

Time Varying Heat Conduction in Solids

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1. Introduction

People experiences heat propagation since ancient times. The mathematical foundations of this phenomenon were established nearly two centuries ago with the early works of Fourier [Fourier, 1952]. During this time the equations describing the conduction of heat in solids have proved to be powerful tools for analyzing not only the transfer of heat, but also an enormous array of diffusion-like problems appearing in physical, chemical, biological, earth and even economic and social sciences [Ahmed & Hassan, 2000]. This is because the conceptual mathematical structure of the non-stationary heat conduction equation, also known as the heat diffusion equation, has inspired the mathematical formulation of several other physical processes in terms of diffusion, such as electricity flow, mass diffusion, fluid flow, photons diffusion, etc [Mandelis, 2000; Marín, 2009a]. A review on the history of the Fourier's heat conduction equations and how Fourier's work influenced and inspired others can be found elsewhere [Narasimhan, 1999].

But although Fourier's heat conduction equations have served people well over the last two centuries there are still several phenomena appearing often in daily life and scientific research that require special attention and carefully interpretation. For example, when very fast phenomena and small structure dimensions are involved, the classical law of Fourier becomes inaccurate and more sophisticated models are then needed to describe the thermal conduction mechanism in a physically acceptable way [Joseph & Preziosi, 1989, 1990]. Moreover, the temperature, the basic parameter of Thermodynamics, may not be defined at very short length scales but only over a length larger than the phonons mean free paths, since its concept is related to the average energy of a system of particles [Cahill, et al., 2003; Wautelet & Duvivier, 2007]. Thus, as the mean free path is in the nanometer range for many materials at room temperature, systems with characteristic dimensions below about 10 nm are in a nonthermodynamical regime, although the concepts of thermodynamics are often used for the description of heat transport in them. To the author's knowledge there is no yet a comprehensible and well established way to solve this very important problem about the definition of temperature in such systems and the measurement of their thermal properties remains a challenging task. On the other hand there are some aspects of the heat conduction through solids heated by time varying sources that contradict common intuition of many people, being the subject of possible misinterpretations. The same occurs with the understanding of the role of thermal parameters governing these phenomena.

Thus, this chapter will be devoted to discuss some questions related to the above mentioned problems starting with the presentation of the equations governing heat transfer for different cases of interest and discussing their solutions, with emphasis in the role of the thermal parameters involved and in applications in the field of materials thermal characterization.

The chapter will be distributed as follows. In the next section a brief discussion of the principal mechanisms of heat transfer will be given, namely those of convection, radiation and conduction. Emphasis will be made in the definition of the heat transfer coefficients for each mechanism and in the concept of the overall heat transfer coefficient that will be used in later sections. Section 2 will be devoted to present the general equation governing non-stationary heat propagation, namely the well known (parabolic) Fourier's heat diffusion equation, in which further discussions will be mainly based. The conditions will be discussed under which this equation can be applied. The modified Fourier's law, also known as Cattaneo's Equation [Cattaneo, 1948], will be presented as a useful alternative when the experimental conditions are such that it becomes necessary to consider a relaxation time or build-up time for the onset of the thermal flux after a temperature gradient is suddenly imposed on the sample. Cattaneo's equation leads them to the hyperbolic heat diffusion equation. Due to its intrinsic importance it will be discussed with some detail. In Section 3 three important situations involving time varying heat sources will be analyzed, namely: (i) a sample periodically and uniformly heated at one of its surfaces, (ii) a finite sample exposed to a finite duration heat pulse, and (iii) a finite slab with superficial continuous uniform thermal excitation. In each case characteristic time and length scales will be defined and discussed. Some apparently paradoxical behaviors of the thermal signals and the role playing by the characteristic thermal properties will be explained and physical implications in practical fields of applications will be presented too. In Section 4 our conclusions will be drawn.

2. Heat transfer mechanisms

Any temperature difference within a physical system causes a transfer of heat from the region of higher temperature to the one of lower. This transport process takes place until the system has become uniform temperature throughout. Thus, the flux of heat, Φ (units of W), should be some function of the temperatures, T_1 and T_2 , of both the regions involved (we will suppose that $T_2 > T_1$). The mathematical form of the heat flux depends on the nature of the transport mechanism, which can be convection, conduction or radiation, or a coupling of them. The dependence of the heat flux on the temperature is in general non linear, a fact that makes some calculations quite difficult. But when small temperature variations are involved, things become much simpler. Fortunately, this is the case in several practical situations, for example when the sun rays heat our bodies, in optical experiments with low intensity laser beams and in the experiments that we will describe here later.

Heat convection takes place by means of macroscopic fluid motion. It can be caused by an external source (forced convection) or by temperature dependent density variations in the fluid (free or natural convection). In general, the mathematical analysis of convective heat transfer can be difficult so that often the problems can be solved only numerically or graphically [Marín, et al., 2009]. But convective heat flow, in its most simple form, i.e. heat transfer from surface of wetted area A and temperature T_2 , to a fluid with a temperature T_1 ,

for small temperature differences, $\Delta T = T_2 - T_1$, is given by the (linear with temperature) Newton's law of cooling,

$$\Phi_{\text{conv}} = h_{\text{conv}} \Delta T \quad (1)$$

The convective heat transfer coefficient, h_{conv} , is a variable function of several parameters of different kinds but independent on ΔT .

On the other hand heat radiation is the continuous energy interchange by means of electromagnetic waves. For this mechanism the net rate of heat flow, Φ_{rad} , radiated by a body surrounded by a medium at a temperature T_1 , is given by the Stefan-Boltzmann Law.

$$\Phi_{\text{rad}} = \sigma e A (T_2^4 - T_1^4) \quad (2)$$

where σ is the Stefan-Boltzmann constant, A is the surface area of the radiating object and e is the total emissivity of its surface having absolute temperature T_2 .

A glance at Eq. (2) shows that if the temperature difference is small, then one should expand it as Taylor series around T_1 obtaining a linear relationship:

$$\Phi_{\text{rad}} = 4\sigma e A T_1^3 (T_2 - T_1) = h_{\text{rad}} \Delta T \quad (3)$$

If we compare this equation with Eq. (1) we can conclude that in this case $h_{\text{rad}} = 4\sigma e A T_1^3$ can be defined as a radiation heat transfer coefficient.

On the other hand, heat can be transmitted through solids mainly by electrical carriers (electrons and holes) and elementary excitations such as spin waves and phonons (lattice waves). The stationary heat conduction through the opposite surfaces of a sample is governed by Fourier's Law

$$\Phi_{\text{cond}} = -kA \nabla T \quad (4)$$

The thermal conductivity, k (W/mK), is expressed as the quantity of heat transmitted per unit time, t , per unit area, A , and per unit temperature gradient. For one-dimensional steady state conduction in extended samples of homogeneous and isotropic materials and for small temperature gradients, Fourier's law can be integrated in each direction to its potential form. In rectangular coordinates it reads:

$$\Phi_{\text{cond}} = kA \frac{T_2 - T_1}{x_2 - x_1} = \frac{kA \Delta T}{L} = \frac{\Delta T}{R_{\text{cond}}} = h_{\text{cond}} \Delta T \quad (5)$$

Here T_1 and T_2 represent two planar isotherms at positions x_1 and x_2 , respectively, $L = x_2 - x_1$, and

$$R_{\text{cond}} = \frac{L}{Ak} = \frac{1}{h_{\text{cond}}} \quad (6)$$

is the thermal resistance against heat conduction (thermal resistance for short) of the sample. The Eq. (5) is often denoted as Ohm's law for thermal conduction following analogies existing between thermal and electrical phenomena. Comparing with Eq. (1) we see that the parameter h_{cond} has been incorporated in Eq. (6) as the conduction heat transfer coefficient.

Using

$$H = h_{\text{conv}} + h_{\text{rad}} = 1/R \quad (7)$$

heat transfer scientists define the dimensionless Biot number as:

$$B_i = \frac{H}{h_{cond}} = \frac{R_{cond}}{R} \quad (8)$$

as the fraction of material thermal resistance that opposes to convection and radiation heat losses.

3. The heat diffusion equation

Eq. 4 represents a very simple empirical law that has been widely used to explain heat transport phenomena appearing often in daily life, engineering applications and scientific research. In terms of the heat flux density ($q = \Phi/A$) it lauds:

$$\vec{q} = -k\vec{\nabla}T \quad (9)$$

When combined with the law of energy conservation for the heat flux

$$\frac{\partial E}{\partial t} = -\text{div}(\vec{q}) + Q \quad (10)$$

where Q represents the internal heat source and

$$\partial E/\partial t = \rho c \partial T/\partial t \quad (11)$$

is the temporal change in internal energy, E , for a material with density ρ and specific heat c , and assuming constant thermal conductivity, Fourier's law leads to another important relationship, namely the non-stationary heat diffusion equation also called second Fourier's law of conduction. It can be written as:

$$\nabla^2 T - \frac{1}{a} \frac{\partial T}{\partial t} = -\frac{Q}{k} \quad (12)$$

with

$$a = k/\rho c \quad (13)$$

as the thermal diffusivity.

Fourier's law of heat conduction predicts an infinite speed of propagation for thermal signals, i.e. a behavior that contradicts the main results of Einstein's theory of relativity, namely that the greatest known speed is that of the electromagnetic waves propagation in vacuum. Consider for example a flat slab and apply at a given instant a supply of heat to one of its faces. Then according to Eq. (9) there is an instantaneous effect at the rear face. Loosely speaking, according to Eq. (9), and also due to the intrinsic parabolic nature of the partial differential Eq. (12), the diffusion of heat gives rise to infinite speeds of heat propagation. This conclusion, named by some authors the paradox of instantaneous heat propagation, is not physically reasonable.

This contradiction can be overcome using several models, the most of them inspired in the so-called CV model.

This model takes its name from the authors of two pioneering works on this subject, namely that due to Cattaneo [Cattaneo, 1948] and that developed later and (apparently) independently by Vernotte [Vernotte, 1958]. The CV model introduces the concept of the

relaxation time, τ , as the build-up time for the onset of the thermal flux after a temperature gradient is suddenly imposed on the sample.

Suppose that as a consequence of the temperature existing at each time instant, t , the heat flux appears only in a posterior instant, $t + \tau$. Under these conditions Fourier's Law adopts the form:

$$\vec{q}(r, t + \tau) = -k\vec{\nabla}T(r, t) \quad (14)$$

For small τ (as it should be, because otherwise the first Fourier's law would fail when explaining every day phenomena) one can expand the heat flux in a Taylor Serie around $\tau = 0$ obtaining, after neglecting higher order terms:

$$\vec{q}(r, t + \tau) = \vec{q}(r, t) + \tau \frac{\partial \vec{q}(r, t)}{\partial t} \quad (15)$$

Substituting Eq. (15) in Eq. (14) leads to the modified Fourier's law of heat conduction or CV equation that states:

$$\tau \frac{\partial \vec{q}}{\partial t} + \vec{q} = -k\vec{\nabla}T. \quad (16)$$

Here the time derivative term makes the heat propagation speed finite. Eq. (16) tells us that the heat flux does not appear instantaneously but it grows gradually with a build-up time, τ . For macroscopic solids at ambient temperature this time is of the order of 10^{-11} s so that for practical purposes the use of Eq. (1) is adequate, as daily experience shows.

Substituting Eq. (17) into the energy conservation law (Eq. (10)) one obtains:

$$\nabla^2 T - \frac{1}{\alpha} \frac{\partial T}{\partial t} - \frac{1}{u^2} \frac{\partial^2 T}{\partial t^2} = -\frac{1}{k} \left(Q + \tau \frac{\partial Q}{\partial t} \right). \quad (17)$$

Here $u = (\alpha/\tau)^{1/2}$ represents a (finite) speed of propagation of the thermal signal, which diverges only for the unphysical assumption of $\tau = 0$.

Eq. (16) is a hyperbolic instead of a parabolic (diffusion) equation (Eq. (12)) so that the wave nature of heat propagation is implied and new (non-diffusive) phenomena can be advised. Some of them will be discussed in section (3.1).

The CV model, although necessary, has some disadvantages, among them: i). The hyperbolic differential Eq. (7) is not easy to be solved from the mathematical point of view and in the majority of the physical situations has non analytical solutions. ii) The relaxation time of a given system is in general an unknown variable. Therefore care must be taken in the interpretation of its results. Nevertheless, several examples can be found in the literature.

As described with more detail elsewhere [Joseph & Preziosi, 1989, 1990] other authors [Band & Meyer, 1948], proposed exactly the same Eq. (7) to account for dissipative effects in liquid He II, where temperature waves propagating at velocity u were predicted [Tisza, 1938; Landau, 1941; Peshkov, 1944] and verified. Due to these early works the speed u is often called the second sound velocity. More recently Tzou reported on phenomena such as thermal wave resonance [Tzou, 1991] and thermal shock waves generated by a moving heat source [Tzou, 1989]. Very rapid heating processes must be explained using the CV model too, such as those taking place, for example, during the absorption of energy coming from ultra short laser pulses [Marín, et al., 2005] and during the gravitational collapse of some stars [Govender, et al., 2001]. In the field of nanoscience and nanotechnology thermal time

constants, τ_c , characterizing heat transfer rates depend strongly on particle size and on its thermal diffusivity. One can assume that for spherical particles of radius R , these times scale proportional to R^2/α [Greffet, 2004; Marín, 2010; Wolf, 2004]. As for condensed matter the order of magnitude of a is $10^{-6}\text{m}^2/\text{s}$, for spherical particles having nanometric diameters, for example between 100 and 1 nm, we obtain for these times values ranging from about 10 ns to 1 ps, which are very close to the above mentioned relaxation times. At these short time scales Fourier's laws do not work in their initial forms.

In the next sections some interesting problems involving time varying heat sources will be discussed assuming that the conditions for the parabolic approach are well fulfilled, and, when required, these conditions will be deduced.

4. Some non-stationary problems on heat conduction

While the parabolic Fourier's law of heat conduction (4) describes stationary problems, with the thermal conductivity as the relevant thermophysical parameter, time varying heat conduction phenomena, which appear often in praxis, are described by the heat diffusion equation (12), being the thermal diffusivity the important parameter in such cases. Thermal conductivity can be measured using stationary methods based in Eq. (4), whose principal limitation is that precise knowledge of the amount of heat flowing through the sample and of the temperature gradient in the normal direction to this flow is necessary, a difficult task when small specimens are investigated. Therefore the use of non-stationary or dynamic methods becomes many times advantageous that allow, in general, determination of the thermal diffusivity. Thus knowledge of the specific heat capacity (per unit volume) is necessary if the thermal conductivity is to be obtained as well, as predicted by Eq. (13). Although this can be a disadvantage, often available specific heat data are used, so that it is not always necessary to determine experimentally it in order to account for the thermal conductivity. This is because specific heat capacity is less sensitive to impurities and structure of materials and comparatively independent of temperature above the Debye temperature than thermal conductivity and diffusivity. More precise, C is nearly a constant parameter for solids. In a plot of thermal conductivity versus thermal diffusivity we can see that solid materials typically fall along the line $C \approx 3 \times 10^6 \text{ J/m}^3\text{K}$ at room temperature. This experimentally proved fact is a consequence of the well known Dulong's and Petit's classical law for the molar specific heat of solids and of the consideration that the volume occupied per atom is about $1.4 \times 10^{-29} \text{ m}^3$ for almost all solids. In other words, the almost constant value of C can be explained by taking into account its definition as the product of the density (ρ) and the specific heat (c). The specific heat is defined as the change in the internal energy per unit of temperature change; thus, if the density of a solid increases (or decreases) the solid can store less (or more) energy. Therefore, as the density increases, the specific heat must decrease and then the product $C = \rho c$ stays constant and, according to Eq. 13, the behavior of the thermal conductivity is similar to that of the diffusivity. In accurate work, however, particularly on porous materials and composites, it is highly recommendable to measure also C . This is because some materials have lower-than-average volumetric specific heat capacity. Sometimes this happens because the Debye temperature lies well above room temperature and heat absorption is not classical. Deviations are observed in porous materials too, whose conductivity is limited partially by the gas entrapped in the porosity, in low density solids, which contain fewer atoms per unit volume so that ρC becomes low, and in composites due to heat fluxes through series and parallel arrangements of layers and

through embedded regions from different materials that strongly modified their effective thermal properties values, as has been described elsewhere [Salazar, 2003]. Although there are several methods for measurement of C their applications are often limited because they involve temperature variations that can affect thermal properties during measurement, in particular in the vicinity of phase transitions and structural changes. Fortunately there is another parameter involved in non-stationary problems and that can be also measured using dynamic techniques. While thermal diffusivity is defined as the ratio between the thermal conductivity and the specific heat capacity, this new parameter, named as thermal effusivity, e , but also called thermal contact parameter by some authors [Boeker & van Grondelle, 1999], is related to their product, as follows:

$$\varepsilon = (k C)^{1/2} \quad (18)$$

It is worth to be noticed that while the two expressions contain the same parameters, they are quite different. Diffusivity is related to the speed at which thermal equilibrium can be reached, while effusivity is related to the heated body surface temperature and it is the property that determines the contact temperature between two bodies in touch to one another, as will be seen below. Measuring both quantities provides the thermal conductivity without the need to know the specific heat capacity (note that Eqs. (13) and (18) lead to $k = \varepsilon a^{1/2}$). Dynamic techniques for thermal properties measurement can be divided in three classes, namely those involving pulsed, periodical and transient heat sources. There are also phenomena encounter in daily life that also involve these kinds of heating sources. This section will be devoted to analyze and discuss the solution of the heat diffusion equation in the presence of these sources. In each case characteristic time and length scales will be presented, the role playing by the characteristic thermal properties will be discussed as well as physical implications in practical fields of applications.

4.1 A sample periodically and uniformly heated at one of its surfaces

Consider an isotropic homogeneous semi-infinite solid, whose surface is heated uniformly (in such a way that the one-dimensional approach used in what follows is valid) by a source of periodically modulated intensity $I_0(1+\cos(\omega t))/2$, where I_0 is the intensity, $\omega = 2\pi f$ is the angular modulation frequency, and t is the time (this form of heating can be achieved in praxis using a modulated light beam whose energy is partially absorbed by the sample and converted to heat [Almond & Patel, 1996] but other methods can be used as well, e.g. using joule's heating [Ivanov et al., 2010]). The temperature distribution $T(x,t)$ within the solid can be obtained by solving the homogeneous (parabolic) heat diffusion equation, which can be written in one dimension as

$$\frac{\partial^2 T(x,t)}{\partial x^2} - \frac{1}{\alpha} \frac{\partial T(x,t)}{\partial t} = 0 \quad (19)$$

The solution of physical interest for most applications (for example in photothermal (PT) techniques [Almond & Patel, 1996]) is the one related to the time dependent component. If we separate this component from the spatial distribution, the temperature can be expressed as:

$$T(x, t) = \text{Re}[\theta(x)\exp(i\omega t)] \quad (20)$$

Substituting Eq. (20) into Eq. (19) leads to

$$\frac{\partial^2 \theta(x,t)}{\partial x^2} - q^2 \theta(x) = 0 \quad (21)$$

where

$$q = \sqrt{\frac{i\omega}{\alpha}} = \frac{(1+i)}{\mu} \quad (22)$$

is the thermal wave number and μ represents the thermal diffusion length defined as

$$\mu = \sqrt{\frac{2\alpha}{\omega}} \quad (23)$$

Using the boundary condition

$$k \frac{\partial \theta(x,t)}{\partial x} \Big|_{x=0} = \text{Re} \left[\frac{I_0}{2} \exp(i\omega t) \right], \quad (24)$$

the Eq. (21) can be solved and Eq. (19) leads to

$$T(x,t) = \frac{I_0}{2\varepsilon\sqrt{\omega}} \exp\left(-\frac{x}{\mu}\right) \text{Re} \left\{ \exp \left[-i \left(\frac{x}{\mu} + \omega t + \frac{\pi}{4} \right) \right] \right\} \quad (25)$$

This solution represents a mode of heat propagation through which the heat generated in the sample is transferred to the surrounding media by diffusion at a rate determined by the thermal diffusivity. Because this solution has a form similar to that of a plane attenuated wave it is called a thermal wave. Although it is not a real wave because it does not transport energy as normally waves do [Salazar, 2006], the thermal wave approach has demonstrated to be useful for the description of several experimental situations, as will be seen later.

Suppose that we have an alternating heat flux, related to a periodic oscillating temperature field. The analogy between thermal and electrical phenomena described in Section 1 when dealing with Fourier's law can be followed to define the thermal impedance Z_t as the temperature difference between two faces of a thermal conductor divided by the heat flux crossing the conductor. Then the thermal impedance becomes the ratio between the change in thermal wave amplitude and the thermal wave flux. At the surface of the semi-infinite medium treated with above one gets,

$$Z_t = \frac{T(x=0,t) - T_{amb}}{-k \frac{dT(x,t)}{dx} \Big|_{x=0}} \quad (26)$$

where T_{amb} is the ambient (constant) temperature (it was settled equal to zero for simplicity). Substituting Eq. (25) in Eq. (26) one obtains after a straightforward calculation [Marín, 2009b]:

$$Z_t = \frac{1-i}{\varepsilon\sqrt{\omega}} = \frac{1}{\varepsilon\sqrt{\omega}} \exp\left(-i\frac{\pi}{4}\right) \quad (27)$$

Note that, contrary to thermal resistance (see Eq. (6)), which depends on thermal conductivity, in the thermal impedance definition the thermal effusivity becomes the relevant parameter.

Using Eq. (27) the Eq. (25) can be rewritten as:

$$T(x,t) = \frac{I_0}{2} Z_t \exp\left(-\frac{x}{\mu}\right) \cos\left(\frac{x}{\mu} + \omega t\right) \quad (28)$$

Eq. (25) shows that the thermal diffusion length, μ , gives the distance at which an appreciable energy transfer takes place and that there is a phase lag between the excitation and the thermal response of the sample given by the term $x/\mu + \pi/4$ in the exponential term. Note that the thermal wave behaviour depends on the values of both, thermal effusivity, which determines the wave amplitude at $x=0$, and the thermal diffusivity, from which the attenuation and wave velocity depend.

Among other characteristics [Marín et al., 2002] a thermal wave described by Eq. (24) has a phase velocity, $v_f = \omega\mu = (2\alpha\omega)^{1/2}$. Because Eq. (21) is a linear ordinary differential equation describing the motion of a thermal wave, then the superposition of solutions will be also a solution of it (often, as doing above, the temperature distribution is approximated by just the first harmonic of that superposition because the higher harmonics damp out more quickly due to the damping coefficient increase with frequency). This superposition represents a group of waves with angular frequencies in the interval $\omega, \omega + d\omega$ travelling in space as "packets" with a group velocity $v_g = 2v_f$ [Marín et al., 2006]. It is worth to notice that both, phase and group velocities depend on the modulation frequency in such a way that if ω tends to infinite, they would approach infinite as well, what is physically inadmissible. This apparent contradiction can be explained using the same arguments given in section 2. Starting from the hyperbolic heat diffusion equation (Eq. (17)) without internal heat sources, and making the separation of variables given by Eq. (20), the equation to be solve becomes

$$\frac{\partial^2 \theta(x,t)}{\partial x^2} - q'^2 \theta(x) = 0 \quad (29)$$

with the boundary condition at the surface [Salazar, 2006]

$$-k \frac{\partial \theta(x,t)}{\partial x} \Big|_{x=0} = \text{Re} \left[\frac{I_0}{2} (1 + i\omega t) \right] \quad (30)$$

Eq. (29) is similar to Eq. (21) but with the complex wave number given by [Marín, 2007a]

$$q' = \omega \sqrt{\frac{\tau}{\alpha}} \sqrt{i \frac{\omega_l}{\omega} - 1} \quad (31)$$

where

$$\omega_l = \frac{1}{\tau} \quad (32)$$

Two limiting cases can be examined. First, for low modulation frequencies so that $\omega \ll \omega_l$ the wave number becomes equal to q (Eq. (22)) and the solution becomes a thermal wave given by Eq. (25). But for high frequencies, i.e. for $\omega \gg \omega_l$, the wave number becomes $q = i\omega/\alpha$, and the solution of the problem has the form [Salazar, 2006.]

$$T(x, t) = \frac{I_0}{2cu} \exp\left(-\frac{xu}{2\alpha}\right) \cos\left(\frac{\omega}{u}x - \omega t\right) \quad (33)$$

Thus according to the hyperbolic solution the amplitude of the surface temperature does not depend on the modulation frequency and depends on the specific heat capacity and the propagation velocity $u = (\alpha/\tau)^{1/2}$. There is not a phase lag, i.e. the excitation source and the surface temperature are in phase. Moreover, the penetration depth becomes also independent on the modulation frequency and depends on the wave propagation velocity. This case represents a form of heat transfer, which takes place through a direct coupling of

vibrational modes (i.e. the acoustic phonon spectrum) of the material. At these high frequencies (short time scale) ballistic transport of heat can be dominant.

Measurement of the periodical temperature changes induced in a sample by harmonic heating is the basis of the working principle of the majority of the so-called photothermal (PT) techniques [Marín, 2009c]. These are methods widely used for thermal characterization because the thermal signal is dependent on thermal properties such as thermal diffusivity (see Eq. (29)). As mentioned above the time constant τ in condensed matter is related to the phonon relaxation time, which is in the picosecond range, so that the limiting frequency becomes about $\omega_1=10^{12}$ Hz. Typical modulation frequencies used in PT experiments are between some Hz and several kHz, i.e. $\omega \ll \omega_1$, so that the more simpler parabolic approach is valid. This offers several advantages related with their use for thermal characterization of materials in situations where heat transport characteristic times are comparable to the relaxation time, τ [Marín, 2007b].

Following the above discussion in what follows the parabolic thermal wave approach will be used to explain a particular phenomenon observed in some experiments realized with PT techniques, which contradicts intuitively expectation. Suppose that a solid sample is subjected to periodical modulated heating at certain frequencies. Using different detection schemas some authors [Caerels et al., 1998] 2452-2458 ; Sahraoui et al., 2002; S. Longuemart et al., 2002; Deprueter et al., 2005; Lima et al., 2006; Marin et al., 2010] have observed that when a sample is in contact with a liquid the resulting sample's temperature may be larger than that due to the bare sample, for certain values of the modulation frequencies. This contradicts the expected behavior that in the presence of a liquid the developed heat will always flow through the sample to the liquid, which acts as a heat sink.

In the PT techniques the periodical heating is mainly generated by impinging intensity modulated light (e.g. a laser beam) on a sample. When light energy is absorbed and subsequently totally or partially transformed into as heat, it results in sample heating, leading to temperature changes as well as changes in the thermodynamic parameters of the sample and its surroundings. Measurements of these changes are ultimately the basis for all photothermal methods. The temperature variations could be detected directly using a pyroelectric transducer in the so called Photopyroelectric (PPE) method [Mandelis & Zver, 1985]. The sample's temperature oscillations can be also the cause of periodical black body infrared electromagnetic waves that are radiated by the sample and that can be measured using an appropriate sensor in the PT radiometry [Chen et al., 1993] In the photoacoustic (PA) method the sample is enclosed in a gas (example air) tight cell. The temperature variations in the sample following the absorption of modulated radiation induce pressure fluctuations in the gas, i.e. acoustic waves, that can be detected by a sensitive microphone already coupled to the cell [Vargas & Miranda, 1980]. Other detection schemes have been devised too.

Consider the experimental setup schematically showed in Fig. 1. A glass sample covers one of the two openings of a PA cell, while the other is closed by a transparent glass window through which a modulated laser light beam impinges on the inner, metal coated (to warrant full absorption of the light) sample's surface, generating periodical heating (the so-called thermal waves) and hence a pressure fluctuation in a PA air chamber, which is detected with a microphone already enclosed in the PA cell. The microphone signal is fed into a Lock-in amplifier for measurement of its amplitude as a function of the modulation frequency, f .

Using this schema Lima *et al* [Lima et al., 2006];] and Marín *et al* [Marin et al., 2010]] have measured the PA signal as a function of the modulation frequency for a bare glass substrate, and then they have deposited about 100 μL of liquid and repeated the same measurement.

In Fig. 2 (a) the normalized signal amplitude (the ratio of the amplitude signal due to the substrate-liquid system and that due to the bare substrate) is showed as a function of f for the case of a distilled water liquid sample. One can see that in certain frequency intervals the normalized signal becomes greater than 1, a fact that, as discussed before, contradicts the intuitively awaited behavior.

In order to explain this apparent paradox the mentioned authors resorted to the thermal wave model supposing that, as other kind of waves do, they experiences reflection, refraction and interference. Consider two regions, 1 and 2, and a plane thermal wave (Eq. (24)) incident from region 1 that is partially reflected and transmitted at the interface.

One can show that for normal incidence the reflection and transmission coefficients can be written as [Bennett & Patty, 1982]:

$$R_{12} = \frac{1-b_{12}}{1+b_{12}} \quad (34)$$

and

$$T_{12} = \frac{2}{1+b_{12}} \quad (35)$$

where

$$b_{12} = \frac{\varepsilon_1}{\varepsilon_2} \quad (36)$$

is the ratio of the media thermal effusivities. Thermal effusivity can be also regarded, therefore, as a measure of the thermal mismatch between the two media.

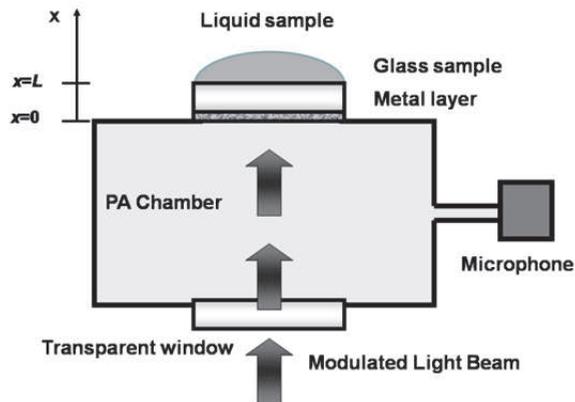


Fig. 1. Schema of a photothermal experimental setup with photoacoustic detection. In the experiment described here the glass substrate was 180 μm thick and it was coated with a 2 μm thick layer of Cu deposited by thermal evaporation in vacuum. The PA cell cylindrical cavity have a 5mm diameter and is 5 mm long. The light source was an Ar-ion laser beam of 50 mW modulated in intensity at 50% duty cycle with a mechanical chopper. An electret microphone was coupled to the cell through a 1mm diameter duct located at the cell wall.

Denoting with s the glass substrate of thickness L , which is sandwiched between two regions, namely 1 (the metallic coating) and 2 (the liquid sample or air). Supposing also that the surface of region 1 opposite to s is heated uniformly (so that a one dimensional analysis can be valid) by a light source of periodically modulated intensity, I_0 . Because its thickness is much smaller than L it can be also supposed that region 1 acts only as a thin superficial light absorbing layer, where a thermal wave will be generated following the periodical heating and launched through the glass substrate. Consider the propagation of a thermal wave described by Eq. (25) through the substrate. The so-called thermal wave model shows that the thermal waves will propagate towards the interface between the sample and region 2 and back towards the sample's surface, 1. On striking the boundaries they will be partially reflected and transmitted, so that interference between the corresponding wave trains takes place. Because the PA signal will be proportional to the temperature at the glass-metal interface the interest is in the resulting temperature at $x=0$, which can be obtained by summing all the waves arriving at this point. The result is [Marín et al., 2010] (the time dependent second exponential term of Eq. (25) will be omitted from now on for sake of simplicity):

$$T(0) = T_0 \left[1 + R_{s2} \frac{\exp(-2q_s L)}{1 - \gamma \exp(-2q_s L)} \right] \quad (37)$$

where T_0 is a frequency dependent term, $\gamma = R_{s1} R_{s2}$, and R_{s1} and R_{s2} are the normal incidence thermal wave reflection coefficients at the s -1 and s -2 interfaces respectively.

The solid line in Fig. 2 represents the normalized signal as a function of f calculated using the above expression for the system composed of a glass substrate ($L=180\mu\text{m}$, $\epsilon_s=1480 \text{Ws}^{1/2}\text{m}^{-2}\text{K}^{-1}$, $\alpha_s=3.5 \times 10^{-6} \text{m}^2/\text{s}$), a Copper ($\epsilon_{\text{Cu}}=37140 \text{Ws}^{1/2}\text{m}^{-2}\text{K}^{-1}$) metallic layer as region 1, and water ($\epsilon_w=1580 \text{Ws}^{1/2}\text{m}^{-2}\text{K}^{-1}$) as a liquid sample (for air the value $\epsilon_a=5.5 \text{Ws}^{1/2}\text{m}^{-2}\text{K}^{-1}$ was taking). The theoretical obtained results for higher frequencies [Marín et al., 2010] showed in part (b) of the same figure that the frequency intervals with amplitude ratios greater than unity are awaited to appear in a periodical manner, a typical result for wave phenomena.

A similar result has been reported by Depriester *et al* [Depriester et al., 2005] in the context of the photothermal infrared radiometry technique, and by Caerels *et al* [Caerels et al., 1998], Longuemart *et al* [Longuemart et al., 2002] and Sahraoui *et al* [Sahraoui et al., 2002] using a photopyroelectric (PPE) technique. The measurement configuration is very similar as that described above for the PA method. The analyzed sample is placed in intimate thermal contact with one of the metal coated surfaces of the sensor (usually a polyvinylidene difluoride (PVDF) polymer film with metalized surfaces serving as electrodes or a pyroelectric ceramic crystal (e.g., LiTaO_3), while a periodical intensity modulated light beam impinges on its opposite metalized side, which acts as a light absorber. Following the absorption of light energy, the PE sensor temperature fluctuates periodically at the modulation frequency of the incident beam thereby generating a voltage, whose amplitude at a given frequency can be measured using a Lock-in amplifier. Recently Marin *et al* [Marín et al., 2011] used this last approach in order to explain the increase of the normalized PPE signal above unity for some frequencies.

The good agreement between experiment and theory shows that the described behavior can be explained as caused by a thermal wave interference phenomenon. The thermal wave

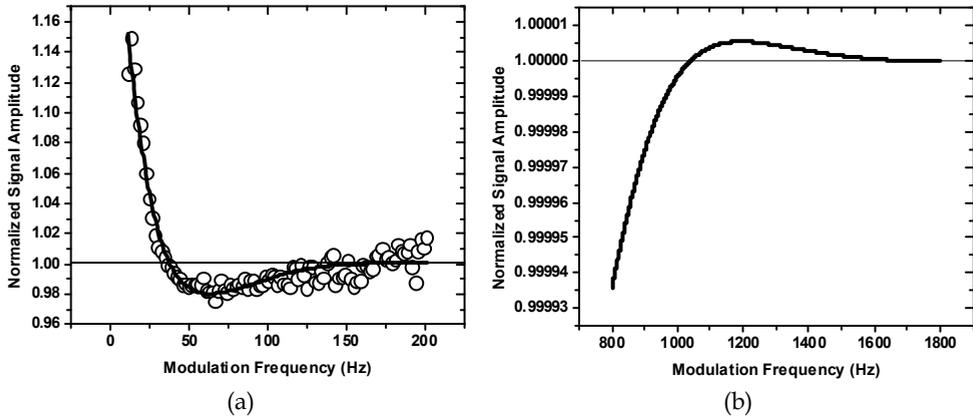


Fig. 2. (a): Normalized signal amplitude as a function of f . Circles: experimental points. Solid curve and (b): Result of theoretical simulation using Eq. (37). Reproduced from [Central Eur. J. Phys, 2010, 8, 4, 634-638].

approach could be helpful not only in the field of PA and PT techniques but it can be also used for the analysis of the phenomenon of heat transfer in the presence of modulated heat sources in multilayer structures, which appear frequently in men’s made devices (for example semiconductor heterostructures lasers and LEDs driven by pulsed, periodical electrical current sources).

4.2 A finite sample exposed to a finite duration heat pulse

Considering a semi-infinite homogeneous medium exposed to a sudden temperature change at its surface at $x=0$ from T_0 to T_1 . For the calculation of the temperature field created by a heat pulse at $t=0$ one has to solve the homogeneous heat diffusion equation (19) with the boundary conditions

$$T(x = 0, t \geq 0) = T_1 ; T(x > 0, t=0) = T_0. \tag{38}$$

The solution for $t>0$ is [Carslaw & Jaeger 1959]:

$$T(x, t) = T_1 + (T_0 - T_1)erf\left(\frac{x}{2\sqrt{at}}\right) \tag{39}$$

where erf is the error function.

Using Fourier’s law (Ec. (9)) one may obtain from the above equation for the heat flow

$$q(x, t) = \frac{\varepsilon(T_1-T_0)}{\sqrt{\pi t}} exp\left(-\frac{x^2}{4\mu_t^2}\right) \tag{40}$$

This expression describes a Gaussian spread of thermal energy with characteristic width

$$\mu_t = 2\sqrt{at} \tag{41}$$

This characteristic distance is the thermal diffusion length (for pulsed excitation) and has a similar meaning as the thermal diffusion length defined by Eq. (23).

If Eq. (40) is scaled to three dimensions one can show that after a time t has elapsed the heat outspread over a sphere of radius μ . Suppose that a spherical particle of radius R is heated in the form described above by a heat pulse at its surface. The particle requires for cooling a time similar to that the necessary for the heat to diffuse throughout its volume. The heat flux at the opposite surface of the particle could be expressed as

$$q(r = 2R) = q_0 \exp \left[- \left(\frac{\tau_c}{t} \right)^2 \right] \quad (42)$$

with q_0 as a time independent constant and a characteristic thermal time constant given by

$$\tau_c = \frac{R^2}{\alpha} \quad (43)$$

This time depend strongly on particle size and on its thermal diffusivity, α [Greffet, 2007; Wolf, 2004; Marín, 2010]. As for most condensed matter samples the order of magnitude of α is 10^{-6} m^2/s , for a sphere of diameter 1 cm one obtain $\tau_c=100$ s and for a sphere with a radius of 6400 km, such as the Earth, this time is of around 10^{12} years, both values compatible with daily experience. But for spheres having diameters between 100 and 1 nm , these times values ranging from about 10 ns to 1 ps, i.e. they are very close to the relaxation times, τ , for which Fourier's Law of heat conduction is not more valid and the hyperbolic approach must be used as well. The above equations enclose the basic principle behind a well established method for thermal diffusivity measurement known as the Flash technique [Parker et al., 1961]. A sample with well known thickness is rapidly heated by a heat pulse while its temperature evolution with time is measured. From the thermal time constant the value of α can be determined straightforwardly. Care must be taken with the heat pulse duration if the parabolic approach will be used accurately. For time scales of the order of the relaxation time the solutions of the hyperbolic heat diffusion equation can differ strongly from those obtained with the parabolic one as has been shown elsewhere [Marín, et al.2005].

Now, coming back to Eq. (40), one can see that the heat flux at the surface of the heated sample ($x=0$) is

$$q(x = 0, t) = \frac{\varepsilon(T_1 - T_0)}{\sqrt{\pi t}} \quad (44)$$

Thus the heat flow is not proportional to the thermal conductivity of the material, as under steady state conditions (see Eq. (23)), but to its thermal effusivity [Bein & Pelzl, 1989]. If two half infinite materials with temperatures T_1 and T_2 ($T_1 > T_2$) touch with perfect thermal contact at $t=0$, the mutual contact interface acquires a contact temperature T_c in between. This temperature can be calculated from Eq. (44) supposing that heat flowing out from the hotter surface equals that flowing into the cooler one, i.e.

$$\frac{\varepsilon_1(T_1 - T_c)}{\sqrt{\pi t}} = \frac{\varepsilon_2(T_2 - T_c)}{\sqrt{\pi t}} \quad (45)$$

or

$$T_c = \frac{\varepsilon_1 T_1 + \varepsilon_2 T_2}{\varepsilon_1 + \varepsilon_2} \quad (46)$$

According to this result, if $\varepsilon_1 = \varepsilon_2$, T_c lies halfway between T_1 and T_2 , while if $\varepsilon_1 > \varepsilon_2$, T_c will be closer to T_1 and if $\varepsilon_1 < \varepsilon_2$, T_c will be closer to T_2 . The Eq. (46) shows that our perception of the

temperature is often affected by several variables, such as the kind of material we touch, its absolute temperature and the time period of the “experiment”, among others (note that the actual value of the contact temperature can be affected by factors such as objects surfaces roughness that have not taking into account in the above calculations). For example, at room temperature wooden objects feels warmer to the rapidly touch with our hands than those made of a metal, but when a sufficient time has elapsed both seem to be at the same temperature. Many people have the mistaken notion that the relevant thermophysical parameter for the described phenomena is the thermal conductivity instead of the thermal effusivity, as stated by Eq. (46). The source of this common mistake is the coincidence that in solids, a high effusivity material is also a good heat conductor. The reason arises from the almost constancy of the specific heat capacity of solids at room temperature explained at the beginning of this section. Using Eq. (13) the Eq. (18) can be written as $\varepsilon^2=Ck$. Then if ε^2 is plotted as a function of k for homogeneous solids one can see that all points are placed close to a straight line [Marín, 2007].

If we identify region 1 with our hand at $T_1=37^\circ\text{C}$ and the other with a touched object at a different temperature, T_2 , the contact temperature that our hand will reach upon contact can be calculated using Eq. (46) and tabulated values of the thermal effusivities. Calculation of the contact temperature between human skin at 37°C and different bodies at 20°C as a function of their thermal effusivities show [E Marín, 2007] that when touching a high thermal conductivity object such as a metal (e.g. Cu), as $\varepsilon_{\text{metal}} \gg \varepsilon_{\text{skin}}$, the temperature of the skin drops suddenly to 20°C and one sense the object as being “cold”. On the other hand, when touching a body with a lower thermal conductivity, e.g. a wood’s object ($\varepsilon_{\text{wood}} < \varepsilon_{\text{skin}}$) the skin temperature remains closest to 37°C , and one sense the object as being “warm”. This is the reason why a metal object feels colder than a wooden one to the touch, although they are both at the same, ambient equilibrium temperature. This is also the cause why human foot skin feels different the temperature of floors of different materials which are at the same room temperature and the explanation of why, when a person enters the cold water in a swimming pool, the temperature immediately felt by the swimmer is near its initial, higher, body temperature [Agrawal, 1999].

In Fig. 3 the calculated contact temperature between human skin at 37°C and bodies of different materials at 1000°C (circles) and 0°C (squares) are represented as a function of their thermal effusivities. One can see that the contact temperature tends to be, in both cases, closer than that of the skin. This is one of the reasons why our skin is not burning when we make a suddenly (transient) contact to a hotter object or freezing when touching a very cold one (despite we fill that the object is hotter or colder, indeed).

Before concluding this subsection the following question merits further analysis. How long can be the contact time, τ_l , so that the transient analysis performed above becomes valid? The answer has to do with the very well known fact that when the skin touches very hot or cold objects a very thin layer of gas (with thickness L) is produced (e.g. water vapour exhaled when the outer layers of the skin are heated or evaporated from ice when it is heated by a warmer hand). This time can be calculated following a straightforward calculation starting from Eq. (44) and Fourier’s law in the form given by Eq. (5). It lauds [Marín, 2008]:

$$\tau_l = \frac{\varepsilon_2 L^2}{\pi k_{\text{gas}}^2} \left(\frac{T_2 - T_c}{T_c - T_1} \right)^2 \quad (47)$$

It is represented in Fig. (4) for different thicknesses of the gas (supposed to be air) layer using for the skin temperature the value $T_2=37^{\circ}\text{C}$.

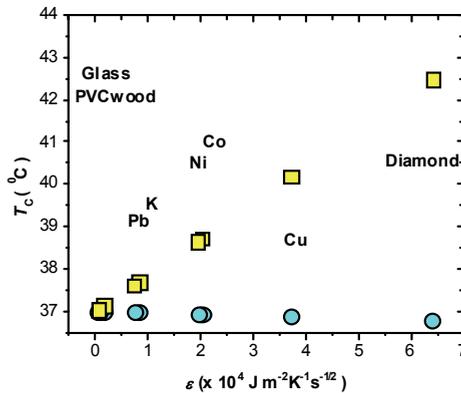


Fig. 3. Contact temperatures as a function of thermal effusivity calculated using Eq. (45) when touching with the hand at 37°C objects of different materials at 0°C (circles) and 1000°C (squares). Reproduced with permission from [Latin American Journal of Physics Education 2, 1, 15-17 (2008)]. Values of the thermal effusivities have been taken from [Salazar, 2003]

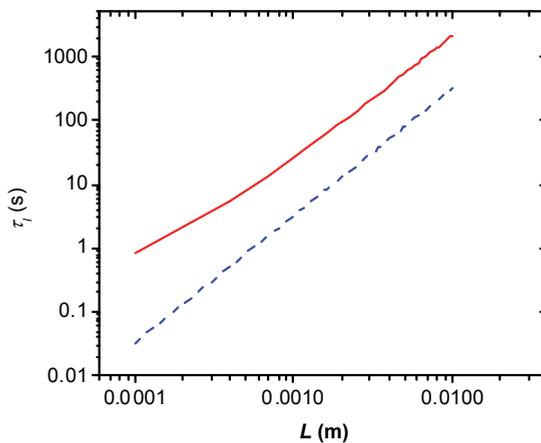


Fig. 4. The time required for the skin to reach values of the contact temperature of 0°C and 100°C without frostbitten or burning up respectively (see text), as a function of the hypothetical thickness of the gas layer evaporated at its surface. The solid and dotted curves correspond to the case of touching a cold (-196°C) and a hot (600°C) object, respectively. Reproduced with permission from [Latin American Journal of Physics Education 2, 1, 15-17 (2008)].

The solid curve corresponds to the case of a cold touched object and the dotted line to that of the hotter ones. For the temperature of a colder object the value $T_1=-196^{\circ}\text{C}$ (e.g. liquid

Nitrogen) was taking. The corresponding limiting contact temperature will be $T_c=0^\circ\text{C}$ (Eq. (46)). In the case of the hot object the value $T_1=600^\circ\text{C}$ ($T_c=100^\circ\text{C}$) was taking. From the figure one can conclude that for gas layer thicknesses smaller than 1mm the time required to heat the skin to 100°C by contact with an object at 600°C is lower than 3s, a reasonable value. On the other hand, for the same layer thickness, liquid Nitrogen can be handled safely for a longer period of time which, in the figure, is about 25 s. These times are of course shorter, because the generated gas layers thicknesses are in reality much shorter than the here considered value.

The above examples try to clarify the role played by thermal effusivity in understanding thermal physics concepts. According to the definition of thermal conductivity, under steady-state conditions a good thermal conductor in contact with a thermal reservoir at a higher temperature extracts from it more energy per second than a poor conductor, but under transient conditions the density and the specific heat of the object also come into play through the thermal effusivity concept. Thermal effusivity is not a well known heat transport property, although it is the relevant parameter for surface heating or cooling processes.

4.3 A finite slab with superficial continuous uniform thermal excitation

The following phenomenon also contradicts common intuition of many people: As a result of superficial thermal excitation the front surface of a (thermally) thick sample reaches a higher equilibrium temperature than a (thermally) thin one [Salazar et al., 2010; Marín et al., 2011]. Consider a slab of a solid sample with thickness L at room temperature, T_0 , is uniformly and continuously heated at its surface at $x=0$. The heating power density can be described by the function:

$$P = \begin{cases} 0 & t < 0 \\ P_0 & t > 0 \end{cases} \quad (48)$$

where P_0 is a constant.

The temperature field in a sample, $T(x, t)$, can be obtained by solving the one-dimensional heat diffusion problem (Eq. (19)) with surface energy losses, i.e., the third kind boundary condition:

During heating the initial condition lauds

$$\Delta T_\uparrow(x, t = 0) = T_\uparrow(x, t = 0) - T_{amb} = 0 \quad (49)$$

and the boundary conditions are:

$$H\Delta T_\uparrow(0, t) - k \left. \frac{\partial \Delta T_\uparrow(x, t)}{\partial x} \right|_{x=0} = P_0 \quad (50)$$

and

$$H\Delta T_\uparrow(L, t) - k \left. \frac{\partial \Delta T_\uparrow(x, t)}{\partial x} \right|_{x=L} = 0 \quad (51)$$

The heat transfer coefficients at the front (heated) and at the rear surface of the sample have been assumed to be the same and are represented by the variable H (see Eq. (7)).

When heating is interrupted, the equations (49) to (60) become

$$\Delta T_\downarrow(x, t = 0) = T_\downarrow(x, t = 0) - T_{amb} = T_{eq} \quad (52)$$

$$H\Delta T_{\downarrow}(0, t) - k \left. \frac{\partial \Delta T_{\downarrow}(x, t)}{\partial x} \right|_{x=0} = 0 \quad (53)$$

and

$$H\Delta T_{\downarrow}(L, t) - k \left. \frac{\partial \Delta T_{\downarrow}(x, t)}{\partial x} \right|_{x=L} = 0 \quad (54)$$

respectively, where T_{eq} is the equilibrium temperature that the sample becomes when thermal equilibrium is reached during illumination, being the initial sample temperature when illumination is stopped.

The solution of this problem is [Valiente et al., 2006]

$$\Delta T_{\downarrow}(x, t) = -\sum_{n=1}^{\infty} A_n e^{-\lambda_n a^2 t} \left[\frac{k\sqrt{\lambda_n}}{h_1} \cos \sqrt{\lambda_n} x + \sin \sqrt{\lambda_n} x \right] \quad (55)$$

and

$$\Delta T_{\uparrow}(x, t) = \frac{-P_0 x + (L+k/H)P_0}{2k+HL} + \sum_{n=1}^{\infty} A_n e^{-\lambda_n a^2 t} \left[\frac{k\sqrt{\lambda_n}}{h_1} \cos \sqrt{\lambda_n} x + \sin \sqrt{\lambda_n} x \right] \quad (56)$$

where $\alpha = a^2$,

$$\lambda_n = \left(\frac{\mu_n}{L} \right)^2 \quad (57)$$

$$\tan \mu = \frac{\frac{2L}{kH}\mu}{L - \left(\frac{\mu}{H} \right)^2} \quad (58)$$

and

$$A_n = -\frac{1}{\|X_n\|} \int_0^L w(\xi) X_n(\xi) d\xi \quad (59)$$

with

$$\|X_n\|^2 = \int_0^L \left[\frac{k\sqrt{\lambda_n}}{h_1} \cos \sqrt{\lambda_n} x + \sin \sqrt{\lambda_n} x \right]^2 dx \quad (60)$$

In order to examine under which condition a sample can be considered as a thermally thin and thick slab the thermodynamic equilibrium limit must be analyzed, i.e. the limit of infinitely long times.

Introducing the Biot Number defined in Eq. (8) and taking $t \rightarrow \infty$ after a straightforward calculation the following results are obtained:

At $x=0$:

$$\Delta T_{\uparrow}(0, \infty) = \frac{P_0}{H} \frac{B_i + 1}{B_i + 2} \quad (61)$$

and

$$\Delta T_{\uparrow}(L, \infty) = \frac{P_0}{H} \frac{1}{B_i + 2} \quad (62)$$

Two limiting cases can be analyzed:

a. Very large Biot number ($B_i \gg 2$):

In this case Eq. (61) becomes

$$\Delta T_{\uparrow}(0, \infty) = \frac{P_0}{H} \quad (63)$$

while from Eq. (62) one has

$$\Delta T_{\uparrow}(L, \infty) = \frac{P_0}{H} \frac{1}{Bi} \quad (64)$$

For their quotient one can write

$$\frac{\Delta T_{\uparrow}(L, \infty)}{\Delta T_{\uparrow}(0, \infty)} = \frac{1}{Bi} \quad (65)$$

There is a thermal gradient across the sample so that the rear sample temperature becomes k/LH times lower than the front temperature. Note that the temperature difference will decrease as the heat losses do, as awaited looking at daily experience.

b. Very small Biot number ($Bi \ll 1$):

In this case both Eq. (61) and Eq. (62) lead to

$$\Delta T_{\uparrow}(0, \infty) = \Delta T_{\uparrow}(L, \infty) = \frac{P_0}{2H} \quad (66)$$

Thus, the equilibrium temperature becomes the same at both sample's surfaces. The sample can be considered thin enough so that there is not a temperature gradient across it. Thus, the condition for a very thin sample is just:

$$Bi \ll 1 \quad (67)$$

With words, following the Biot's number definition given in section 1, the temperature gradient across the sample can be neglected when the conduction heat transfer through its opposite surfaces of the sample is greater than convection and radiation losses.

The results presented above explain the phenomenon that the equilibrium temperature becomes greater for a thicker sample. Denoting the front (heated side) sample's temperature of a thick sample ($Bi \gg 1$) at $t \rightarrow \infty$ as $u_{\uparrow thick}$, and that of a thin ones ($Bi \ll 1$) as $u_{\uparrow thin}$. Their quotient is:

$$\frac{u_{\uparrow thick}(0, \infty)}{u_{\uparrow thin}(0, \infty)} = 2 \quad (68)$$

Here L_{thick} means that this is a thickness for which the sample is thermally thick. This means that after a sufficient long time the front surface temperature of a thick sample becomes two times higher than that for a thin sample. As discussed elsewhere [Marín et al., 2011]

The here presented results can have practical applications in the field of materials thermal characterization. When the thermally thin condition is achieved, the rise temperature becomes [Salazar et al., 2010; Valiente et al., 2006]

$$\Delta T_{\uparrow} = \frac{P_0}{2H} \left[1 - \exp\left(-\frac{t}{\tau_r}\right) \right] \quad (69)$$

while when illumination is interrupted the temperature decreases as

$$\Delta T_{\downarrow} = \frac{P_0}{2H} \exp\left(-\frac{t}{\tau_r}\right) \quad (70)$$

where

$$\tau_r = CL_{thin}/2H \tag{71}$$

and L_{thin} means that the sample thickness is such that it is thermally thin. If the front and/or rear temperatures (remember that both are the same for a thermally thin sample) are measured as a function of time during heating (and/or cooling) the value of τ_r can be determined by fitting to the Eq. (69) (and/or Eq. (70)) and then, using Eq. (71), the specific heat capacity can be calculated if the sample's thickness is known. This is the basis of the so-called temperature relaxation method for measurement of C [Mansanares et al., 1990]. As we see from Eq. (71) precise knowledge of H is necessary.

On the other hand, from Eq. (65) follows that measurement of the asymptotic values of rear and front surface temperatures of a thermally thick sample leads to:

$$r = C \frac{u_1(L,\infty)}{u_1(0,\infty)} = \frac{1}{Bi} = \frac{k}{L_{thick}H} \tag{72}$$

from which thermal conductivity could be determined. Note that the knowledge of the H value is here necessary too.

From Eqs. (71) and (72) the thermal diffusivity value can be determined straightforwardly without the necessity of knowing H , i.e. it can be calculated from the quotient [Marín et al., 2011]:

$$\frac{r}{\tau} = \frac{2k}{CL_{thin}L_{thick}} = 2 \frac{\alpha}{L_{thin}L_{thick}} \tag{73}$$

Fig. 5 shows a kind of Heisler Plot [Heisler, 1947] of the percentile error associated to the thermally thick approximation as a function of the sample's thickness using a typical value of $H=26 \text{ W/m}^2$ [Salazar et al., 2010] for a sample of plasticine ($k=0.30 \text{ W/mK}$) and for a sample of cork ($k=0.04 \text{ W/mK}$).

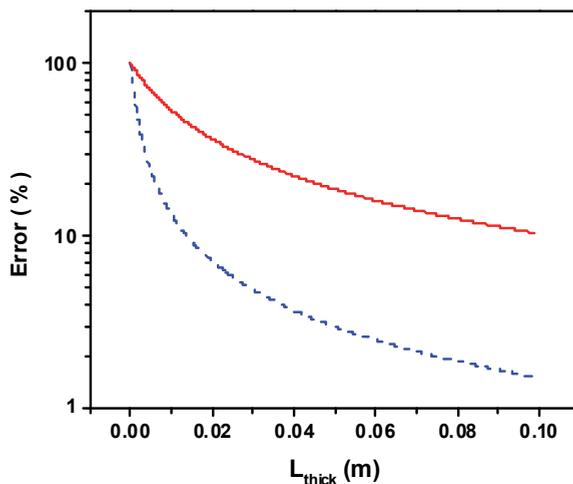


Fig. 5. Heisler Plots for Plasticine (solid line) and Cork (dashed line).

Note that for a 5 cm thick plasticine sample this error becomes about 20 %, while a considerable decrease is achieved for a low conductivity sample such as cork with the same thickness. These errors become lower for thicker samples, but rear surface temperature measurement can become difficult. Thus it can be concluded that practical applications of this method for thermal diffusivity measurement can be achieved better for samples with thermal conductivities ranging between 10^{-2} and 10^{-1} W/mK. Although limited, in this range of values are included an important class of materials such as woods, foams, porous materials, etc. For these the thermally thick approximation can be reached with accuracy lower than 10 % for thicknesses below about 2-3 cm.

Thermal diffusivity plays a very important role in non-stationary heat transfer problems because its value is very sensible to temperature and to structural and compositional changes in materials so that the development of techniques for its measurement is always impetuous. The above described method is simple and inexpensive, and renders reliable and precise results [Lara-Bernal et al., 2011]. The most important achievement of the method is that it cancels the influence of the heat losses by convection and radiation which is a handicap in other techniques because the difficulties for their experimental quantification.

5. Conclusion

Heat conduction in solids under time varying heating is a very interesting and important part of heat transfer from both, the phenomenological point of view and the practical applications in the field of thermal properties characterization. In this chapter a brief overview has been given for different kinds of thermal excitation. For each of them some interesting physical situations have been explained that are often misinterpreted by a general but also by specialized people. The incompatibility of the Fourier's heat conduction model with the relativistic principle of the upper limit for the propagation velocity of signals imposed by the speed of light in vacuum was discussed, with emphasis of the limits of validity this approach and the corrections needed in situations where it is not applicable. Some applications of the thermal wave's analogy with truly wave fields have been described as well as the principal peculiarities of the heat transfer in the presence of pulsed and transient heating. It has been shown that although the four fundamental thermal parameters are related to one another by two equations, each of them has its own meaning. While static and stationary phenomena are governed by parameters like specific heat and thermal conductivity respectively, under non-stationary conditions the thermal effusivity and diffusivity are the more important magnitudes. While the former plays a fundamental role in the case of a body exposed to a finite duration short pulse of heat and in problems involving the propagation of oscillating wave fields at interfaces between dissimilar media, thermal diffusivity becomes the most important thermophysical parameter to describe the mathematical form of the thermal wave field inside a body heated by a non-stationary Source. It is worth to be noticed that the special cases discussed here are not the only of interest for thermal science scientists. There are several open questions that merit particular attention. For example, due to different reasons (e.g. the use of synchronous detection in PT techniques and consideration of only the long-term temperature distribution once the system has forgotten its initial conditions in the transient methods), in the majority of the works the oscillatory part of the generated signal and the transient contribution have been analyzed

separately, with no attention to the combined signal that appears due to the well known fact that when a thermal wave is switched on, it takes some time until phase and amplitude have reached their final values. Nevertheless, it is expected that this chapter will help scientists who wish to carry out theoretical or experimental research in the field of heat transfer by conduction and thermal characterization of materials, as well as students and teachers requiring a solid formation in this area.

6. Acknowledgment

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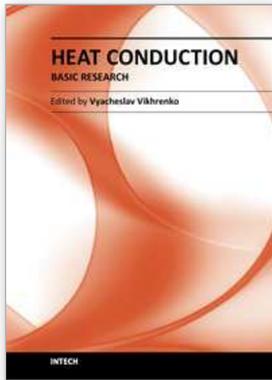
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The content of this book covers several up-to-date approaches in the heat conduction theory such as inverse heat conduction problems, non-linear and non-classic heat conduction equations, coupled thermal and electromagnetic or mechanical effects and numerical methods for solving heat conduction equations as well. The book is comprised of 14 chapters divided into four sections. In the first section inverse heat conduction problems are discussed. The first two chapters of the second section are devoted to construction of analytical solutions of nonlinear heat conduction problems. In the last two chapters of this section wavelike solutions are attained. The third section is devoted to combined effects of heat conduction and electromagnetic interactions in plasmas or in pyroelectric material elastic deformations and hydrodynamics. Two chapters in the last section are dedicated to numerical methods for solving heat conduction problems.

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