Application of Hagedoorn's Plus-Minus Method to Hydrology Study

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1. Introduction

The Memphis aquifer has been the major source of water for the City of Memphis municipal, industrial, and commercial uses for the past 100 years, and is considered to be among the highest quality water reservoirs in the nation. Above the Memphis aquifer are the confining unit (aquitard) of the Memphis aquifer and the surficial aquifer (Figure 1). The surficial aquifer is exposed to the surface and is prone to pollution due to industrial and human activities. The potential for contamination of the Memphis aquifer is exacerbated in areas where the aquitard is missing or thin. Recent studies indicated that the drinking aquifer might be at risk for contamination due to aquitard breaches existing in the confining unit of the Memphis aquifer. Aquitard breaches in the Memphis area have been identified through the correlation of stratigraphic picks from borehole data (Parks and Mirecki, 1992). The lack of uniform data coverage has restricted the study of breaches in Shelby County to areas proximal to the well fields. Although accurate, direct and reliable, the study does not provide crucial information about aquitard breaches, such as their extent, orientation, origination, and matrix characterization. Indirect methods (e.g. shallow seismic methods) can provide critical information that can help identify the possible causes responsible for the formation of the breaches (Ge et al., 2010, Part II). In this paper, the Hagedoorn's (1959) plus-minus method was applied to the seismic refraction data acquired in a walkaway test to map the top of the confining unit and identify possible aquitard breaches.

2. Hagedoorn's plus-minus method

The Hagedoorn's (1959) plus-minus method provides a simple and fast tool to interpret refraction data and calculate the geometry and velocity of the first refractor. The procedure is remarkably straightforward: the arrival times of the refracted waves from two reciprocal shots are simply added to find the depth to the refractor at all geophone stations and subtracted to find the velocity of the wave propagating through the refractor (Overmeeren, 2001). The Hagedoorn method has been shown to be a cost-effective and efficient means of mapping the shallow subsurface velocity structure (Overmeeren, 2001). Overmeeren (2001) used Hagedoorn's plus-minus method in a regional groundwater study and found that this method not only can provide a detailed section, but also produces additional information to reduce ambiguity in the interpretation of other geophysical data (e.g., vertical electrical soundings). In Hagedoorn's (1959) classic paper, he utilized wave front reconstruction,

usually by graphical means, to demonstrate the principle of the method. The derivation started from a model of one horizontal layer with velocity V_1 over a half space with velocity V_2 ($>V_1$). Two shots, A and B, (Figure 2A) are so far enough from the receiver spread that the first arrivals at each receiver are all from refracted waves (not direct waves). The red

Quarternary	Holocene		Alluvium	0-51m	Soil	Surficial Aquifer
	Pleistocene		Loess	0-19m	Clayey Silt	
	Pleistocene + Pliocene		Fluvial deposit	0-30m	Sand, Coarsening downward with some gravel and clay	
Tertiary	Eocene	Oligocene	Jackson Formation	Jackson Upper Claiborne confining Unit	0 | 110 m	Clay with interspersed sand lenses	Aquitard
		The Claiborne Group	Corkfield Formation				
			Cook Mtn. Formation				
			The Memphis Sands	Memphis Sands	200 | 270 m	Medium to fine sand, occasional silt and clay	The Memphis Aquifer

Fig. 1. Geology stratigraphy, lithology, and hydrologic significances in the Memphis area (modified from Parks and Mirecki, 1992).
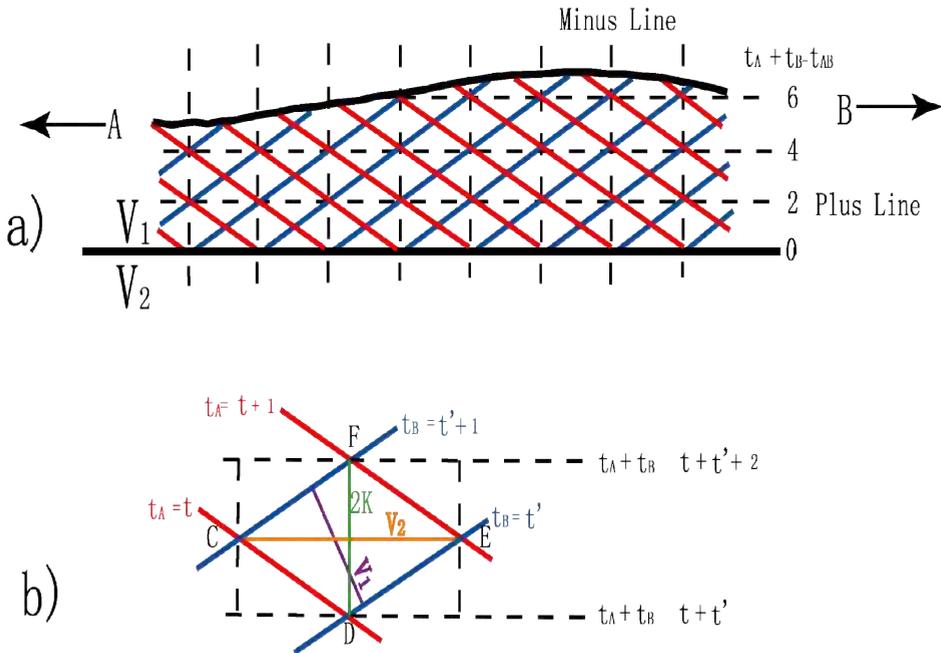
Fig. 2. Schematic wave fronts used in the Hagedoorn's plus-minus method (Hagedoorn, 1959). A) One layer over halfspace model with wave fronts drawn from two reciprocal shots; B) wave fronts composing the diamond-shaped region showing the constant plus value along the plus line. See text for details.

lines in the figure represent wave fronts generated by source A and propagating to the right; the blue lines represent wave fronts generated from source B and propagating to the left. The time intervals between all the neighboring wave fronts from each source are all the same, and regarded as unit time 1. Since the refractor is horizontal, the intersecting wave fronts drawn form diamond-shaped figures (Figure 2B). For the wave fronts propagating from source A to the right (CD and EF), the traveltimes are t and t+1, respectively; for the wave fronts propagating from source B to the left (DE and CF), the traveltimes are t'and t'+1, respectively. Note that vertex C is the intersection of wave fronts CD and CF; the summation of traveltimes of the two wave fronts at this intersection is t+t'+1. For intersection E, the summation is also t+t'+1. Hence, for the horizontal vertices (intersections) of the diamonds, the summation of travel times from the two wave fronts is constant. This is true for all the diamonds and results in what Hagedoorn called the "plus" lines, drawn as horizontal dashed lines in Figures 2A and B. The plus value is calculated by adding the two travel times at each intersection and subtracting t_{AB}, the travel time from source A to source B. The resulting values equal 0 on the refractor, 2 on the horizontal line through the first set of intersections vertically above those defining the refractor, 4 on the next line up, and so on. Note that any of the "plus lines" can be used to plot the refractor shape (structure). The plus values can be calculated on the surface. At each receiver station, t_A and t_B are the first arrival times picked from the two reciprocal shot records. The travel time from source A to source B, t_{AB}, may not be recorded in a typical refraction survey, but since t_{AB} is a constant for the

two reciprocal shot records, it does not affect the shape of the refractor if t_{AB} is not included in the calculation. As shown in diamond CDEF, the distance between the two wave fronts CF and DE is v_1 (because the time difference between the two neighboring wave fronts is unit time) and similarly the length of CE is v_2. Let $2k$ be the length of DF (the distance between the horizontal, dashed 'plus' lines), then $2k$ can be calculated from equation 1 (Hagedoorn, 1959),

$$2k = \frac{v_1}{\sqrt{1 - (v_1^2 / v_2^2)}} \tag{1}$$

Since the length of DF corresponds to a difference of one time unit for both wave fronts and the plus value difference between D and F is 2, consequently, the product of k and its "Plus" value is the actual height of a point above the boundary (Hagedoorn, 1959). Consequently, the product of k and the difference of two plus values at two points gives the actual distance between the two points.

Similarly, the difference between the travel times between shot A and B at an intersection is called the "minus" value. The minus value is constant along vertical lines passing through the intersections of wave fronts. In Figure 2, the minus lines are shown as vertical dashed lines spaced at a distance interval equal to the value of the velocity below the boundary, V_2 (because the time intervals between the neighboring wave fronts is the unit time) and their minus values differ by two time units. Hagedoorn (1959) also demonstrated this method for more specific cases (e.g., a refractor with a change in velocity and curved refractors).

3. Application

Hagedoorn's method was applied to the data collected in the study area to model the arrivals from the first refractor. The arrival from the first refractor observed on shot gathers has an apparent velocity of ~1458 m/s (Figure 3), which corresponds to the velocity of the confining unit in this area (Liu et al., 1997). According to Liu et al., (1997), the P-wave velocity (Figure 4) increases abruptly across this layer, giving rise to the first refracted energy observed in Figure 3A. In the Memphis area, the Pliocene strata directly overlies the confining unit (Eocene and Oligocene) and Miocene deposits are missing, indicating that after the deposition of the Jackson formation (the upper stratigraphic element of the confining unit), the area may have undergone significant erosion within the fluvial depositional system (Van Arsdale and TenBrink, 2000), and that erosional features (e.g. paleochannels) might be preserved at the top of the confining unit.

Based on the ~21 m crossover distance observed on shot gathers, 6 reciprocal shots were selected to perform the calculation. Each pair of reciprocal shots was located at both sides of the spread and at the same distance from the center of the spread. First arrival times were manually picked on unprocessed shot gathers for each shot pair and plotted in Figure 5. No data were recorded from one reciprocal shot location to the other (i.e. t_{AB}), and the summation of the reciprocal first arrival times was used to plot the geometry of the first refractor. Figure 6 shows the shape of the refractor calculated from the 6-shot pairs. Note that although the first arrival picks are very scattered (Figure 5), the shape of the refractor obtained from different shot pairs was very consistent, corroborating the robustness of this method. However, since t_{AB} is not available, the absolute depth cannot be calculated. The

interpretation of the geometry so obtained (Figure 6) can only be regarded as the general trend or shape of the refractor. The small scale oscillations on Figure 6 should not be interpreted as the detailed structure of the refractor. A comparison between the first arrival picks and the geometry of the refractor (Figure 5 and Figure 6) suggests that these scattered
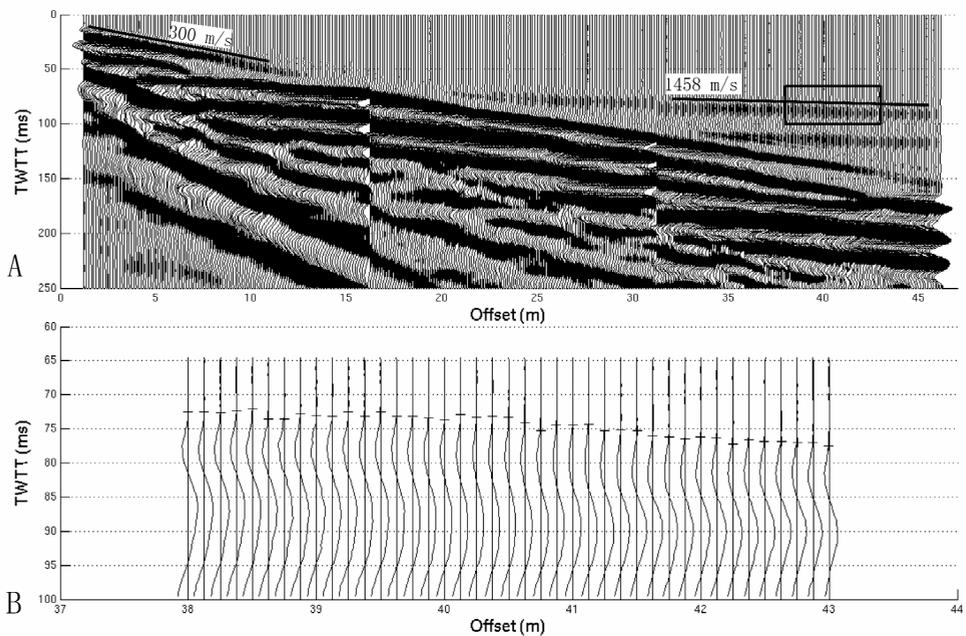

Fig. 3. A: composite shot gather, showing the refracted arrival with an apparent velocity of 1458 m/s, interpreted as the top of the Upper Claiborne clay layer (aquifer); B: close up of the rectangular area in A showing the data quality and the first break picks.
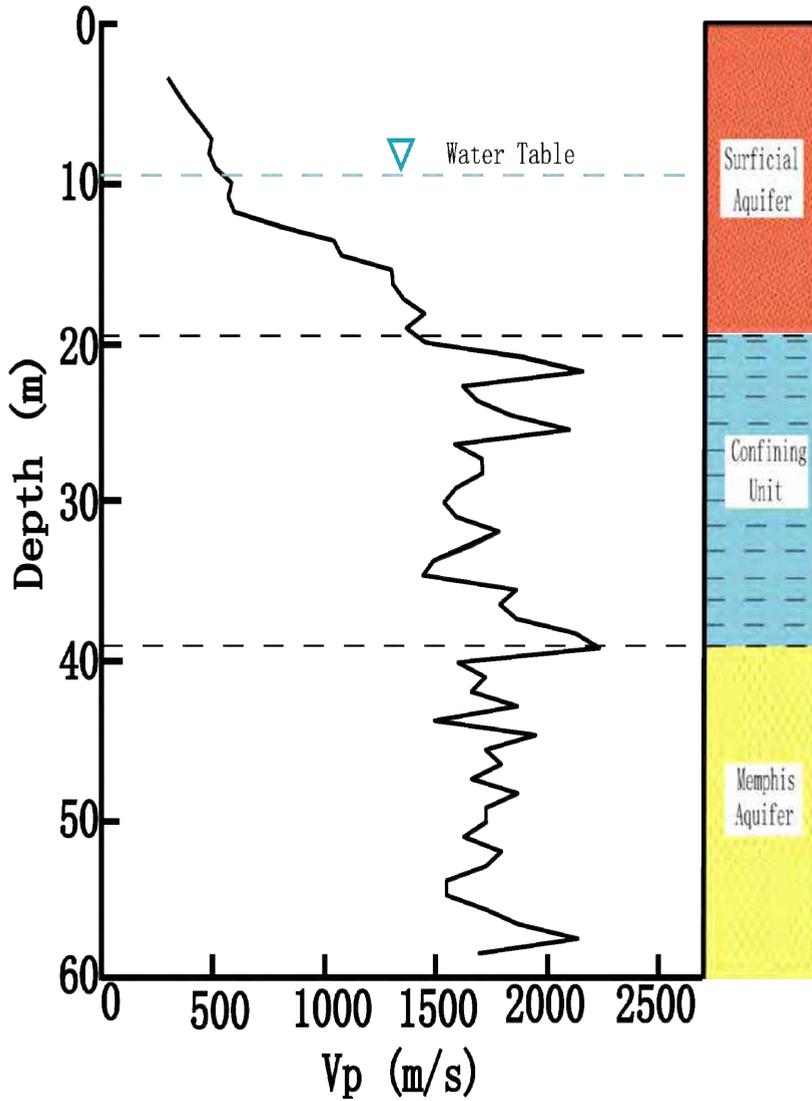
Fig. 4. VSP P-wave velocity profile and hydrological units for a borehole in Shelby County (Liu et al., 1997).
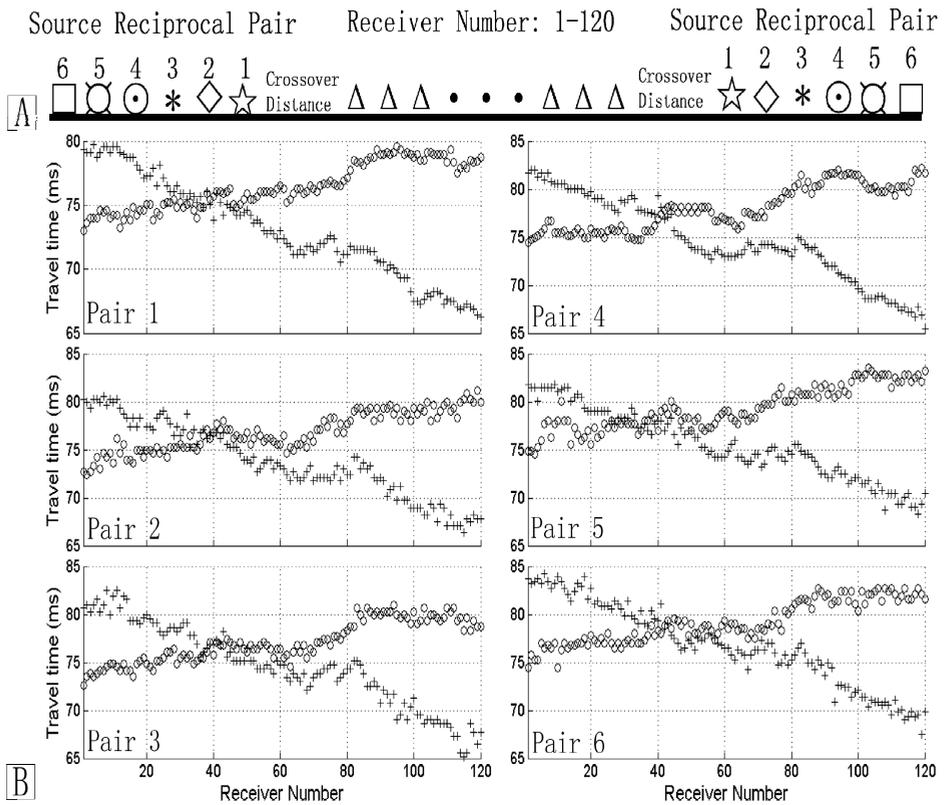
Fig. 5. A: geometry of the 6 reciprocal shot pairs selected for the analysis; B: first arrival picks for the first refracted arrival for the 6 reciprocal shots.
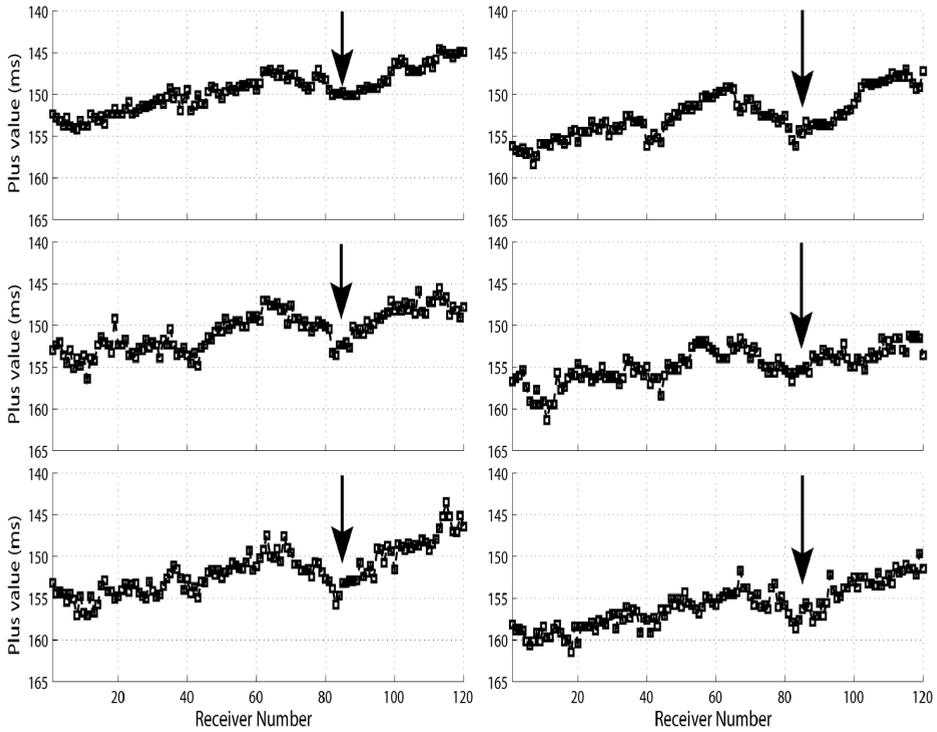
Fig. 6. Geometry of the first refractor resulting from the plus-minus method applied to the 6 reciprocal shots in Figure 5. Arrows show location of the depression.

saw-shape details are likely due to the scattered character of the first arrival picks (see Figure 5). In the field, the 120-receiver spread formed a straight line with a receiver spacing of 0.125 m (Ge et al., 2010, Part I). The clear hyperbolic moveout of the reflection events in Figure 3A indicates that the geophones were planted in the proper positions (all the receivers formed a straight line with a receiver spacing of 0.125 m) in the field. The scattered first break picks from the refracted wave field thus are not due to any inaccurate positioning of geophones but probably resulted from the low S/N ratio (the weaker amplitude of the refracted waves and the relatively higher amplitudes of background noise) and the heterogeneity of the surficial layer. The general trend of the geometry of the first refractor observed in all the reciprocal shots, shows a depression of about 9 ms around receiver number 85 (Figure 6). The velocity of the first layer can be estimated by measuring the slope of the direct wave in Figure 3A, which gives the first layer velocity (v_1) of around 300 m/s. The velocity of the refractor, v_2 , is about 1458 m/s (Figure 3A). Using equation 1, k can be calculated to be 153.3, which results in a depth of the depression of about 1.4 m (153.3 * 0.009). The width of the depression visible on the reciprocal shot pairs is ~6 m. The apparent velocity of the first layer was calculated using the offset gather of 1.25 m because the information derived from the plus-minus method is relative to the refractor right below the receiver spread, not elsewhere. The velocity of the surficial layer below the receiver spread was therefore used to estimate the depth of the refractor. The observation that the depression is visible across all of the pairs and occurs at the same place suggests that the result obtained from different shot pairs is reliable and that this method is robust in imaging the geometry of the first refractor. If the pick error is 2 samples (corresponding to 1 ms), the maximum error of the plus value will be 2 ms. By using the same procedure used to calculate the depth of the depression, the corresponding uncertainty is calculated to be about 0.3 m.

Based on the geometry of the first refractor, which corresponds in this area to the top of the confining unit, and considering the fluvial depositional environment that characterized the study area in the Pliocene, the observed V-shaped depression is interpreted as a paleochannel resulting from river erosion likely associated with the Wolf river fluvial system, a branch of Mississippi river and a major river system in the study area.

4. Conclusions

Hagedoorn's plus-minus method was applied to the dataset to map the first refractor, represented by the top of the confining unit. Although the first arrival picks from different pairs of shots are very scattered, the calculated geometry of the top of the aquitard is consistent among the reciprocal shots. This suggests that this method is robust in mapping the structure of the first refractor. The geometry of the mapped first refractor reveals the presence of a depression that is interpreted as a paleochannel, consistently with the fluvial depositional environment and the presence of extensive erosional events that postdate the sedimentation of the Jackson formation.

This study shows that Hagedoorn's plus-minus method can provide a simple and fast tool to interpret refraction data and calculate the geometry and velocity of the first refractor. It has proved to be a cost-effective and efficient geophysical method in hydrology and ground water studies.

5. References

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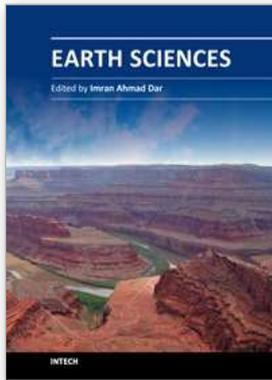
Earth Sciences

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