

# Non-Linear Analysis of Point Processes Seismic Sequences in Guerrero, Mexico: Characterization of Earthquakes and Fractal Properties

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## 1. Introduction

The purpose of this work is to reveal the efficiency of some statistical non-linear methods so as to characterize a seismic zone linked to subduction in Mexico. The Pacific plate subducting into the North American plate produces an important number of earthquakes (EQs), whose magnitudes exceed  $M_n = 5$ . This region comprises the following States: Jalisco, to the northwest, Michoacan, Guerrero, and Oaxaca, to the southeast; it extends along roughly 1350 km (Figure 1). Therefore, the characterization of this region - in all scopes - is very important. Here, we focus on the application of non-linear methods in the Guerrero State, because it displays an important number of EQs (their magnitudes rise up to 6) and it has a different slip inclination to the rest of the subduction zone, and some authors (Singh et al., 1983; Pardo and Suarez, 1995) have considered that there are some lags of seismicity. The assumptions of the non-linear methods analyzed in this work are: that EQs are stochastic point processes; that the Fano Factor (FF) reveals the fractality of EQs; and that the NHGPPP adjusts to extreme events. The application of these methods to the Guerrero seismic sequence allows us to explain the phenomenological behaviour in the subduction zone.

Traditionally, studies to characterize earthquakes' processes focus on the tectonics mechanism, basically following deterministic approaches. Recently, some studies have investigated the time scale properties of seismic sequences with non-linear statistical approaches so as to understand the dynamics of the process. The deep comprehension of the correlation time structures governing observational time-series can provide information on the dynamical characterization of seismic processes and the underlying geodynamical mechanisms (Telesca et al., 2001).

Scale-invariant processes provide relevant statistical features for characterizing seismic sequences. Since 1944, Gutenberg and Richter have found that earthquake magnitude size follows a power-law distribution. Other scale-invariant features were determined in Kagan (1992, 1994) and Kagan and Jackson (1991). A theory to explain the presence of scale-invariance was proposed by Bak et al. (1988); they introduced the idea of self-organized criticality (SOC), beginning from a simple cellular automaton model, namely a sand pile (Turcotte, 1990; Telesca et al., 2001).

Sieh (1978) and Stuart and Mavko (1979) proposed that earthquakes are due to a stick-slip process involving the sliding of the crust of the earth along faults. When slip occurs at some location, the strain energy is released and the stress propagates in the vicinity of that position. As such, the SOC concept is well-suited for rationalizing observations of the occurrences and magnitudes of earthquakes (Bak and Tang, 1989). An important part of the relaxation mechanism of the crust of the earth is submitted to inhomogeneous increasing stresses accumulating at continental-plate borders (Sornette and Sornette, 1989). The use of scaling laws concerning earthquakes has been especially used to develop models of seismogenesis, and the efforts of the characterization of EQs in Guerrero state on the part of some authors have been devoted to the shape of waves' propagation in order to reduce the uncertainty in magnitude determination and location (Singh et al., 1983; Pardo and Suarez, 1995).

In addition, many authors using several statistical techniques for specific volcanoes have carried out some studies of volcanic eruption time-series. Most of them have been developed within the scope of statistical distributions. Some of the earliest (Wickman, 1965, 1976; Reymont, 1969; Klein, 1982) employed stochastic principles to analyze eruption patterns. Further studies included the transition probabilities of Markov chains (Carta et al., 1981; Aspinall et al., 2006; Bebbington, 2007), Bayesian analysis of volcanic activity (Ho, 1990; Solow, 2001; Newhall and Hoblitt, 2002; Ho et al., 2006; Marzocchi et al., 2008), homogeneous and non-homogeneous Poisson processes applied to volcanic series (De la Cruz-Reyna, 1991; Ho, 1991), a Weibull renewal model (Bebbington and Lai, 1996a, b), geostatistical hazard-estimation methods (Jaquet et al., 2000; Jaquet and Carniel, 2006), a mixture of Weibull distributions (Turner et al., 2008) and, finally, non-homogeneous statistics to link geological and historical eruption time-series (Mendoza-Rosas and De la Cruz-Reyna, 2008). An exhaustive list of the available literature on this subject is made in Mendoza-Rosas and De la Cruz-Reyna (2009).

Along the same research lines, several distributions have been used to model seismic activity. Among these, the Poisson distribution - which implies the independence of each event from the time elapsed since the previous event - is the most extensively used, since in many cases and for large events a simple discrete Poisson distribution provides a good fit (Boschi et al., 1995).

Like some random phenomena, such as noise and traffic in communication systems (Ryu and Meadows, 1994), biological ion-channel openings (Teich, 1989), trapping times in amorphous semiconductors (Lowen and Teich, 1993a,b), seismic events occur at random locations in time. A stochastic point process is a mathematical description which represents these events as random points on the time axis (Cox and Isham, 1980). Such a process may be called fractal if some relevant statistics display scaling, characterized by power-law behaviour - with related scaling coefficients - that indicates that the represented phenomenon as containing clusters of points over a relatively large set of time scales (Lowen and Teich, 1995). Kagan (1994) and Telesca et al. (1999, 2000a,b, 2010) maintain that an earthquake's occurrence might be characterized by clustering properties with both short and long timescales with temporal correlation among the seismic events.

In this paper, we discuss the estimating of the fractality of a point process modelling a seismic sequence, corresponding to the Guerrero coast (the most seismically active area of the southern coast of Mexico), analyzing the performance of the Fano factor. Afterwards, we look at the extreme-value theory applied to NHGPPP so as to quantitatively evaluate the probabilities of extreme EQ occurrences. This work is organized as follows: first, we present

the theoretical concepts of stochastic point processes, fractal analysis by Fano factors and NHGPPP; then we present the EQ data series of the Guerrero region; and finally, we show the results of the analysis of this data when treated as stochastic point processes.

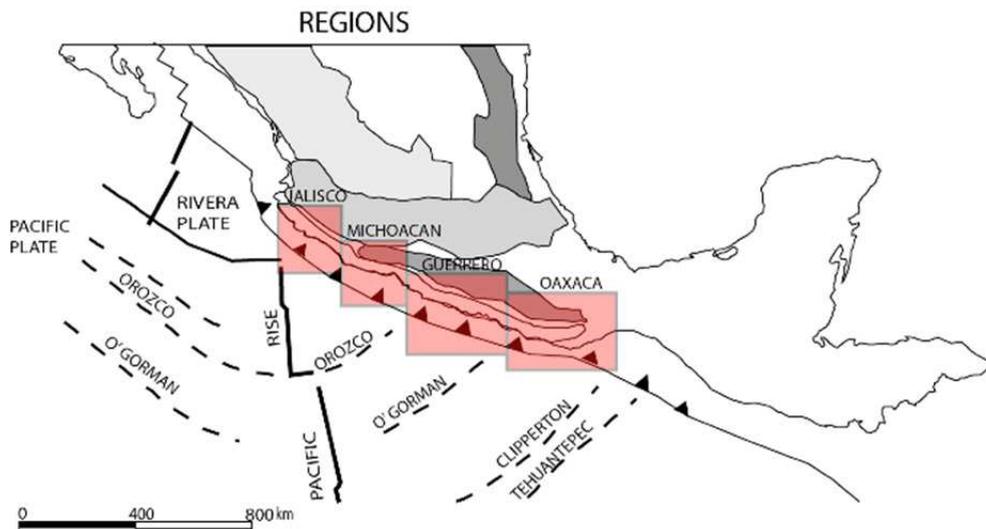


Fig. 1. Four seismicity regions dividing southern Mexico along the Mexican subduction zone, based on the seismicity and shape of the subduction (modified from Singh et al., 1983).

## 2. Point process

A stochastic point process was described by some authors (Telesca et al., 2001; Cox and Isham, 1980; Lowen and Teich, 1995) in terms of a mathematical description which represents the events as random points on the time axis (Cox and Isham, 1980). Such a process may be called fractal if some relevant statistics display scaling, characterized by power-law behaviour - with related scaling coefficients - that indicates that the represented phenomenon as containing clusters of points over a relatively large set of timescales (Lowen and Teich, 1995).

In this work, any earthquake sequence is assumed to be a realization of a point process, with events occurring at some random locations in time, and it is completely defined by the set of event times - or equivalently - by the set of inter-event intervals. Over a continuous time process, events can occur anywhere on the time axis. In a discrete time point process, the occurrence of events occurs at equally spaced increments. The continuous time point process is a simple Poisson process. If the point process is Poissonian, the occurrence times are uncorrelated; for this memoryless process, the inter-event interval probability density function  $P(t)$  behaves as a decreasing exponential function  $P(t) = \lambda e^{-\lambda t}$ , for  $t \geq 0$ , with  $\lambda$  as the mean rate of the process.

If the point process is characterized by fractal behaviour, the inter-event interval probability density function  $P(t)$  generally decreases as a power-law function of the inter-event time,  $P(t) = kt^{-(1+\alpha)}$ , with  $\alpha$  the so-called fractal exponent (Thurner et al., 1997). The exponent  $\alpha$  measures the strength of the clustering and represents the scaling coefficient of the

decreasing power-law spectral density of the process  $S(f) \propto f^{-\alpha}$  (Lowen and Teich, 1993a,b). The power spectral density furnishes information about how the power of the process is concentrated at various frequency bands (Papoulis, 1990) and it provides information about the nature of the temporal fluctuations of the process.

In recent studies, some authors (Bodri, 1993; Luongo et al., 1996) have focused their attention on the observational evidence of time-clustering properties in earthquake sequences of different seismic areas, demonstrating the existence of a range of time scales with scaling behaviour. The method that they used - the Cantor dust method (Mandelbrot, 1983) - consists of dividing the time interval  $T$ , over which  $N$  earthquake occur, into a series of  $n$  smaller intervals of length  $t = T/n$  with  $n = 2, 3, 4, \dots$  and computing the number  $R$  of intervals of length  $t$  which contain at least one event. If the distribution of events has a fractal structure (Smalley et al., 1987) then  $R \approx t^{1-D}$ , where  $D$  is the fractal dimension, which has sub-unitary values: the clustering is higher as  $D$  approaches to 0, while a value of 1 corresponds to an uniform distribution (events equally spaced in time). But the parameter  $R$  does not give information about the temporal fluctuations, because it is not directly correlated to the power spectral density  $S(f)$  of the process itself.

### 3. Fractal analysis (Fano factor)

The self-organized critical systems reach the critical steady state with temporary fluctuations in their events characterized by the energy they release. To detect the presence of clustering of events in a time series, several methods can be used among which is the Fano factor calculation, which estimates the value of the fractal exponent  $\alpha$  of the study process.

According to such authors as Telesca et al. (2004), assuming a sequence of events is the result of a point process defined by the set of occurrence times. You can use a statistical measure such as the Fano factor  $FF(\tau)$  to characterize the process.

For fractal process, that displays clustering properties,  $P(t)$  generally behaves as a power-law function of the inter-event time  $t$  with exponent  $(1+\alpha)$ , were  $\alpha$  is called fractal exponent, which characterizes the clustering of the process.

The representation of a point process is given by dividing the time axis into equally spaced contiguous counting windows of duration  $\tau$ , and producing a sequence of counts  $\{N_k(\tau)\}$ , with  $N_k(\tau)$  denoting the number of earthquakes in the  $k$ th window:

$$N_k(\tau) = \int_{t_{k-1}}^{t_k} \sum_{j=1}^n \delta(t-t_j) dt \quad (1)$$

The sequence is a discrete-random process of natural numbers.

The  $FF(\tau)$  (Thurner et al., 1997) is a measure of correlation over different timescales. It is defined as the variance of the number of events in a specified counting time  $\tau$  divided by the mean number of events in that counting time, that is:

$$FF(\tau) = \frac{\langle N_k^2(\tau) - N_k(\tau) \rangle^2}{\langle N_k(\tau) \rangle} \quad (2)$$

where  $\langle \rangle$  denotes the expectation value.

The  $FF$  varies as a function of counting time  $\tau$ . The exception is the Homogeneous Poisson Point Process (HPP). For an HPP, the variance-to-mean ratio is always unity for any counting time  $\tau$ . Any deviation from unity in the value of  $FF(\tau)$  therefore indicates that the point process in question is not a homogenous Poisson in nature. An excess greater than the unit reveals that a sequence is less ordered than an HPP, while values below the unit signify sequences that are more ordered.

The  $FF(\tau)$  of a fractal point process with  $0 < \alpha < 1$  varies as a function of counting time  $\tau$  as:

$$FF(\tau) = 1 + \left( \frac{\tau}{\tau_0} \right)^\alpha \tag{3}$$

The monotonic power-law increase is representative of the presence of fluctuations on many timescales (Lowen and Teich, 1995);  $\tau_0$  is the fractal onset time and it marks the lower limit for significant scaling behavior in the  $FF(\tau)$  (Teich et al., 1996). Therefore a straight-line fit to an estimate of  $FF(\tau)$  vs.  $\tau$  on a doubly logarithmic plot can also be used to estimate the fractal exponent. However, the estimated slope of the  $FF$  saturates at unity so that this measure finds its principal applicability for processes with  $\alpha < 1$ .

#### 4. The NHGPPP analysis

Within self-organized critical systems there are a great number of small events; however, the main changes of the system are associated with extreme events. The theory of extreme values is an area of statistics that is devoted to developing statistical models and techniques for estimating the performance of the unusual. These rare events are those which are far from the bulk of the distribution. However, there is no formal definition of extreme events in many cases, being defined as those events that exceed some threshold of magnitude, though they can also be defined as the maximum or minimum of one variable over a certain period. From a statistical standpoint, the problem of extreme value theory is a problem of extrapolation. The basic idea that leads to such extrapolation is that of finding a good parametric model for the tail of the distribution of the data generated by the process that can then be adjusted for extreme observations.

Overall, there are two approaches to the topic of Extreme Value Theory (EVT), a group of older models, known as Block top models and a new group of models known as "Peaks Over Threshold" (POT). The latter group corresponds to a pre-fixed high threshold models (Coles, 2001; Beguería, 2005). EVT focused on peak values above a value  $u$ , with these values being distributed as a Generalized Pareto Distribution.

The method characterizes the exceeding of a threshold based on the assumption that the occurrence of excesses on a strict threshold of a series characterized by an independent identically distributed random variable has a Poisson behaviour, and that the excesses have an exponential distribution or - more generally - a Generalized Pareto (GP) (Davison and Smith, 1990; Coles, 2001).

The distribution of excess  $F_u$  represents the probability of exceeding the threshold " $u$ ", in at most an amount of " $y$ ", which is conditioned by the information that has already exceeded the threshold (Cebrian, 1999, Lang et al. 1999; McNeal, 1999).

Definition: a distribution function with two parameters is known as the Generalized Pareto Distribution (GPD).

$$G_{k,a}(y) = \begin{cases} 1 - \left(1 - \frac{ky}{a}\right)^{\frac{1}{k}}, & k \neq 0 \\ 1 - e^{-\frac{y}{a}}, & k = 0 \end{cases} \quad (4)$$

Where  $a > 0$  and  $k$  is arbitrary, the range of  $y$  is:  $0 < y < a/k$  if  $k \leq 0$ ,  $0 < y < \infty$ , if  $k > 0$ .

The  $k < 0$  case is just a re-parameterization of one or more forms of the Pareto distribution, but the extension  $k \leq 0$  was proposed by Pickands (1975). The case  $k = 0$  is interpreted as the limit when  $k \rightarrow 0$ , (i.e. the exponential distribution).

#### 4.1 Properties for stability threshold

*Property 1.* If  $Y$  is a GPD  $u > 0$  a threshold, then the conditional distribution for excesses over a threshold - the conditional distribution  $Y - u$  given  $Y > u$  - is also distributed as a Generalized Pareto Distribution.

*Property 2.* If  $N$  has a Poisson distribution with conditional on  $N$ , where  $N$  is the number of the exceedances of a threshold and  $Y_1, Y_2, \dots, Y_N$  are independent random variables identically distributed as a GP, then  $\max(Y_1, Y_2, \dots, Y_N)$  for each  $N$  follows a Generalized Extreme Value Distribution. Thus, the exceedances satisfy a Poisson process, with excess distributed as a GPD that implies the Classical Distribution of Extreme Values.

Both properties characterize the GPD in the sense that does not exist another family that has these properties.

The excesses of a variable with GPD also follow a GPD - by Property 1 - allowing it to obtain the value(s) of (the) threshold(s) that rise to the extreme values, which also represents a distribution whose parameter values are constant.

Davison and Smith (1990) apply this idea to the expected value of the excess over a threshold  $u$  in the case where the GPD is a linear function of the threshold (Diaz, 2003; Beguería, 2005; Lin, 2003).

If  $k > -1$ ,  $u > 0$  and  $a - uk > 0$  then

$$E(x - u, x > u) = \frac{a - uk}{1 + k} \quad (5)$$

On the values above the threshold at which the GPD is adequate, the mean of excess of the sample is,

$$x_u = \frac{\sum_{i, x_i > u} (x_i - u)}{N_u} \quad (6)$$

This should be approximately a linear function of  $u$ , where  $N_u$  is the number of the exceedance above a predetermined threshold (McNeil and Saladin, 1997; Martínez, 2003, Lin 2003).

Consider the graph of the mean excesses  $z_u$  (the sum of positive differences in the magnitude of the fixed threshold and the EQ magnitude that exceeds that threshold, by the number of excesses) against the threshold  $u$ :

$$z_u = \frac{\sum_{i, x_i > u} x_i}{N_u} \tag{7}$$

If the assumption that it behaves as a GPD is correct, then the plot should follow a straight line with the intercept  $\frac{a}{1+k}$  and slope  $\frac{k}{1+k}$ . Therefore, it is enough to fit a straight line so as to obtain both parameters  $(a, k)$ . This is a relatively simple method for corroborating the linear relationship between the mean excess and the threshold  $u$  (Davison and Smith, 1990; Coles, 2001; Beguería, 2005, Lin 2003).

The parameters of the GPD for each year are given in Table 2, and the average parameters are calculated and compared in terms of how well they fit with the data annually and how well they make a global settlement for all years. Once obtained, the parameters can be estimated as a GPD function that adjusts the excesses.

Finally the probability of the excesses is obtained by using a non-homogeneous Poisson Pareto process, and for this we need the rate of occurrence and the GPD as a function of the intensity of the NHGPPP, namely:

$$\theta(e) = \frac{N_u}{t} \left[ 1 - \frac{k(e-u)}{a} \right]^{\frac{1}{k}} \tag{8}$$

The general methodology - as described earlier - is to obtain the variable rate of occurrence of a Non-Homogeneous Poisson Process that we will use for the analysis of the EQs that occurred in Guerrero.

### 5. Data processing and analysis

In the Mexican Republic - including its territorial sea - five tectonic plates converge: the North American, the Pacific, the Caribbean, the Cocos plate and the plate Rivera (Nava, 1987; Kostoglodov et al., 2001).

The subduction zone includes the entire Pacific coast between Puerto Vallarta in the state of Jalisco to Tapachula in Chiapas state. This extension has produced the largest earthquakes to have occurred this century in Mexico (Kostoglodov et al., 2001). Subduction earthquakes occur mainly in the coastal state of Guerrero. This type of earthquakes is rated as the most dangerous and they deform the ocean floor and generate tsunamis. According to the above description, it is possible to locate Guerrero in a territorial space latent to earthquakes' presence, not only with it being built on ground plate convergence but also by the presence of seismic gaps.

The Guerrero gap is one of the main concerns of researchers, being a rupture zone which for more than ninety years has not recorded an earthquake. Considering the magnitude of past earthquakes, these have ranged between 7.5 and 7.9 degrees and so - according to the

elapsed time - an earthquake is expected to peak at 8.4 degrees, which would represent a greater seismic event than occurred in 1985, which is why in this study we have provided the data epicentres located off the coast of Guerrero - in the area between 15.5 -17.5 N and 98.0-102.0 W for a period of 16 years from 1990 to 2005, as provided by the National Seismological Service (NSS, Servicio Sismologico Nacional).

Figure 2 represents the 4700 epicentres located in the study area. The figure shows that the number of small and medium earthquakes increases towards the state of Oaxaca and we can distinguish a swarm near the Guerrero gap. Note also that the EQs have a greater magnitude away from the coast.

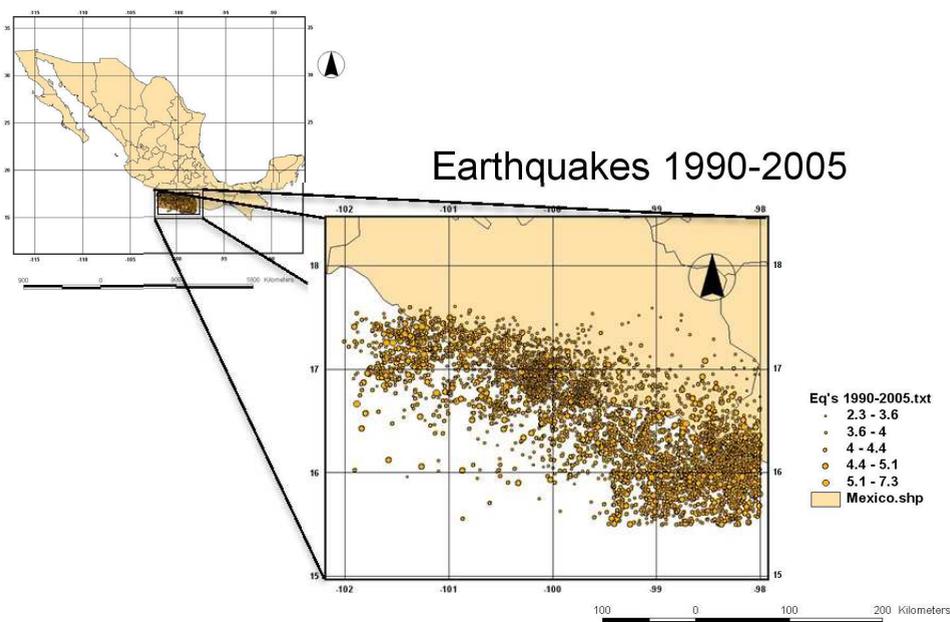


Fig. 2. Location of the 4700 epicenters in the study area (15.5-17.5 N and 98.0-102.0 W) for the 16 year period, 1990-2005, such that the size of each circle is proportional to the magnitude of the event.

### 5.1 Magnitudes of seismic data for years

First we made a preliminary analysis of the data per year (Fig. 3 shows two years as an example) in order to determine the extreme events of the series.

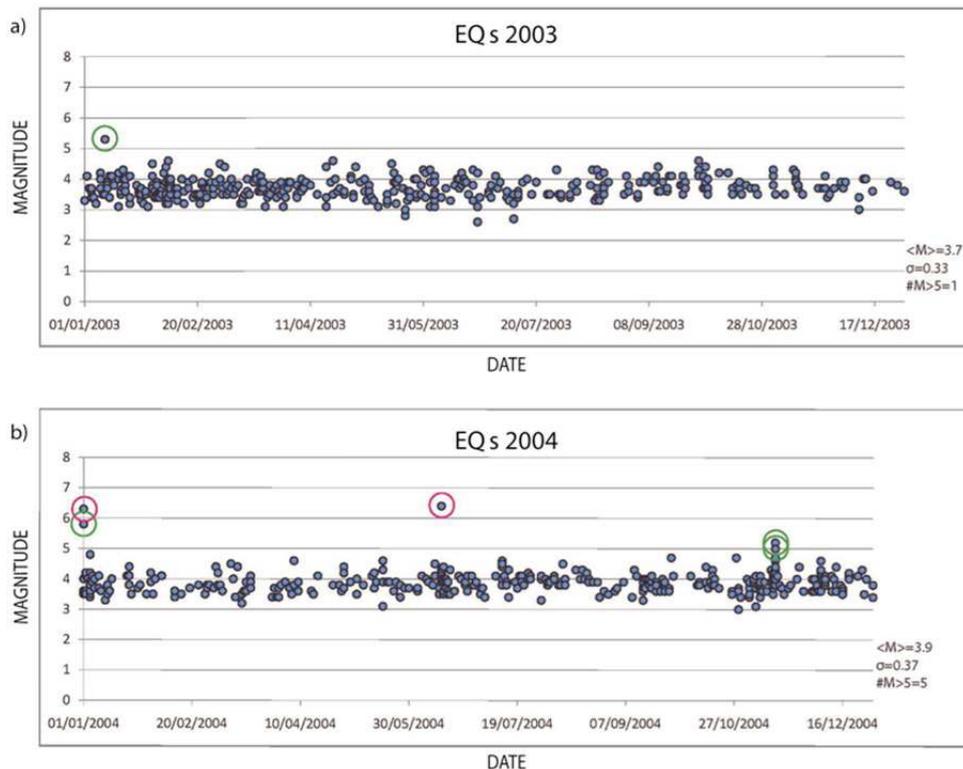


Fig. 3. An example of the EQs' series occurring per year in terms of its magnitude. The selected years are: a) 2003 b) 2004. The green open circles focus the EQs with  $5.0 \leq M_n < 6.0$  and magenta circles shows EQs with  $M_n \geq 6.0$ .

The analysis made of the data finds that the overall average magnitude earthquakes can be considered as  $3.9 \pm 0.4$ , and the number of EQs with a magnitude greater than the threshold 5 is between 0-7 per year. However, the years 1997 and 2002 have 9 and 14 events respectively, which doubles the number of events of  $M_n > 5$  occurring in the area. The influence of the type of instrumentation used to record the earthquakes is obvious because we observe that events which have a magnitude of less than two are not registered. Equally, as to the implementation of broadband seismographs in 1992, the number of records of EQ magnitudes of less than 3 increased significantly. We also observed that for the analyzed period there are no EQs with  $M_n < 7.5$ , so we decided to take thresholds corresponding to magnitudes: 3, 4, 5, 6, and 7 for this analysis. This data can be seen concentrated in Table 1, which also shows the dependence between the existence of large-scale EQs and the total number of events.

Year	$M_n > 5$ (year)	No. Max EQ (month)
1990	5	27
1991	5	36
1992	7	24
1993	7	32
1994	3	27
1995	3	98
1996	7	54
1997	9	78
1998	6	57
1999	0	34
2000	0	30
2001	7	201
2002	14	94
2003	1	57
2004	5	58
2005	1	44

Table 1. Extreme events and maximum number of EQs occurring per month and per year

## 5.2 Temporal clustering analysis of EQs

The self-organized critical systems reach this condition due to temporary fluctuations in their events, where they release much of their energy. For this reason, it is necessary to obtain the Fano factor for calculating the fractal exponent ( $\alpha$ ), so that it can detect the temporal clustering of events characteristic of the type of event detected.

The exponent of Fano is an estimate of the fractal exponent  $\alpha$  of the power law that characterizes the density spectrum of a process with scaling properties. The value of  $\alpha$  indicates the degree of clustering in a process according to Thurner et al. (1997). Over long time scales, the curve behaves essentially as  $\sim T^\alpha$  and the curve can be fit by a straight line of the slope  $\alpha$ . When  $\alpha \approx 0$ , the point process is a Homogeneous Poisson and the occurrence times are uncorrelated. However, if  $\alpha \neq 0$ , the point process is a Non-Homogeneous Poisson and has scaling properties. The value of  $\alpha$  obtained for the studied area - as is shown in figure 4 ( $\alpha = 0.6653$ ) - indicates, as expected, the presence of scaling behavior of the occurrence of the earthquakes.

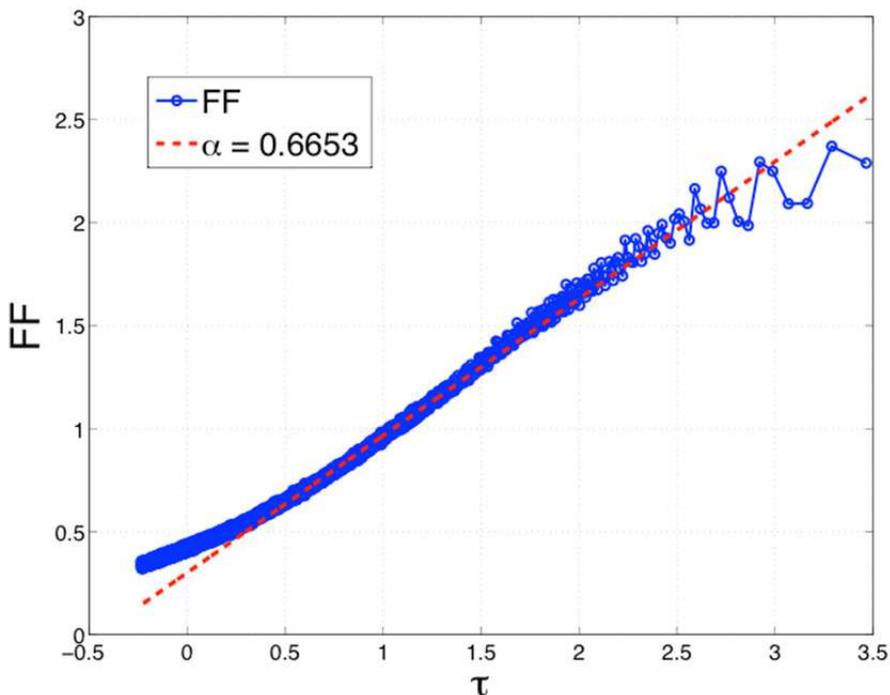


Fig. 4.  $FF$  curve for  $\alpha = 0.6653$ . The estimation of the fractal exponent  $\alpha$  - indicative of the degree of clusterization - is carried out estimating the slope of the curve plotted in bi-logarithmic scale in the linear range (in red) of the counting times.

### 5.3 Setting a process of Non-Homogeneous Poisson Pareto

Since the temporal clustering analysis of the EQ series indicates that it is a Non-homogeneous Poisson process, then the methodology - discussed in section 3 - for the analysis NHGPPP is appropriated so as to apply to these data series and in order to calculate the probability of the occurrence of extreme EQs. For the property of the stability threshold it is known that the excesses can be fitted to a GPD, and to validate the method we proceed to make an analysis of the excesses.

To compute the mean excess (that is, the sum of positive differences in the magnitude of the fixed threshold and the magnitude of earthquakes that exceed the threshold, per number of excesses), and the mean exceedance (the sum of the magnitude of the earthquakes that exceed the threshold fixed by the number of the exceedance) we used equations 6 and 7, respectively.

First, you get the graph average exceedance over a threshold and check the feasibility of the linear fit of the observed data with  $R^2$  so as to be close to the unit, in this case for the mean exceedance  $R^2 = 0.9832$ , which is indicative of the reliability and applicability of the proposed method. Next, we proceed to obtain the shape and scale parameters of the GPD (i.e.  $k$  and  $a$ , respectively), and for this - as shown in Figure 5 - we fit a straight line by a linear regression in the plot of the mean of the excesses against the magnitude of the fixed threshold.

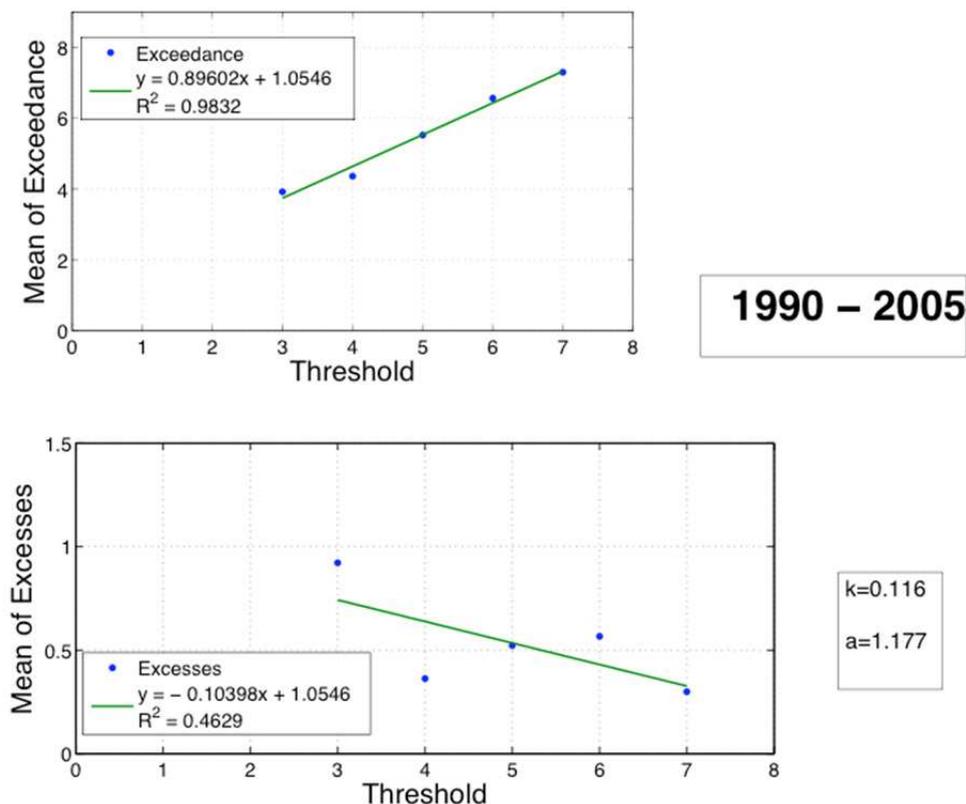


Fig. 5. Fitting of the mean exceedance for the period 1990-2005, with  $R^2 = 0.9832$  for the mean of exceedance and  $R^2 = 0.4629$  for the mean of excesses.

Once the linear regression is fitted, we get the shape and scale parameters. This procedure was done for earthquakes occurring by year, and the parameters of the GPD for each year of the studied period are given in Table 2; the parameters were also calculated for the whole period - repeating the procedure already explained - and obtaining  $k$  and  $a$  for the whole period. The comparison between the parameters computed annually and those for the global setting allows us to conclude that the global  $k$  and  $a$  can be used for all the data in the studied region, as it is shown in figure 6.

The obtained parameters  $k$  and  $a$  were used to fit the GPD to the excess. Figure 6 presents this comparison between the distribution obtained by  $k$  and  $a$  per year and for whole period for two selected years as an example of good fitting (year 2004) and the worst of them (year 2003).

In most of the years that were analyzed, the parameters of the GPD average reproduce the behaviour of the data; however, the years 2002, 2003 and 2005 show clear differences in the settings, as these years have atypical features of the studied area, so we will proceed to

calculate the probabilities of the whole area for periods of up to 100 years, with the parameters obtained from the full term.

Year	<i>k</i>	<i>a</i>
1990	0.1765	1.5294
1991	0.2195	1.7073
1992	0.25	1.75
1993	0.0695	1.0695
1994	0.1628	1.3953
1995	0.0616	1.1677
1996	0.1381	1.409
1997	0.0891	1.042
1998	0.1635	1.3369
1999	0.1451	1.1776
2000	0.1364	1.1364
2001	0.1905	1.4286
2002	0.0516	0.8448
2003	0.087	0.837
2004	0.1236	1.236
2005	0.2318	1.6239
1990-2005	0.116	1.177

Table 2. Parameters *k* and *a* obtained of linear fitting per year and for the complete data series.

To calculate the intensity distribution of the NHGPPP and to obtain the probabilities of the EQs' occurrence, we use equation 8. Since the approach of exceedance implicitly assumes that the scale inherent to the phenomena is open, we force the magnitude scale to ends at  $M_n < 9$  and we then subtract the probabilities of the EQs exceeding that magnitude from the probabilities of the lower magnitudes. As it is, the probabilities of occurrence lower than one event of  $M_n < 5$  for a period of 100 years is always 1. Using the same equation (8) we compute the values of the intensity function for a threshold of  $M_n > 5$  for a period of 100 years, and we obtain  $P(M_n > 7) = 1$ .

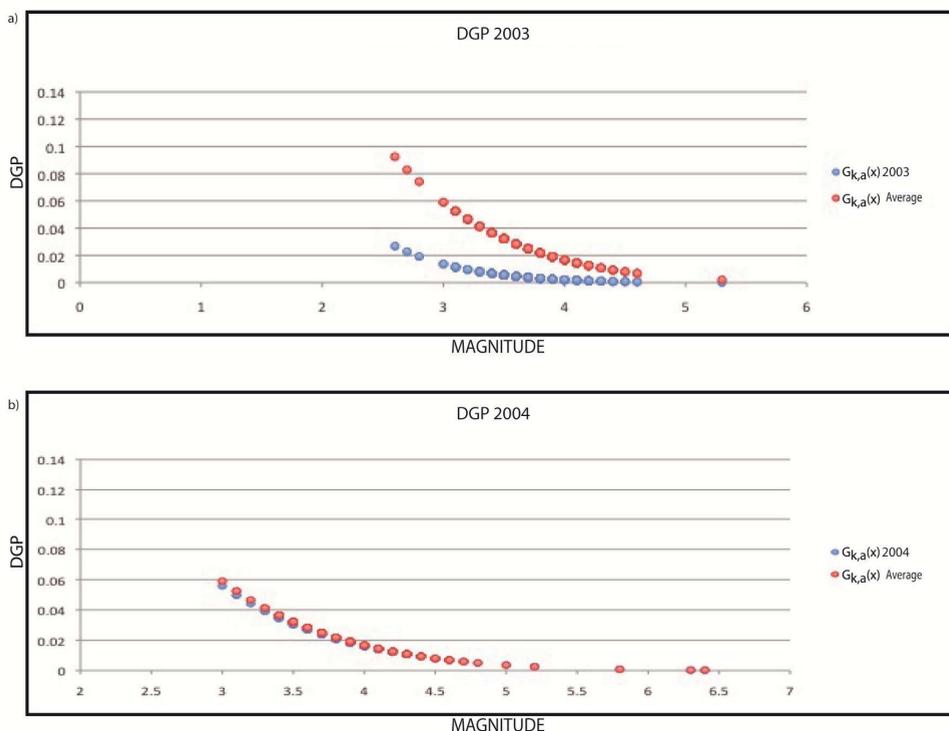


Fig. 6. Graph of the GPD fitted to the excesses using the parameters for each year compared to the excesses GPD adjusted for the period of 16 years, for the years: a) 2003 and b) 2004, providing examples of good and worst adjust.

## 6. Discussions

One of the goals in describing geological processes is to be able to predict their future behaviour. However, this has not been possible despite the many efforts being made in science. So, a small step is made by the characterization of such processes as earthquakes and volcanic eruptions, allowing for a step towards prediction.

As has been indicated, the data analysis was conducted over a period of 16 years - from 1990 to 2005 - looking at the earthquakes of the Guerrero state recorded by the NSS seismic web.

From the preliminary analysis of the EQ series, it was observed that the influence of the kind of instrumentation used to record EQ events is evident throughout the years, as with the implementation of broadband seismographs in 1992 where the registration of the number of EQ magnitudes of less than 3 increases considerably - and so the behaviour of the data is affected. The mean EQ magnitudes of  $3.9 \pm 0.4$  was computed for all the data, and the number of EQ with  $M_n > 5$  is between 0 - 7 per year; however, the years 1997 and 2002 have 9 and 14 events respectively, which doubles the number of events of that magnitude which occurred in the area. In addition, the number of small and medium

earthquakes increases towards the state of Oaxaca and we can distinguish a swarm near the Guerrero gap; also, we note that the EQs are of a greater magnitude away from the coast. Following this, in order to characterize the EQ events as punctual point process series, the clusterization of this data was allowed by the Fano factor, which indicates an  $\alpha$  value of 0.6653. This points out that the EQs follow a Non-Homogeneous Poisson Process.

Next, for the property of the stability threshold, it is known that the excesses can be fitted to a GPD and so we next proceed to an analysis of the excess, obtaining the parameters  $k$  and  $a$  to fit again to a GPD. The adjusted parameters were computed in two ways: one analysis of the EQs covered each year and the other the whole period (16 years). In most of the studied years, the adjustment of the mean of the GPD parameters reproduced the behaviour of all the data, except for the years 2002, 2003 and 2005, which show clear differences in the settings (as these years have atypical features of the area, the calculation was done with the average parameters).

The last step was to calculate the intensity distribution of the NHGPPP and to obtain the probabilities of the EQs' occurrence with the specific magnitudes in which we are interested. Because the approach of exceedance implicitly assumes an open scale of the phenomena, we assume that the magnitude scale ends at  $M_n < 9$ , and we compute the values of the intensity function for a threshold of  $M_n > 7$  for a period of 100 years, and so we obtain  $P(M_n > 7) = 1$ .

## 7. Conclusions

Finally we can conclude that the parameters determined from the seismic data for the 16 year period allows us to calculate the fractal dimension of the data -  $\alpha = 0.6653$  - it demonstrates the presence of temporal clusterization and variations in values linked to the occurrence of a high magnitude events. This fact allows us to say that the analyzed data follows a Non-Homogeneous Poisson Process and we can use a NHGPPP to characterize the distribution intensity of the data. As such, a good approximation of the probability of the occurrence of earthquakes of magnitudes greater than 7 for a period of 100 years in the area of Guerrero sees a finding of  $P(M_n > 7) = 1$ .

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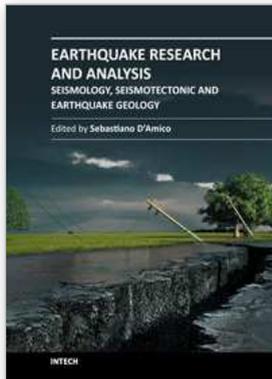
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