

Research Progress of Complex Electric Power Systems: Graph Theory Approach

Yagang Zhang, Zengping Wang and Jinfang Zhang
*State Key Laboratory of Alternate Electrical Power System
with Renewable Energy Sources
(North China Electric Power University),
China*

1. Introduction

Electric power system is one of the most complex artificial systems in this world, which safe, steady, economical and reliable operation plays a very important part in guaranteeing socioeconomic development, even in safeguarding social stability. In early 2008, the infrequent disaster of snow and ice that occurred in the south of China had confirmed it again. The complexity of electric power system is determined by its characteristics about constitution, configuration, operation, organization, etc., which has caused many disastrous accidents, such as the large-scale blackout of America-Canada electric power system on August 14, 2003, the large-scale blackout of Italy electric power system on September 28, 2003. In order to resolve this complex and difficult problem, some methods and technologies that can reflect modern science and technology level have been introduced into this domain, such as computer and communication technology, control technology, superconduct and new materials technology and so on. Obviously, no matter what we adopt new analytical method or technical means, we must have a distinct recognition of electric power system itself and its complexity, and increase continuously analysis, operation and control level (Yuan, 2007; Ye, 2003; Xue, 2002).

A fault is defined as a departure from an acceptable range of an observed variable or calculated parameter associated with systems. It may arise in the basic technological components or in its measurement and control instruments, and may represent performance deterioration, partial malfunctions or total breakdowns. Fault analysis implies the capability of determining, either actively or passively, whether a system is functioning as intended or as modeled. The goal of fault analysis is to ensure the success of the planned operations by recognizing anomalies of system behavior. A system with faults does not necessarily imply that the system is not functioning. Detecting a fault involves identifying a characteristic of the system, which when a fault occurs, can be distinguished from other characteristics of the system. According to nonlinear complex systems, we have carried out large numbers of basic researches (Zhang et al., 2010; Zhang et al., 2006; Zhang et al., 2007; Zhang & Wang, 2008). In this chapter, basing on graph theory and multivariate statistical analysis theory, we will discuss the complexity in electric power system.

The fault in electric power system can not be completely avoided. When electric power system operates from normal state to failure or abnormal operates, its electric quantities (current, voltage and their angles, etc.) may change significantly. In our researches, after some accidents, utilizing real-time measurements of phasor measurement unit (PMU) (Phadke & Thorp, 2008; Wang et al., 2007; Rakpenthai et al., 2005; Peng et al., 2006), basing on graph theory and multivariate statistical analysis theory, we are using mainly Breadth-first search (BFS), Depth-first search (DFS) and cluster analysis technology (Arifin & Asano, 2006; Otazu & Pujol, 2006; Park & Baik, 2006; Tola et al., 2008; Zhao et al., 2008; Templ et al., 2008), and seeking after for the uniform laws of electrical quantities' marked changes. Then we can carry out fast and exact analysis of fault component. Finally we can accomplish fault isolation.

2. Electric circuit theory

Let's consider a circuit with resistors(R), inductors (L), and capacitors(C), which has one element of each connected in a loop (Robinson, 2004). The part of the circuit containing one element is a branch. The points where the branches connect are nodes. There are three branches and nodes in Figure. 1.

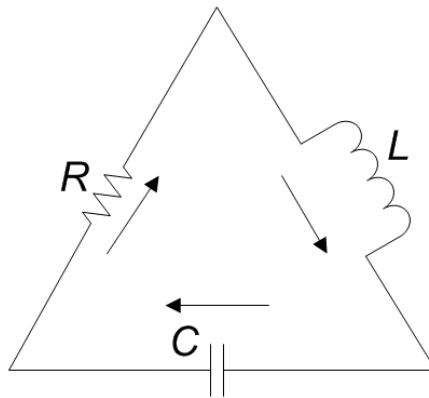


Fig. 1. RLC electric circuit

Let i_R , i_L and i_C be the current in the resistor, inductor and capacitor respectively, and let v_R , v_L and v_C be the voltage drop across the three branches of the circuit. Kirchhoff's voltage law states that the sum of the voltage drops around any loop is zero, that is, $v_R + v_L + v_C = 0$. Kirchhoff's current law states that the total current flowing into a node must equal the current flowing out of that node, namely, $|i_R| = |i_L| = |i_C|$ with the correct choice of signs.

A resistor is determined by a relationship between the current i_R and voltage v_R . Here, a linear resistor given by $v_R = R i_R$, where $R > 0$ is a constant, which is determined by Ohm's law. An inductor is characterized by giving the time derivative of the current $\frac{di_L}{dt}$, Faraday's law has proved that,

$$L \frac{di_L}{dt} = v_L \quad (1)$$

where the constant $L > 0$ is called the inductance. A capacitor is characterized by giving the time derivative of the voltage $\frac{dv_C}{dt}$, in terms of the current i_C ,

$$C \frac{dv_C}{dt} = i_C \quad (2)$$

where the constant $C > 0$ is called the capacitance.

Furthermore, let's define two variables $x = i_R = i_L = i_C$, $y = v_C$, and $v_R = R x$, $v_L = -v_C - v_R = -y - R x$. So, this system can be expressed as,

$$\frac{d}{dt} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -\frac{R}{L} & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \quad (3)$$

Its characteristic equation is,

$$\lambda^2 + \frac{R}{L} \lambda + \frac{1}{LC} = 0 \quad (4)$$

which has two roots:

$$\lambda = -\frac{R}{2L} \pm \sqrt{\frac{R^2 C - 4L}{4L^2 C}} \quad (5)$$

Actually this system always has eigenvalues with negative real parts. If $R^2 \geq \frac{4L}{C}$, then the eigenvalues are real; else $R^2 < \frac{4L}{C}$, they are complex.

3. Search principles in graph theory

Many real world situations can conveniently be described by means of a diagram consisting of a set of points together with lines joining certain pairs of these points. In mathematics and computer science, graph theory is the study of graphs: mathematical structures used to model conjugated relations between objects from a certain collection. A graph is an abstract notion of a set of nodes and connection relations between them, that is, a collection of vertices or nodes and a collection of edges that connect pairs of vertices. A graph may be undirected, meaning that there is no distinction between the two vertices associated with each edge, or its edges may be directed from one vertex to another.

Applications of graph theory are primarily, but not exclusively, concerned with labeled graphs and various specializations of these. Structures that can be represented as graphs are

ubiquitous, and many problems of practical interest can be represented by graphs. For example, in electric circuit theory, the Kirchhoff's voltage law and Kirchhoff's current law are only concerned with the structures and properties of the electric circuit. Then, any concrete electric circuit can be abstracted as a graph (Bondy & Murth, 1976). Here, let's give a simple electric circuit (See Figure 2), and its structure can be expressed as a graph (See Figure 3).

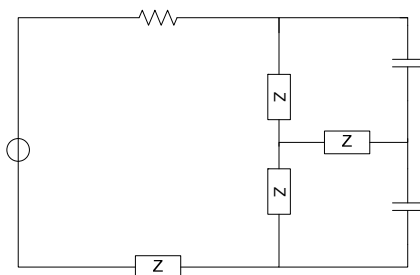


Fig. 2. A simple electric circuit

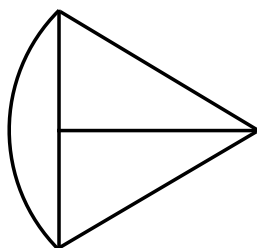


Fig. 3. A graph based on the simple electric circuit

Graph theory can be used to model many different physical and abstract systems such as transportation and communication networks, models for business administration, political science, and psychology and so on. Efficient storage and algorithm design techniques based on the graph representation make it particularly useful for utilizing computer. There are many algorithms that can be applied to resolve different kinds of problems, such as Breadth-first search, Depth-first search, Bellman-Ford algorithm, Dijkstra's algorithm, Ford-Fulkerson algorithm, Kruskal's algorithm, Nearest neighbor algorithm, Prim's algorithm, etc. Hereinto, Breadth-first search (BFS) is a graph search algorithm that begins at the root node and explores all the neighboring nodes. Then for each of those nearest nodes, it explores their unexplored neighbor nodes, and so on, until it finds the goal.

BFS is an uninformed search method that aims to expand and examine all nodes of a graph or combinations of sequence by systematically searching through every solution. In other words, it exhaustively searches the entire graph or sequence without considering the goal until it finds it. From the standpoint of the algorithm, all child nodes obtained by expanding a node are added to a first-in, first-out (FIFO) queue. In typical implementations, nodes that have not yet been examined for their neighbors are placed in some container (such as a

queue or linked list) called “open” and then once examined are placed in the container “closed” (Knuth, 1997).

BFS can be used to solve many problems in graph theory, for example:

- Testing whether graph is connected, and finding all connected components in a graph;
- Computing a spanning forest of graph;
- Computing, for every vertex in graph, a path with the minimum number of edges between start vertex and current vertex or reporting that no such path exists;
- Computing a cycle in graph or reporting that no such cycle exists.

The Depth-first search (DFS) is an algorithm for traversing or searching a tree, tree structure, or graph. One starts at the root and explores as far as possible along each branch before backtracking (Thomas et al., 2001).

In formal way, DFS is an uninformed search that progresses by expanding the first child node of the search tree that appears and going deeper and deeper until a goal node is found, or until it reaches a node which has no child node. Then the search backtracks, and it will return to the most recent node that it has not finished exploring. The space complexity of DFS is much lower than BFS. It also lends itself much better to heuristic methods of choosing a likely-looking branch. Time complexity of both algorithms is proportional to the number of vertices plus the number of edges in the graphs they traverse.

4. Cluster analysis

Theories of classification come from philosophy, mathematics, statistics, psychology, computer science, linguistics, biology, medicine, and other areas. Cluster analysis can also be named classification, which is concerned with researching the relationships within a group of objects in order to establish whether or not the data can be summarized validly by a small number of clusters of similar objects. That is, cluster analysis encompasses the methods used to:

- Identify the clusters in the original data;
- Determine the number of clusters in the original data;
- Validate the clusters found in the original data.

Cluster analysis is commonly applied for statistical analyses of large amounts of experimental data exhibiting some kind of redundancy, which allows for compression of data to amount feasible for further exploration. This permits further mining of each cluster independently or, alternatively, constructing a high level view of the data set by replacing each cluster with its best single representative. Cluster analysis has great strength in data analysis and has been applied successfully to the researches of various fields. The effectiveness of a cluster approach depends on many choices. These include the choice of a cluster algorithm, an appropriate feature subspace, and a similarity metric defined over this subspace. In addition, cluster algorithms typically have a set of tunable parameters inherent to them that can heavily influence their performance. For example, many algorithms require the number of clusters desired, the maximum number of iterations, learning rate, its change schedule, etc. While some of these choices are obvious for simple artificial datasets. The most common clustering algorithm choices are hierarchical cluster analysis.

5. Fault analysis based on BFS and DFS

Now let us consider IEEE9-Bus system, Figure 4 is presents the IEEE 9-Bus system electric diagram. In the structure of electric power system, Bus1 appears single-phase to ground fault. Through simulation experiments, using these actual measurement data of corresponding variables, we can carry through fault analysis of fault component and non-fault component.

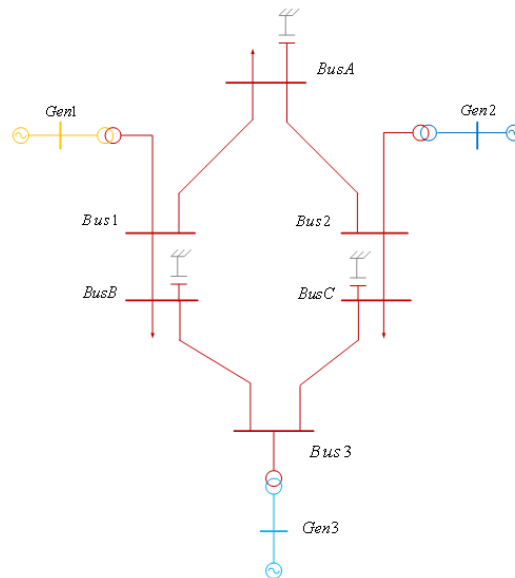


Fig. 4. Electric diagram of IEEE 9-Bus system

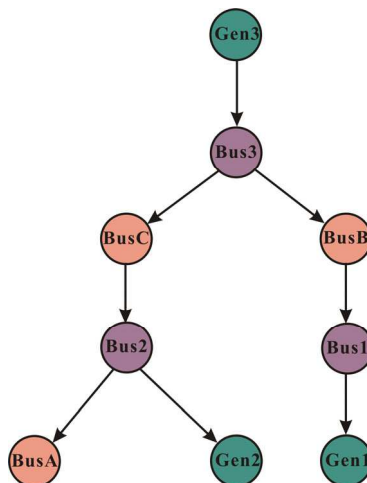


Fig. 5. BFS diagram of IEEE 9-Bus system

The adjacency matrix of IEEE9-Bus system can be expressed as follows,

$$\begin{matrix}
 & \text{Bus1} & \text{Bus2} & \text{Bus3} & \text{BusA} & \text{BusB} & \text{BusC} & \text{Gen1} & \text{Gen2} & \text{Gen3} \\
 \text{Bus1} & \left(\begin{matrix} 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \end{matrix} \right) \\
 \text{Bus2} & \left(\begin{matrix} 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \end{matrix} \right) \\
 \text{Bus3} & \left(\begin{matrix} 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{matrix} \right) \\
 \text{BusA} & \left(\begin{matrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} \right) \\
 \text{BusB} & \left(\begin{matrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} \right) \\
 \text{BusC} & \left(\begin{matrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} \right) \\
 \text{Gen1} & \left(\begin{matrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} \right) \\
 \text{Gen2} & \left(\begin{matrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} \right) \\
 \text{Gen3} & \left(\begin{matrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} \right)
 \end{matrix}$$

By the simulation experiments, we can get node phase voltage at T_{-1}, T_0 (Fault), T_1, T_2 and T_3 five times, see Table 1.

Bus	T_{-1}	T_0 (Fault)	T_1	T_2	T_3
Gen1	1.0100	0.7275	0.6924	0.6814	0.6747
Gen2	1.0100	0.8762	0.8476	0.8327	0.8134
Gen3	1.0100	0.8449	0.8071	0.7909	0.7710
Bus1	1.0388	0	0	0	0
Bus2	1.0430	0.7622	0.7350	0.7217	0.7049
Bus3	1.0534	0.7600	0.7275	0.7134	0.6960
BusA	1.0319	0.7540	0.7248	0.7114	0.6944
BusB	1.0222	0.2512	0.2404	0.2356	0.2294
BusC	1.0061	0.2470	0.2381	0.2336	0.2276

Table 1. The Node Phase Voltage At T_{-1}, T_0 (Fault), T_1, T_2 And T_3 Five Times

Figure.5 is the BFS process of IEEE9-Bus system. In this diagram, Gen1 is the first generator node, it is also one of the terminals of BFS, and Bus1 is just the only node that connects with it. Combined the information characters of electrical measurements that have marked changes, the difference of Bus1 and other Buses is distinct. At the beginning, Bus1 has just been set as single-phase to ground, which is a typical bus-bar fault. In the final analysis, both of these two aspects are consistent, and we can identify effectively fault location based on BFS.

Figure.6 is the DFS process of IEEE9-Bus system. In this diagram, the difference of Bus1 and other Buses is more distinct. Gen1 is the terminal of DFS, and Bus1 is just the only node that connects with it. In the beginning, we have set the Bus1 as single-phase to ground fault. Both of these two aspects are consistent. So, we can also identify effectively fault location based on DFS.

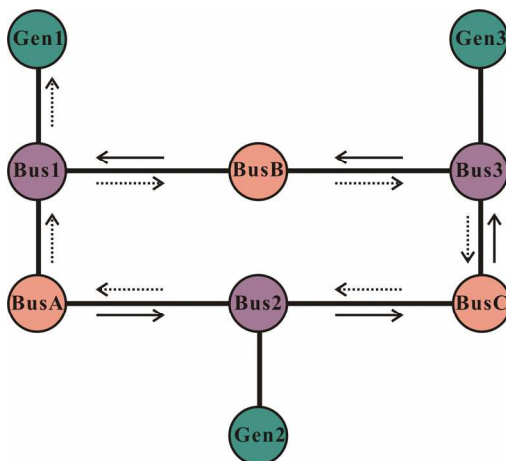


Fig. 6. DFS diagram of IEEE 9-Bus system

6. Fault analysis based on hierarchical cluster analysis

The hierarchical cluster analysis does not require us to specify the desired number of clusters K , instead affording a cluster dendrogram. In practice, the choice can be based on some domain specific and often have subjective components. There are three steps to hierarchical cluster analysis. First, we must identify an appropriate proximity measure. Second, we need to identify the appropriate cluster method for the data. Finally, an appropriate stopping criterion is needed to identify the number of clusters in the hierarchy. The distance or similarity metric used in cluster is crucial for the success of the cluster method. Euclidean distance and Pearson correlation are among the most frequently used.

Now let us continue to consider IEEE9-Bus system. According to the results of the simulation experiments (Table 1), basing on node phase voltage, we carry out hierarchical cluster analysis. Figure.7 is the dendrogram of hierarchical cluster analysis based on node phase voltage.

Let us explain the entire process of cluster analysis in detail. The entire cluster analysis process is carried out according to the principle of similarity from high to low (distance from near to far), the order is,

- Steps 1: BusC is combined with BusB to form the new BusB;
- Steps 2: Bus3 is combined with Bus2 to form the new Bus2;
- Steps 3: BusA is combined with Bus2 to form the new Bus2;
- Steps 4: Bus2 is combined with Gen1 to form the new Gen1;
- Steps 5: Gen3 is combined with Gen2 to form the new Gen3;
- Steps 6: Gen2 is combined with Gen1 to form the new Gen1;
- Steps 7: BusB is combined with Bus1 to form the new Bus1;
- Steps 8: Gen1 is combined with Bus1 to form the new Bus1.

From the entire hierarchical cluster process analysis, Bus1 has the lowest similarity to other nodes (the farthest distance to other nodes). It can also be found easily out from Figure.7

that Bus1 has remarkable difference with other buses, and the fault characteristic is obvious. These results are entirely identical to the fault location set in advance, so we can also confirm exactly fault location by the hierarchical cluster analysis.

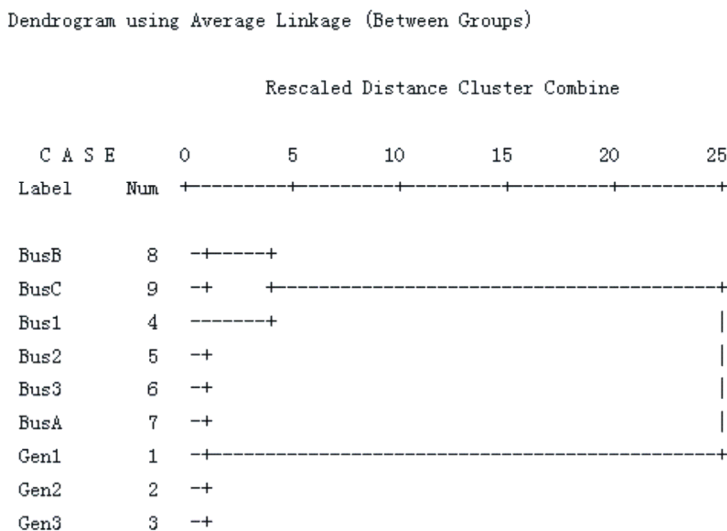


Fig. 7. The dendrogram of hierarchical cluster analysis based on node phase voltage

7. New approach for finding connected routes of power network

The connectivity analysis is the essence of the power system topology analysis, which is also playing the basic function in many kinds of advanced application software for power system analysis and calculation (Monticelli, 1999). The actual power network structures are shown by the result of the power system topology analysis processor, as the bus/branch model. Hence, the aim of topology analysis is to map the bus-section/switching-device model in the physical equipments level into the bus/branch model for a number of advanced functions in Energy Management System (EMS) or Distribution Management System (DMS) (Monticelli, 1999), such as power flow calculation, state estimation, dispatcher training simulator (DTS) and so on.

The main methods used in connectivity analysis of power system are based on the Graph theory, including tree-search based method and matrix-based method (Zhu et al., 2002). The former one can also be classified into two algorithms according to the different search patterns, named as Depth First Search (DFS) and Breadth First Search (BFS). By now, this topology analysis method has a widely application in power system analysis software (e.g. EMS or DMS). However, if the loop structure exists in the current network, the efficiency of tree-search based method will be low (Zhu et al., 2002). Matrix-based method is a systematic method based on adjacent matrix (Goderya et al., 1980), which can clearly depict the connected relationship between the two nodes belonged to the same branch. As the aim of connectivity analysis, whether any two nodes is connected or not and how many connected pieces all nodes

could be mapped into will be the ultimate target of topology analysis. Therefore, the complete connected matrix will be needed to gain a global connectivity information among nodes, which can be obtained by the self-multiplying of the above mentioned adjacent matrix with the number of operations no more than $n - 1$ (where, the n is the total number of the nodes). Obviously, the calculation burden of the matrix-based method will increase sharply as the expanding of the network scale. So in the substation or plants of power system where the number of nodes is not large, the matrix-based one also can play the role in grouping the connected physical nodes and then mapping the connected pieces into buses. Substation configuration has taken the most part of the total time needed in topology analysis (Zhang et al., 2010). Therefore, the substation configuration will be paid more attention in this paper.

The basic object of the above two methods is the vertexes/edges model mapped from the physical connections, in which the edges are corresponding to the switch devices, and the vertex will be the electrical connected points or physical buses. Therefore, once the state of the switches is changed, the connectivity analysis process will be restarted. Neither in tree-search based method nor in matrix-based method, the repeat search and calculation can not be avoided, which seriously effects the efficiency of the tracking of the status change happened in switch devices. In order to reduce the on-line topology analysis burden, the reference (Zhang et al., 2010) has established one method based on the graphic characteristic of the main connections, in which the each element of the complete connected matrix is represented by a set of connected routes with the open-close state of edges as variables. If the status of the each edge is determined, the value of these connected routes could be gained and the relationship between the mentioned nodes is confirmed. Hence, the large amount of repeat calculation is avoided. However, the connected routes finding algorithm is based on the type of electrical main connections used as the rule-based method, which is not systematic method and not suit for the irregular connections.

In this section, a new connected routes finding algorithm is proposed based on the adjoint matrix of the symbolic adjacent matrix, and a simplifying method is also applied to extract the connected routes information readily. Compare to the graphic feature based method, the new finding algorithm is an systematic one, which is suitable for various network connection structures.

8. The new connected routes finding algorithm

According to the Graph Theory (Diestel, 2000), the matrix could be used as the representation of one special graph, which is easily be analyzed and calculated by the computer. The two kinds of matrix usually used in analysis are incident matrix A_{inc} and adjacent matrix A_{adj} respectively. The A_{inc} depicts the connected relationship between the nodes and edges, and the A_{adj} gives a description of the relationship between two nodes. The aim of connectivity analysis is to group the nodes into different connected pieces, A_{adj} could meet this requirement and usually be used to do connectivity analysis.

For a special simple network structure as shown in Figure.8, the vertex set is $V(G) = [v1, v2, v3, v4, v5, v6]$, and the edge set is $E(G) = [e1, e2, e3, e4, e5, e6]$. The mapping relationship set between the vertexes and edges is $\varphi(G): E \rightarrow V \times V$, such as $\varphi(e1) = \{v1, v2\}$, $\varphi(e2) = \{v2, v3\}$, ..., $\varphi(e6) = \{v5, v6\}$.

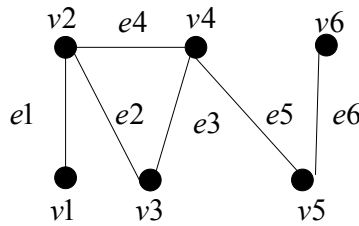


Fig. 8. A simple graph G with six vertexes and six edges

The adjacent matrix A_{adj} of graph G in Figure.8 is following:

$$A_{adj} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \tag{6}$$

Where, if there is a edge between vertex i and vertex j , the value of $A_{adj}(i, j)$ is equal to 1; otherwise, $A_{adj}(i, j) = 0$. Especially, the diagonal element of A_{adj} is set to 1. Obviously, this matrix A_{adj} can not describe the relationship between any two nodes, which are not incident to the same edge. Therefore, the completed connected matrix A is needed, which can depict the connected relationship between any two nodes in one graph. Corresponding to connected graph G in Figure. 8, the elements of matrix A are equal to 1, as following:

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \tag{7}$$

In a summary, the matrix-based method for the connectivity analysis is to find the completed connected matrix A from original adjacent matrix A_{adj} . According to the Graph Theory, the matrix A is obtained from the formulation (8):

$$A = A_{adj}^{n-1} \tag{8}$$

where, n is the number of vertexes in the graph G. In this process, the computation burden centers on the square of the matrix. It is worth to point out that the repeat calculation has taken the most part of the total time for the determination process of connected relationship among nodes.

8.1 The basic principle of new algorithm

Different from the traditional matrix-based method, the edges in the graph G are treated as the independent variables with the open-close two statuses; that is, if the edge is opened, the value of this corresponding variable is set to 0, otherwise, the value is equal to 1. Therefore, the adjacent matrix A_{adj} is the function of the current edge status, as following based on the Figure. 8.

$$A_{adj}(E) = \begin{bmatrix} 1 & e1 & 0 & 0 & 0 & 0 \\ e1 & 1 & e2 & e4 & 0 & 0 \\ 0 & e2 & 1 & e3 & 0 & 0 \\ 0 & e4 & e3 & 1 & e5 & 0 \\ 0 & 0 & 0 & e5 & 1 & e6 \\ 0 & 0 & 0 & 0 & e6 & 1 \end{bmatrix} \quad (9)$$

where, E is organized as a vector, such as $[e1, e2, e3, e4, e5, e6]$. Once the state of the each edge is determined, the vector E will be established and the matrix $A_{adj}(E)$ is also formed by substituting the current edge status. Take Figure.8 as an example, all the edges are closed and the current edge status vector E is equal to $[1, 1, 1, 1, 1, 1]$.

The adjoint matrix of A_{adj} is defined as A_{adj}^* , and corresponding to formulation (9), the detailed element representations in the first row of A_{adj}^* are extracted for the in-depth analysis, which are as following:

$$\begin{aligned} A_{adj}^*(1,1) &= e2^2 e5^2 + e2^2 e6^2 - e2^2 - 2e2e3e6^2 e4 \\ &\quad + 2e4e3e2 + e3^2 e6^2 - e6^2 + e6^2 e4^2 \\ &\quad - e3^2 - e5^2 + 1 - e4^2 \\ A_{adj}^*(1,2) &= e1e3^2 - e1e3^2 e6^2 + e1e5^2 + e1e6^2 - e1 \\ A_{adj}^*(1,3) &= -e1e2e5^2 - e1e2e6^2 + e1e2 - e1e4e3 \\ &\quad + e1e3e6^2 e4 \\ A_{adj}^*(1,4) &= e1e6^2 e2e3 - e1e2e3 - e1e6^2 e4 + e1e4 \\ A_{adj}^*(1,5) &= e1e5e2e3 - e1e5e4 \\ A_{adj}^*(1,6) &= -e1e6e5e2e3 + e1e6e5e4 \end{aligned} \quad (10)$$

The detailed analysis of the above representations in (10) is:

As to non-diagonal element, the representation has included all the routes between the two nodes indexed by the column and row number. For instance, in Figure.8, there is only one route between node $v1$ and node $v2$, which is $e1$. Comparing to the $A_{adj}^*(1,2)$, the route $e1$ is included in this element's representation as one individual term stamped by one rectangle,

$$A_{adj}^*(1,2) = e1e3^2 - e1e3^2e6^2 + e1e5^2 + e1e6^2 - \boxed{e1} \tag{11}$$

Similarly, there are two connected routes between node $v1$ and $v4$, respectively which are $e1e4$ and $e1e2e3$. As a result in the representation of $A_{adj}^*(1,4)$, these two routes are playing the roles as two terms signed by two rectangles,

$$A_{adj}^*(1,4) = e1e6^2e2e3 - \boxed{e1e2e3} - e1e6^2e4 + \boxed{e1e4} \tag{12}$$

As to diagonal element, there is not route between the node and itself. As shown in $A_{adj}^*(1,1)$, the representation is very complicated. However, in the practical analysis, the value of the diagonal element is set to 1. Obviously, this term is also existing in the $A_{adj}^*(1,1)$ as

$$\begin{aligned} A_{adj}^*(1,1) = & e2^2e5^2 + e2^2e6^2 - e2^2 - 2e2e3e6^2e4 \\ & + 2e4e3e2 + e3^2e6^2 - e6^2 + e6^2e4^2 \\ & - e3^2 - e5^2 + \boxed{1} - e4^2 \end{aligned} \tag{13}$$

In a summary, the non-diagonal element of the adjoint matrix A_{adj} has contained all the routes between the relevant two nodes. However, it is very low efficient to finding all the connected routes between the nodes just by the observation method. Therefore, in the next subsection, one simplifying method is proposed to make a rapidly and accurately finding.

8.2 Simplifying method

Having taken many different network structures cases into consideration, the needed terms which represent the connected routes between the mentioned two nodes can be extracted by the following three steps:

As to one special representation of the element in adjoint matrix A_{adj}^* ,

Step1: Firstly, if the highest power of one term in the polynomial is not less than 2, this term should be removed;

Step2: Secondly, if the absolute value of coefficient of one term is not equal to 1, this term also should be removed;

Step3: Thirdly, the coefficient of all the left terms is set to its absolute value, which is 1 in fact.

After the above mentioned three steps, the representation of element in adjoint matrix A_{adj} can be simplified into the needed connected routes formation. In order to make a detailed depiction, the $A_{adj}^*(1,2)$ is picked and taken as example.

For $A_{adj}^*(1,2) = e1e3^2 - e1e3^2e6^2 + e1e5^2 + e1e6^2 - e1$:

These terms such as $e1e3^2$, $-e1e3^2e6^2$, $e1e5^2$, $e1e6^2$ will be removed in the first step, because the highest power of them are equal to 2 respectively; and the simplified result $A_{adj}^*(1,2)_{s1}$ in this step is as following:

$$A_{adj}^*(1,2)_{-s1} = s_1(A_{adj}^*(1,2)) = -e1 \quad (14)$$

In the second step, no term will be removed, because the left term $-e1$ does not meet the condition that absolute value of coefficient is not equal to 1. Hence, there is:

$$A_{adj}^*(1,2)_{-s2} = s_2(A_{adj}^*(1,2)_{-s1}) = -e1 \quad (15)$$

In the final step, the coefficients of terms in $A_{adj}^*(1,2)_{-s2}$ are substituted by their absolute values, and ultimate organization formation of connected routes is obtained as following:

$$A_{adj}^*(1,2)_{-s3} = s_3(A_{adj}^*(1,2)_{-s2}) = e1 \quad (16)$$

Similarly, the first row in the adjoint matrix A_{adj} can be transformed into the connected routes formation as following

$$\begin{aligned} A_{adj}^*(1,1)_{-s3} &= 1 \\ A_{adj}^*(1,2)_{-s3} &= e1 \\ A_{adj}^*(1,3)_{-s3} &= e1e2 + e1e4e3 \\ A_{adj}^*(1,4)_{-s3} &= e1e2e3 + e1e4 \\ A_{adj}^*(1,5)_{-s3} &= e1e5e2e3 + e1e5e4 \\ A_{adj}^*(1,6)_{-s3} &= e1e6e5e2e3 + e1e6e5e4 \end{aligned} \quad (17)$$

According to the above simplifying method, the elements in the other rows of the adjoint matrix A_{adj} also can be simplified and organized as the connected routes form. When the matrix A_{adj}_{-s3} is formed as the ultimate result, the current edge statuses can be substituted as individual variables and the completed connected matrix A also can be obtained equally. Obviously, the following formulation will be established:

$$A = A_{adj}_{-s3}(E) \quad (18)$$

9. The application for substation configuration of power system topology analysis

In substation configuration, the network structure is more complicated than the network connection among substations or plants. The number of nodes in one substation or plants is not large, so the matrix-based method can be adopted for the operation of low dimension matrix. Generally, one electrical connection can be mapped into a graph, in which the switch device is corresponding to the edge, and the physical bus and electrical connected point are transformed into vertexes.

In this part, the more complicated electrical main connection—angle connection with four angles is taken as an example. One typical connection of this type is show in Figure. 9.

The corresponding adjacent matrix A_{adj} is:

$$A_{adj} = \begin{bmatrix} 1 & e1 & 0 & e2 \\ e1 & 1 & e3 & 0 \\ 0 & e3 & 1 & e4 \\ e2 & 0 & e4 & 1 \end{bmatrix} \tag{19}$$

As the adjoint matrix of the symmetrical matrix A_{adj} , the upper triangular of A_{adj}^* is:

$$A_{adj}^* = \begin{bmatrix} -e4^2 + 1 - e3^2 & e1e4^2 - e1 - e4e3e2 & e1e3 + e2e4 & -e1e3e4 - e2 + e3^2e2 \\ & -e4^2 + 1 - e2^2 & -e3 - e2e1e4 + e2^2e3 & e4e3 + e2e1 \\ & & -e2^2 + 1 - e1^2 & -e4 + e1^2e4 - e1e2e3 \\ & & & -e3^2 + 1 - e1^2 \end{bmatrix} \tag{20}$$

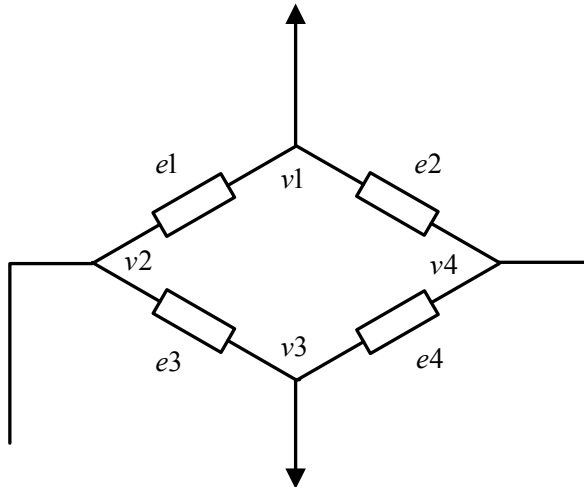


Fig. 9. A typical angle connection with four angles

After the treatment of the simplifying method proposed in Section II, the final connected routes matrix is represented by the upper triangular as following:

$$A_{adj_s3}^* = \begin{bmatrix} 1 & e1 + e4e3e2 & e1e3 + e2e4 & e1e3e4 + e2 \\ & 1 & e3 + e2e1e4 & e4e3 + e2e1 \\ & & 1 & e4 + e1e2e3 \\ & & & 1 \end{bmatrix} \tag{21}$$

Hence, the completed connected matrix A can be determined according to matrix $A_{adj_s3}^*$ with the current edges status vector E .

If the e_1 and e_4 are opened with the other two edges closed, the current edge status vector $E = [1, 0, 0, 1]$, then

$$A = A_{adj}^* - s_3(E) = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad (22)$$

Therefore, the vertex v_1 and v_2 are grouped into one connected pieces, and the left two vertexes are mapped into the second bus.

Sometimes, the edge e_4 is opened from closed, and the edge e_2 and e_3 are closed, the current edge status vector has become as $E = [1, 1, 1, 0]$; therefore, the complete connected matrix A is equal to:

$$A = A_{adj}^* - s_3(E) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad (23)$$

In other words, all the four vertexes are connected with each, and only one connected piece is formed with the current edge statuses. In this way, this connection is mapped into one bus.

10. Conclusion

Electric power system is one of the most complex artificial systems in this world, which safe, steady, economical and reliable operation plays a very important part in guaranteeing socioeconomic development, even in safeguarding social stability. The complexity of electric power system is determined by its characteristics about constitution, configuration, operation, organization, etc. However, no matter what we adopt new analytical method or technical means, we must have a distinct recognition of electric power system itself and its complexity, and increase continuously analysis, operation and control level.

The characteristic of the adjoint matrix has been given an in-depth analysis, which is deduced from the adjacent matrix represented as the function of current edge status in the network. By the use of the simplifying method, the needed connected routes information could be extracted readily and accurately. Combination the original analysis of the adjoint matrix and the connected routes extracting process, a novel connected routes finding algorithm is established. In this way, the complete connected matrix is finally formed as the function of the edge state, and each element could be represented by a set of connected routes provided by the proposed new algorithm. Once the status of each edge is determined, the connectivity between any two nodes is easily to be confirmed by substituting the current statuses into the corresponding connected routes set. These connected routes set could be found in the off-line way based on the novel routes finding algorithm, which will be able to save the on-line analysis time, especially for the topology

analysis of power system. Finally, the case study on the angle connection in substation has validated the efficiency of the established novel routes finding algorithm.

The fault in electric power system can not be completely avoided. When electric power system operates from normal state to failure or abnormal operates, its electric quantities (current, voltage and their angles, etc.) may change significantly. In our researches, utilizing real-time measurements of PMU, we are using mainly graph theory and multivariate statistical analysis theory to seek after for the uniform laws of electrical quantities' marked changes. Then we can carry out fast and exact analysis of fault component. Finally we can accomplish fault isolation.

These researches have proven that the complexity of electric power system can be explored successfully by analysis and calculation based on graph theory and multivariate statistical analysis theory.

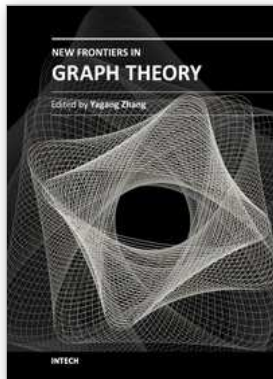
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51000 Rijeka, Croatia
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中国上海市延安西路65号上海国际贵都大饭店办公楼405单元
Phone: +86-21-62489820
Fax: +86-21-62489821

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