks (sect.4.3.1(7)/EN-1998.01). The position of all floor mass centres (CM) coincides with the geometric centre of the floor rigid diaphragms. Next, the static eccentricities are calculated as $e_{o,I} = 1.67$ & $e_{o,II} = 4.54$ metres, along the fictitious principal I & II-axes, respectively. The two torsional-stiffness radii r_I & r_{II} are calculated as $r_I = 9.35$ & $r_{II} = 7.90$ metres on level $0.8H$ of the building (namely, on the 3rd floor; H is the total height of the building) according to the theory of *Dynamics of Structures* (Makarios, 2008;), and along I & II-axes, respectively. Note that, in the case of a single-storey building, the torsional-stiffness radius r_I is for example calculated by the square root of the ratio of the torsional stiffness k_{III} of the building around its Stiffness Centre to the lateral stiffness k_{II} along principal II-axis (i.e. $r_I = \sqrt{k_{III}/k_{II}}$, $r_{II} = \sqrt{k_{III}/k_I}$). Also, the radius of gyration of a typical floor rigid diaphragm is calculated as $l_s = \sqrt{J_m/m} = \sqrt{26265.47/400} = 8.10$ metres (see data in Fig.1). In order to calculate the final design eccentricities for loading P_I along the principal I-axis of the building, we have:

$$\max e_{II} = e_{f,i} + e_{ai} = 1.50 \cdot e_{o,II} + 0.05 \cdot L_{II} = 1.50 \cdot 4.54 + 0.05 \cdot 24.11 = 8.02 ,$$

$$\min e_{II} = e_{r,i} + e_{ai} = 0.50 \cdot e_{o,II} - 0.05 \cdot L_{II} = 0.50 \cdot 4.54 - 0.05 \cdot 24.11 = 1.06$$

The envelope of the two individual static loading states, using the above final design eccentricities, produces the results E_I of the first seismic component (along principal I-axis). The results E_{II} of the second seismic component (along principal II-axis) are obtained in a similar way.

Please note the following important point: According to sect.4.2.3.2(6)/EN-1998.01, the building does not satisfy the criterion of regularity in plan, because $e_{o,II} = 4.54$ is not lower than $0.3 \cdot r_{II} = 0.3 \cdot 7.90 = 2.37$. Furthermore, $r_{II} = 7.90$ is lower than $l_s = 8.10$ and, according to sect.4.3.3.1(8)d/EN-1998.01, the condition $r_{II}^2 > l_s^2 + e_{o,II}^2$ is not true. Consequently, the present building has to be simulated as a fully spatial model, according to both the elastic analysis (sect.4.3.3.1(7)/EN-1998.01) and the non-linear static (pushover) analysis (sect.4.3.3.4.2.1(2)P/EN-1998.01).

To conclude, the real/fictitious principal elastic Cartesian system $P_o(I,II,III)$ of the asymmetric multi-storey building is initially calculated. In order to calculate the real/fictitious principal elastic Cartesian system $P_o(I,II,III)$ of the asymmetric multi-storey r/c building, all structural elements (columns, walls, beams, cores) of the building model must have effective flexural and shear stiffness, constant along their whole length,

corresponding to 50% of their geometric section values. This effective stiffness is more rational, since it leads to a realistic "total lateral stiffness" of the building in the post-elastic, non-linear area. However, if we use much lower values of effective stiffness (i.e. equal to $0.20EI$ or less), then an artificial increase of the eigen-periods occurs. In such a case, one possible result is that the building model is inadequately seismically loaded, because the state of co-ordination between the fundamental eigen-period of the building and the fundamental period of the actual seismic ground strong motion is removed. Existing views about the use of very small effective flexural stiffness values stem from the following assumption, that "…*plastic hinges appeared simultaneously at both ends of each structural element of the multi-storey building.*" However, this is rather incorrect, because many structural elements do not yield, even in the "near collapse state" of the building, as we can observe from the seismic NLRHA. On the other hand, cracks appeared on the plastic hinges of the r/c elements with yielding steel bars, while at the next moment, when the bending of the elements has the opposite sign, some cracks on the plastic hinges cannot fully close, because the yielded steel bars obstruct their closure. Additionally, in the frame of the static pushover analysis of irregular in the plan, asymmetric multi-storey buildings, a Cartesian system $CM(xyz)$ that has the same orientation with the known fictitious elastic system $P_o(I,II,III)$ of the building must be adopted as a global reference system. Thus, the lateral floor static loads, according to the static pushover analyses, must be oriented along the two horizontal principal I & II–axes of the building. Otherwise, it is well-known from the linear area that it is not possible to calculate the envelope of the floor displacements/stress of an asymmetric building. In other words, the loading along the principal axes of a building creates the most unfavourable state for static methods. Moreover, as a general conclusion, final design eccentricities ($\max e_I$, $\min e_I$, $\max e_{II}$ & $\min e_{II}$) must always be taken into account in pushover analysis, and thus the present non-linear static pushover procedure is a natural extension of the equivalent linear static method, with mathematical consistency.

With reference to the simulation for the non-linear analysis, we consider that each structural column/beam consists of two equivalent sub-cantilevers (Fig.2a,b). Each sub-cantilever is represented by an elastic beam element with a non-linear spring at its base. This inelastic spring represents the inelastic deformations that are lumped at the end of the element. In order to identify the characteristics of a plastic hinge, it is assumed that each member (beam/column) deforms in antisymmetric bending. The point of contraflexure is approximately located at the middle of the element and for this reason, two sub-cantilevers appear for each member. Therefore, each sub-cantilever has a "shear length" $L_{s,i}$, where its chord slope rotation θ characterizes the Moment-Chord Slope Rotation ($M - \theta$) diagram of the non-linear spring in the end section. In practice, the two critical end-sections of the clear length of the beams and columns, as well as the critical section at the base of the walls/cores of the building model, must be provided with suitable non-linear springs. In order to calculate the $M - \theta$ diagram of a plastic hinge, two methods are frequently implemented: firstly, the most rational method involves the use of suitable semi-empirical, non-linear diagrams of Moment-Chord Slope Rotation ($M - \theta$) of the "shear length" of each sub-cantilever, taken from a large database of experimental results, despite their considerable scattering (Panagiotakos & Fardis 2001). Such relations are proposed by Eurocode EN-1998.03 (Annex A: sections A.3.2.2 to A.3.2.4). Secondly, the calculation of $M - \theta$ diagrams can alternatively be achieved via Moment-Curvature

(M - φ) diagrams, which can be calculated using the "fiber elements" (i.e. XTRACT, 2007;). Next, using a suitable experimental lengh L_p for each plastic hinge, $M - \theta$ diagrams are calculated (Fig.2c) by the integral of curvatures in length L_p. More specifically, the yielding rotation can be calculated by $\theta_y = \varphi_y \cdot L_s / 3$ and the plastic rotation by $\theta_p \approx \left(\varphi_u - \varphi_y \right) \cdot L_p$, where φ_y is the yielding curvature of the end section and φ_u is the ultimate one by "fiber element" method, whilst the ultimate rotation θ_u is always calculated from the sum of both the yielding rotation θ_y and the plastic rotation θ_p. However, using the second method, only flexural deformations are taken into account, while other sources of inelastic properties are ignored. For example, shear & axial deformations, slippage and extraction of the main longitudinal reinforcement and opened cracks with yielding steel bars, should also be taken into account. In order to avoid all these issues, the initial method of semi-empirical Moment-Chord Slope Rotation ($M - \theta$) diagrams, either a combination of the two methods, is often selected. Moreover, the maximum available ultimate rotation of a plastic hinge in Fig.(2a) very closely approximates (in practice) the ultimate chord slope rotation θ of the equivalent sub-cantilever in Fig.(2b). On the other hand, other simulations of plastic hinges, such as the one based on the "dual component model", where each member is replaced by an elastic element (central length of the element) and an elasto-plastic element (plastic hinge) in parallel, are not suitable for analyzing reinforced concrete members, because then the "stiffness degradation" for repeated dynamic seismic loading cycles is ignored. Also, in many cases, and especially for existing r/c structures, a "shear failure" on structural members can occur because the mechanism of the shear forces in the plastic hinges is destroyed, and therefore doubts arise concerning the validity of the well-known modified "Morsch's truss-model" after the yielding. Therefore, the available plastic rotation θ_p of a plastic hinge is reduced (by up to 50% in several cases); (see Eq.(A.12) of EN-1998.03). Also, in the case where plastic hinges appeared simultaneously at both ends of an structural element, the flexural effective stiffness of the section is calculated by $EI_{eff} = M_y \cdot L_s / 3\theta_y$, (arithmetic mean value of the two antisymmetric bending states of the element).

Last but not least, in order to calculate the non-linear Moment-Chord Slope Rotation ($M - \theta$) diagrams, in contrast to the initial design state, the mean values of the strength of the materials must be used (namely, $f_{cm} = f_{ck} + 8$ in MPa for concrete and $f_{sm} = 1.10 f_{sk}$ for steel, where f_{ck} and f_{sk} are the characteristic values). If some structural elements have a low available ductility ($\mu < 2$), such as low-walls with a shear span ratio $a_s < 1.50$ at their base, columns/beams with inadequate strength in shear stress, brittle members such as short-columns with $a_s < 2.50$ etc, then these must be checked in relation to stress only. With reference to the "shear length L_s" of a sub-cantilever, we can note that it is calculated by $L_s = M/V$, where M is the flexural yielding moment and V is the respective shear force on a plastic hinge. In the case of tall-walls, the "shear length" L_s can be calculated by the distance of the zero-moment point to the base in elevation, from a temporary lateral static floor force.

The "shear span ratio" is calculated by $a_s = M/Vh = L_s/h$, where h is the dimension of the section that is perpendicular to the axis of the flexural moment M. Moreover, the yielding moment M of the diagram $M - \theta$ cannot be greater than the value $\max M = V_R L_s$, where V_R is calculated by Eq.(A.12) of Eurocode EN/1998.03, with the axial force N of the column at zero (i.e. due to the action of the vertical seismic building vibration of vertical and horizontal seismic components) and the plastic ductility of the element is equal to $\mu_\Delta^{pl} = \theta_p/\theta_y = 5$ (conservative value), because then the "brittle shear failure" appears prematurely, due to the doubts of the validity of the modified "Morsch's truss-model" after the yielding.
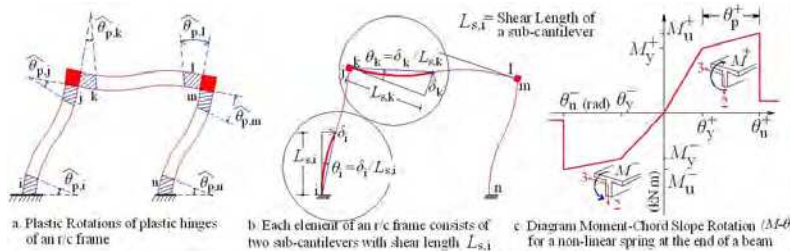

Fig. 2. a. Plastic rotations of plastic hinges; b. Two equivalent sub-cantilevers of each structural column/beam; c. Diagram $M - \theta$ of inelastic springs at the ends of the elements (where θ is the chord slope rotation).

On the other hand, in the case of seismic Non-Linear Response History Analysis (NLRHA), we have to use suitable pairs (or triads) of uncorrelated accelerograms as seismic action. However, the seismic demands of asymmetric multi-storey buildings are often unreliable for the following reasons:

i.   It is possible for the accelerograms to be deemed unsuitable (they often do not have the necessary frequency content or are inadequate as regards the number of strong cycles of the dynamic loading or the strong motion duration). In order to cope with this problem, according to contemporary seismic codes, at least seven triads (or pairs, in the case where the vertical seismic component is ignored) of accelerograms must be taken into consideration and then, the average of the response quantities from all these analyses should be used as the "final design values" of the seismic action effect. Note that, for each pair (or triad) of accelerograms, the latter must act simultaneously (sect.3.2.3.1.1(2)P/EN-1998.01) and be "statistically independent". This means that the "correlation factor" among these accelerograms must be zero (uncorrelated accelerograms). However, each accelerogram of each pair must be represented by the same response acceleration spectrum (sect.3.2.2.1(3)P/EN-1998.01). The acceleration response spectra (with 0.05 equivalent viscous damping ratio) of artificial (or recorded) accelerograms should match the elastic response spectra for 0.05 equivalent viscous damping ratio. This should occur over a wide range of periods (or at least between 0.2T and 2T, where T is the fundamental period of the structure), as is defined by sect.3.2.3.1.2(4c)/EN-1998.01. Also, no value of the "mean elastic spectrum", which is calculated based on all the used accelerograms, should be less than 90% (or greater than 110%) of the corresponding value of the tagret elastic response spectrum. The peak ground acceleration of each accelerogram must always be equal to $a_g \cdot S$, where a_g is the design ground acceleration of the local area and

S is the soil factor according to EN-1998.01. Simultaneously, for strong earthquakes, the minimum duration of the strong motion of each accelerogram must be sufficient (about 15s in common cases), whilst, from personal observations, I consider that there should be a minimum number of approximately ten "large" and thirty-five "significant" loading loops, due to the dynamic seismic cyclic loading (i.e. a "large" cycle has an extreme ground acceleration of over $0.75PGA$ and a "significant" cycle has an extreme ground acceleration of between $0.30PGA$ and $0.75PGA$). However, the subject of the exact definition of the Design Basis Earthquake and the Maximum Capable Earthquake of a seismic hazard area is open to question, while recently, a new framework for the simulation and definition of the seismic action of Design Earthquake Basis for the inelastic single degree of freedom system, using the Park & Ang damage index on the structures, has been developed (Moustafa, 2011;).

ii.   It is possible for the numerical models from the building simulation to be inadequate, because many false assumptions about the non-linear dynamic properties of plastic hinges can be inserted into the building model. For example, each non-linear spring must possess a suitable dynamic model $M - \theta$ for cyclic dynamic seismic loading (Dowell et al, 1998; Takeda et al, 1970; Otani, 1981; Akiyama, 1985;), where after each loading cycle, a suitable "stiffness degradation" must be taken into account (Fig.3).
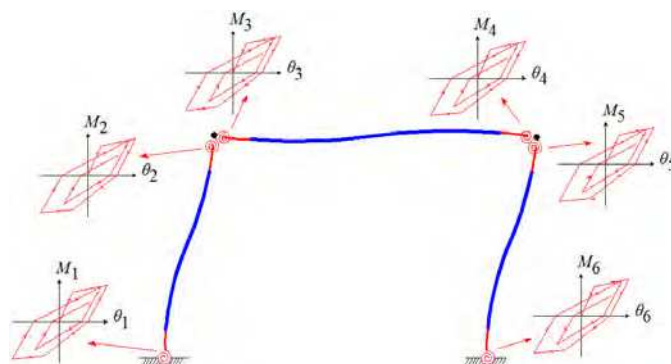

Fig. 3. Non-linear model of a planar frame, where each spring possesses a suitable diagram for cyclic dynamic seismic loading.

iii. It is possible for the numerical integration method to be inadequate, as regards accuracy & stability.

iv. The critical dynamic loading orientation of the pair of horizontal seismic components is unknown or does not exist and leads to the examination of various other orientations (at least one more orientation with a rotation of 45 degrees relative to the initial principal horizontal orientation must be examined). Alternative, a special methodology about the examination of various orientations usimg pairs of accelerograms, gives the envelope of the most unfavourable results (in the linear area only) was presented by Athanatopoulou (2005). Note that, for each pair of accelerograms, all possible sign combinations of seismic components must be taken into account, resulting in four combinations for each pair. Moreover, seven (7) pairs must be taken into account. In addition, due to the accidental eccentricities, four different positions of the floor mass centres must be considered. Therefore, (2 orientations)x(4 combinations of signs per pair)x(7 pairs)x(4 positions of the

mass centres)=224 NLRH Analyses. Alternatively, in order to reduce the number of said analyses, the action of accidental eccentricities can be taken into account using equivalent external floor moments $M_{m,i}$ around the vertical axis, which are calculated by the floor lateral static forces, $M_{\prime i} = \pm F_{\prime i} \times e_{ai,m}$, where $F_{\prime i}$ is the lateral floor static force and $e_{ai,m}$ is a mean accidental eccentricity ($e_{ai,m} = \sqrt{e_{ai,I}^2 + e_{ai,II}^2}$) for the case where the base shear is same for both principal directions of the building. Otherwise, the external moments can derive by $M_{m,i} = \pm \sqrt{M_{I,i}^2 + M_{II,i}^2} = \sqrt{\left(F_{\prime I,i} \times e_{ai,II}\right)^2 + \left(F_{\prime II,i} \times e_{ai,I}\right)^2}$. In this case, we arrive at a total number of fifty six (56) NLRH Analyses.

Consequently, the most important issues that we come across during the static pushover analysis are the following: (1) Why is the use of a single degree of freedom system that represents the real structure required? (2) How can the spatial action of the two horizontal seismic components during the static pushover analysis be taken into account? (3) What is the most suitable monitoring point in the case of asymmetric buildings? (4) Which distribution of lateral floor static forces is most suitable? and (5) At which point, in the plan of an asymmetric building, should the lateral floor static forces be applied? The design dynamic eccentricities and accidental eccentricities must also be taken into account, which means that the real or fictitious elastic axis of the building with the real or fictitious horizontal main elastic axes, where the lateral static forces must be oriented during the static pushover analysis, must be calculated. On the other hand, the Non-Linear Response History Procedure has a high computational cost and, simultaneously, many of its results present a high sensitivity. For these reasons, in practice, the mathematically documented static non-linear (pushover) analysis, presented below in this chapter, is a reliable alternative approximate method, which can envelope the accurate seismic demand floor inelastic displacements by the above Non-Linear Response History Procedure, in a rational way. However, it is known that the remaining seismic demands, such as the inter-storey drift ratios, real distribution of the plastic hinges on the building, yielding/failure mechanism of the building, etc, are only predicted correctly by using Non-Linear Response History Analysis.

2. The equivalent non-linear SDF system of asymmetric multi-storey buildings in non-linear static pushover analysis

Let us consider an asymmetric multi-storey building consisting of a rigid deck (diaphragm) on each floor, where the total mass $m_{,i}$ of each floor is concentrated at the floor geometric centre $CM_{,i}$ of the diaphragm, and the multi-storey building has a vertical mass axis, where all floor concentrated masses are located. Each centre $CM_{,i}$ therefore has the translational mass $m_{,i}$ for any horizontal direction, while it has the floor mass inertia moment $J_{m,i} = m_{,i} \, l_{s,i}^2$ due to its diaphragm (where $l_{s,i}$ is the floor inertia radius). Every diaphragm is supported on massless, axially loaded columns and structural walls, in reference to the global Cartesian mass system $CM(x,y,z)$ that is parallel to the known fictitious principal Cartesian reference system $P_o\left(I,II,III\right)$, as presented in the previous paragraph. Next, the original point CM has three degrees of freedom for each floor; two horizontal displacements $u_{x,i}$ and $u_{y,i}$ relative to the ground, along axes x and y respectively, and a rotation $\theta_{z,i}$ around the vertical z-axis (Fig.4). Moreover, for the needs of this mathematical analysis, we consider that this N-storey building is loaded with the 3N-dimensional loading vector **P** of external lateral floor static loading,

where the floor forces are oriented in parallel to one fictitious principal axis (i.e. along the *II*-axis) of the building. The loading vector **P** relating to the 3N-dimensional vector **u** of the degrees of freedom of the floor masses of the asymmetric multi-storey building is formed according to the pure translational (along the principal *II*-axis) mode-shape's distribution of lateral floor static forces in elevation (Eqs.2a,b). For the needs of this analysis, the lateral floor static forces of the loading vector **P** in the plan, are located at $\max e_I$ distance from the fictitious stiffness centre P_o to the mass centre of the building (Fig.4a). Next, a primary static pushover analysis of the asymmetric N-storey building is performed, using this increased static loading vector **P**, until the collapse of the building model.

$$\mathbf{u} = \begin{Bmatrix} \mathbf{u}_{,1} \\ \mathbf{u}_{,2} \\ ... \\ \mathbf{u}_{,i} \\ ... \\ \mathbf{u}_{,N} \end{Bmatrix}, \quad \mathbf{P} = \begin{Bmatrix} \mathbf{P}_{,1} \\ \mathbf{P}_{,2} \\ ... \\ \mathbf{P}_{,i} \\ ... \\ \mathbf{P}_{,N} \end{Bmatrix} \tag{2a,b}$$

where $\mathbf{u}_{,i} = \begin{Bmatrix} u_{x,i} \\ u_{y,i} \\ \theta_{z,i} \end{Bmatrix}$ and $\mathbf{P}_{,i} = \begin{Bmatrix} P_{x,i} \\ P_{y,i} \\ M_{z,i} \end{Bmatrix} = \begin{Bmatrix} 0 \\ P_{,i} \\ P_{,i} \left(\max e_{I,i} - e_{o,I} \right) \end{Bmatrix}$ for each floor *i* of the building.

Next, let us consider an intermediate step of this pushover analysis near the middle of the inelastic area. On each floor *i*, the vector of inelastic displacements of the mass centre of level *i* of the building is $\mathbf{u}_{,i,o} = \left\{ u_{x,i,o} \quad u_{y,i,o} \quad \theta_{z,i,o} \right\}^T$, while the global vector of inelastic displacements of all floor mass centres constitutes the 3N-dimensional vector $\mathbf{u}_o$, Eq.(3). It follows that this vector constitutes the initial state (index "o") of displacements in the following mathematical analysis:

$$\mathbf{u}_o = \begin{Bmatrix} u_{x,1,o} \\ u_{y,1,o} \\ \theta_{z,1,o} \\ u_{x,2,o} \\ u_{y,2,o} \\ \theta_{z,2,o} \\ ... \\ ... \\ u_{x,N,o} \\ u_{y,N,o} \\ \theta_{z,N,o} \end{Bmatrix} = \begin{Bmatrix} u_{x,1,o}/u_{y,N,o} \\ u_{y,1,o}/u_{y,N,o} \\ \theta_{z,1,o}/u_{y,N,o} \\ u_{x,2,o}/u_{y,N,o} \\ u_{y,2,o}/u_{y,N,o} \\ \theta_{z,2,o}/u_{y,N,o} \\ ... \\ ... \\ u_{x,N,o}/u_{y,N,o} \\ 1.00 \\ \theta_{z,N,o}/u_{y,N,o} \end{Bmatrix} u_{y,N,o} = \begin{Bmatrix} \psi_{x,1,y,N} \\ \psi_{y,1,y,N} \\ \psi_{z,1,y,N} \\ \psi_{x,2,y,N} \\ \psi_{y,2,y,N} \\ \psi_{z,2,y,N} \\ ... \\ ... \\ \psi_{x,N,y,N} \\ 1.00 \\ \psi_{z,N,y,N} \end{Bmatrix} u_{y,N,o} = \mathbf{\Psi}_{y,N,o} \cdot u_{y,N,o} \tag{3}$$
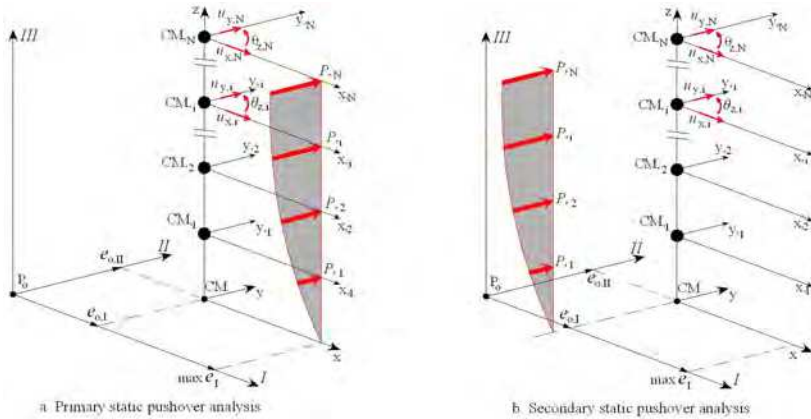
Fig. 4. Spatial Asymmetric Multi-storey System. Primary and Secondary pushover analysis.

Next, consider the known pure translational mode shape distribution **Y** of the lateral floor forces in elevation (from the linear modal analysis), where, Eq.(4);

$$\mathbf{Y} = \left\{ Y_1 \quad Y_2 \quad ... \quad Y_i \quad ... \quad Y_N \right\}^{\mathrm{T}} \tag{4}$$

with $Y_N = 1.00$.

In order to define the optimum equivalent Non-Linear SDF system, the following two assumptions are set (Makarios, 2009):

1st Assumption: Vector $\mathbf{\psi}_{y,N,o}$ of the inelastic displacement distribution of the floor mass centres of the asymmetric multi-storey building is a 'notional mode-shape'.

2nd Assumption: The lateral force **P** causes translational and torsional accelerations on the concentrated mass of the asymmetric multi-storey building. Therefore, if we consider that the vector loading **P** of the external loads is a function of time *t*, i.e. **P**(*t*) is a dynamic loading, this function is written as follows in Eq.(5):

$$\mathbf{P} = \left\{ \begin{array}{c} P_{x,1} \\ P_{y,1} \\ M_{z,1} \\ P_{x,2} \\ P_{y,2} \\ M_{z,2} \\ ... \\ ... \\ P_{x,N} \\ P_{y,N} \\ M_{z,N} \end{array} \right\} = \left\{ \begin{array}{c} 0.00 \\ P_{,1} \\ (\,\mathrm{max}e_{\mathrm{I},1} - e_{\mathrm{o},\mathrm{I}}\,)\cdot P_{,1} \\ 0.00 \\ P_{,2} \\ (\,\mathrm{max}e_{\mathrm{I},2} - e_{\mathrm{o},\mathrm{I}}\,)\cdot P_{,2} \\ ... \\ ... \\ 0.00 \\ P_N \\ (\,\mathrm{max}e_{\mathrm{I},N} - e_{\mathrm{o},\mathrm{I}}\,)\cdot P_{,N} \end{array} \right\} \mathrm{f}(t) = \left\{ \begin{array}{c} 0.00 \\ Y_1 \\ (\,\mathrm{max}e_{\mathrm{I},1} - e_{\mathrm{o},\mathrm{I}}\,)\cdot Y_1 \\ 0.00 \\ Y_2 \\ (\,\mathrm{max}e_{\mathrm{I},2} - e_{\mathrm{o},\mathrm{I}}\,)\cdot Y_2 \\ ... \\ ... \\ 0.00 \\ 1.00 \\ (\,\mathrm{max}e_{\mathrm{I},N} - e_{\mathrm{o},\mathrm{I}}\,) \end{array} \right\} P_N \cdot \mathrm{f}(t) = \mathbf{i}\,P_N \cdot \mathrm{f}(t)\,, \tag{5}$$

where $\mathrm{f}(t)$ is a known, suitable, increasing, linear, monotonic time function (i.e. $\mathrm{f}(t) = \mathrm{a} \cdot t + \mathrm{b}$, where a & b are coefficients) and $P_{,i}$ is the value of the floor lateral force.

Therefore, the equation of the motion of the masses of the multi-storey building, which are loaded with the dynamic loading $\mathbf{P}(t)$, is described by the mathematical system of linear second order differential N-equations, which can be written in matrix form, Eq.(6):

$$\mathbf{M}\,\ddot{\mathbf{u}}_{o}(t) + \mathbf{C}\,\dot{\mathbf{u}}_{o}(t) + \mathbf{K}\,\mathbf{u}_{o}(t) = \mathbf{P}(t) \tag{6}$$

where $\mathbf{u}_{o}(t)$ is the 3N-dimensional vector of displacements from Eq.(3), $\mathbf{K}$ and $\mathbf{M}$ are the 3Nx3N-dimensional square symmetric matrices (Eqs.7a,b) of lateral elastic stiffness and masses respectively, while $\mathbf{C}$ is the damping 3Nx3N matrix.

$$\mathbf{K} = \begin{bmatrix} \mathbf{k}_{,1} & \mathbf{k}_{,12} & \dots & \mathbf{k}_{,1i} & \dots & \mathbf{k}_{,1N} \\ & \mathbf{k}_{,2} & \dots & \mathbf{k}_{,2i} & \dots & \mathbf{k}_{,2N} \\ & & \dots & \dots & \dots & \dots \\ & & & \mathbf{k}_{,i} & \dots & \mathbf{k}_{,iN} \\ & & & & \dots & \dots \\ & & & & & \mathbf{k}_{,N} \end{bmatrix}, \ \mathbf{M} = \begin{bmatrix} \mathbf{M}_{,1} & & & & & \\ & \mathbf{M}_{,2} & & & & \\ & & \dots & & & \\ & & & \mathbf{M}_{,i} & & \\ & & & & \dots & \\ & & & & & \mathbf{M}_{,N} \end{bmatrix} \tag{7a,b}$$

where, $\mathbf{k}_{,i} = \begin{bmatrix} k_{xx,i} & k_{xy,i} & k_{xz,i} \\ k_{yx,i} & k_{yy,i} & k_{yz,i} \\ k_{zx,i} & k_{zy,i} & k_{zz,i} \end{bmatrix}$, $\mathbf{M}_{,i} = \begin{bmatrix} m_i & 0 & 0 \\ 0 & m_i & 0 \\ 0 & 0 & J_{m,i} \end{bmatrix}$ for each floor i

and where $J_{m,i} = m_i\, l_{s,i}^2$ is the mass inertia moment of the diaphragm (floor) around the vertical axis passing through the mass centre, and $l_{s,i}$ is the floor inertia radius. The floor lateral stiffness sub-matrix $\mathbf{k}_{,i}$, as well as the global lateral stiffness matrix $\mathbf{K}$ of the building, are calculated with reference to the global Cartesian mass system CM(x,y,z) using a suitable numerical technique pertaining to the "stiffness condensation" of the total stiffness matrix. Alternatively, we enforce a displacement (that is equal to one) of a floor mass centre along one degree of freedom and simultaneously all the remaining degrees of freedom of the building are fixed. The generated reactions on the degrees of freedom provide the coefficients of a column of the global lateral stiffness matrix $\mathbf{K}$, and this procedure is repeated for all other mass degrees of freedom of the building.

Using Eqs.(2), (3), (5) & (7), Eq.(6) is written as follows:

$$\mathbf{M}\,\boldsymbol{\psi}_{y,N,o} \cdot \ddot{\mathbf{u}}_{y,N,o}(t) + \mathbf{C}\,\boldsymbol{\psi}_{y,N,o} \cdot \dot{\mathbf{u}}_{y,N,o}(t) + \mathbf{K}\,\boldsymbol{\psi}_{y,N,o} \cdot \mathbf{u}_{y,N,o}(t) = \mathbf{i}\ P_N \cdot \mathrm{f}(t) \tag{8}$$

Next, we pre-multiply Eq.(8) by the $\boldsymbol{\psi}_{y,N,o}^{T}$ vector:

$$\boldsymbol{\psi}_{y,N,o}^{T}\mathbf{M}\,\boldsymbol{\psi}_{y,N,o} \cdot \ddot{\mathbf{u}}_{y,N,o}(t) + \boldsymbol{\psi}_{y,N,o}^{T}\mathbf{C}\,\boldsymbol{\psi}_{y,N,o} \cdot \dot{\mathbf{u}}_{y,N,o}(t) + \boldsymbol{\psi}_{y,N,o}^{T}\mathbf{K}\,\boldsymbol{\psi}_{y,N,o} \cdot \mathbf{u}_{y,N,o}(t) =$$

$$= \boldsymbol{\psi}_{y,N,o}^{T}\,\mathbf{i} \cdot P_N \cdot \mathrm{f}(t) \tag{9}$$

where $\boldsymbol{\psi}_{y,N,o}^{T}\,\mathbf{i}$ is a factor.

In this case, it is clear that the base shear $V_{oy}(t)$ of the building, for every time t, is given as follows:

$$V_{oy}(t) = P_N \cdot f(t) \cdot \sum_{i=1}^{N} Y_i \qquad (10)$$

Therefore, the lateral force $P_N \cdot f(t)$ at the top of the frame is given by:

$$P_N \cdot f(t) = \frac{V_{oy}(t)}{\sum_{i=1}^{N} Y_i} \qquad (11)$$

Eq.(9) is divided by the number $\boldsymbol{\psi}_{y,N,o}^{T}\,\mathbf{i}$ and by inserting Eq.(3), we arrive at:

$$\frac{\boldsymbol{\psi}_{y,N,o}^{T}\,\mathbf{M}\,\boldsymbol{\psi}_{y,N,o}\cdot\sum_{i=1}^{N}Y_i}{\boldsymbol{\psi}_{y,N,o}^{T}\,\mathbf{i}}\cdot\ddot{u}_{y,N,o}(t) + \frac{\boldsymbol{\psi}_{y,N,o}^{T}\,\mathbf{C}\,\boldsymbol{\psi}_{y,N,o}\cdot\sum_{i=1}^{N}Y_i}{\boldsymbol{\psi}_{y,N,o}^{T}\,\mathbf{i}}\cdot\dot{u}_{y,N,o}(t) +$$

$$+ \frac{\boldsymbol{\psi}_{y,N,o}^{T}\,\mathbf{K}\,\boldsymbol{\psi}_{y,N,o}\cdot\sum_{i=1}^{N}Y_i}{\boldsymbol{\psi}_{y,N,o}^{T}\,\mathbf{i}}\cdot u_{y,N,o}(t) = V_{oy}(t) \qquad (12)$$

Eq.(12) presents an equation of the motion of a single-degree-of-freedom (SDF) system. This equation must be transformed, so that the "optimum equivalent Non-linear SDF system" represents the initial asymmetric multi-storey building. As we can see in Eq.(12), the degree of freedom is the displacement $u_{y,N}$ of the mass centre CM, while loaded with the base shear V_{oy}. Thus, the "effective lateral stiffness" k^* of this SDF system must be obtained through an additional, special, secondary, non-linear (pushover) analysis of the initial, asymmetric multi-storey building, where the lateral static force is applied to the mass centre CM along the y-axis (Fig.4b). Hence, the lateral stiffness k^* of the first branch and the lateral stiffness $\alpha \cdot k^*$ of the second branch of the bilinear diagram V_{oy}-$u_{y,N}$ are the result of this special, secondary pushover analysis (Fig.5a). Therefore, we can consider that the slopes k^* and $\alpha \cdot k^*$ are already known from the secondary pushover analysis. Thus, for reasons of convergence between the known analytic SDF system of Eq.(12) and the (unknown at present) required optimum equivalent NLSDF system of the asymmetric multi-storey building, Eq.(12) is multiplied with the coefficient $L = k^*/k_o$, hence:

$$m^* \cdot \ddot{u}_{y,N,o}(t) + c^* \cdot \dot{u}_{y,N,o}(t) + k^* \cdot u_{y,N,o}(t) = L \cdot V_{oy}(t) \qquad (13)$$

where,

$$m^* = \frac{k^*}{k_o} \cdot \frac{\boldsymbol{\psi}_{y,N,o}^T \, \mathbf{M} \, \boldsymbol{\psi}_{y,N,o} \cdot \sum_{i=1}^{N} Y_i}{\boldsymbol{\psi}_{y,N,o}^T \, \mathbf{i}} \tag{14}$$

$$k_o = \frac{\boldsymbol{\psi}_{y,N,o}^T \, \mathbf{K} \, \boldsymbol{\psi}_{y,N,o} \cdot \sum_{i=1}^{N} Y_i}{\boldsymbol{\psi}_{y,N,o}^T \, \mathbf{i}} \tag{15}$$

$\boldsymbol{\psi}_{y,N,o}^T \, \mathbf{M} \, \boldsymbol{\psi}_{y,N,o}$ = Coefficient, $\boldsymbol{\psi}_{y,N,o}^T \, \mathbf{i}$ = Coefficient,

$\boldsymbol{\psi}_{y,N,o}^T \, \mathbf{K} \, \boldsymbol{\psi}_{y,N,o}$ = Coefficient, $\sum_{i=1}^{N} Y_i$ = Coefficient,

Therefore, the optimum equivalent NLlinear SDF system is represented by Eq.(13) and is presented in Fig.(6a). In addition, we can assume that the NLSDF system possesses the equivalent viscous damping c^*, hence:

$$c^* = \frac{k^*}{k_o} \cdot \frac{\boldsymbol{\psi}_{y,N,o}^T \, \mathbf{C} \, \boldsymbol{\psi}_{y,N,o} \cdot \sum_{i=1}^{N} Y_i}{\boldsymbol{\psi}_{y,N,o}^T \, \mathbf{i}} = 2 \cdot m^* \cdot \omega^* \cdot \xi \tag{16}$$

where, ω^* is the circular frequency (in rad/s) of the equivalent vibrating SDF system in the linear range:

$$\omega^* = \sqrt{k^*/m^*} \tag{17}$$

and ξ is the equivalent viscous damping ratio that corresponds to the critical damping of the SDF system (about 5% for reinforced concrete).

In order to transform the diagram $V_{oy} \text{-} u_{y,N}$ of Fig.(5a,b) into the capacity curve $P^* \text{-} \delta^*$ of Fig.(5b) of the equivalent NLSDF system, factor Γ is directly provided by:

$$\Gamma = m_{tot}/m^* \tag{18}$$

where,

$$m_{tot} = \boldsymbol{\iota}_y^T \, \mathbf{M} \, \boldsymbol{\iota}_y = \left\{ \boldsymbol{\iota}_{,1} \quad \boldsymbol{\iota}_{,2} \quad \dots \quad \boldsymbol{\iota}_{,i} \quad \dots \quad \boldsymbol{\iota}_{,N} \right\}^T \begin{bmatrix} \mathbf{M}_{,1} & & & & & \\ & \mathbf{M}_{,2} & & & & \\ & & \dots & & & \\ & & & \mathbf{M}_{,i} & & \\ & & & & \dots & \\ & & & & & \mathbf{M}_{,N} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\iota}_{,1} \\ \boldsymbol{\iota}_{,2} \\ \dots \\ \boldsymbol{\iota}_{,i} \\ \dots \\ \boldsymbol{\iota}_{,N} \end{Bmatrix}$$

with　$\mathbf{l}_{,i}^{T} = \{0 \quad 1 \quad 0\}^{T}$

Moreover, $\mathbf{l}_{y}$ is the global 'influence vector', that represents the displacements of the masses resulting from the static application of a unit horizontal ground displacement of the building along the seismic loading (i.e. y-direction). Hence, the optimum equivalent NLSDF system is characterized by Eq.(13) and has a bilinear capacity curve (P^{*}-δ^{*}) according to Fig.(5b).
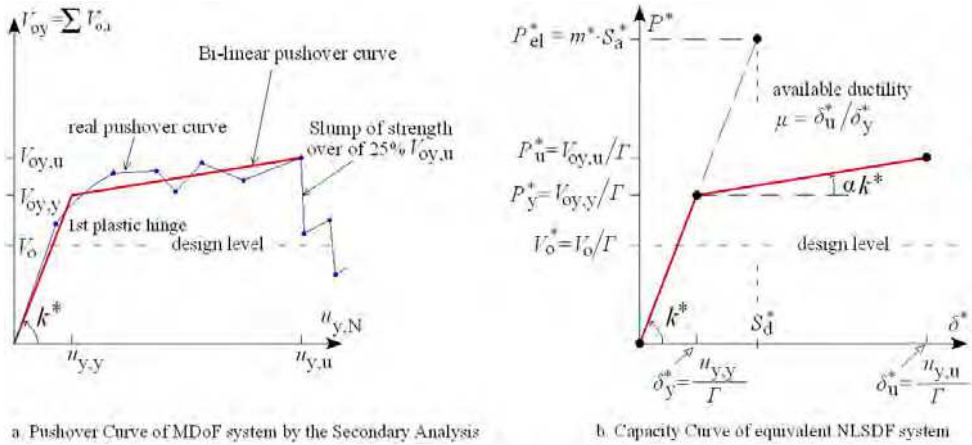

a. Pushover Curve of MDoF system by the Secondary Analysis　　　b. Capacity Curve of equivalent NLSDF system

Fig. 5. Transformation of the Pushover Curve of the Multi Degree of Freedom system into the Capacity Curve of the optimum equivalent NLSDF system.

The maximum elastic base shear P_{el}^{*} of the infinitely elastic SDF system, the yielding base shear P_{y}^{*} of the respective equivalent non-linear SDF system, its ultimate base shear P_{u}^{*}, its yielding displacement δ_{y}^{*} and its ultimate displacement δ_{u}^{*}, are given (Fig.5b):

$$P_{el}^{*} = m^{*} \cdot S_{a}^{*} \, , \; P_{y}^{*} = \frac{V_{oy,y}}{\Gamma} \, , \; P_{u}^{*} = \frac{V_{oy,u}}{\Gamma} \qquad \text{(19a,b,c)}$$

$$\delta_{y}^{*} = \frac{u_{y,y}}{\Gamma} \, , \; \delta_{u}^{*} = \frac{u_{y,u}}{\Gamma} \qquad \text{(20a,b)}$$

The "effective period" T^{*} of the optimum equivalent NLSDF system is given by Eq.(21), Fig.(6a):

$$T^{*} = 2\pi \sqrt{\frac{m^{*}}{k^{*}}} = 2\pi \sqrt{\frac{m^{*} \cdot \delta_{y}^{*}}{P_{y}^{*}}} \qquad \text{(21)}$$

Next, in order to evaluate the demand seismic displacement of the asymmetric multi-storey r/c building, we must first calculate the seismic target-displacement $\delta^*_{t,inel}$ of the optimum equivalent NLSDF system using the known Inelastic Response Spectra, since this is the only credible option; otherwise, a Non-Linear Response History procedure must be performed. Therefore, the inelastic seismic target-displacement $\delta^*_{t,inel}$ is calculated directly using Eq.(22), Fig.(6b). Secondly, we must transform the inelastic seismic target-displacement $\delta^*_{t,inel}$ into the target-displacement of the monitoring point (namely, the centre mass at the top level of the real initial asymmetric multi-storey building, along the examined direction), multiplying it with factor Γ, Eq.(28). Finally, the maximum required seismic displacements of the other positions of the multi-storey building can be obtained from that step in the known primary pushover analysis, where the seismic target-displacement of Eq.(28) of the monitoring point appears.
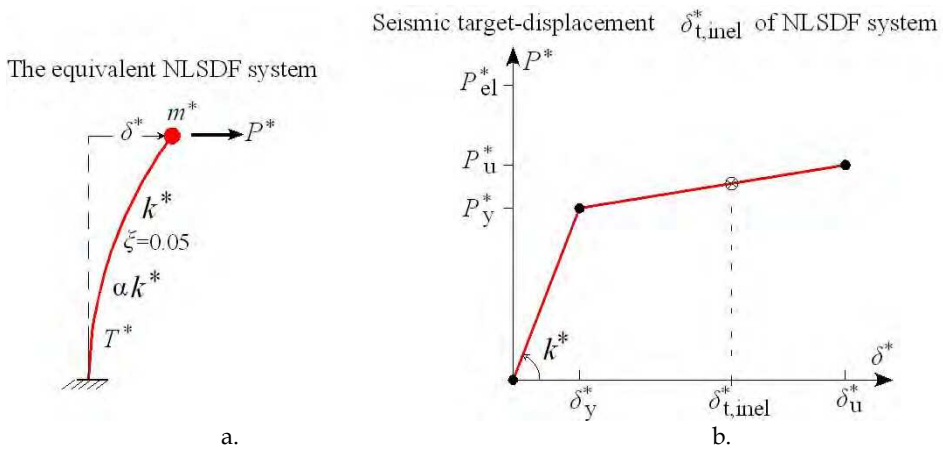

Fig. 6. a. The optimum equivalent NLSDF system, b. The seismic target-displacement of the equivalent NLSDF system.

More specifically, with reference to the optimum equivalent NLSDF system, its elastic spectral acceleration S^*_a is calculated by the elastic response acceleration spectrum, for the known period T^*. Consequently, the maximum required inelastic seismic displacement (target-displacement) δ_t of the new equivalent NLSDF system arises from the inelastic spectrum of a known earthquake, according to the following equations:

If $\dfrac{P^*_y}{m^*} < S^*_a$, then the response is non-linear and post-elastic and thus the target-displacement is inelastic, because this means that the yielding base shear P^*_y of the NLSDF system is less than its elastic seismic base shear; therefore this system yields:

$$\delta^*_{t,inel} = L \cdot \frac{\mu_d}{R_y} \cdot \frac{S^*_a}{(\omega^*)^2} = L \cdot \frac{\mu_d}{R_y} \cdot \frac{S^*_a}{4\pi^2} \cdot \left(T^*\right)^2 = L \cdot \frac{\mu_d}{R_y} \cdot S^*_d \qquad (22)$$

where,

$S_a^* = S_a(T^*)$ and $S_d^* = S_d(T^*)$ are the "elastic spectral acceleration" and the "elastic spectral displacement" from the elastic response acceleration and displacement spectrum respectively, for equivalent viscous damping ratio $\xi = 0.05$ for r/c structures,

$R_y = P_{el}^*/P_y^*$ is the reduction factor of the system (Fig.5b)

$L = k^*/k_o$ the "convergence factor" between the initial Multi Degree of Freedom system and the NLSDF

μ_d is the demand ductility of this non-linear SDF system (Eq.23):

$$\mu_d = 1 + \frac{\left(R_y^c - 1\right)}{c} \tag{23}$$

c is a coefficient due to the second branch slope from Eqs.(24, 25, 26), using a suitable slope ratio $\alpha = 0\%, 2\%, 10\%$ of the equivalent NLSDF system (Fig.5b), according to Krawinkler & Nassar (1992):

$$c = \frac{T^*}{1+T^*} + \frac{0.42}{T^*} \text{ for the second branch slope } \alpha = 0\% \tag{24}$$

$$c = \frac{T^*}{1+T^*} + \frac{0.37}{T^*} \text{ for the second branch slope } \alpha = 2\% \tag{25}$$

$$c = \frac{\left(T^*\right)^{0.8}}{1+\left(T^*\right)^{0.8}} + \frac{0.29}{T^*} \text{ for the second branch slope } \alpha = 10\% \tag{26}$$

Note that the demand ductility μ_d by Eq.(23) simultaneously satisfies both the rule of "equal energies" and the rule of "equal displacements" according to Veletsos & Newmark (1960) and Newmark & Hall (1982).

If $\frac{P_y^*}{m^*} \geq S_a^*$, then the response is linear elastic and thus the target-displacement $\delta_{t,el}^*$ is

elastic, because this means that the yielding base shear P_y^* of the NLSDF system is greater than the elastic seismic base shear and therefore this system remains in the linear elastic area,

$$\delta_{t,el}^* = L \cdot \frac{S_a^*}{(\omega^*)^2} = L \cdot \frac{S_a^* \cdot \left(T^*\right)^2}{4\pi^2} \tag{27}$$

It is worth noting that, in order to achieve adequate seismic stability of the building, the demand ductility μ_d must be less than the available ductility $\mu = \delta_u^*/\delta_y^*$. Next, in the first

approach (index "1"), the maximum required seismic displacement $u_{y,N,1}$ of the mass at the top level of the asymmetric multi-storey building, along y-direction, is directly given by:

$$u_{y,N,1} = \Gamma \cdot \delta_t^*$$ (28)

Finally, as mentioned above, the other maximum required seismic displacements $u_{x,i}$ and $\theta_{z,i}$ of the mass of the multi-storey building are obtained from that step in the known primary pushover analysis, where the displacement $u_{y,N,1}$ appears. If an optimum approach is required, then we can repeat the calculation of Eqs.(3-28), using the new displacements $u_{x,i}, u_{y,i}, \theta_{z,i}$ instead of $u_{x,i,o}$ $u_{y,i,o}$ $\theta_{z,i,o}$; the third approach is not usually needed. Finally, for each lateral loading, we follow the same methodology with a suitable index alternation (i.e. if the lateral floor forces are parallel to x-axis, then $\mathbf{u}_o = \boldsymbol{\psi}_{x,N,o} \cdot u_{x,N,o}$, $\mathbf{P}_{,i} = \left\{ P_{x,i} \quad 0 \quad M_{z,i} \right\}^T$, $\mathbf{l}_{,i}^T = \left\{ 1 \quad 0 \quad 0 \right\}^T$, etc).

To sum up, the above-mentioned optimum equivalent Non-Linear Single Degree of Freedom System allows us to provide the definition of the behavior factor of the multi-storey building. Indeed, it is well known that the global behavior factor q of a system is mathematically defined by the single degree of freedom system only. Therefore, in order to define the behavior factor of an asymmetric multi-storey building, an equivalent SDF system (such as the above optimum equivalent NLSDF system) must be estimated at first, and then its behavior factor, which refers to the asymmetric multi-storey building, is easy to determine. According to recent research (Makarios 2010), the "available behavior factor" q_{av} of the optimum equivalent NLSDF system indirectly represents the "global available behavior factor" of the asymmetric multi-storey building, while q_{av} is given by Eq.(29). Note that, in order for a building to cope with seismic action, the total "available behavior factor" q_{av} has to be higher than the "design behavior factor" q, that was used for the initial design of the building.

$$q_{av} = q_o \cdot q_m \cdot q_p$$ (29)

where q_o is the "*overstrength partial behavior factor*" that is part of the q_{av} and is related to the "extent of static-indefiniteness / superstaticity" of the asymmetric multi-storey building, as well as the constructional provisions of the Reinforced Concrete Code used, and can be calculated by Eq.(30), where V_o^* is the design base shear of the NLSDF system ($V_o^* = m^* \cdot S_a$):

$$q_o = P_y^* / V_o^*$$ (30)

Furthermore, q_m is the "*ductility partial behavior factor*" that is another part of q_{av} that is related to the ductility of the equivalent elasto-plastic SDF system, and is calculated by Eq.(31), which is an inversion of Eq.(23):

$$q_m = \left[c(\mu - 1) + 1 \right]^{1/c}$$ (31)

where $\mu = \delta_u^* / \delta_y^*$ is now the "available ductility" of the NLSDF system.

Next, q_p is the "*post-elastic slope partial behavior factor*" that is the last part of the q_{av} and is related to the slope of the post-elastic (second) branch of the bilinear capacity curve of the equivalent NLSDF system and is calculated by Eq.(32):

$$q_p = P_u^* / P_y^* \tag{32}$$

Note that, if the second (post-elastic) branch has a negative slope (i.e. intense action of second order phenomena), then the "*post-elastic slope partial behavior factor*" q_p is less than one.

Next, with reference to the spatial seismic action (signs of seismic components, simultaneous action of the two horizontal seismic components) in the presented non-linear static (pushover) procedure, we can observe the following issues:

Being aware, from the above mathematical analysis, that two static pushover analyses are needed for one location of lateral floor static forces, we can conclude that, if we use the final four design eccentricities, then twelve (12) individual static pushover analyses are required, as following:

- For the first seismic horizontal component along principal *I*-axis, six load cases are needed, Fig.(7). Note that from each primary analysis (where one "final design eccentricity" out of the four is used), the demand seismic displacements of the asymmetric multi-storey building are obtained, whilst from the secondary analysis (where the lateral static forces are located on the mass centres) the effective lateral stiffness k^* of its NLSDF is defined. Then, the twelve individual pushover analyses are analytically provided:

  1st case: Lateral static loads along positive principal (+)*I*-axis, with final design eccentricity $\max e_{\mathrm{II}}$ (primary pushover analysis, E_1).

  2nd case: Lateral static loads along negative principal (–)*I*-axis, with final design eccentricity $\max e_{\mathrm{II}}$ (primary pushover analysis, E_2).

  3rd case: Lateral static loads along positive principal (+)*I*-axis, with final design eccentricity $\min e_{\mathrm{II}}$ (primary pushover analysis, E_3).

  4th case: Lateral static loads along negative principal (–)*I*-axis, with final design eccentricity $\min e_{\mathrm{II}}$ (primary pushover analysis, E_4).

  5th case: Lateral static loads that are located on the floor mass centres along positive principal (+)*I*-axis (secondary pushover analysis, useful for the 1st and 3rd cases).

  6th case: Lateral static loads that are located on the floor mass centres along negative principal (–)*I*-axis (secondary pushover analysis, useful for the 2nd and 4th cases).

  The envelope of the results from the previous primary pushover analyses (1st, 2nd, 3rd & 4th) is symbolized as $E_{,I}$; it represents the demands of the first seismic component along principal *I*-axis:

$$E_{,I} = \max\left(\left| E_1 \right|, \ \left| E_2 \right|, \ \left| E_3 \right|, \ \left| E_4 \right| \right) \tag{33}$$

- For the second seismic horizontal component along principal *II*-axis, six more load cases
  are needed, Fig.(8):

  7th case: Lateral static loads along positive principal (+)*II*-axis, with final design
  eccentricity $\max e_I$ (primary pushover analysis, E_7).

  8th case: Lateral static loads along negative principal (–)*II*-axis, with final design
  eccentricity $\max e_I$ (primary pushover analysis, E_8).

  9th case: Lateral static loads along positive principal (+)*II*-axis, with final design
  eccentricity $\min e_I$ (primary pushover analysis, E_9).

  10th case: Lateral static loads along negative principal (–)*II*-axis, with final design
  eccentricity $\min e_I$ (primary pushover analysis, E_{10}).

  11th case: Lateral static loads that are located on the floor mass centres along positive
  principal (+)*II*-axis (secondary pushover analysis, useful for the 7th & 9th cases).

  12th case: Lateral static loads that are located on the floor mass centres along negative
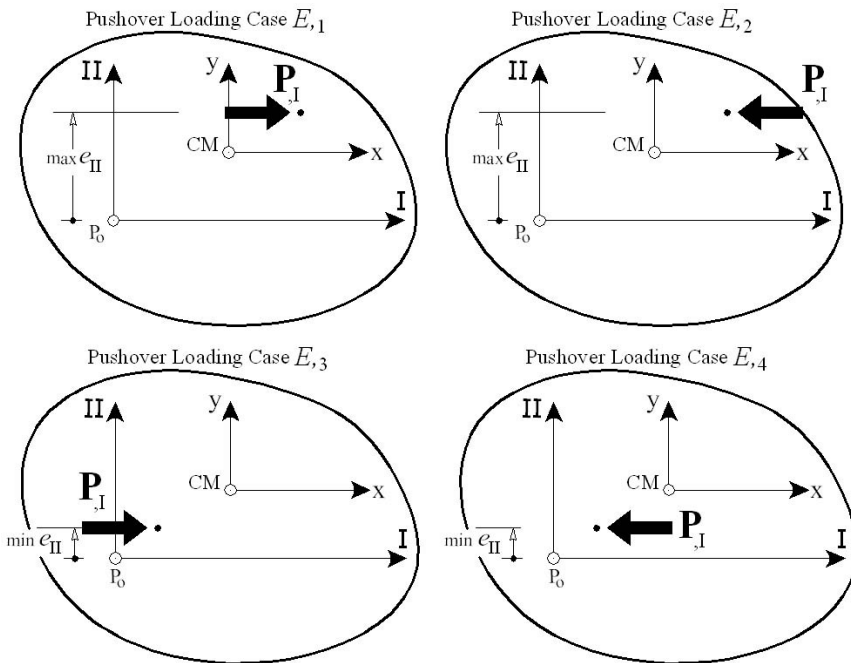 principal (–)*II*-axis (secondary pushover analysis, useful for the 8th & 10th cases).

Fig. 7. The envelope of these four pushover analyses represents the results of the first
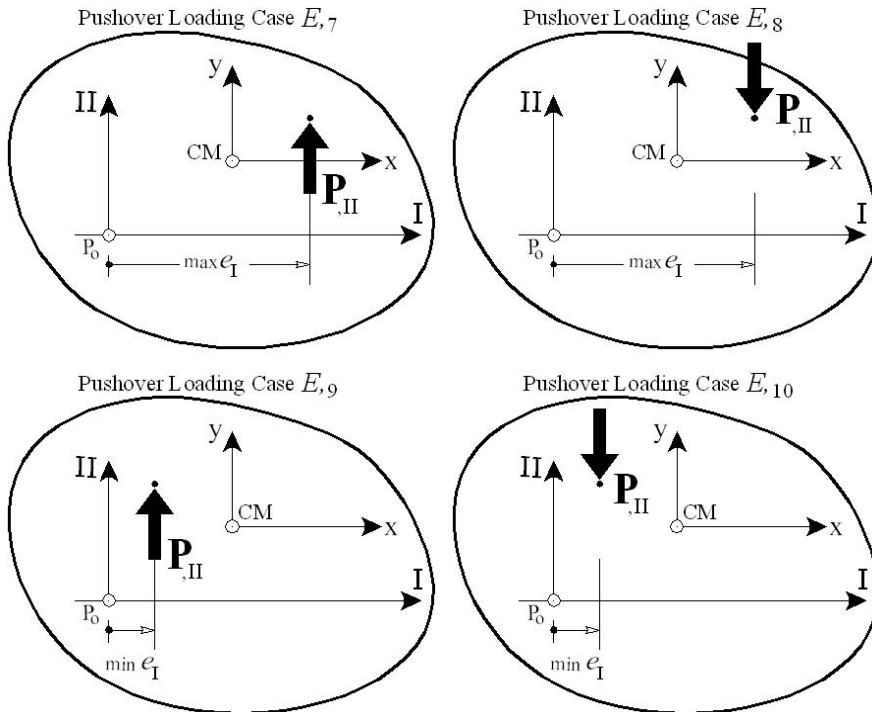seismic component $E_{,I}$.

Fig. 8. The envelope of these four pushover analyses represents the results of the second seismic component $E_{,\mathrm{II}}$.

The envelope of the results from the pushover analyses (7th, 8th, 9th & 10th) is symbolized as $E_{,\mathrm{II}}$ and represents the demands of the second seismic component along principal *II*-axis:

$$E_{,\mathrm{II}} = \max\left(\left|E_7\right|, \ \left|E_8\right|, \ \left|E_9\right|, \ \left|E_{10}\right|\right) \tag{34}$$

Note that, the use of the two signs of seismic components is necessary, because superpositions are generally forbidden in the non-linear area (sect.4.3.3.4.1(7)P/EN-1998.01). With reference to the simultaneous action of the two horizontal seismic components, Eq.(1) must be applied in order to obtain the final result of the extreme seismic demands by the spatial seismic action (where $E_{,\mathrm{I}}$ & $E_{,\mathrm{II}}$ are calculated by Eqs.(33-34). Note that suitable numerical examples, which demostrate the applicability of the proposed method, have been provided in other papers (Makarios, 2009 & 2011), where the correctness of these final results has been verified by the Non-Linear Response History Analysis.

In addition, contemporary Sesmic Codes, such as Eurocode EN-1998, do not give details and quidance regarding the static seismic pushover analyis of irregular in plan, asymmetric, multistorey r/c buildings. In reality, Annex B of EN-1998.01 refers to the calculation of the seismic target-displacement of planar frames only, whilst for spatial irregular in plan multi-storey buildings there are no relevant provisions. The present work intends to precisely cover this gap. The present non-linear static (pushover) procedure is a natural extension of the established equivalent linear (simplified spectral) static method, recommended by the Seismic Codes (EAK/2003, NBCC/1995), which are adequately documented as regards torsional provisions. The twelve static pushover analyses of the irregular asymmetric building involve an important computational cost, but on the other hand, this is much lower compared to the two hundred and twenty-four (224) non-linear dynamic analyses that the Non-Linear Response History Procedure requires. Thus, the presented non-linear static (pushover) procedure is a very attractive option, despite its twelve non-linear static analyses. In addition, within the framework of the proposed procedure, there is also natural supervision, which constitutes a very important issue.

3. Conclusions

Consequently, the most important issues that we come across during the static pushover analysis, in order to avoid the exact non-linear response history analysis, have been noted. More specifically, we have explained (1) why it is necessary to use a single degree of freedom system that represents the real structure; (2) how the spatial action of the two horizontal seismic components during the static pushover analysis can be taken into account; (3) what is the most suitable monitoring point in the case of asymmetric buildings; (4) which distribution of the lateral floor static forces is suitable; and (5) at which point in the plan of an asymmetric building, lateral floor static forces should be applied. Design dynamic eccentricities plus accidental eccentricities must be taken into account. This point leads to the calculation of the real or fictitious elastic axis of the building and the real or fictitious horizontal main elastic axes, where the lateral static forces must be oriented during the static pushover analysis (Fig.1). Extensive guidance has been given about the simulation of plastic hinges using Eurocode EN/1998.03. Moreover, various important issues have been discussed concerning seismic non-linear response history analysis. Next, a documented proposal of the optimum
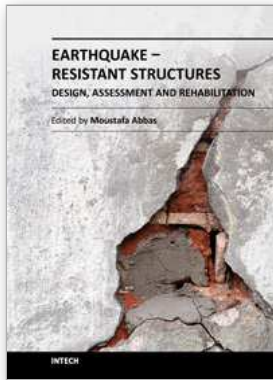
equivalent NLSDF system that represents asymmetric multi-storey buildings is presented. The definition of the NLSDF system is mathematically derived, by studying suitable dynamic loadings on the masses of each r/c system and using simplified assumptions. The coupling of the translational and the torsional degrees-of-freedom of the asymmetric building has also been taken into consideration, without dividing the asymmetric building into various, individual (two planar) subsystems. This NLSDF system is used in combination with the inelastic design spectra in order to calculate the seismic demands. The natural meaning of the characteristics of the asymmetric building, such as the fundamental eigenperiod, is not distorted by the optimum equivalent NLSDF system, since both the fundamental period of the multi-storey building and the period of the NLSDF system are very close, due to the fact where the total methodology is derived in a mathematical way. Extended numerical comparisons have verified the correctness of the use of the equivalent NLSDF system (Makarios, 2009 & 2011;). However, on the other hand, it is a known fact that, the sequence of plastic hinges on a building and the yielding/failure mechanism of multi-storey buildings are not correctly calculated by the various pushover analyses, which can only provide approximate results; nevertheless, this question is beyond the scope of the present chapter.

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