10

Energy Transfer in Pyroelectric Material

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1. Introduction

Smart materials are different from the usual materials and can sense their environment and respond, in the flexibility of its properties that can be significantly altered in a controlled fashion by external stimuli, such as stress, temperature, electric and magnetic fields. Fig. 1 shows the general relationship in smart materials among mechanical, electrical, and thermal fields. Such characteristics enable technology applications across a wide range of sectors including electronics, construction, transportation, agriculture, food and packaging, health care, sport and leisure, white goods, energy and environment, space, and defense.

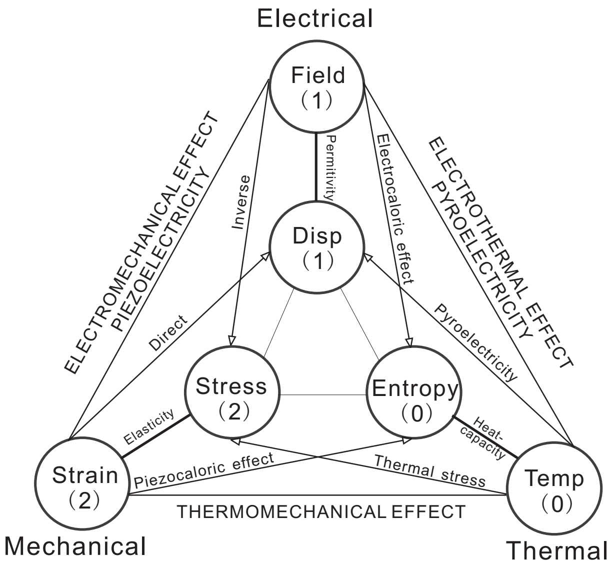


Fig. 1. The relationship among mechanical, electrical, and thermal fields.

The most widely used smart materials are piezoelectric ceramics, which expand or contract when voltage is applied. Pyroelectric material is a kind of smart materials and can be electrically polarized due to the temperature variation. Fig. 2 indicates the relationship between pyroelectrics and other smart materials. It follows that a pyroelectric effect cannot exist in a crystal possessing a center of symmetry. Among the 21 noncentrosymmetrical crystalline classes only 10 may theoretically show pyroelectric character, (Cady, 1946; Eringen & Maugin, 1990; Nelson, 1979). It has many applications which occur both in technology (i.e. infrared detection, imaging, thermometry, refrigeration, power conversion, memories, biology, geology, etc...) and science (atomic structure of crystals, anharmonicity of lattice vibrations etc...(Hadni, 1981)). Recently, advanced technical developments have increased the efficiency of devices by scavenging energy from the environment and transforming it into electrical energy. When thermal energy is considered and spatial thermal gradients are present, thermoelectric devices can be used. When thermal fluctuations are present, the pyroelectric effect can be considered, see (Cuadras et al., 2006; Dalola et al., 2010; Fang et al., 2010; Gael & et al., 2009; Guyomar et al., 2008; Khodayari et al., 2009; Olsen et al., 1984; Olsen & Evans, 1983; Shen et al., 2007; Sodano et al., 2005; Xie et al., 2009). The thermal wave, also called temperature wave, is also found to be a good method to probe in a remote way near surface boundaries, to measure layer thicknesses and to locate faults (Busse, 1991).

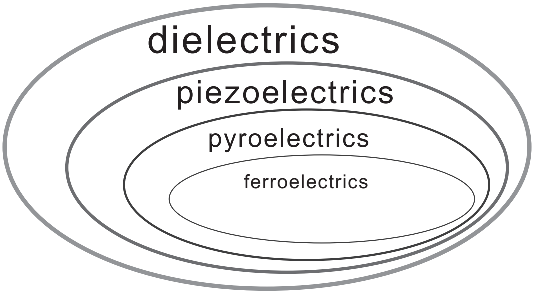


Fig. 2. The relationship of dielectrics, piezoelectrics, pyroelectrics and ferroelectrics.

Therefore, pyroelectric medium can be transformer among mechanical, electrical and thermal energies. It is with this feature in mind that we have to do research to cover the coupling even if only one type energy is needed. In this chapter the following works are performed to exploit pyroelectric material.

Firstly, the general theory of inhomogeneous waves in pyroelectric medium is addressed. Majhi (Majhi, 1995) studied the transient thermal response of a semi-infinite piezoelectric rod subjected to a local heat source along the length direction, by introducing a potential function and applying the Lord and Shulman theory. Sharma and Kumar (Sharma & Kumar, 2000) studied plane harmonic waves in piezo-thermoelastic materials. He, Tian and Shen (He et al., 2002) discussed various thermal shock problems of a piezoelectric plate. Baljeet (Baljeet, 2005) formulated the governing differential equations for generalized thermo-piezoelectric solid by using both L-S and G-L theories and found that the velocities of these plane waves depend upon properties of material and the angle of propagation. Sharma and Pal (Sharma & Pal, 2004) discussed the propagation of plane harmonic waves in transversely isotropic generalized piezothermoelastic materials and found four dispersive modes. The propagation of Rayleigh waves in generalized piezothermoelastic half-space is investigated by Sharma and Walis (Sharma & Walia, 2007). Topics of homogeneous and inhomogeneous waves, reflection/transmission and energy problems in pyroelectrics are firstly researched by authors (Kuang, 2009; 2010; Kuang & Yuan, 2010; Yuan, 2009; Yuan & Kuang, 2008; 2010).

The speciality of pyroelectric material lies in its relaxation in corresponding thermal field. Introduction of relaxation time into the heat conduction theory is about 50 years ago. Cattaneo (Cattaneo, 1958) and Vernotte (Vernotte, 1958) originally proposed the relaxation time for heat flux in the heat conduction theory, on basis of which the governing equations of thermoelasticity with relaxation time were deduced by Kaliski (Kaliski, 1965), and independently by Lord and Shulman (Lord & Shulman, 1967). Notwithstanding, this theory is usually called L-S theory. Several years later, Green and Lindsy (Green & Lindsay, 1972) gave another form of governing equations for thermoelasticity called G-L theory. Further, Joseph and Preziosi (Joseph & Preziosi, 1989) used two relaxation times: one for heat flux and the other for temperature gradient, and also obtained a system of equations of thermoelasicity. Kuang (Kuang, 2009; 2010) proposed an inertial entropy theory and got the governing equations for thermoelasticity which is different from L-S and G-L theories. For pyroelectrics the effects of relaxation times on wave velocities and attenuation are estimated by (Kuang, 2009; 2010; Yuan, 2009; Yuan & Kuang, 2008; 2010).

Taking account of the relaxation, we introduce the inhomogeneous wave into pyroelectric medium here. The difference from the homogeneous wave is that the wave propagation vector is not coincident with the attenuation vector. The attenuation angle, defined by the angle between wave propagation vector and attenuation vector, is found to be limited in the range of (-90°,90°). It is found that increasing the attenuation angle will introduce more dissipation and anisotropy. In our work, four wave modes are found in pyroelectric medium, which are temperature, quasitransverse I, II and quasilongitudinal due to the coupling state relationship. Though there is no independent wave mode for the electric field, it can still propagate with other wave modes. The variations of phase velocities and attenuations with propagation angle and attenuation angle are discussed. Phase velocity surfaces on anisotropic and isotropic planes are presented for different attenuation angle. It is found that attenuation angle almost doesn't influence the phase velocities of elastic waves in both anisotropic and isotropic planes. In contrast, the roles it plays on temperature wave are obvious. The effects of the positive and negative attenuation angles are not the same in anisotropic plane.

The propagation of a wave in any medium is associated with the movement of energy. Therefore, the energy process in pyroelectrics is researched for the first time.

The energy process especially the dissipation energy is one of the most important dynamic characteristics of continuous media. Many researches were conducted on this problem. Umov (Umov, 1874) introduced the concept of the energy flux vector and found the first integral of energy conservation equations of elasticity theory. Fedorov (Fedorov, 1968) used this theory and discussed the energy flux, energy density and the energy transport velocity of plane waves in the elastic theory. In paper of (Kiselev, 1982), the energy fluxes of complex fields in inhomogeneous media were considered. Based on Umov's theory of energy flux, he represented analogous results for complex fields which are characterized by the pair of complex vector fields. On the basis of the results, the Lagrangian density and Umov vector were derived. At the same time, the question of additivity of the Umov flux vectors of longitudinal and transverse waves was also discussed.

For the class of plane inhomogeneous waves propagating in linear viscoelastic media, Buchen (Buchen, 1971) gave a detailed description of the physical properties and energy associated with these inhomogeneous waves. The paralleled paper by Borcherdt (Borcherdt, 1973) adopted a different derivation from Buchen's and discussed the mathematical framework for describing plane waves in elastic and linear inelastic media. The expressions for the energy flux, energy densities, dissipated energy, stored energy were derived from an explicit energy conservation relation. Based on the motion equation and its integral form, Červený (Cerveny & Psencik, 2006) discussed three different types of energy fluxes in anisotropic dissipative media. The relationships among them, especially their applications in the interface between dissipative media, were researched in detail. In the field of piezoelectrics, Auld (Auld, 1973) derived the energy flux in the electromagnetic field and also its form in the piezoelectric media. Baesu (Baesu et al., 2003) considered non-magnetizable hyperelastic dielectrics which conduct neither heat nor electricity and also obtained the energy flux with the linearized theory.

In this chapter, the energy process in pyroelectric medium with generalized heat conduction theory is studied firstly. According to the derived energy conservation law, the energy densities, energy dissipated and energy flux are defined. Generally there are several type velocities in wave theory, such as phase velocity, group velocity and energy velocity. The phase velocity is related to the phase of the wave. Owing to damping, the usual definition of group velocity of waves become meaningless and this issue can be solved by considering the energy of the physical phenomenon of wave propagation (Mainardi, 1973). Regarding the propagation of the energy, the energy flux may be used in order to quantify the energy velocity vector and they have the same direction. The energy flux vector has a dynamical definition and consequently, polarization of the wave (the amplitudes of displacements, temperature and electric potential) is taken into account. In particular the phase velocity and energy velocity are compared in the results and discussion section.

We shall use the operation rules: the dot above a letter denotes the time derivative, the index following the comma in the subscript denotes the partial derivative with respect to relevant Cartesian coordinate, and the asterisk in the superscript denotes the complex conjugate.

2. The inhomogeneous waves in pyroelectric medium

2.1 The governing equations and state equations

The pyroelectric medium can be influenced by the mechanical, electric and thermal fields. These fields have their own governing equations. The physical quantities of pyroelectric medium in these fields are not independent, because they are related by the state equations. The known fundamental equations for the pyroelectric medium are listed as follows.

1. Mechanical field equations in $\Re^3$
   Equation of motion:
   $$\sigma_{ij,j} + b_i = \rho \ddot{u}_i \tag{1}$$
   Geometric property:
   $$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{2}$$
   where u_i is the displacement vector, σ_{ij} the stress tensor, b_i the body force per unit volume, ρ the density and ε_{ij} the strain tensor.
2. Electrical field equations under the quasi-static assumption $\Re^3$
   Gauss equation:
   $$D_{i,i} = \varrho_e \tag{3}$$
   where D_i is the electric displacement. The absence of free charge requires $\varrho_e = 0$. In quasi-static approximation, the electric field **E** is derivable from a potential, that is

$$(\nabla \times \mathbf{E})_i = 0, E_i = -\varphi_{,i} \tag{4}$$

where φ is the scalar quasi-static electric potential.

3. Thermal field equations in $\Re^3$
   If the temperature disturbance $\theta \ll T_0$, the entropy equation is

$$\rho T_0 \dot{\eta} = -q_{i,i} \tag{5}$$

in which T_0 is the initial temperature, η is the entropy per unit volume. The thermal flux vector q_i is related to the temperature disturbance $\theta = T - T_0$ by

$$Lq_i = -\kappa_{ij}\theta_{,j} \tag{6}$$

in which L is an operator defined by

$$L = 1 + \tau \frac{\partial}{\partial t}$$

Equation (6) is called the generalized Fourier heat conduction equation. In these two equations, κ_{ij} indicates the heat conduction constant and τ is the relaxation time.
In the above individual field introduces physical quantities, and they are not independent and should satisfy the state equations, which play roles in two aspects: 1. physically they reflect the real world interactions among the three fields; 2. they are useful to formulate a solvable equation system mathematically. The constitutive equations (Yuan & Kuang, 2008) can be expressed by

$$\begin{aligned} \sigma_{ij} &= c_{ijkl}\varepsilon_{kl} - e_{kij}E_k - \gamma_{ij}\theta \\ D_k &= e_{kij}\varepsilon_{ij} + \lambda_{ik}E_i + \xi_k\theta \\ \rho\eta &= \gamma_{ij}\varepsilon_{ij} + \xi_i E_i + \frac{\rho C\theta}{T_0} \end{aligned} \tag{7}$$

In this system of equations, c_{ijkl} denotes the elastic stiffness; e_{kij} the piezoelectric tensor; γ_{ij} the thermo-mechanical tensor; ρ the density; λ_{ik} the dielectric permittivity tensor; ξ_k the pyroelectric constants'; T_0 the initial temperature; C is the specific heat capacity.
Inserting these state equations into Equations (1), (4) and (5) and using Equations (2) and (6), we obtain

$$\begin{aligned} & c_{ijkl}u_{k,lj} + e_{kij}\varphi_{,kj} + \gamma_{ij}\theta_{,j} = \rho\ddot{u}_i \\ & e_{kij}u_{i,jk} - \lambda_{ik}\varphi_{,ik} + \xi_k\theta_{,k} = 0 \\ & T_0\gamma_{ij}\left(\dot{\varepsilon}_{ij} + \tau\ddot{\varepsilon}_{ij}\right) + T_0\xi_i\left(\dot{E}_i + \tau\ddot{E}_i\right) + \rho C\left(\dot{\theta} + \tau\ddot{\theta}\right) = \kappa_{ij}\theta_{,ij} \end{aligned} \tag{8}$$

which is a system of equations in the unknown fundamental functions: the displacements u_k, the electric potential φ, the temperature disturbance θ. There are 7 equations in this system and also the same number of unknowns, therefore it can be solved.

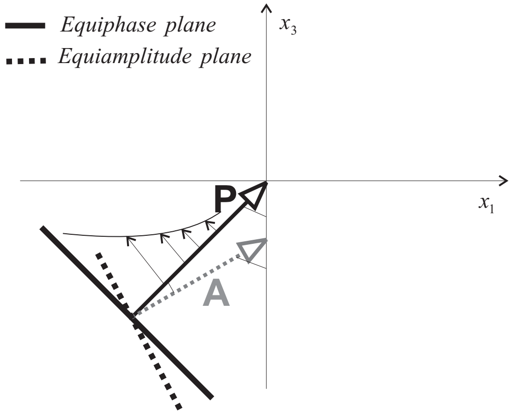


Fig. 3. Equiphase plane, equiamplitude plane and exponential variation of the amplitude along the phase propagation direction.

2.2 The fundamental concepts of inhomogeneous wave theory

When the wave vector is complex, generally speaking, the propagation direction (normal to the equiphase plane) is different from the attenuation direction (normal to the equiamplitude plane), see Fig. 3. Any plane wave can be expressed as

$$\begin{aligned} &f = f_0 e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)} = f_0 e^{i(k_m x_m - \omega t)}, \quad \mathbf{k} = [k_1, k_2]^T = \mathbf{P} + i\mathbf{A} \\ &\mathbf{P} = P\mathbf{n}, \quad \mathbf{A} = A\mathbf{m}, \; k_j = P_j + iA_j, \; k^2 = \mathbf{k}\cdot\mathbf{k} = P^2 - A^2 + 2i\mathbf{P}\cdot\mathbf{A} \end{aligned} \tag{9}$$

where $\mathbf{P}$ is the propagation vector, P is its module, and $\mathbf{n}$ is the unit vector along the propagation direction; $\mathbf{A}$ is the attenuation vector, A is its module, and $\mathbf{m}$ is the unit vector perpendicular to the plane of constant amplitude. When $\mathbf{n} = \mathbf{m}$, we call it homogeneous wave, otherwise inhomogeneous wave. Hereafter, we assume that θ, transportation angle, is the angle between $\mathbf{n}$ and x_2; γ, attenuation angle, is the angle between $\mathbf{n}$ and $\mathbf{m}$; and $\vartheta(=\theta+\gamma)$ is the angle between $\mathbf{m}$ and x_2. Using Equation (9), we obtain

$$\begin{aligned} &\mathbf{n} = [\sin\theta, \cos\theta]^T, \; \mathbf{m} = [\sin(\theta+\gamma), \cos(\theta+\gamma)]^T, \; \mathbf{n}\cdot\mathbf{m} = \cos\gamma \\ &k_1 = P_1 + iA_1 = Pn_1 + iAm_1, \; k_2 = P_2 + iA_2 = Pn_2 + iAm_2 \\ &P = \sqrt{P_1^2 + P_2^2}, \; A = \sqrt{A_1^2 + A_2^2} \end{aligned}$$

Due to $\mathbf{n} = \mathbf{m}$ and $\gamma = 0$ in homogeneous wave, we have $k_1 = (P+iA)\sin\theta$, $k_2 = (P+iA)\cos\theta$. Therefore, $\mathbf{k}$ is determined by one complex number and a real propagation angle θ, but in inhomogeneous wave $\mathbf{n} \neq \mathbf{m}$, we have to use four parameters (P, A, θ, γ) to determine wave vector.

Unlike propagation angle θ, γ has its boundary to guarantee the waves are of attenuation. On the basis of non-negative dissipation rate of linear viscoelastic media, Buchen (Buchen, 1971) verified that γ is in the range of 0° to 90° and the same conclusion can also be seen in reference (Borcherdt, 1973). In the present paper, the boundary of attenuation angle γ is determined by the condition that waves should be attenuate physically.

2.3 The propagation of inhomogeneous plane waves in infinite medium

For the solution to Equation 8, the general monochromatic plane waves are assumed as

$$\begin{aligned} u_k &= U_k \exp\left[i(x_i k_i - \omega t)\right] \\ \theta &= \Theta \exp\left[i(x_i k_i - \omega t)\right] \\ \varphi &= \Psi \exp\left[i(x_i k_i - \omega t)\right] \end{aligned} \tag{10}$$

where k_i is the complex-valued wave vector, ω is the circular frequency, t is the time variable and U_j, Θ and Ψ are generally the complex amplitudes (or polarizations) of displacements, temperature and electric potential respectively. The subscript i, k equal to 1, 2, 3. It is noted that in Equation (10), $\exp\left[i(x_i k_i - \omega t)\right]$ is used, which is different from homogeneous wave with $\exp\left[i(kn_i x_i - \omega t)\right]$. In other words, in the inhomogeneous wave, $k_i x_i$ can't be expressed as $kn_i x_i$.

Inserting Equation (10) into Equation (8) yields a system of Christoffel algebraic equations in amplitude vector $\mathbf{U}$

$$\boldsymbol{\Lambda}\left(\mathbf{k}, \omega\right)\mathbf{U} = \mathbf{0}, \qquad \mathbf{U} = \left[U_1, U_2, U_3, \Psi, \Theta\right]^T \tag{11}$$

$$\boldsymbol{\Lambda}\left(k, \omega, \mathbf{n}\right) = \begin{bmatrix} \Gamma_{11}\left(\mathbf{k}\right) - \rho\omega^2 & \Gamma_{12}\left(\mathbf{k}\right) & \Gamma_{13}\left(\mathbf{k}\right) & i\alpha_1^*\left(\mathbf{k}\right) & e_1^*\left(\mathbf{k}\right) \\ \Gamma_{21}\left(\mathbf{k}\right) & \Gamma_{22}\left(\mathbf{k}\right) - \rho\omega^2 & \Gamma_{23}\left(\mathbf{k}\right) & i\alpha_2^*\left(\mathbf{k}\right) & e_2^*\left(\mathbf{k}\right) \\ \Gamma_{31}\left(\mathbf{k}\right) & \Gamma_{32}\left(\mathbf{k}\right) & \Gamma_{33}\left(\mathbf{k}\right) - \rho\omega^2 & i\alpha_3^*\left(\mathbf{k}\right) & e_3^*\left(\mathbf{k}\right) \\ e_1^*\left(\mathbf{k}\right) & e_2^*\left(\mathbf{k}\right) & e_3^*\left(\mathbf{k}\right) & -i\xi_k k_k & \lambda^*\left(\mathbf{k}\right) \\ \gamma_1^*\left(\mathbf{k}\right)\omega & \gamma_2^*\left(\mathbf{k}\right)\omega & \gamma_3^*\left(\mathbf{k}\right)\omega & \kappa^*\left(\mathbf{k}\right) & \xi^*\left(\mathbf{k}\right) \end{bmatrix} \tag{12}$$

where

$$\begin{aligned} &\Gamma_{ik}\left(\mathbf{k}\right) = C_{ijkl} k_j k_l, e_i^*\left(\mathbf{k}\right) = e_{kij} k_k k_j, \gamma_i^*\left(\mathbf{k}\right) = T_0 \gamma_{ij} k_j \left(\omega - i\tau\omega^2\right) \\ &\xi^*\left(\mathbf{k}\right) = T_0 \xi_i k_i \left(-\omega + i\tau\omega^2\right), \lambda^*\left(\mathbf{k}\right) = \lambda_{ik} k_i k_k,\ \kappa^*\left(\mathbf{k}\right) = \kappa_{ij} k_i k_j - \rho C\left(i\omega + \tau\omega^2\right) \end{aligned} \tag{13}$$

Nontrivial solutions for U_i, Θ and Ψ require

$$\det \boldsymbol{\Lambda}\left(\mathbf{k}, \omega\right) = 0. \tag{14}$$

which is complex equation in wave vector $\mathbf{k}$ for given ω. Decomposing the equation into the real and imaginary parts, we can obtain a solvable equations in P and A:

$$\begin{cases} D_{\Re}(P, A) = 0 \\ D_{\Im}(P, A) = 0 \end{cases} \text{and } P, A \in 0 \cup \mathbb{R}^+ \tag{15}$$

Due to that the equations are very tedious, we would not present them in explicit forms. Equation (15) are nonlinear and coupling equations in (P, A). According to the definitions of P and A in Equation (9), the right solution of P and A should be real valued. Therefore, the domain of θ and γ are determined by the condition that P and A are nonnegative real numbers(only one direction of wave propagation is considered). The wave propagates with the velocity c_p (=ω/P), with non-negative value in attenuation A. This condition agrees with the Sommerfeld radiation condition; i.e., vanishing at infinity. When A and P are obtained for given θ and γ, we can use Equations (9) to determine the inhomogeneous wave vector **k.** For each k_i, we can get a corresponding amplitude vector $\mathbf{U}$ with one undetermined component. Generally, there are four roots of (**P**,**A**) to Equation (15) corresponding to four wave vector $\mathbf{k}$. For every $\mathbf{k}$, $\mathbf{P}$ and $\mathbf{A}$, we have two components $(k_{i\alpha}, P_{k\alpha}, A_{k\alpha})$, in which $i = 1, 2, 3, 4$ and $\alpha = 1, 2$. They are related to three elastic waves and one temperature wave; The electric field doesn't have its own wave mode, but, through the constitutive relations, it can propagate with other four wave modes. After P, A are solved, the phase velocity can be given by

$$c_p = \frac{\omega}{P} \tag{16}$$

and also the attenuation A.

Therefore, general solutions in pyroelectric medium equal to the sum of four wave modes, which are

$$u_k = \sum_{j=1}^{4} U_k^{(j)} e^{i\left(k_m^{(j)} x_m - \omega t\right)} = \sum_{j=1}^{4} U_k^{(j)} e^{i\left[\left(P^{(j)}\mathbf{n}^{(j)} + iA^{(j)}\mathbf{m}^{(j)}\right)\cdot\mathbf{x} - \omega t\right]} \tag{17}$$

$$\theta = \sum_{j=1}^{4} \Theta^{(j)} e^{i\left(k_m^{(j)} x_m - \omega t\right)} \quad \varphi = \sum_{j=1}^{4} \Psi^{(j)} e^{i\left(k_m^{(j)} x_m - \omega t\right)}$$

in which j indicates the wave mode.

2.4 Quantitative analysis of pyroelectric media

The material under study is transversely isotropic $BaTiO_3$, in which the isotropic plane is x_1-x_2 and the anisotropic plane is x_1-x_3 plane. All the physical constants are rewritten with the help of Voigt notation, whose rule is that the subscript of a tensor is transformed by $\{11 \rightarrow 1, 22 \rightarrow 2, 33 \rightarrow 3, 23 \rightarrow 4, 31 \rightarrow 5, 12 \rightarrow 6\}$.

Coordinate index	(11)	(12)	(13)	(33)	(44)	(66)	(15)
Elastic moduli $E(10^{10}\text{Pa})$	15.0	6.6	6.6	14.6	4.4	4.3	
Piezoelectric Charge constant $e(\text{C/m}^2)$			-4.35	17.5			11.4
Electric permittivity $\lambda(10^{-9}\text{f/m})$	9.867			11.15			
Thermal expansion tensor $\alpha(10^{-6}1/\text{K})$	8.53		1.99				
Pyroelectric constant $\xi(10^{-4}\text{C/m}^2\text{K})$				5.53			
Thermal conductivity tensor $\kappa(\text{J/m}\cdot\text{K}\cdot\text{s})$	1.1	1.1		3.5			

Table 1. Material properties of $BaTiO_3$

The material constants of $BiTiO_3$ studied in this paper are shown in Table 1. The specific heat capacity C is 500 (J/K·Kg); the relaxation times $\tau = 10^{-10}$s for L-S theory; density $\rho = 5700$ kg/m^3; the prescribed circular frequency $\omega = 2\pi \times 10^6$ s^{-1}; the thermo-mechanical coupling coefficients γ_{ij} are given by

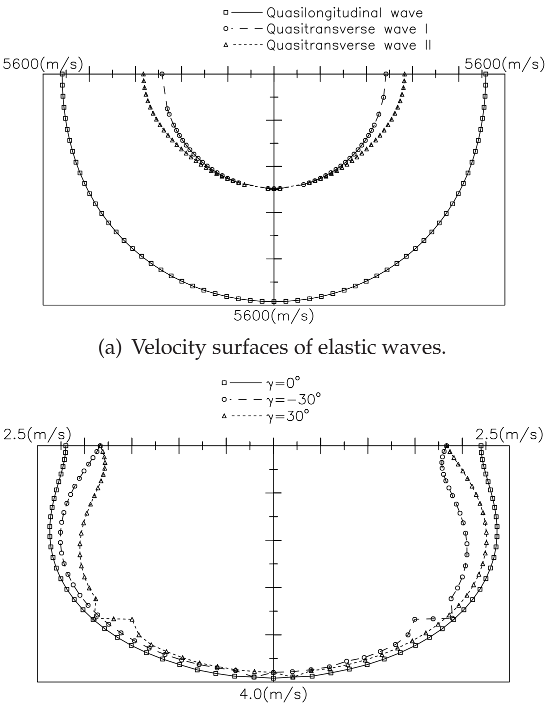
$$\gamma_{11} = \gamma_{22} = (c_{11} + c_{12})\alpha_{11} + (c_{13} + e_{31})\alpha_{33}, \ \gamma_{33} = 2c_{13}\alpha_{11} + (c_{33} + e_{33})\alpha_{33}$$

2.4.1 Determination of the boundary of attenuation angle

The condition in Equation (15) requires that the attenuation angle γ should be limited in the range of $(-90^0, 90^0)$ to get attenuate wave, by which we can obtain four wave modes: quasilongitudinal, quasitransverse I, II and temperature waves. This conclusion is consistent with previous researchers (Borcherdt, 1973; Buchen, 1971; Kuang, 2002). Their studies demonstrated that attenuation angle γ is confined in the range of $(0^0, 90^0)$ for isotropic viscoelastic medium. This result can be arrived at by ours, that the positive and negative attenuation angles come to the same results for isotropic medium. But the influences of positive and negative attenuation angles on waves in the anisotropic plane for the transverse material are different. Attenuation angle introduces more dissipation and anisotropy (Carcione & Cavallini, 1997).

2.4.2 The velocity surfaces

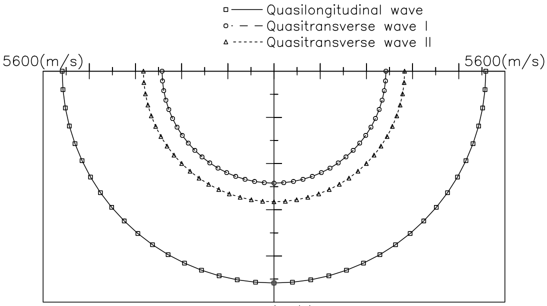
With the material constants shown in Table 1, the phase velocity surface sections are calculated. Fig. 4(a),(b) show the sections of phase velocity surfaces in the anisotropic x_1-x_3 plane and isotropic x_1-x_2 plane. It is demonstrated that the attenuation angle γ almost doesn't change the phase velocities of elastic waves, therefore only the case at $\gamma = 0$ is presented. The elastic wave velocity surfaces, including quasilongitudinal, quasitransverse I,II waves, show the anisotropic behaviors in the anisotropic x_1-x_3 plane. It is seen that, in Fig. 4(a), the quasi-longitudinal waves are the fastest, while the thermal wave are the slowest and the quasi-transversal waves stand in between them and all of them are related to propagation angle θ. Instead the role played by attenuation angle γ on temperature wave is obvious as shown in Fig. 4(b). The influences of the positive and negative attenuation angles are different in anisotropic x_1-x_3 plane, but both can reduce the velocity of temperature wave.
On the isotropic x_1-x_2 plane, Fig. 5(b) implies that the negative and positive attenuation angle have the same role. Velocities of all waves in isotropic plane don't depend on the propagation angle.



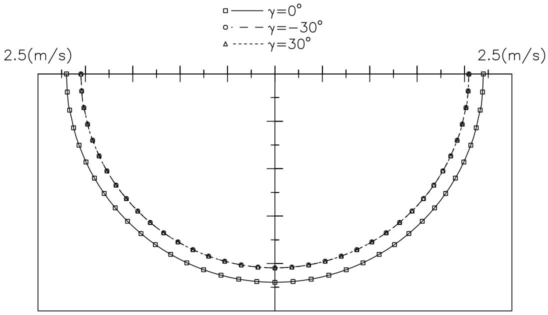
(a) Velocity surfaces of elastic waves.

(b) Velocity surfaces of temperature wave.

Fig. 4. Sections of the velocity surfaces in (x_1,x_3) plane at different attenuation angle γ.



(a) Velocity surfaces of elastic waves.



(b) Velocity surfaces of temperature waves.

Fig. 5. Sections of the velocity surfaces in (x_1,x_2) plane at different attenuation angle γ.

3. Dynamic energy balance law in pyroelectric medium

We shall formulate the energy balance laws as consequences of the governing equations presented in the previous section, see (Yuan, 2010).
I. We consider the scalar product of the velocity $\dot{u}_i$ with the motion equation. Multiplying Equation (1) by $\dot{u}_i$ results in

$$\sigma_{ij,j}\dot{u}_i + \rho b_i \dot{u}_i = \rho \ddot{u}_i \dot{u}_i$$

Taking account of the identity

$$(\sigma_{ij}\dot{u}_i)_{,j} = \sigma_{ij,j}\dot{u}_i + \sigma_{ij}\dot{u}_{i,j}$$

and, by considering a region Ω with surface element $\partial\Omega$ in the configuration of the body, applying the volume integral and Gaussian Theorem to the previous equation, we obtain

$$\int_{\partial\Omega} \sigma_{ij}\dot{u}_i n_j dS + \int_{\Omega} \rho b_i \dot{u}_i dV = \int_{\Omega} \sigma_{ij}\dot{u}_{i,j} dV + \int_{\Omega} \rho \ddot{u}_i \dot{u}_i dV$$

where n_j is the unit outward normal of dS. Let $t_i = \sigma_{ij}n_j$ in the surface integral, and substituting σ_{ij} with the constitutive equation Equation $(7)_1$ within the integral operator, this equation can be rewritten as

$$\int_{\partial\Omega} t_i\dot{u}_i dS + \int_{\Omega} \rho b_i \dot{u}_i dV = \int_{\Omega} \left(c_{ijkl}u_{k,l}\dot{u}_{i,j} - e_{kij}E_k\dot{u}_{i,j} - \gamma_{ij}\theta\dot{u}_{i,j}\right) dV + \int_{\Omega} \rho\ddot{u}_i\dot{u}_i dV \tag{18}$$

which can be of the form

$$\int_{\partial\Omega} t_i\dot{u}_i dS + \int_{\Omega} \rho b_i \dot{u}_i dV = \int_{\Omega} \left(\dot{W}_e - e_{kij}E_k\dot{u}_{i,j} - \gamma_{ij}\theta\dot{u}_{i,j}\right) dV + \int_{\Omega} \dot{K} dV \tag{19}$$

where

$$\dot{W}_e = c_{ijkl}u_{k,l}\dot{u}_{i,j} = \frac{1}{2}\frac{\partial}{\partial t}(c_{ijkl}u_{k,l}u_{i,j})$$

which represents the rate of mechanical potential energy density.

$$\dot{K} = \rho\ddot{u}_i\dot{u}_i = \frac{1}{2}\frac{\partial}{\partial t}\left(\rho\dot{u}_i\dot{u}_i\right)$$

which is the rate of kinetic energy density.

II. Multiplying φ by the time derivative of Equation (3), integrating the resulting expression over volume Ω and using the identity equation $\left(\dot{D}_k\varphi\right)_{,k} = \dot{D}_{k,k}\varphi + \dot{D}_k\varphi_{,k}$ and Gaussian Theorem, we have

$$-\int_{\partial\Omega} \varphi\dot{D}_k n_k dS - \int_{\Omega} E_k\dot{D}_k dV = 0$$

where n_k is the unit outward normal of dS.

Substitution the constitutive equation Equation $(7)_2$ into the above equation yields

$$-\int_{\partial\Omega} \varphi\dot{D}_k n_k dS - \int_{\Omega} E_k \left(e_{kij}\dot{\varepsilon}_{ij} + \lambda_{ik}\dot{E}_i + \xi_k\dot{\theta}\right) dV = 0$$

which is of the form

$$-\int_{\partial\Omega} \varphi\dot{D}_k n_k dS - \int_{\Omega} \left(e_{kij}\dot{\varepsilon}_{ij}E_k + \dot{W}_E + \xi_k\dot{\theta}E_k\right) dV = 0 \tag{20}$$

where the rate of electric energy density is defined as

$$\dot{W}_E = \frac{1}{2}\frac{\partial}{\partial t}(\lambda_{ik}E_iE_k)$$

The addition of Equation (19) and Equation (20) yields

$$\int_{\partial\Omega} t_i\dot{u}_i dS + \int_{\Omega} \rho b_i\dot{u}_i dV + \int_{\Omega} \gamma_{ij}\theta\dot{u}_{i,j} dV - \int_{\partial\Omega} \varphi\dot{D}_k n_k dS - \int_{\Omega} E_k\xi_k\dot{\theta} dV = \int_{\Omega} \left(\dot{W}_e + \dot{K} + \dot{W}_E\right) dV \tag{21}$$

III. Taking the time differential on Equation $(7)_3$ and using Equation (5), we get

$$T_0\gamma_{ij}\dot{\varepsilon}_{ij} + T_0\xi_i\dot{E}_i + \rho C\dot{\theta} = -q_{i,i}$$

Applying the operator L on both sides of this equation and using Equation (6) yields

$$\kappa_{ij}\theta_{,ij} - L(T_0\gamma_{ij}\dot{\varepsilon}_{ij} + T_0\xi_i\dot{E}_i + \rho C\dot{\theta}) = 0 \tag{22}$$

Multiplying Equation (22) by θ and apply volume integral on this expression, we obtain

$$\int_\Omega \kappa_{ij}\theta_{,ij}\theta dV - \int_\Omega T_0\gamma_{ij}L\left(\dot{\varepsilon}_{ij}\right)\theta dV - \int_\Omega T_0\xi_i L\left(\dot{E}_i\right)\theta dV - \int_\Omega \rho CL\left(\dot{\theta}\right)\theta dV = 0 \tag{23}$$

Using the identity $(\theta\theta_{,i})_{,j} = \theta_{,j}\theta_{,i} + \theta\theta_{,ij}$ and Gaussian Theorem, then we have

$$\int_\Omega \kappa_{ij}\theta_{,ij}\theta dV = \int_\Omega \kappa_{ij}\left[(\theta\theta_{,i})_{,j} - \theta_{,j}\theta_{,i}\right]dV = \int_{\partial\Omega} \kappa_{ij}n_j\theta\theta_{,i}dS - \int_\Omega \kappa_{ij}\theta_{,j}\theta_{,i}dV$$

Inserting this relation into Equation (23) and expanding the result by using the entropy equation Equation $(7)_3$, we get

$$\frac{1}{T_0}\int_{\partial\Omega} \kappa_{ij}\theta_{,i}\theta n_j dS - \frac{1}{T_0}\int_\Omega \kappa_{ij}\theta_{,j}\theta_{,i}dV = \int_\Omega \gamma_{ij}\dot{\varepsilon}_{ij}\theta dV + \int_\Omega \xi_i\dot{E}_i\theta dV + \int_\Omega \frac{\rho C}{T_0}\dot{\theta}\theta dV + \tau\int_\Omega \frac{\rho\ddot{\eta}\theta}{T_0}dV \tag{24}$$

Thus the rate of thermal energy density $\dot{W}_\theta$ can be expressed as

$$\dot{W}_\theta = \frac{\rho C}{T_0}\dot{\theta}\theta = \frac{1}{2}\frac{\partial}{\partial t}\left(\frac{\rho C}{T_0}\theta^2\right)$$

Combining Equation (21) and Equation (24) by eliminating $\int_\Omega \gamma_{ij}\dot{\varepsilon}_{ij}\theta dV$, finally we obtain

$$\begin{aligned}
&\int_{\partial\Omega} t_i\dot{u}_i dS - \int_{\partial\Omega} \varphi\dot{D}_k n_k dS + \frac{1}{T_0}\int_{\partial\Omega} \kappa_{ij}\theta_{,i}\theta n_j dS \\
&= \int_\Omega \tau\frac{\rho\ddot{\eta}\theta}{T_0}dV + \frac{1}{T_0}\int_\Omega \kappa_{ij}\theta_{,j}\theta_{,i}dV + \int_\Omega \frac{\partial}{\partial t}\left(E_i\xi_i\theta\right)dV + \frac{\partial}{\partial t}\int_\Omega \left(W_e + K + W_E + W_\theta\right)dV
\end{aligned} \tag{25}$$

which is the energy balance law for pyroelectric medium with generalized Fourier conduction law for arbitrary time dependent wave field.
As the general energy balance states:

$$\int_\Omega Q dV = -\oint_{\partial\Omega} P_i n_i dS - \frac{\partial}{\partial t}\int_\Omega W dV \tag{26}$$

which is the law governing the energy transformation. The physical significance of Equation (26) is that the rate of heat or dissipation energy Q equals to the reduction of the rate of entire energy $\dot{W}$ within the volume plus the reduction of this energy flux outward the surface bounding the volume. P_i is called the energy flux vector(also called the Poyting vector, Poyting-Umov vector) and its direction indicates the direction of energy flow at that point, the length being numerically equal to the amount of energy passing in unit time through unit area perpendicular to **P**.
In this chapter, important conclusions can be made from Equation (25): the energy density W in the the pyroelectric medium:

$$\begin{aligned} W &= W_e + K + W_E + W_\theta \\ W_e &= \frac{1}{2} c_{ijkl} u_{k,l} u_{i,j},\ K = \frac{1}{2}\rho \dot{u}_i \dot{u}_i, \\ W_E &= \frac{1}{2}\lambda_{ik} E_i E_k,\ \ W_\theta = \frac{1}{2}\frac{\rho C}{T_0}\theta^2 \end{aligned} \tag{27}$$

which is sum of the mechanical potential energy density W_e, the kinetic energy density K, the electric energy density W_E, the heat energy density W_θ .
The physical meaning of $E_i\xi_i\theta$ can be seen from constitutive equation in Equation $(7)_3$, from which $E_i\xi_i$ is found to contribute entropy. Therefore the result $E_i\xi_i\theta$, by its multiplication with temperature disturbance θ, is the dissipation due to the pyroelectric effect. Therefore Q the rate of energy dissipation per unit volume is represented by

$$Q = \tau \frac{\rho \ddot{\eta} \theta}{T_0} + \frac{1}{T_0}\kappa_{ij}\theta_{,j}\theta_{,i} + \frac{\partial}{\partial t}\left(E_i \xi_i \theta\right) \tag{28}$$

in which the energy dissipated by the heat conduction is $\frac{1}{T_0}\kappa_{ij}\theta_{,j}\theta_{,i}$, the dissipation energy generated by the relaxation is $\tau\frac{\rho\ddot{\eta}\theta}{T_0}$ and the last term is due to pyroelectric effect.
The energy flux vector(also called the Poyting vector, Poyting-Umov vector) P_i is defined as

$$P_i = -\sigma_{ji}\dot{u}_j + \varphi \dot{D}_i - \kappa_{ij}\theta_{,j}\frac{\theta}{T_0} \tag{29}$$

If the temperature effect is not taken account of, Equations (27), (29) can be degenerated into the forms in reference (Baesu et al., 2003).

3.1 Energy balance law for the real-valued inhomogeneous harmonic wave

In previous section, we derived the energy balance equation for the pyroelectric medium and defined the total energy, dissipation energy and energy flux vector explicitly. Keeping in mind that the real part is indeed the physical part of any quantity, and considering Equation (10), we can define the corresponding fundamental field functions as

$$\begin{aligned} u_i &= \frac{1}{2}\left[U_i \exp(i x_s k_s)\exp(i\omega t) + U_i^* \exp(-i x_s k_s^*)\exp(-i\omega t)\right] \\ \theta &= \frac{1}{2}\left[\Theta \exp(i x_i k_i)\exp(i\omega t) + \Theta^* \exp(-i x_i k_i^*)\exp(-i\omega t)\right] \\ \varphi &= \frac{1}{2}\left[\Psi \exp(i x_i k_i)\exp(i\omega t) + \Psi^* \exp(-i x_i k_i^*)\exp(-i\omega t)\right] \end{aligned} \tag{30}$$

which are the real-valued inhomogeneous harmonic waves assumed on the basis of the pair of complex vector fields for Equation (8).
The velocity of plane of constant phase is defined by

$$\mathbf{v}_p = \omega \mathbf{P} / \left\|\mathbf{P}\right\|^2 \tag{31}$$

and the maximum attenuation is $\|\mathbf{A}\|$, where $\|\ \|$ indicates the norm(or length) of a vector.
The quantities of the rate of energy density, the dissipation energy and the energy flux vector can be expressed by inserting Equation (30) into Equations (27), (28) and (29).

The mechanical potential energy density W_e

$$W_e = \frac{1}{2}c_{ijkl}\mathrm{Re}\left[U_k k_l U_i^* k_j^*\right]\exp\left(-2x_s A_s\right) - \frac{1}{2}c_{ijkl}\mathrm{Re}\left[U_k k_l U_i k_j \exp\left(2ix_s k_s\right)\exp\left(2i\omega t\right)\right] \tag{32}$$

The first term on the right-hand side of this equation is time-independent and the second term is time harmonic with frequency 2ω. The first term, expressed as $\langle W_e \rangle$ afterwards, represents the result of W_e averaged over one period. From now on, we shall use $\langle\ \rangle$ indicates the mean quantity over one period. The notation Re stands for the real part and Im the imaginary part. Similarly, the kinetic energy density K takes the form

$$K = \frac{1}{2}\rho\omega^2 U_i U_i^* \exp\left(-2x_s A_s\right) - \frac{1}{2}\rho\omega^2 \mathrm{Re}\left[U_i U_i \exp\left(i2x_s k_s\right)\exp\left(i2\omega t\right)\right] \tag{33}$$

The electric energy density W_E

$$W_E = \frac{1}{2}\lambda_{ik}\mathrm{Re}\left[k_i k_k^* \Psi\Psi^* \exp\left(-2x_s A_s\right)\right] - \frac{1}{2}\lambda_{ik}\mathrm{Re}\left[k_i k_k \Psi\Psi \exp\left(i2x_s k_s\right)\exp\left(i2\omega t\right)\right] \tag{34}$$

The heat energy density W_θ

$$W_\theta = \frac{1}{2}\frac{\rho C}{T_0}\left\{\Theta^*\Theta \exp\left(-2x_s A_s\right) + \mathrm{Re}\left[\Theta\Theta \exp\left(i2x_i k_i\right)\exp\left(i2\omega t\right)\right]\right\} \tag{35}$$

The rate of energy dissipation density

$$Q = Q^{(\kappa)} + Q^{(\tau)} + Q^{(\xi)}$$

where $Q^{(\kappa)}$ due to the heat conduction

$$Q^{(\kappa)} = \frac{1}{T_0}\kappa_{ij}\mathrm{Re}\left[k_i k_j^* \Theta^*\Theta \exp\left(-2x_s A_s\right)\right] - \frac{1}{T_0}\kappa_{ij}\mathrm{Re}\left[k_i k_j \Theta\Theta \exp\left(i2x_s k_s\right)\exp\left(i2\omega t\right)\right] \tag{36}$$

$Q^{(\tau)}$ because of the relaxation

$$\begin{aligned} Q^{(\tau)} &= \tau\frac{\rho}{T_0}\left(\gamma_{ij}\ddot{\varepsilon}_{ij}\theta + \xi_i \ddot{E}_i \theta + \frac{\rho C}{T_0}\ddot{\theta}\theta\right) \\ &= \tau\frac{\rho}{T_0}\omega^2\left\{\gamma_{ij}\mathrm{Im}\left(U_i k_j \Theta^*\right)\exp\left(-2x_s A_s\right) + \gamma_{ij}\mathrm{Im}\left[U_i k_j \Theta \exp\left(i2x_s k_s\right)\exp\left(i2\omega t\right)\right] + \right. \\ &\quad + \xi_i \mathrm{Im}\left(k_i^*\Psi^*\Theta\right)\exp\left(-2x_s A_s\right) + \xi_i Im\left[k_i^*\Psi^*\Theta^*\exp\left(i2x_s k_s\right)\exp\left(i2\omega t\right)\right] + \\ &\quad \left. - \Theta\Theta^*\exp\left(-2x_s A_s\right) - \mathrm{Re}\left[\Theta\Theta \exp i(2x_i k_i)\exp\left(i2\omega t\right)\right]\right\} \end{aligned} \tag{37}$$

At last, $Q^{(\xi)}$ attributed by the pyroelectric effect

$$Q^{(\xi)} = 2\xi_i Re(k_i\omega\Psi\Theta^*)\exp\left(-2x_s A_s\right) + 2\xi_i Re\left[(k_i\omega\Psi\Theta)\exp(2ix_s k_s)\exp\left(i2\omega t\right)\right] \tag{38}$$

The energy flux vector P_i consists of three different parts: $P_i^{(u)}$ is generated in the elastic field; $P_i^{(\varphi)}$ in the electric field; $P_i^{(\theta)}$ in the thermal field, which are expressed as

$$\begin{aligned} P_j^{(u)} &= -\sigma_{ji}\dot{u}_j \\ &= -\omega c_{jikl}\left\{\mathrm{Re}\left(U_i^* U_k k_l\right)\exp\left(-2x_s A_s\right) + \mathrm{Re}\left[U_i U_k k_l \exp\left(i2x_s k_s\right)\exp\left(i2\omega t\right)\right]\right\} + \\ &\quad \omega e_{kji}\left\{-\mathrm{Re}\left(k_k U_i^*\Psi\right)\exp\left(-2x_s A_s\right) + \mathrm{Re}\left[k_k \Psi U_i \exp\left(i2x_s k_s\right)\exp\left(i2\omega t\right)\right]\right\} + \\ &\quad \omega\gamma_{ji}\left[\left[\mathrm{Im}\left(U_i^*\Theta\right)\exp\left(-2x_s A_s\right) - \omega \mathrm{Im} U_i \Theta \exp\left(i2x_s k_s\right)\exp\left(i2\omega t\right)\right]\right] \end{aligned} \tag{39}$$

In the electric field, $P_j^{(\varphi)}$

$$P_j^{(\varphi)} = \varphi \dot{D}_j$$
$$= -\omega e_{jmn} \{\mathrm{Re}\,(U_m k_n \Psi^*) \exp[-2x_s A_s] + \mathrm{Re}\,[\Psi U_m k_n \exp[i(2x_s k_s)] \exp i2\omega t]\} + \quad (40)$$
$$\omega \lambda_{mj} \{\mathrm{Re}\,(k_m \Psi\Psi^*) \exp(-2x_s A_s) + \mathrm{Re}\,[k_m \Psi\Psi \exp(i2x_s k_s) \exp(i2\omega t)]\} +$$
$$\omega \xi_j \{\mathrm{Im}\,(\Theta^*\Psi) \exp(-2x_s A_s) - \mathrm{Im}\,[\Theta\Psi \exp[i(2x_i k_i)] \exp(i2\omega t)]\}$$

In the thermal field, $P_j^{(\theta)}$

$$P_j^{(\theta)} = -\kappa_{ij}\theta_{,i}\frac{\theta}{T_0} = \frac{1}{T_0}\left\{-\Theta\Theta^*\kappa_{ij}\mathrm{Im}\,(k_i)\exp(-2x_s A_s) + \mathrm{Im}\left[\kappa_{ij}k_i\Theta\Theta\exp(i2x_s k_s)\exp(i2\omega t)\right]\right\} \quad (41)$$

It is to be noted that the mean quantities still satisfy Equation (25) of energy balance equation for pyroelectric medium.

Since the energy flux and the energy density have the dimensions of watt per square meter and joule per cubic meter respectively, their ratio gives a quantity with dimension of velocity. This energy velocity $\mathbf{v}_E$ is defined as the radio of the mean energy flux to the mean energy density over one period, that is

$$\mathbf{v}_E = \langle \mathbf{P} \rangle \,/\, \langle W \rangle \quad (42)$$

which corresponds to the average local velocity of energy transport. From an experimental point of view, it is more interesting to define velocity from averaged quantities (Deschamps et al., 1997).

We can substitute the expressions in Equations (32)-(35) and (39)-(41) into (42), which yields a lengthy formulation. Comparing the expression of phase velocity in Equation (31) with the energy velocity in Equation (42), it is obvious that they are different from each other in moduli as well as directions.

3.2 Results and discussion

According to previous studies, it is already known that there are waves of four modes, which are quasilongitudinal, quasitransverse I, II and temperature. In this section, we'd like to discuss phase velocity $\mathbf{v}_p$, energy velocity $\mathbf{v}_E$ related to the four mode waves. They are studied as functions defined in propagation angle θ and attenuation angle γ. After wave vector $\mathbf{k}$ is determined, Equations (31) and (42) yield the phase velocity and energy velocity respectively. The material constants under study is transversely isotropic material, see Section 2.4.

The variation of phase and energy velocity of quasilongitudinal wave is presented in Fig. 6 (a) which shows that the phase velocity does not vary with attenuation angle γ , while the corresponding energy velocity can be influenced by γ. With γ increasing, the energy velocity turns small. It is also noted that the phase velocity is a little bigger than the energy velocity for quasilongitudinal wave mode.

The case of temperature wave is shown in Fig. 6 (b). Different from quasilongitudinal wave, the phase velocity and energy velocity of temperature wave are influenced by propagation angle θ and attenuation angle γ. Both phase velocity and energy velocity decay with γ. For given γ, the phase velocity is also bigger than energy velocity.

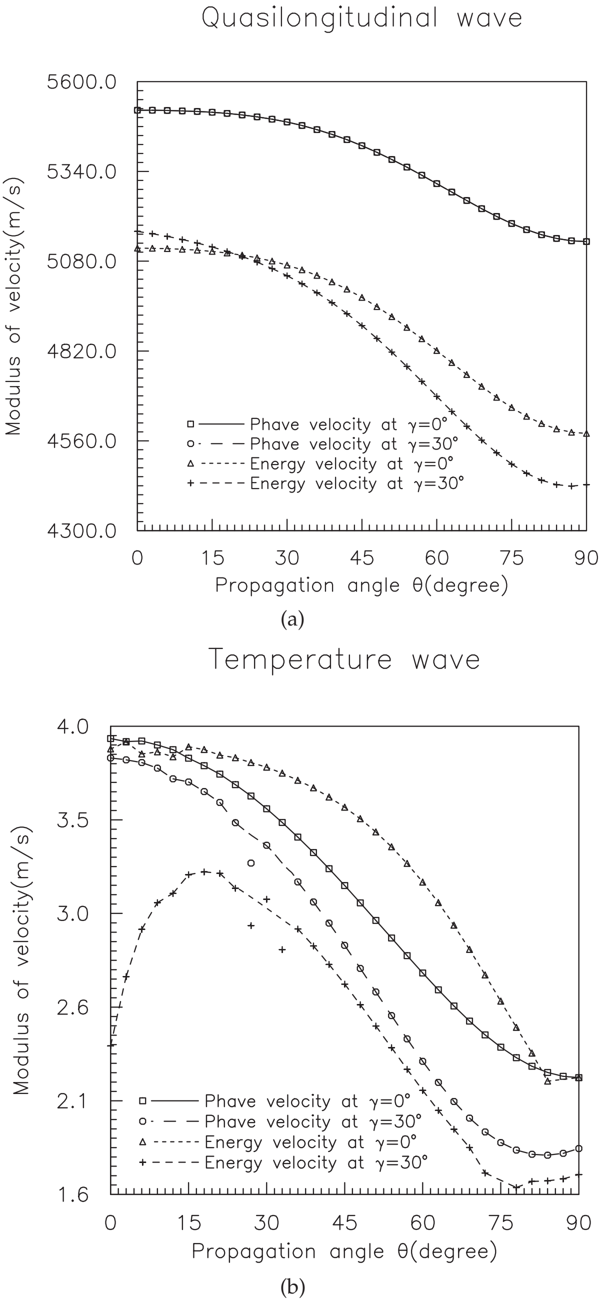
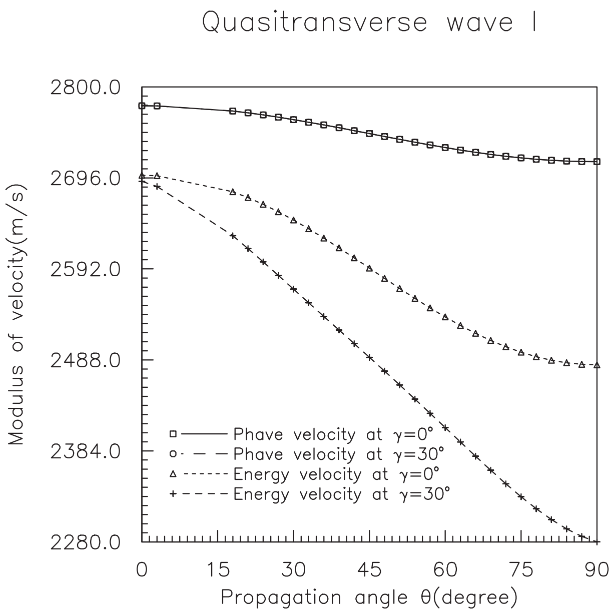
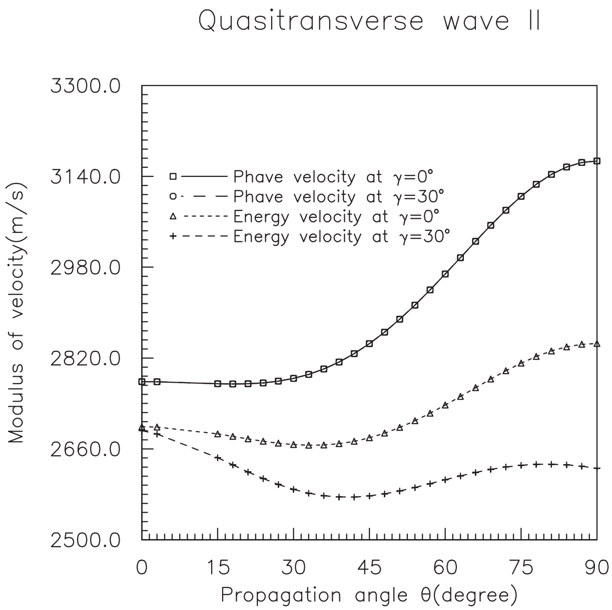


Fig. 6. Variations of velocity with propagation angle θ at γ=0°, 30°.



(a)



(b)

Fig. 7. Variations of velocity with propagation angle θ at γ=0°,30°.

Plots of the computed velocities of quasitransverse wave I and II are given in Fig. 7. The phase velocities of both wave modes are almost independent of γ and the energy velocity become small with γ increasing.

4. Conclusion

In this chapter, the energy process of the pyroelectric medium with generalized heat conduction theory is addressed in the framework of the inhomogeneous wave results originally. The characters of inhomogeneous waves lie in that its propagation direction is different from the biggest attenuation direction. The complex-valued wave vector is determined by four parameters. The range of attenuation angle should be confined in (-90°,90°) to make waves attenuate. Further analysis demonstrates that, in anisotropic plane, the positive and negative attenuation angle have different influences on waves, while, in the isotropic plane, they are the same. Based on the governing equations and state equations, the dynamic energy conservation law is derived. The energy transfer, in an arbitrary instant, is described explicitly by the energy conservation relation. From this relation, it is found that energy density in pyroelectric medium consists of the electric energy density, the heat energy density, the mechanical potential energy density, the kinetic energy density. The heat loss or dissipation energy is equal to the reduction of the entire energy within a fixed volume plus the reduction of this energy flux outward the surface bounding this volume. The dissipation energy in pyroelectric medium are attributed by the heat conduction, relaxation time and pyroelectric effect. The energy flux is obtained and it can not be influenced by the relaxation time. The phase velocity and energy velocity of four wave modes in pyroelectric medium are studied. Results demonstrate that the attenuation angle almost doesn't influence phase velocity of quasilongitudinal, quasitransverse I, II wave modes, while plays large role on the temperature wave. The energy velocities of the four wave modes all decay with the attenuation angle.

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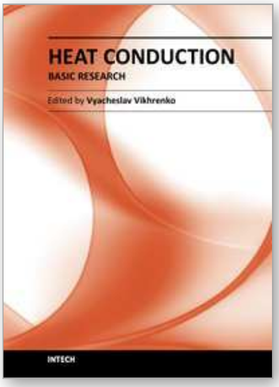

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The content of this book covers several up-to-date approaches in the heat conduction theory such as inverse heat conduction problems, non-linear and non-classic heat conduction equations, coupled thermal and electromagnetic or mechanical effects and numerical methods for solving heat conduction equations as well. The book is comprised of 14 chapters divided into four sections. In the first section inverse heat conduction problems are discuss. The first two chapters of the second section are devoted to construction of analytical solutions of nonlinear heat conduction problems. In the last two chapters of this section wavelike solutions are attained.The third section is devoted to combined effects of heat conduction and electromagnetic interactions in plasmas or in pyroelectric material elastic deformations and hydrodynamics. Two chapters in the last section are dedicated to numerical methods for solving heat conduction problems.

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