

Loop Transfer Recovery for the Grape Juice Concentration Process

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1. Introduction

It is necessary to ensure the quality of concentrated, because it is highly used in the food industry in juices, drinks, sweets, etc. Its application is in full development because it can compete with any other constituent, it is a natural product, and considering that is a very important regional industry, marketing greatly affects the regional economy. Because of this, that it is extremely important to ensure quality and quantity concentrate.

Argentina is one of the principal producers and exporters of concentrated clear grape juices in the world. They are produced mainly in the provinces of San Juan and Mendoza (Argentine Republic) from virgin grape juice and in the major part from sulfited grape juices. The province of San Juan's legislation establishes that a portion of the grapes must be used for making concentrated clear grape juices. This product has reached a high level of penetration in the export market and constitutes an important and growing productive alternative.

An adequate manufacturing process, a correct design of the concentrate plants and an appropriate evaluation of their performance will facilitate optimization of the concentrated juices quality parameters. The plant efficiency is obtained from knowledge of the physics properties of the raw material and products. These properties are fundamental parameters that are used in the designing and calculations on all the equipment used and also in the control process.

The multi-step evaporation (M-SE) is the most important unit operation used in the food industry to concentrate juices of grapes and apples. Even when the main objective of this process is to produce a concentrated product, it should also possess certain organoleptic properties that are critical with respect to its quality and acceptance grade by the customers. Product requirements and the complex characteristics of the process such as non-linear behavior, input and output constraints, time delays and loop interactions justify the use of an advanced control system.

The rheological behavior influences directly the heat transfer coefficient (Pilati, 1998; Rubio, 1998) and therefore its knowledge is essential together with the influence of temperature on

its value. The juices (concentrate and intermediate products) physical properties, such as density, viscosity, boiling point elevation, specific heat and coefficient of thermal expansion, are affected by their solid content and their temperature. For this reason, it is necessary to know the physical properties values, as a function of the temperature and the solids content, during the manufacture process.

The principal solids constituents of clear grape juices are sugars and its concentration affects directly the density, viscosity and refraction index. Tables were developed to relate reducing sugar contents, refractometric values and density of pure solutions, at 20°C, for concentrate ranges from 0% to 85% w/w and sucrose solutions for different range concentrations 0% to 70% and a temperature range from 0 to 100°C (AOAC, 1995).

Barbieri (1980) worked with white concentrated clear grape juice in a falling film multiple effect evaporators. They obtained 18.2, 27.3, 38.6, 48.6 and 64.6°Brix samples. They measured density, viscosity and boiling point elevation as a function of soluble solids concentration and temperature. They presented the results in plots with predictive equations for the properties which were studied.

Di Leo (1988) published density, refraction index and viscosity data for a rectified concentrated grape juice and an aqueous solution of a 1:1 glucose/levulose mixture, for a soluble solids concentrate range from 60 to 71% (in increments of 0.1%) and 20°C.

Pandolfi, (1991) studied physical and chemical characteristics of grape juices produced in Mendoza and San Juan provinces, Argentina. They determined density at 20°C in sulfited grape juices of 20–22°Bx and concentrated grape juices of 68–72 °Bx. They obtained no information on intermediate concentrations or other temperatures.

In general, the clarified juice concentrates have a Newtonian behavior (Ibarz 1993; Rao 1984; Saenz, 1986; Saravacos, 1970), although some authors have found a small pseudoplasticity in the flow of grape concentrates, from the variety Concord (*Vitis labrusca*) for concentrations above 55°Bx. It has been attributed to the presence of some soluble solids, mostly pectins and tartrates (Moressi, 1984; Saravacos, 1970). Other authors consider the juice concentrates as Newtonian, even at high soluble solids concentrations of 60–70°Bx (Barbieri, 1980; Di Leo, 1988; Rao, 1984; Schwartz, 1986).

If we analyze the temperature influence on this product's viscosity, it seems which is directly related with soluble solids concentration; the higher the concentration, the higher is the variation of the viscosity with temperature (Rao, 1984; Saravacos, 1970; Bayindirli, 1992; Crapiste, 1988; Constela, 1989).

Schwartz (1986) determined clear grape juice viscosity at 20, 30, 40 and 50°C, for 30, 40, 50, 60 and 66% soluble solids concentration, but did not publish the experimental data. These authors presented the correlation constants values of the Arrhenius equation for temperature, a potential and an exponential model between viscosity and solids concentration for each temperature studied.

The physical property that represents density change in a material, due to an increase in its temperature at constant pressure, is called the coefficient of thermal expansion. The importance of this parameter can be seen in the effect that density change in the product can have over heat transfer during the process. There is not publish data on the coefficient of the thermal expansion for grape juices and their concentrates. The existing information did not cover all the temperature and concentration ranges that are used in the evaporation process, or else cover to pure sugar solutions, or grape juices of other varieties and/ or originating in other geographical zones.

On the other hand, sensitivity theory, originally developed by Bode (Bode, 1945), has regained considerable importance, due to the recent work developed by many researchers. This research effort has made evident the fundamental role played by sensitivity theory to highlight design tradeoffs and to analyze, qualitatively, control system performance. One of the fields of active research is the analysis of different design strategies from the point of view of sensitivity properties. From this perspective one of the richest strategies is the optimal linear quadratic regulator (LQR). It is well known that the sensitivity of a LQR Loop is always less than one (Anderson, 1971). However, it is also known that when the state is not directly fed back but reconstructed through an observer this property is normally lost. In fact, the situation more general, since the recovery problem appears every time a control design based on state feedback design is implemented through observers.

It has been shown that when the plant is minimum-phase, a properly design Kalman filter provides complete recovery of the input sensitivity achieved by LQR with full state feedback (Doyle, 1979). Either full or partial-order filter may be used. On the other hand it is also known that it is generally impossible to obtain LTR if we use observers for a plant with unstable zeros. An exception to this rule arises in MIMO Systems when input directions are orthogonal to non-minimum phase zero directions (Zhang, 1990).

On the other hand, it is known that the only way to obtain full recovery for a general non-minimum-phase plant is to increase the number of independent measurements. This idea has been suggested in conjunction with the use of reduced-order Kalman filters (Friedland, 1989).

The additional independent measurements are used to modify the structure of the open loop transfer function. The standard LTR procedure is applied and it is implemented combining the resulting full-order Kalman filter with the additional measurements optimally weighted. The idea is obviously to feed back only a subset of the state, for that reason we speak of 'partial' state feedback. The basic approach assumes that all states are available for measurement. However in this paper, it is also shown how to do L_2 - optimization on the amount of recovery of the input sensitivity when a given set of measurements is available. This situation is important since in many additional situations there are limitations regarding which variables can be measured and how many additional sensors can be used. This connects the recovery theory with the issue of additional measurements raised in the context of practical ideas for control design, as illustrated in the control of the inverted pendulum; see (Middleton, 1990). The theory supporting the proposal is built on some important technical results which allow for computing the amount of recovery, as a function of frequency (Zhang, 1990).

2. Process description

Figure 1 shows the input and output streams in a vertical generic effect evaporator with long tubes. The solution to be concentrated circulates inside the tubes, while the steam, used to heat the solution, circulates inside the shell around the tubes. The evaporator operates in parallel mode. The solution to be concentrated and the steam are fed to the first effect by the bottom and by the upper section of the shell, respectively. Later on, the concentrated solution from the first effect is pumped to the bottom of the second effect, and so on until the fourth effect. On the other hand, the vaporized solvent from each effect serves as heater in the next one. Each effect has a baffle in the upper section that serves as a drops splitter for the solution dragged by the vapor. The vapor from the fourth effect is sent to a condenser

and leaves the process as a liquid. Finally, the solution leaving the fourth effect attains the desired concentration and the solution is sent to a storage tank.

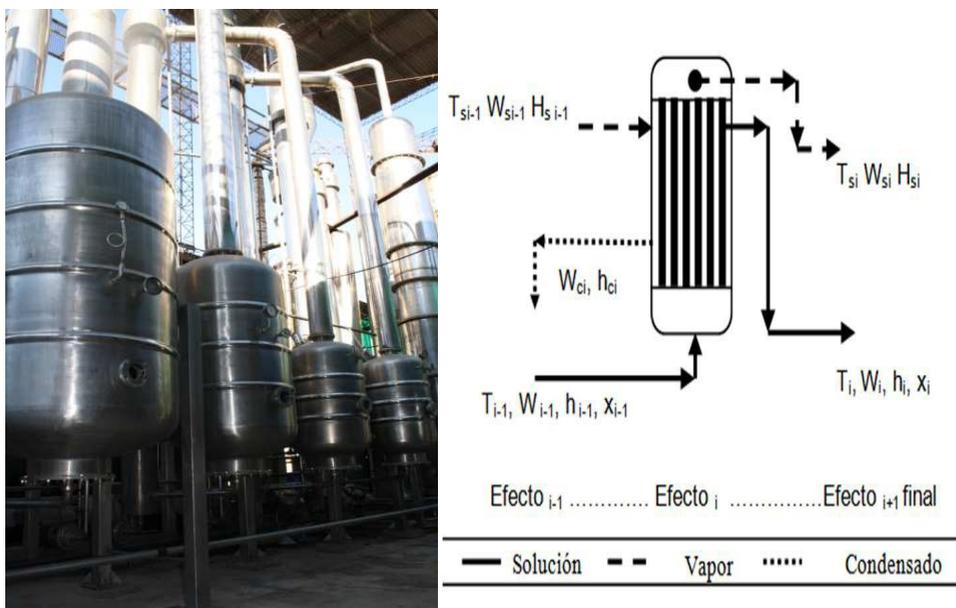


Fig. 1. Photo of evaporator and scheme of effect i in the four-stage evaporator flow sheet. $i = 1, \dots, 4$.

3. Phenomenological model

Stefanov (2005) has developed a rigorous model with distributed parameters based on partial differential equations for a falling-film evaporator, in which the open-loop stability of the model to disturbances is verified. On the other hand, various methods have been proposed in order to obtain reduced-order models to solve such problems (Armaou, 2002; Camacho, 1999; El-Farra, 2003; Zheng, 2002). However, there is not a general framework yet, which assure an effective implementation of a control strategy in a multiple effect evaporator.

In practice, due to a lack of measurements to characterize the distributed nature of the process and actuators to implement such a solution, the control of systems represented by PDE in the grape juice evaporator, is carried out neglecting the spatial variation of parameters and applying lumped systems methods. However, a distributed parameters model must be developed in order to be used as a real plant to test advance control strategies by simulation.

The mathematical model of the evaporator is obtained by application of the mass and energy balances to each effect:

a. Global mass balances in each effect:

$$\frac{dM_i}{dt} = W_{i-1} - W_{si} - W_i \quad (1)$$

in this equations $W_i, i = 1, \dots, 4$, are the solution mass flow rates leaving the effects 1 to 4, respectively. W_0 is the input mass flow rate that is fed to the equipment. $W_{si}, i = 1, \dots, 4$, are the vapor mass flow rates coming from effects 1 to 4, respectively. $dM_i/dt, i = 1, \dots, 4$, represent the solution mass variation with the time for each effect.

b. Solute mass balances for each effect:

$$\frac{d(M_i X_i)}{dt} = W_{i-1} X_{i-1} - W_i X_i \tag{2}$$

where, $X_i, i = 1, \dots, 4$, are the concentrations of the solutions that leave the effects 1 to 4, respectively. X_0 is the concentration of the fed solution.

c. Energy balances:

$$\frac{d(M_i h_i)}{dt} = W_{i-1} h_{i-1} - W_i h_i - W_{si} H_{si} + A_i U_i (T_{si-1} - T_i) \tag{3}$$

where, $h_i, i = 1, \dots, 4$, are the liquid stream enthalpies that leave the corresponding effects, h_0 is the feed solution enthalpy, and $H_{si}, i = 1, \dots, 4$, are the vapor stream enthalpies that leave the corresponding effects and, A_i represents the heat transfer area in each effect. The model also includes algebraic equations. The vapor flow rates for each effect are calculated neglecting the following terms: energy accumulation and the heat conduction across the tubes. Therefore:

$$W_{si} = \frac{U_i A_i (T_{si-1} - T_i)}{H_{si-1} - h_{ci}} \tag{4}$$

For each effect, the enthalpy can be estimated as a function of temperatures and concentrations (Perry, 1997), where:

$$H_{si} = 2509.2888 + 1.6747 \cdot T_{si} \tag{5}$$

$$h_{ci} = 4.1868 \cdot T_{si} \tag{6}$$

$$C_{\rho i} = 0.80839 - 4.3416 \cdot 10^{-3} \cdot X_i + 5.6063 \cdot 10^{-4} \cdot T_i \tag{7}$$

$$h_i = 0.80839 \cdot T_i - 4.3416 \cdot 10^{-3} \cdot X_i T_i + 2.80315 \cdot 10^{-4} \cdot T_i^2 \tag{8}$$

$T_i, i = 1, \dots, 4$, are the solution temperatures in each effect, and T_{s0} is the vapor temperature that enters to the first effect. $T_{si}, i = 1, \dots, 4$, are the vapor temperatures that leave each effect. The heat transfer coefficients are:

$$U_i = \frac{490 \cdot (D^{0.57} \cdot W_{si}^{3.6/L})}{\mu_i^{0.25} \cdot \Delta T_i^{0.1}} \tag{9}$$

where, the Arrhenius type equation for the viscosity is:

$$\mu_i = \mu_0 \cdot e^{\frac{A \cdot X_i}{100 - B \cdot X_i}} \tag{10}$$

$$A = C_1 + \frac{C_2}{T_i} \tag{11}$$

$$B = C_3 + C_4 \cdot T_i \tag{12}$$

The global heat-transfer coefficients are directly influenced by the viscosity and indirectly by the temperature and concentration in each effect. The constants C_1, C_2, C_3, C_4 depend on the type of product to be concentrated (Kaya, 2002).

Although the model could be improved, the accuracy achieved is enough to incorporate a control structure.

4. Standard LTR procedure

4.1 The basic approach

Consider a linear time-invariant system with state characterization is given by:

$$\dot{x} = Ax(t) + Bu(t) + v(t) \tag{13}$$

$$y(t) = Cx(t) \tag{14}$$

where $x(t) \in R^n, u(t) \in R^m, v(t) \in R^m, y(t) \in R^l, A, B$ and C have consistent dimensions. We further assume that is a wide sense stationary process with covariance matrix Q . We then have that the system transfer matrix function is given by:

$$G(s) = C(sI - A)^{-1}B \tag{15}$$

If the state feedback law:

$$u(t) = -Fx(t) \tag{16}$$

is applied, we obtain an input sensitivity function is given by:

$$S_0(s) = |I + H(s)|^{-1} \tag{17}$$

where:

$$H(s) = F(sI - A)^{-1}B \tag{18}$$

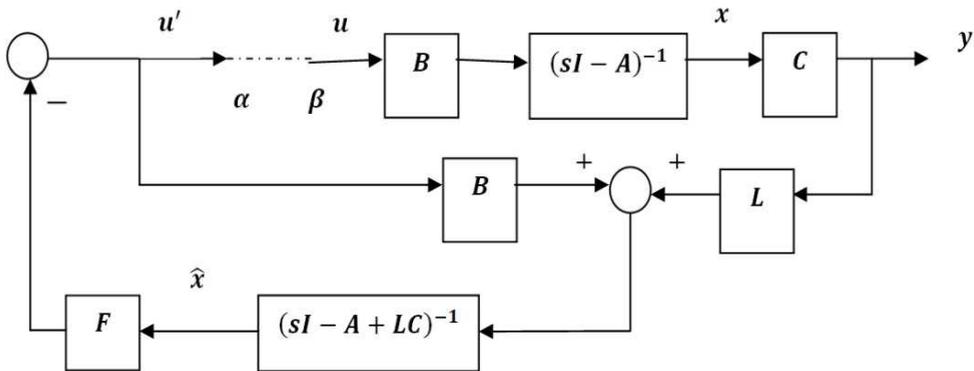


Fig. 2. Optimal LQG Scheme

The complete LQG control system appears in figure 2. The question of loop transfer recovery deals with the problem of keeping the sensitivity given in equation (17) when the

control law is implemented with an observer of the state (Stein, 1987), i.e. the control $\mathbf{u}(t)$ is generated through:

$$\mathbf{u}(t) = -\mathbf{F}\hat{\mathbf{x}}(t) \tag{19}$$

where $\hat{\mathbf{x}}(t)$ is the output of a state observer is given by:

$$\dot{\hat{\mathbf{x}}}(t) = \mathbf{A}\hat{\mathbf{x}}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{L}\{\mathbf{y}(t) - \mathbf{C}\hat{\mathbf{x}}(t)\} \tag{20}$$

If the observer is designed using standard Kalman filter theory, then the filter gain \mathbf{L} satisfies:

$$\mathbf{L} = \Sigma \mathbf{C}^T \tag{21}$$

where Σ is the symmetric nonnegative definite solution of the algebraic Riccati equation:

$$\mathbf{A}\Sigma + \Sigma \mathbf{A}^T - \Sigma \mathbf{C}^T \mathbf{C} \Sigma + \mathbf{Q} = \mathbf{0} \tag{22}$$

In this case the input sensitivity is given by:

$$\mathbf{S}_{obs}(s) = \{\mathbf{I} + [\mathbf{I} + \mathbf{F}(s\mathbf{I} - \mathbf{A} + \mathbf{LC})^{-1}\mathbf{B}]^{-1}\mathbf{F}(s\mathbf{I} - \mathbf{A} + \mathbf{LC})^{-1}\mathbf{LC}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}\}^{-1} \tag{23}$$

After some elementary matrix manipulation, we obtain:

$$\mathbf{S}_{obs}(s) = \mathbf{S}_o(s)\{\mathbf{I} + \mathbf{F}(s\mathbf{I} - \mathbf{A} + \mathbf{LC})^{-1}\mathbf{B}\} \tag{24}$$

Or

$$\mathbf{S}_{obs}(s) = \mathbf{S}_o(s)\{\mathbf{I} + \mathbf{F}\phi[\mathbf{B} - \mathbf{L}(s\mathbf{I} - \mathbf{A} + \mathbf{LC})^{-1}\mathbf{C}\phi\mathbf{B}]\} \tag{25}$$

where:

$$\phi = (s\mathbf{I} - \mathbf{A})^{-1} \tag{26}$$

It becomes then sensible to measure the amount of recovery by the relative sensitivity error (Turan, 1990) given by:

$$\mathbf{E}(s) = \mathbf{S}_o^{-1}\{\mathbf{S}_{obs} - \mathbf{S}_o\} \tag{27}$$

Using the equation (25) we obtain:

$$\mathbf{E}(s) = \mathbf{F}(s\mathbf{I} - \mathbf{A} + \mathbf{LC})^{-1}\mathbf{B} \tag{28}$$

It has been shown (Doyle, 1979) that if $\mathbf{G}(s)$ is a minimum-phase transfer matrix then complete recovery, i.e. $\mathbf{E}(s) = \mathbf{0}$, can be achieved provided that:

- We first augment equation (13) to read:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\{\mathbf{u}(t) + \mathbf{U}\mathbf{w}(t)\} + \mathbf{v}(t) \tag{29}$$

where $\mathbf{w}(t) \in \mathbf{R}^m$ is a wide sense stationary process with covariance matrix $\mathbf{q}^2\mathbf{I}$ and \mathbf{U} is a unitary matrix. We assume that $\mathbf{v}(t)$ and $\mathbf{w}(t)$ are uncorrelated.

- We then solve the Riccati equation (10) substituting \mathbf{Q} by $\mathbf{Q} + \mathbf{q}\mathbf{B}\mathbf{B}^T$
- We finally let $\mathbf{q} \rightarrow \infty$. In this case $\mathbf{L}(\mathbf{q}) \rightarrow \mathbf{q}\mathbf{B}\mathbf{U}$.

The above procedure yields:

$$\mathbf{S}_{obs}(s) \rightarrow \mathbf{S}_o(s) \tag{30}$$

4.2 A factorized form of non-minimum phase plants

It is known that a transfer matrix function $G(s)$ with zeros in C^+ (right-half complex plane) can be described as:

$$G(s) = G_m(s)B_z(s) \tag{31}$$

where $B_z(s)$ is a stable all-pass factor with zeros located at the non-minimum-phase zeros of $G(s)$ and satisfies $B_z(s)B_z^T(s) = I$.

One possible way to build the factorization of equation (31) has been proposed by Enns (Enns, 1984; Zhang, 1990). The main result can be stated as follows.

Lemma 2.1 Given a transfer matrix function $G(s) = C(sI - A)^{-1}B$ with $l \in C^+$ zeros (including multiplicity), z_1, \dots, z_l , there exists a matrix B_m such that:

$$G(s) = C(sI - A)^{-1}B_mB_z(s) \tag{32}$$

where $G(s) = C(sI - A)^{-1}B_m$ is minimum-phase and $B_z(s)$ is an all-pass stable matrix factor. We then have that:

$$B_z(s) = B_{z_1}(s)B_{z_2}(s) \dots B_{z_l}(s) \tag{33}$$

and

$$B_m = B_m^l \tag{34}$$

where, for $i = 1, \dots, l$:

$$B_{z_i}(s) = I - \frac{2\Re\{z_i\}}{s+z_i^*} \eta_i \eta_i^H \tag{35}$$

and

$$B_m^l = B_m^{l-1} - 2\Re\{z_l\} \xi_l \eta_l^H \tag{36}$$

with $B_m^0 = B$, $G_m^i(s) := C(sI - A)^{-1}B_m^i$.

The symbol $\Re\{ \cdot \}$ denotes the real part operator. The vectors η_i and ξ_i are solutions of:

$$\begin{bmatrix} z_i I - A & -B_m^{i-1} \\ -C & 0 \end{bmatrix} \begin{bmatrix} \xi_i \\ \eta_i \end{bmatrix} = 0 \tag{37}$$

□ □ □

We also have the following useful results:

Lemma 2.2 For SISO systems, the sequence $\{B_m^k\}$ can be alternatively computer as:

$$B_m^k = \underbrace{\prod_{i=1}^k \{I - 2\Re\{z_i\}(z_i I - A)^{-1}\}}_{\equiv M_k} B \tag{38}$$

Proof:

A1.- $B_m^0 = B$

A2.- From (36) $B_m^1 = B_m^0 - 2\Re\{z_1\} \xi_1 \eta_1^H$.

But for SISO systems $\eta_1 = \eta_2 = \dots = \mathbf{1}$

and from (37) $\xi_1 = (z_1 I - A)^{-1} B_m^0$

therefore $B_m^1 = \underbrace{\{I - 2\Re\{z_1\}(z_1 I - A)^{-1}\}}_{\equiv M_1} B$

A3.- From (36) $B_m^k = B_m^{k-1} - 2\Re\{z_k\}\xi_k$. But for SISO systems $\eta_k = 1$, and from (37) we have that $B_m^k = \{I - 2\Re\{z_k\}(z_k I - A)^{-1}\}B_m^{k-1}$ then if (37) is satisfied for $k - 1$, the result follows. □ □ □

Corollary:

$$\xi_k = (z_k I - A)^{-1} M_{k-1} B \tag{39}$$

$$M_0 = I \tag{40}$$

$$M_1 = I - 2\Re\{z_1\}(z_1 I - A)^{-1} \tag{41}$$

□ □ □

Theorem 2.1 Consider a non-minimum-phase system (A, B, C) , and its minimum-phase counterpart (A, B_m, C) , with B_m computed according to lemma 2.1. Let L and L_m be the optimal observer gains for these two systems, then $L_m = L$.

Proof: See Zhang & Freudenberg (Zhang, 1990).

4.3 Loop transfer recovery and non-minimum phase plants

Assume now that $G(s)$ is a non-minimum phase plant and that it is factorized as in equation (33). If the standard LTR procedure is applied to recover the input sensitivity, then when $q \rightarrow \infty$ the sensitivity function satisfies:

$$S_{obs}(s) = S_o(s)\{I + E(s)\} \tag{42}$$

where

$$E(s) := F(sI - A)^{-1}\{B - qB_m W(I + qC\phi B_m W)^{-1}C\phi B_m B_z(s)\} \tag{43}$$

then

$$E(s) = F(sI - A)^{-1}\{B - B_m B_z(s)\} \tag{44}$$

It has been also shown that:

$$E(s) = E^l(s) = \sum_{k=1}^l \frac{2\Re\{z_k\}}{s+z_k} F \xi_k \eta_k^H B_z^{k-1}(s) \tag{45}$$

From equation (44) it is evident that for this type of plants the amount of recovery at a frequency w depends on the value of $\|E(jw)\|$, where $\|\cdot\|$ is a suitable norm. As in equation (41), $E(s)$ corresponds to the error of the sensitivity in loop with the LTR observer.

The results of the previous two sections can be appreciated if we consider a SISO system with one zero in $s = z \in R^+$. If the standard LTR procedure is applied we have that:

$$\lim_{q \rightarrow \infty} S_{obs}(s) = S_o(s) \left\{ 1 + \frac{2z}{s+z} H(z) \right\} \tag{46}$$

where

$$H(z) = F(zI - A)^{-1} B \tag{47}$$

One can then notice that if $|H(z)|$ is small, i.e. when the LQR design bandwidth is small in comparison with the magnitude of the C^+ zero, then the recovery is almost complete. This

case will be also the situation for high frequencies since the factor $\frac{2z}{s+z}$ is low-pass filter. The sensitivity resulting from a LQR/LTR applied to a non-minimum phase plant is very significant at low frequencies and decreases as the frequency increases. The inability of the LQR/LTR scheme to recover sensitivity is consistent with some fundamental design tradeoffs for non-minimum phase systems (Freudenberg, 1988).

4.4 LTR procedure with partial state feedback (LTR/PSF)

If we assume that, apart from the system output, there is one or more independent measurements, we can implement a control system originally designed to work with state with state feedback via a mixture of observed and measured states (Aros, 1991). We propose to use the scheme shown in figure 3, where Γ is a diagonal matrix with nonzero diagonal entries for the corresponding measures states. It then becomes clear that $\Gamma = \mathbf{0}$ corresponds to the standard LTR scheme and $\Gamma = \mathbf{I}$ corresponds to the optimal regulator with full state feedback.

A key result to describe the degree of recovery in the non-minimum phase case is given in the next lemma.

Lemma 3.1 Assume that the plant $G(s) = C(sI - A)^{-1}B$ in non-minimum phase with $l \in C^+$ zeros and factorized according to lemma 2.1. Then if we use a LTR/PSF scheme and we let $q \rightarrow \infty$ the sensitivity function is given by:

$$S_{\Gamma}(s) = S_o(s)\{I + \sum_{k=1}^l F(I - \Gamma)\xi_k\eta_k^H W_k(s)\} \tag{53}$$

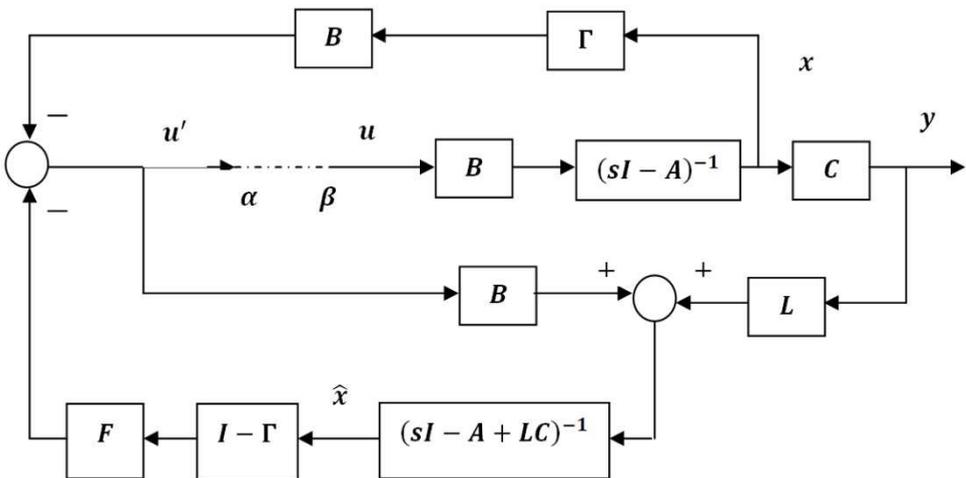


Fig. 3. LTR/PSF scheme.

where:

$$W_k(s) = \frac{2\Re(z_k)}{s+z_k^*} B_z^{k-1}(s) \tag{54}$$

$$B_z^{k-1}(s) = \prod_{i=1}^k B_{z_i}(s) \tag{55}$$

$$B_{z_l}(s) = \frac{-s+z_l}{s+z_l^*} \tag{56}$$

Proof: Straightforward on using lemma 2.1 and equation (45). □ □ □

Remark 3.1 For SISO systems $\eta_k = 1 \forall k$ and consequently (53) simplifies to:

$$S_\Gamma(s) = S_o(s)\{I + \sum_{k=1}^l F(I - \Gamma)\xi_k W_k(s)\} \tag{57}$$

Sufficient and necessary conditions for full recovery in SISO systems are given in the next theorem:

Theorem 3.1 Assume that a plant with transfer function $G(s)$ has $l \in \mathbb{C}^+$ zeros denoted by z_1, \dots, z_l . We apply the LTR/PSF procedure we obtain $E_\Gamma(s) = 0$, if $F(I - \Gamma)$ is orthogonal to ξ_k for $k = 1, 2, \dots, l$ when $q \rightarrow \infty$.

Proof:

- i. Sufficiency: straightforward on inspection of equation (53).
- ii. Necessity: consider equation (57). We first notice that given the fact that functions $W_k(s)$ form a set of linearly independent functions, the only way to nullify the sum $\forall s$ is that the scalar $F(I - \Gamma)\xi_k$ be made equal to zero $\forall k$. □ □ □

The user must then choose (if possible) the matrix Γ to satisfy the orthogonality condition in theorem 3.1. Equivalently, Γ must satisfy:

$$F T \zeta_k = F \xi_k \quad k = 1, 2, \dots, l \tag{58}$$

The computation of Γ is given the next lemma.

Lemma 3.2 Consider a SISO plant as in theorem 3.1. If the LTR/PSF scheme is applied measuring, apart from the output, states $x_1, x_2 \dots, x_\lambda$ then we obtain full recovery of the sensitivity if there exist $\gamma_1, \gamma_2 \dots, \gamma_\lambda$, with $\Gamma = \text{diag}\{\gamma_1, \gamma_2, \dots, \gamma_\lambda, 0, \dots, 0\}$, satisfying:

$$\begin{bmatrix} \alpha_{1_1} & \dots & \alpha_{\lambda_1} \\ \vdots & \ddots & \vdots \\ \alpha_{1_l} & \dots & \alpha_{\lambda_l} \end{bmatrix} \begin{bmatrix} \gamma_1 \\ \vdots \\ \gamma_\lambda \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^n \alpha_{i_1} \\ \vdots \\ \sum_{i=1}^n \alpha_{i_l} \end{bmatrix} \tag{59}$$

where

$$\alpha_{i_k} = f_i \xi_{k_i}; \quad i = 1, 2, \dots, n \tag{60}$$

$$F = [f_1 \quad f_2 \quad \dots \quad f_n] \tag{61}$$

$$\xi_k = [\xi_{k_1} \quad \xi_{k_2} \quad \dots \quad \xi_{k_n}]^T \tag{62}$$

Proof: By straight substitution. □ □ □

Remark 3.2 From equation (53) it appears that full recovery is obtained $\forall \Gamma$ if η_k and ξ_k are orthogonal. This it shows that the LTR/PSF scheme maintains the standard LTR property claimed in Wall (Wall, 1980) and proved in Zhang (Zhang, 1990).

Remark 3.3 A complete analysis of the conditions the existence of none, one or an infinite number of solutions is out of the scope of this work, but some insight can be gained on analyzing the one RHP zero case and L_2 optimization. Both topics will be addressed below.

Theorem 3.2 Consider a scalar plant with transfer function $G(s)$ and one RHP zero located at $s = z$. If the LTR/PSF is applied measuring the input and one additional state variable (not proportional to the output) then $S_\Gamma(s) \rightarrow S_o(s)$ when $q \rightarrow \infty$.

Proof:

From (50) and (46), we can write:

$$\lim_{q \rightarrow \infty} S_\Gamma(s) = S_o(s) \left\{ 1 + \frac{2z}{s+z} H(z) - \frac{2z}{s+z} H'(z) \right\} \tag{63}$$

where $H(s)$ is given in equation (47) and

$$H'(z) = F\Gamma(zI - A)^{-1}B \tag{64}$$

Without loss of generality we can assume that the state variable being fed back is $x_1(t)$. We can thus express Γ as:

$$\Gamma = \gamma\Gamma' \tag{65}$$

where

$$\Gamma' = \text{diag}\{1, 0, \dots, 0\} \tag{66}$$

then full recovery is obtained if γ is chosen to satisfy:

$$\gamma = \frac{F(zI-A)^{-1}B}{F\Gamma'(zI-A)^{-1}B} \tag{67}$$

Remark 3.4 On examining equation (67) we note:

1. If the output is proportional to x_1 there is not solution for γ since then $F\Gamma'(zI - A)^{-1}B = 0$. It certainly agrees with intuition, since nothing can be gained by measuring twice the same variable.
2. There is not solution either when $F\Gamma' = 0$. This case also is intuitive since this situation corresponds to a control law where $x_1(t)$ was not required to be fed back.

These results also apply, mutatis mutandis, to MIMO systems, with the additional complexity which comes from the directionality properties of multivariable systems.

L_2 Optimization

When the designer don't have freedom to choose which state variable can be measured, due either to technical or economical reasons, then the feedback gains for the additional available measurements can be computed by solving an optimization problem. The simplest optimization problem can be posed in L_2 . We examine that case for SISO systems.

Assume first that we measure and feed back the state variables $x_1(t), x_2(t), \dots, x_\lambda(t)$ with gains $[\gamma_1, \gamma_2, \dots, \gamma_\lambda]^T$. Then the L_2 optimization problem consists in finding a vector γ^0 satisfying:

$$\gamma^0 = \arg \min_{\gamma \in \mathbb{R}^\lambda} J(\gamma) \tag{68}$$

where

$$J = \int_0^\infty |E_\Gamma(j\omega)|^2 d\omega \tag{69}$$

$$J = \int_0^\infty \|G_1(j\omega) + \gamma^T G_2(j\omega)\|^2 d\omega \tag{70}$$

with

$$G_1(j\omega) = F(j\omega I - A + LC)^{-1}B \tag{71}$$

$$\gamma^T G_2(j\omega) = F\Gamma(j\omega I - A + LC)^{-1}B \tag{72}$$

This problem has a unique solution if $G(j\omega)$ is not identical to zero. This unique solution is given by:

$$\gamma^0 = -[\int_0^\infty G_2(j\omega)[G_2^*(j\omega)]^T d\omega]^{-1} \Re\{ \int_0^\infty G_1(j\omega)G_2^*(j\omega) d\omega \} \tag{73}$$

for $\lambda = 1$ equation (73) becomes:

$$\gamma^0 = -\frac{\Re\{\int_0^\infty G_1(j\omega)G_2^*(j\omega) d\omega\}}{\int_0^\infty \|G_2(j\omega)\|^2 d\omega} \tag{74}$$

5. Simulatinos results

5.1 Open loop

In figure 4, it shows the response of open loop system, when making a disturbance in one of the manipulated variables such as power flow; it represents the temperature of the first effect and concentration effect of the fourth output.

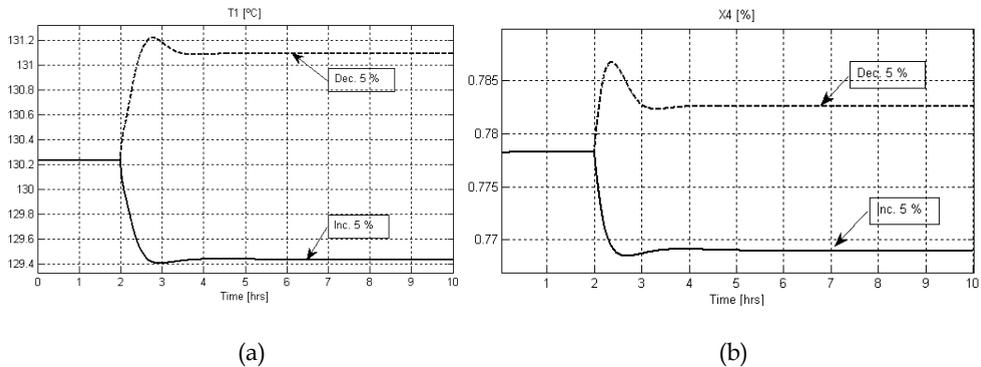


Fig. 4. Behavior of the concentration in the evaporator to a change of a step in the flow of food (up 5% - decrease of 5%)

In figure 5, it shows the response of open loop system, when making a disturbance in the steam temperature is the other manipulated variable; it represents the temperature of the first effect and concentration effect of the fourth output.

In figure 6, it shows the response of open loop system, when performing a step in one of the shocks as the concentration of power; it represents the temperature of the first effect and concentration effect of the fourth output.

In Figure 7, it shows the response of open loop system, when performing a step in the temperature of the food which is the other perturbations of the system; it represents the temperature of the first effect and concentration effect of the fourth output.

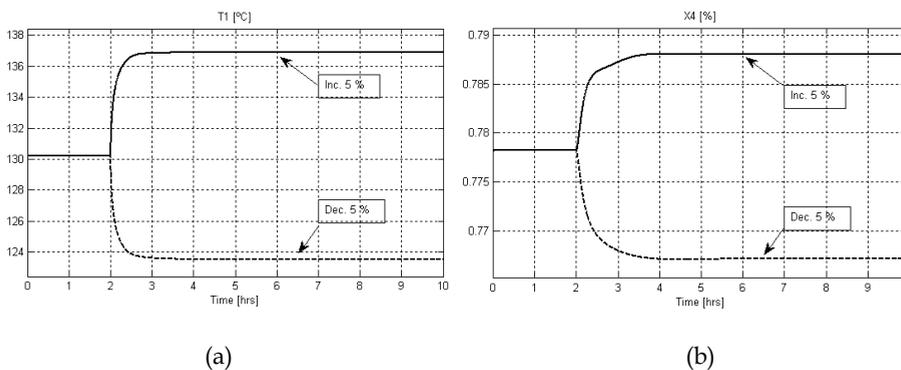


Fig. 5. Behavior of the concentration in the evaporator to a change of a step in the temperature of steam power (up 5% - decrease of 5%)

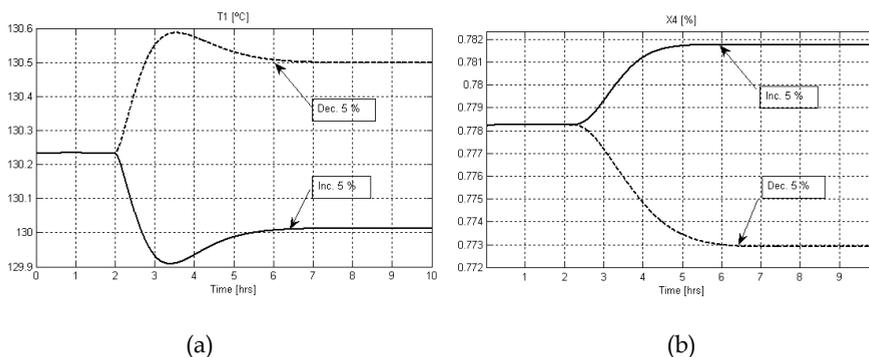


Fig. 6. Behavior of the concentration in the evaporator to a change of a step in the concentration of power (increase of 5% - decrease of 5%)

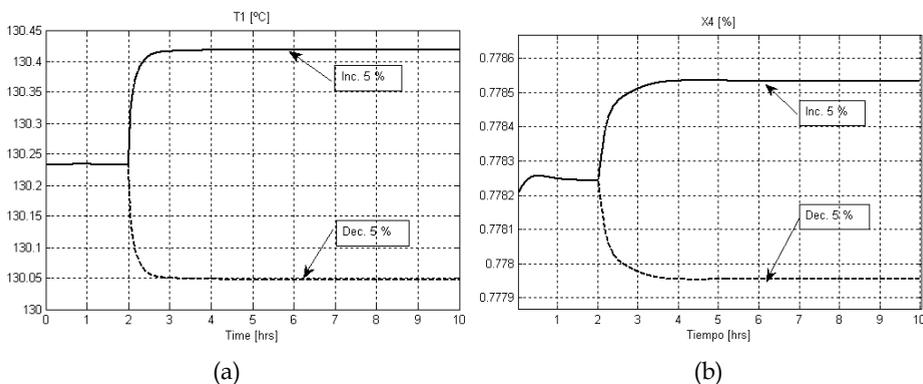
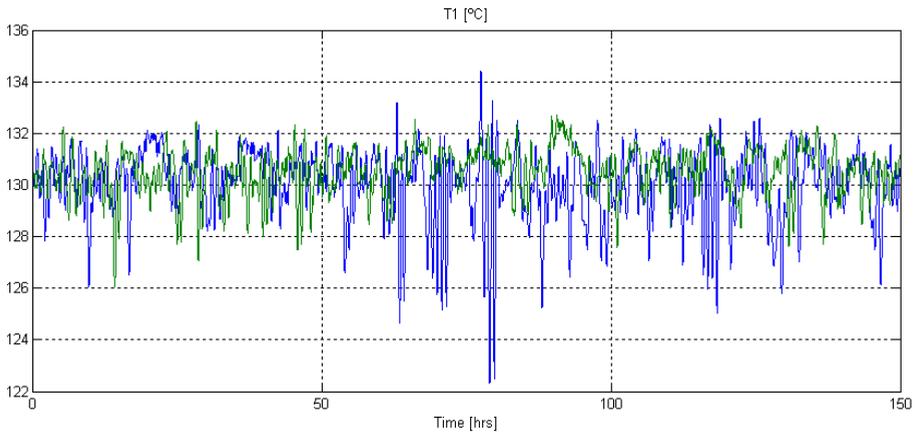


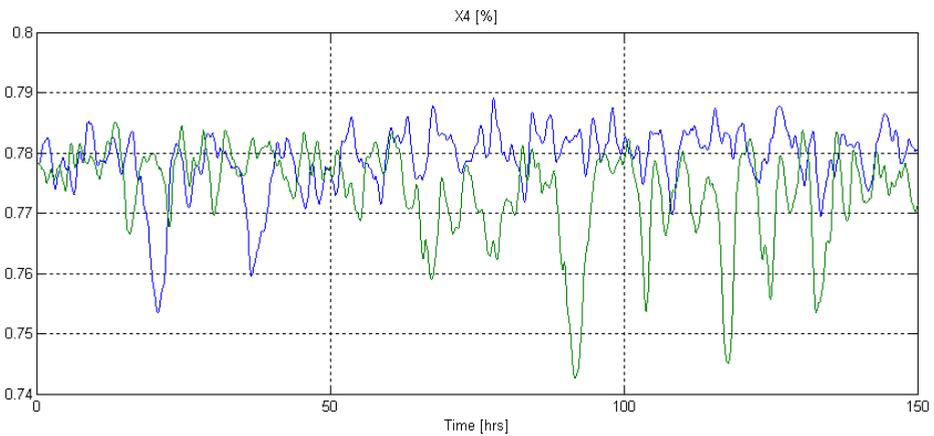
Fig. 7. Behavior of the concentration in the evaporator to a change of a step in the temperature of the input solution (5% increase - decrease of 5%)

5.2 Close loop

Controlled system response for optimal regulator, whereas white noise disturbances, as well as step-like variation to the inlet concentration to 50, and then a step 100 is added to the feed temperature at the entrance.

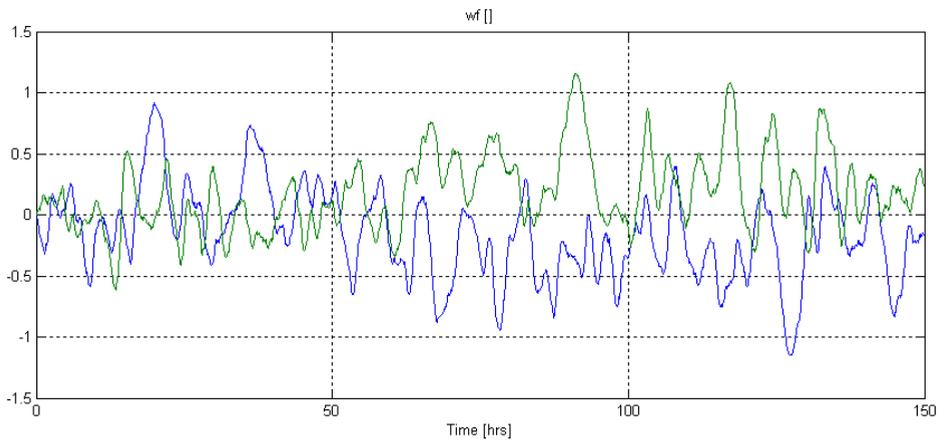


(a)

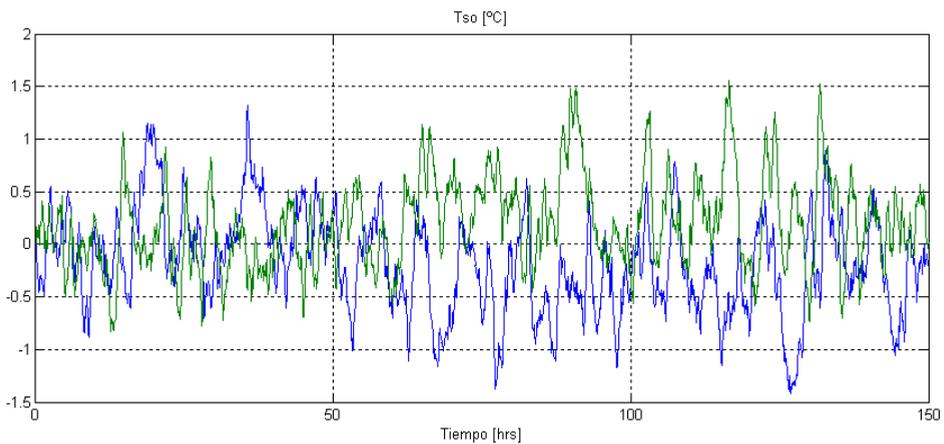


(b)

Fig. 8. Controlled system response to changes in the shocks in type of step and white noise (blue for changes of +5% - green changes -5%)



(a)

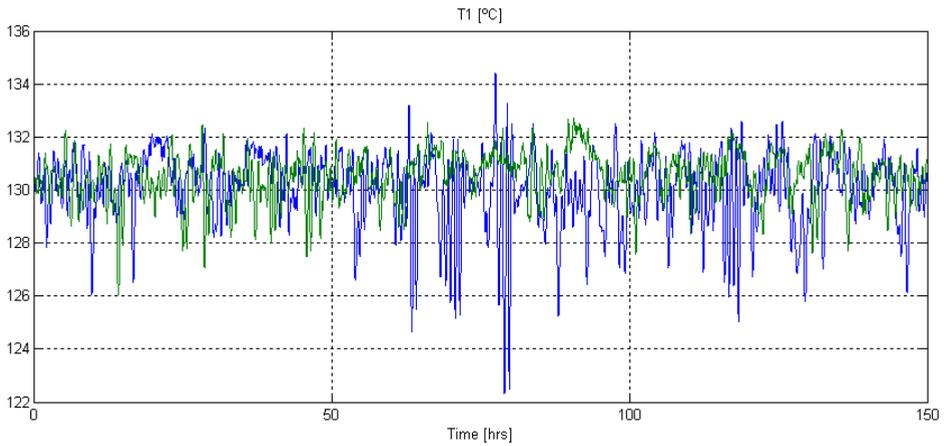


(b)

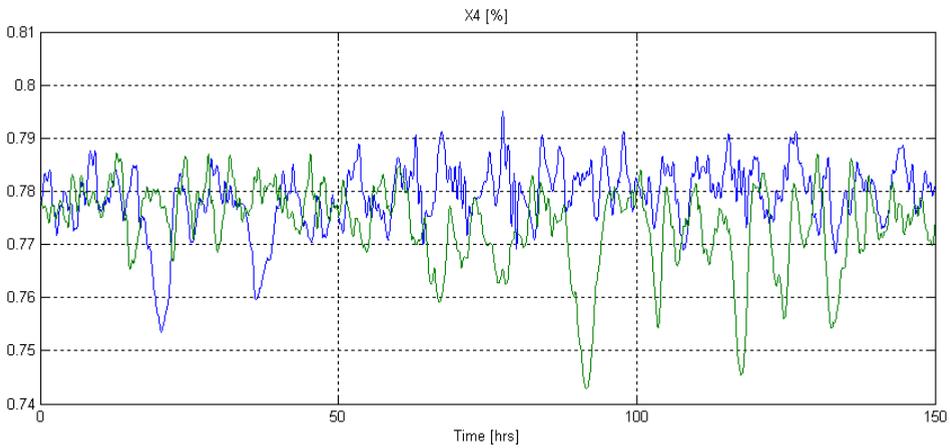
Fig. 9. Efforts to control the controlled system to changes in the shocks in type of step and white noise (blue for changes of +5% - green changes -5%)

5.3 LQG- design

Controlled system response for optimal regulator, whereas white noise disturbances, as well as like step variation of 5% for the inlet concentration to 50 and then to 100 adds a step is 5% of the feed temperature at the entrance.

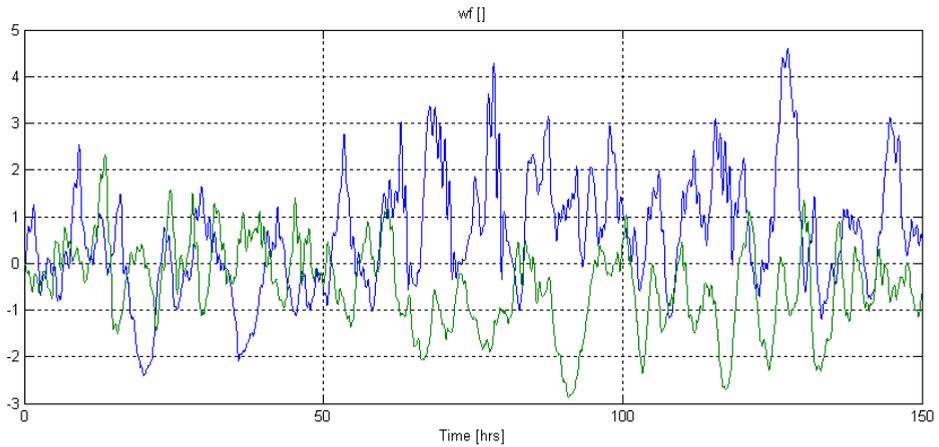


(a)

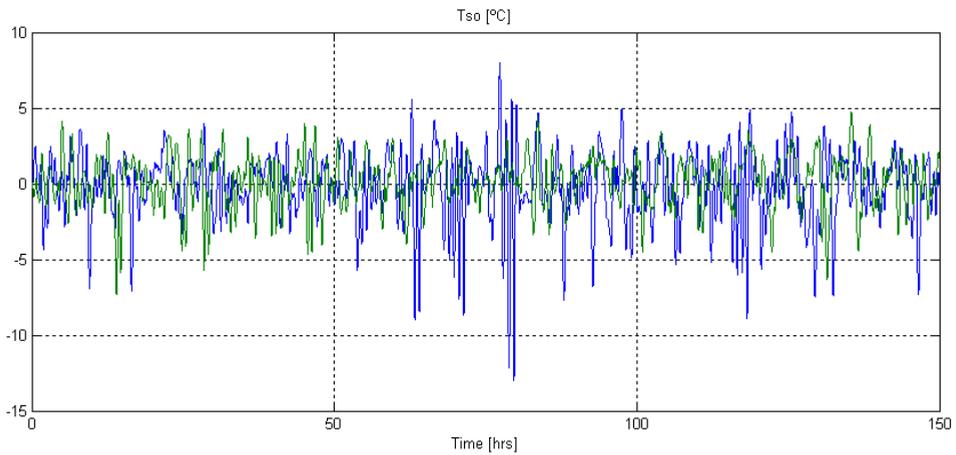


(b)

Fig. 10. LQG controlled system response to changes in the type shocks of step and white noise (blue for changes of +5% - green changes -5%)



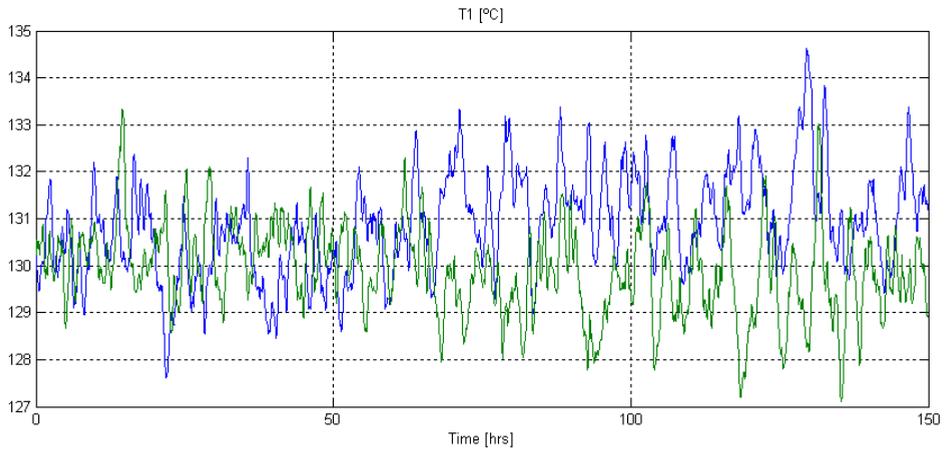
(a)



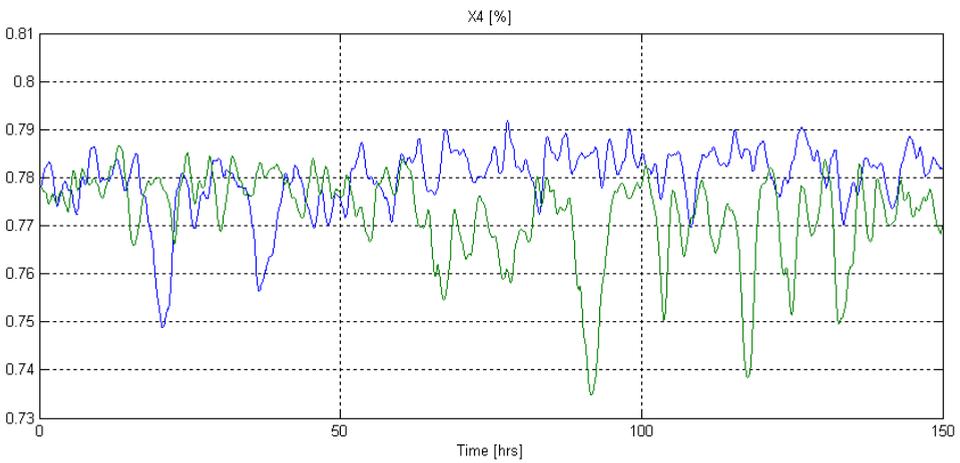
(b)

Fig. 11. Control efforts for the LQG-controlled system to changes in the type shocks of step and white noise (blue for changes of +5% - green changes -5%)

5.4 LQG/LTR



(a)



(b)

Fig. 12. Controlled system response LQG / LTR to changes in the type shocks of step and white noise (blue for changes of +5% - green changes -5%)

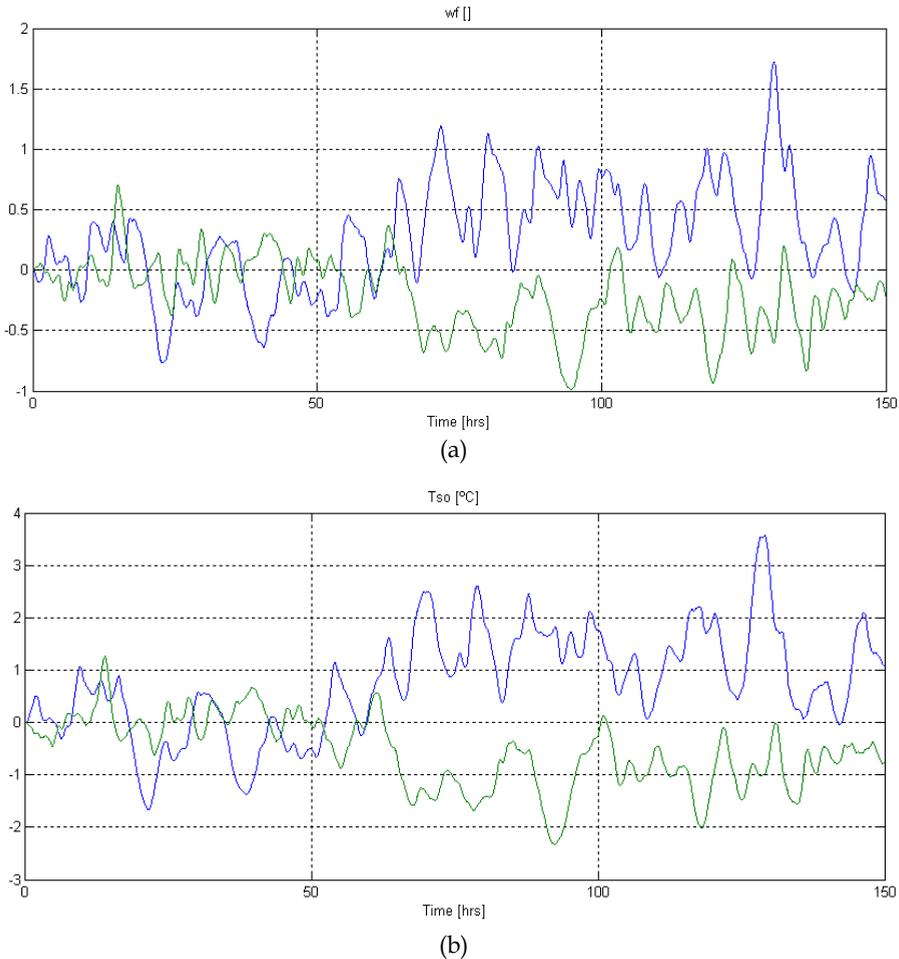


Fig. 13. Efforts to control the controlled system LQG / LTR to changes in the type shocks of step and white noise (blue for changes of +5% - green changes -5%)

6. Conclusions

Looking at the results presented in figures 4 to 7, show that it is appropriate to consider as manipulated variables steam temperature and feed rate of the solution to concentrate, and as measurable disturbance and characteristic of the system to the concentration the solution concentrated and the inlet temperature of food. You can check the analysis of these figures that the evaporation process presents a complex dynamic, high delay, coupling between the variables, high nonlinearities.

From the results shown in figures 8 to 13, on the behavior of the controlled system verifies that the design LQG/LTR has a better performance especially when control efforts are softer. Partly, it validates the robustness of the proposed control system, despite having analyzed only the rejection of disturbances, since these regulatory systems at the show a good response.

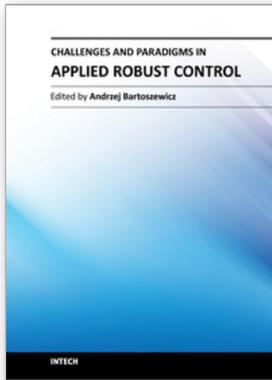
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The main objective of this book is to present important challenges and paradigms in the field of applied robust control design and implementation. Book contains a broad range of well worked out, recent application studies which include but are not limited to H-infinity, sliding mode, robust PID and fault tolerant based control systems. The contributions enrich the current state of the art, and encourage new applications of robust control techniques in various engineering and non-engineering systems.

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