Entransy - a Novel Theory in Heat Transfer Analysis and Optimization

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1. Introduction

It has been estimated that, of all the worldwide energy utilization, more than 80% involves the heat transfer process, and the thermal engineering has for a long time recognized the huge potential for conserving energy and decreasing CO_2 release so as to reduce the global warming effect through heat transfer efficiency techniques (Bergles, 1988, 1997; Webb, 1994; Zimparov, 2002). In addition, since the birth of electronic technology, electricity-generated heat in electronic devices has frequently posed as a serious problem (Arden, 2002; Chein & Huang, 2004), and effective cooling techniques are hence needed for reliable electronic device operation and an increased device lifespan. In general, approaches for heat transfer enhancement have been explored and employed over the full scope of energy generation, conversion, consumption and conservation. Design considerations to optimize heat transfer have often been taken as the key for better energy utilization and have been evolving into a well-developed knowledge branch in both physics and engineering.

During the last several decades and promoted by the worldwide energy shortage, a large number of heat transfer enhancement technologies have been developed, and they have successfully cut down not only the energy consumption, but also the cost of equipment itself. However, comparing with other scientific issues, engineering heat transfer is still considered to be an experimental problem and most approaches developed are empirical or semi-empirical with no adequate theoretical base (Gu et al., 1990). For instance, for a given set of constraints, it is nearly impossible to design a heat-exchanger rig with the optimal heat transfer performance so as to minimize the energy consumption.

Therefore, scientists developed several different theories and methods to optimize heat transfer, such as the constructal theory (Bejan, 1997) and the minimum entropy generation (Bejan, 1982). Then heat transfer processes were optimized with the objective of minimum entropy generation. Based on this method, several researchers (Nag & Mukherjee, 1987; Sahin, 1996; Sekulic et al., 1997; Demirel, 2000; Sara et al., 2001; Ko, 2006) analyzed the influences of geometrical, thermal and flow boundary conditions on the entropy generation in various convective heat transfer processes, and then optimized them based on the premise that the minimum entropy generation will lead to the most efficient heat transfer performance. However, there are some scholars (Hesselgreaves, 2000; Shah & Skiepko, 2004; Bertola & Cafaro, 2008) who questioned whether the entropy generation is the universal irreversibility measurement for heat transfer, or the minimum entropy generation is the general optimization criterion for all heat transfer processes, regardless of the nature of the

applications. For instance, by analyzing the relationship between the efficiency and the entropy generation in 18 heat exchangers with different structures, Shah & Skiepko (2004) demonstrated that even when the system entropy generation reaches the extremum, the efficiency of the heat exchangers can be at either the maximum or the minimum, or anything in between. In addition, the so-called "entropy generation paradox" (Bejan, 1996; Hesselgreaves, 2000) exists when the entropy generation minimization is used as the optimization criterion for counter-flow heat exchanger. That is, enlarging the heat exchange area from zero simultaneously increases the heat transfer rate and improves heat exchanger efficiency, but does not reduce the entropy generation rate monotonously - the entropy generation rate increases at first then decreases. Therefore, it was speculated that the optimization criterion of minimum entropy generation is not always consistent with the heat transfer improvement.

Recently, Guo et al. (2007) introduced the concepts of entransy and entransy dissipation to measure, respectively, the heat transfer capacity of an object or a system, and the loss of such capacity during a heat transfer process. Moreover, Guo et al. (2007) proposed the entransy dissipation extremum and the corresponding minimum entransy dissipation-based thermal resistance as alternative optimization criterions for heat transfer processes not involved in thermodynamic cycles, and consequently, developed the minimum entransy dissipation-based thermal resistance principle to optimize the processes of heat conduction (Guo et al., 2007; Chen et al., 2009a, 2011), convective heat transfer (Meng et al., 2005; Chen et al., 2007, 2008, 2009b), thermal radiation (Cheng & Liang, 2011), and in heat exchangers (Liu et al., 2009; Guo et al., 2010).

This chapter summarizes the entransy theory in heat transfer, such as the definitions of entransy, entransy dissipation and its corresponding thermal resistance with multi-temperatures, the minimum entransy dissipation-based thermal resistance principle for heat transfer, etc., and introduces its applications in heat transfer optimization. Finally, we will make comparisons between entransy optimization and entropy optimization to further examine their applicability to heat transfer optimization in applications of different natures.

2. The origin of entransy

After an intensive study, we found that all transport processes contain two different types of physical quantities due to the existing irreversibility, i.e., the conserved ones and the non-conserved ones, and the loss or dissipation in the non-conserved quantities can then be used as the measurements of the irreversibility in the transport process. Taking an electric system as an example, although both the electric charge and the total energy are conserved during an electric conduction, the electric energy however is not conserved and it is partly dissipated into the thermal energy due to the existence of the electrical resistance. Consequently, the electrical energy dissipation rate is often regarded as the irreversibility measurement in the electric conduction process. Similarly, for a viscous fluid flow, both the mass and the momentum of the fluid, transported during the fluid flow, are conserved, whereas the mechanical energy, including both the potential and kinetic energies, of the fluid is turned into the thermal energy due to the viscous dissipation. As a result, the mechanical energy dissipation is a common measure of irreversibility in a fluid flow process. The above two examples show that the mass, or the electric quantity, is conserved during the transport processes, while some form of the energy associated with them is not. This loss or dissipation of the energy can be used as the measurement of irreversibility in

these transport processes. However, an irreversible heat transfer process seems to have its own particularity, for the heat energy always remains constant during transfer and it doesn't appear to be readily clear what the non-conserved quantity is in a heat transfer process (Chen et al., 2011). Based on the analogy between electrical and heat conductions, Guo et al. (2007) made a comparison between electrical conduction and heat conduction as shown in table 1. It could be found in the table that there is no corresponding parameter in heat conduction for the electrical potential energy in a capacitor, and hence they defined an equivalent quantity, G, that corresponding to the electrical potential energy in a capacitor

$$G = \frac{1}{2}Q_{vh}T \ , \tag{1}$$

which is called entransy. They further derived Eq. (1) according to the similar procedure of the derivation of the electrical potential energy in a capacitor. Entransy was originally referred to as the heat transport potential capacity in an earlier paper by the authors (Guo et al., 2003).

Electrical charge stored in a capacitor	Electrical current	Electrical resistance	Capacitance
Q_{ve} [C]	I [C/s]=[A]	R_e [Ω]	$C_e = Q_{ve}/U_e$ [F]
Thermal energy stored in a body	Heat flow rate	Thermal resistance	Heat capacity
$Q_{vh} = Mc_vT$ [J]	$\dot{Q}_h$ [W]	R_h [K/W]	$C_h = Q_{vh}/T$ [J/K]
Electrical potential	Electrical current density	Ohm's law	Electrical potential energy in a capacitor
U_e [V]	$\dot{q}_e$ [A/m²]	$\dot{q}_e = -K_e\nabla U_e$	$E_e = Q_{ve}U_e/2$
Thermal potential (temperature)	Heat flux density	Fourier's law	Thermal potential in a body
$U_h = T$ [K]	$\dot{q}_h$ [W/m²]	$\dot{q}_h = -K_h\nabla T$	?

Table 1. Analogy between electrical and thermal conductions (Guo et al., 2007)

Entransy represents the heat transfer ability of an object (Guo et al., 2007). It possesses both the nature of "energy" and the transfer ability. If an object is put in contact with an infinite number of heat sinks that have infinitesimally lower temperatures, the total quantity of "potential energy" of heat which can be output is ½$Q_{vh}T$. Biot (1955) suggested a similar concept in the derivation of the differential conduction equation using the variation method. Eckert & Drake (1972) pointed out that "Biot in a series of papers beginning in 1955 formulated from the ideas of irreversible thermal dynamics a variational equivalent of the heat-conduction equation that constituted a thermodynamical analogy to Hamilton's principle in mechanics and led to a Lagrangian formulation of the heat conduction problem in terms of generalized coordinates…, Biot defines a thermal potential $E = \frac{1}{2}\iiint_\Omega \rho cT^2 dV$ ….

The thermal potential E plays a role analogous to a potential energy…". However, Biot did not further explain the physical meaning of thermal potential and its application was not found later except in the approximate solutions of anisotropic conduction problems.

Accompanying the electric charge, the electric energy is transported during electric conduction. Similarly, along with the heat, the entransy is transported during heat transfer too. Furthermore, when a quantity of heat is transferred from a high temperature to a low temperature, the entransy is reduced and some of entransy is dissipated during the heat transport. The lost entransy is called entransy dissipation. Entransy dissipation is an evaluation of the irreversibility of heat transport ability. For instance, let's consider two bodies A and B with the initial temperatures T_A and T_B, respectively. Their initial entransy is

$$G_1 = \frac{1}{2}\left(M_A c_{vA} T_A^2 + M_B c_{vB} T_B^2\right),$$ (2)

where M is the body mass, c_v the specific heat at constant volume, and the subscripts A and B represent the bodies A and B, respectively. When these two bodies contact each other, thermal energy will flow from the higher temperature body to the lower temperature one. After infinite long time, their temperature will be the same and the entransy becomes

$$G_2 = \frac{1}{2}\left(M_A c_{vA} + M_B c_{vB}\right)T_2^2,$$ (3)

where T_2 is the equilibrium temperature

$$T_2 = \frac{M_A c_{vA} T_A + M_B c_{vB} T_B}{M_A c_{vA} + M_B c_{vB}}.$$ (4)

The entransy difference before and after equilibrium is

$$G_1 - G_2 = \frac{1}{2}\frac{M_A M_B c_{vA} c_{vB}\left(T_A - T_B\right)^2}{M_A c_{vA} + M_B c_{vB}} > 0.$$ (5)

Equation (5) proves that the total entransy is reduced after the two bodies getting equilibrium, i.e. there is an entransy dissipation associating with the heat transport.

3. Entransy balance equation and optimization for heat conduction (Guo et al., 2007; Chen et al., 2011)

3.1 Entransy balance equation for heat conduction

In a heat conduction process, the thermal energy conservation equation is expressed as

$$\rho c_v \frac{\partial T}{\partial t} = -\nabla \cdot \dot{q} + \dot{Q},$$ (6)

where ρ is the density, t the time, $\dot{q}$ the heat flow density, and $\dot{Q}$ the internal heat source. Multiplying both sides of Eq. (6) by temperature T gives an equation which can be viewed as the balance equation of the entransy in the heat conduction:

$$\rho c_v T \frac{\partial T}{\partial t} = -\nabla \cdot \left(\dot{q}T\right) + \dot{q} \cdot \nabla T + \dot{Q}T,$$ (7)

that is,

$$\frac{\partial g}{\partial t} = -\nabla \cdot (\dot{q}T) - \phi_h + \dot{g} , \tag{8}$$

where $g=G/V=uT/2$ is the specific entransy, V the volume, u the specific internal energy, $\dot{q}T$ the entransy flow density, $\dot{g}$ the entransy change due to heat source, and ϕ_h can be taken as the entransy dissipation rate per unit volume, expressed as

$$\phi_h = -\dot{q} \cdot \nabla T \tag{9}$$

The left term in either Eq. (7) or (8) is the time variation of the entransy stored per unit volume, consisting of three items shown on the right: the first represents the entransy transferred from one (or part of the) system to another (part), the second term can be considered as the local entransy dissipation during the heat conduction, and the third is the entransy input from the internal heat source. It is clear from Eq. (9) that the entransy is dissipated when heat is transferred from high temperature to low temperature. Thus, heat transfer is irreversible from the viewpoint of entransy, and the dissipation of entransy can hence be used as a measurement of the irreversibility in heat transfer.

3.2 The entransy dissipation extremum principle for heat conduction

Heat transfer optimization aims for minimizing the temperature difference at a given heat transfer rate,

$$\delta(\Delta T) = \delta f(x,y,z,\tau,T,k,q,\rho,c_{v,}\cdots) = 0 , \tag{10}$$

Or maximizing the heat transfer rate at a given temperature difference,

$$\delta(\dot{Q}) = \delta g(x,y,z,\tau,T,k,q,\rho,c_{v,}\cdots) = 0 . \tag{11}$$

In conventional heat transfer analysis, it is difficult to establish the relationship between the local temperature difference, or local heat transfer rate, and the other related physical variables over the entire heat transfer area, so the variational methods in Eqs. (10) and (11) are not practically useable. However, the entransy dissipation in Eq. (9) is a function of the local heat flux and local temperature gradient in the heat transfer area, and thus the variational method will become utilizable if written in terms of the entransy dissipation (Cheng, 2004).

Integrating the balance equation of the entransy Eq. (7) over the entire heat transfer area gives:

$$\iiint\limits_{\Omega} \rho c_v T \frac{\partial T}{\partial t} dV = \iiint\limits_{\Omega} -\nabla \cdot (\dot{q}T) dV + \iiint\limits_{\Omega} \dot{q} \cdot \nabla T dV + \iiint\limits_{\Omega} \dot{Q}T dV . \tag{12}$$

For a steady state heat conduction problem, the left term in Eq. (12) vanishes, i.e.

$$0 = \iiint\limits_{\Omega} -\nabla \cdot (\dot{q}T) dV + \iiint\limits_{\Omega} \dot{q} \cdot \nabla T dV + \iiint\limits_{\Omega} \dot{Q}T dV . \tag{13}$$

If there is no internal heat source in the heat conduction domain, Eq. (13) is further reduced into:

$$\iiint_{\Omega} -\nabla \cdot (\dot{q}T)dV = \iiint_{\Omega} -\dot{q} \cdot \nabla T dV \cdot \tag{14}$$

By transforming the volume integral to the surface integral on the domain boundary according to the Gauss's Law, the total entransy dissipation rate in the entire heat conduction domain is deduced as

$$\Phi_h = \iiint_{\Omega} -\dot{q} \cdot \nabla T dV = \iint_{\Gamma} -\dot{q}T dS = \iint_{\Gamma^+} \dot{q}_{in}T_{in}dS - \iint_{\Gamma^-} \dot{q}_{out}T_{out}dS \cdot \tag{15}$$

where Γ^+ and Γ^- represent the boundaries of the heat flow input and output, respectively. The continuity of the total heat flowing requires a constant total heat flow $\dot{Q}_t$,

$$\dot{Q}_t = \iint_{\Gamma^+} \dot{q}_{in}dS = \iint_{\Gamma^-} \dot{q}_{out}dS \cdot \tag{16}$$

We further define the ratio of the total entransy dissipation and total heat flow as the heat flux-weighted average temperature difference ΔT

$$\Delta T = \frac{\Phi_h}{\dot{Q}_t} = \iint_{\Gamma^+} \frac{\dot{q}_{in}}{\dot{Q}_t} T_{in}dS - \iint_{\Gamma^-} \frac{\dot{q}_{out}}{\dot{Q}_t} T_{out}dS \cdot \tag{17}$$

For one-dimensional heat conduction, Eq. (17) is reduced into $\Delta T = (T_{in} - T_{out})$, exactly the conventional temperature difference between the hot and cold ends. Using the heat flux-weighted average temperature difference defined in Eq. (17) and applying the divergence theorem, a new expression for optimization of a steady-state heat conduction at a given heat flow rate can be constructed as:

$$\dot{Q}_t \delta(\Delta T) = \delta \iiint_{\Omega} -\dot{q} \cdot \nabla T dV = \delta \iiint_{\Omega} k|\nabla T|^2 \, dV = 0 \cdot \tag{18}$$

It shows that when the boundary heat flow rate is given, minimizing the entransy dissipation leads to the minimum in temperature difference, that is, the optimized heat transfer. Conversely, to maximize the heat flow at a given temperature difference, Eq. (18) can be rewritten as:

$$\Delta T \delta \dot{Q}_t = \delta \iiint_{\Omega} -\dot{q} \cdot \nabla T dV = \delta \iiint_{\Omega} \frac{1}{k}|\dot{q}|^2 \, dV = 0 \, , \tag{19}$$

showing that maximizing the entransy dissipation leads to the maximum in boundary heat flow rate.

Likewise, for a steady state heat dissipating process with internal heat source in Eq. (13), the total entransy dissipation rate in the entire heat conduction domain is derived as

$$\Phi_h = \iiint_{\Omega} -\dot{q} \cdot \nabla T dV = \iiint_{\Omega} \dot{Q}T dV - \iint_{\Gamma^-} \dot{q}_{out}T_{out}dS \cdot \tag{20}$$

Since the heat generated in the entire domain will be dissipated through the boundaries, i.e.,

$$\dot{Q}_t = \iiint_{\Omega} \dot{Q}dV = \iint_{\Gamma^-} \dot{q}_{out}dS \cdot \tag{21}$$

Again the heat flux-weighted average temperature is defined as the entransy dissipation over the heat flow rate

$$\Delta T = \frac{\Phi_h}{\dot{Q}_t} = \iiint_\Omega \frac{\dot{Q}}{\dot{Q}_t} T dV - \iint_{\Gamma^-} \frac{\dot{q}_{out}}{\dot{Q}_t} T_{out} dS \,, \tag{22}$$

and thus the optimization of the process is achieved when

$$\dot{Q}_t \delta(\Delta T) = \delta \iiint_\Omega -\dot{q} \cdot \nabla T dV = \delta \iiint_\Omega k |\nabla T|^2 dV = 0 \,, \tag{23}$$

which means that in a heat dissipating process, minimizing the entransy dissipation leads to the minimum averaged temperature over the entire domain.

Based on the results from Eqs. (18), (19) and (23), it can be concluded that the entransy dissipation extremum (EDE) lead to the optimal heat transfer performance at different boundary conditions. This extreme principle can be concluded into the minimum thermal resistance principle defined by entransy dissipation. (Guo et al., 2007)

3.3 Application to a two-dimensional volume-point heat conduction

We will apply our proposed approach to practical cases where heat transfer is used for heating or cooling such as in the so-called volume-point problems (Bejan, 1997) of heat dissipating for electronic devices as shown in Fig. 1. A uniform internal heat source distributes in a two-dimensional device with length and width of L and H, respectively. Due to the tiny scale of the electronic device, the joule heat can only be dissipated through the surroundings from the "point" boundary area such as the cooling surface in Fig. 1, with the opening W and the temperature T_0 on one boundary. In order to lower the unit temperature, a certain amount of new material with high thermal conductivity is introduced inside the device. As the amount of the high thermal conductivity material (HTCM) is given, we need to find an optimal arrangement so as to minimize the average temperature in the device.
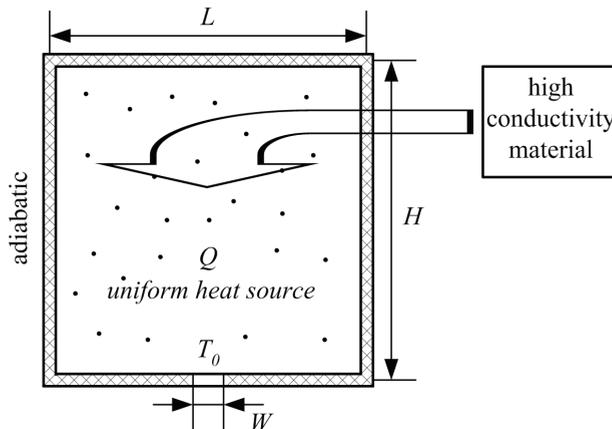

Fig. 1. Two-dimensional heat conduction with a uniformly distributed internal heat source (Chen et al., 2011)

According to the new extremum principle based on entransy dissipation, for this volume-point heat conduction problem, the optimization objective is to minimize the volume-average temperature, the optimization criterion is the minimum entransy dissipation, the optimization variable is the distribution of the HTCM, and the constraints is the fixed amount of the HTCM, i.e.,

$$\iiint_{\Omega} k(x,y) dV = const .$$ (24)

By variational method, a Lagrange function, Π, is constructed

$$\Pi = \iiint_{\Omega} \left(k|\nabla T|^2 + Bk \right) dV .$$ (25)

where the Lagrange multiplier B remains constant due to a given amount of thermal conductivity.

The variation of Π with respect to temperature T gives

$$\iint_{\Gamma} k\nabla T \delta T \cdot \vec{n} dS - \iiint_{\Omega} \nabla \cdot (k\nabla T) \delta T dV = 0 .$$ (26)

Because the boundaries are either adiabatic or isothermal, the surface integral on the left side of Eq. (26) vanishes, that is,

$$\iint_{\Gamma} k\nabla T \delta T \cdot \vec{n} dS = 0 .$$ (27)

Moreover, due to a constant entransy output and a minimum entransy dissipation rate, the entransy input reaches the minimum when

$$\delta \iiint_{\Omega} \dot{Q} T dV = \iiint_{\Omega} \dot{Q} \delta T dV = 0 .$$ (28)

Substituting Eqs. (27) and (28) into Eq. (26) in fact gives the thermal energy conservation equation based on Fourier's Law:

$$\nabla \cdot (k\nabla T) + \dot{Q} = 0 .$$ (29)

This result validates that the irreversibility of heat transfer can be measured by the entransy dissipation rate. The variation of Π with respect to thermal conductivity k gives

$$|\nabla T|^2 = -B = const .$$ (30)

This means that in order to optimize the heat dissipating process, i.e. to minimize the volume-average temperature, the temperature gradient should be uniform. This in turn requires that the thermal conductivity be proportional to the heat flow in the entire heat conduction domain, i.e. the HTCM be placed at the area with the largest heat flux.

As an example, the cooling process in low-temperature environment is analyzed here. For the unit shown in Fig. 1, $L = H = 5$ cm, $Q = 100$ W/cm², $W = 0.5$ cm and $T_0 = 10$ K. The thermal conductivity of the unit is 3 W/(m·K), and that for the HTCM is 300 W/(m·K) occupying 10% of the whole heat transfer area.

Fig. 2(b) shows the distribution of the HTCM according to the extremum principle of entransy dissipation, where the black area represents the HTCM - the same hereinafter. (The implementary steps are as shown in the reference of Chen et al., 2011.) The HTCM with a tree structure absorbs the heat generated by the internal source and transports it to the isothermal outlet boundary - similar in both the shape and function of actual tree roots.
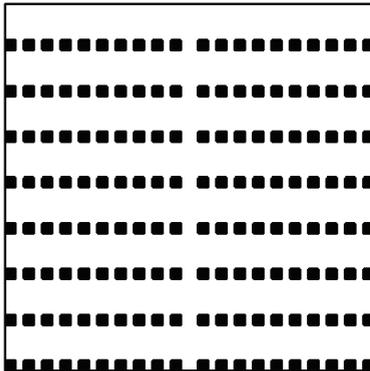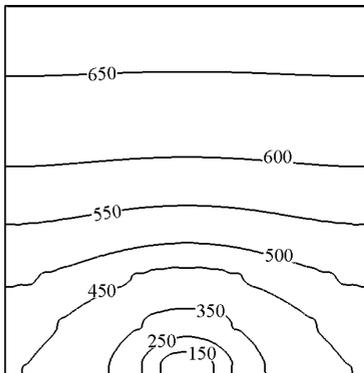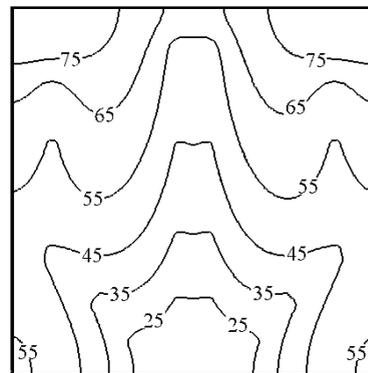

(a) Simple uniform HTCM arrangement
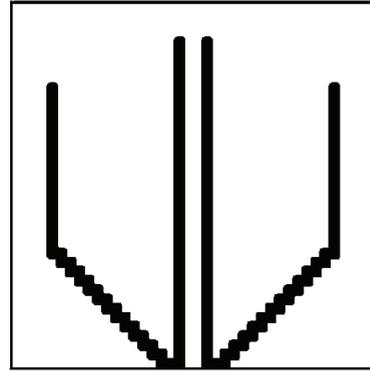
(b) HTCM arrangement using the extremum principle of entransy dissipation

Fig. 2. Different arrangements of HTCM (Chen et al., 2011)

(a) From the uniform HTCM arrangement in Fig. 2(a)

(b) From the optimized arrangement of HTCM in Fig. 2(b)

Fig. 3. The temperature fields obtained from different arrangements of HTCM (Chen et al., 2011)

For a fixed amount of HTCM, Figs. 3(a) and 3(b) compare the temperature distributions between a uniform distribution of HTCM shown in Fig. 2(a), and the optimized distribution in Fig. 2(b) based on the extremum principle of entransy dissipation. The average temperature in the first case is 544.7 K while the temperature in the second optimized case is 51.6 K, a 90.5 % reduction! It clearly demonstrates that the optimization criterion of entransy

dissipation extremum is highly effective for such applications. Furthermore, as shown in Fig. 3(b), the temperature gradient field is also less fluctuating in the optimized case.

In addition, based on the concept of the entransy dissipation rate, we (Chen et al. 2009a) introduced the non-dimensional entransy dissipation rate and employed it as an objective function to analyze the thermal transfer process in a porous material.

4. Entransy balance equation and dissipation for convective heat transfer

4.1 Entransy balance equation for convective heat transfer (Cheng, 2004)

For a convection problem, the energy equation is

$$\rho\left[c_p \frac{DT}{Dt} + P \frac{D}{Dt}\left(\frac{1}{\rho}\right) \right] = \phi_\mu - \nabla \cdot \dot{q} + \dot{Q} \, , \tag{31}$$

where P, t and ϕ_μ are the pressure, time, and viscous dissipation, respectively. Multiply Eq. (31) by T and by some subsequent derivation (Cheng, 2004), there is

$$\rho\left(\frac{\partial g}{\partial t} + U \cdot \nabla g \right) = -\nabla \cdot (\dot{q} T) + \dot{Q} T + \phi_\mu T + \dot{q} \cdot \nabla T \, , \tag{32}$$

where $g = c_p T^2/2$, is the entransy per unit volume of the fluid. On the left, the 1st term is time variation of entransy, and the 2nd term is entransy variation accompanying fluid flow; on the right, the 1st term is entransy variation due to boundary heat exchange, the 2nd term is the entransy change due to heat source, the 3rd term is the entransy variation due to dissipation and the 4th term is the entransy dissipation. It is clear from Eq. (32) that the heat transport ability is reduced when heat is transferred from high temperature to low temperature.

4.2 The entransy dissipation extremum principle for convective heat transfer

Similar to the derivation of EDE principle in heat conduction, for a steady-state convective heat transfer process of a fluid with constant boundary heat flux and ignoring the heat generated by viscous dissipation, integrating Eq. (32), and transforming the volume integral to the surface integral on the domain boundary yields:

$$Q_0 \delta\left(T_w - \frac{T_{in} + T_{out}}{2} \right) = \delta \iiint\limits_\Omega k |\nabla T|^2 \, dV \, . \tag{33}$$

Since the inlet and outlet temperatures, T_{in} and T_{out}, of the fluid are fixed for a given boundary heat flux, Equation (33) means that, the minimum entransy dissipation in the domain is corresponding to the minimum boundary temperature, i.e. the minimum boundary temperature difference.

Similarly, from (32) the maximum entransy dissipation is obtain,

$$\left(T_w - T_{in} \right) \delta Q_0 - \frac{Q_0}{\rho c_p} \delta Q_0 = \delta \iiint\limits_\Omega k |\nabla T|^2 \, dV \, . \tag{34}$$

Because Eq. (33) illustrates that for the range of the boundary heat flux from 0 to its maximum value, $\rho V c_p (T_w\text{-}T_{in})$, the entransy dissipation increases monotonically with the

increasing boundary heat flux, that means maximum entransy dissipation results in the maximum boundary heat flux. Equation (33) together with Eq. (34) is called entransy dissipation extreme (EDE) principle in convective heat transfer. Thus, the entransy dissipation can be used to optimize the flow field with given viscosity dissipation so that the heat transfer rate can be increased most with given cost of pressure lose (Guo et al., 2007; Meng et al. 2005, Chen et al. 2007).

4.3 Application to laminar heat transfer (Meng et al., 2005)

For a steady laminar flow, the viscosity dissipation is

$$
\phi_m = \mu \left[\begin{array}{l} 2\left(\dfrac{\partial u}{\partial x}\right)^2 + 2\left(\dfrac{\partial v}{\partial y}\right)^2 + 2\left(\dfrac{\partial w}{\partial z}\right)^2 \\ + \left(\dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x}\right)^2 + \left(\dfrac{\partial u}{\partial z} + \dfrac{\partial w}{\partial x}\right)^2 + \left(\dfrac{\partial v}{\partial z} + \dfrac{\partial w}{\partial y}\right)^2 \end{array} \right], \tag{35}
$$

where u, v and w are the velocity components along x, y and z directions, respectively. The mechanical work maintaining the fluid flow equals to the integral of the viscous dissipation function over the whole domain,

$$
\Pi_{\phi m} = \iiint_{\Omega} \phi_m dV . \tag{36}
$$

The optimal flow field was established by configuring a Lagrange functional that includes the objective and constraint functions. The established Lagrange function is,

$$
\Pi = \iiint_{\Omega} \left[k\nabla T \cdot \nabla T + C_0 \phi_m + A\left(\nabla \cdot k\nabla T - \rho c_p U \cdot \nabla T\right) + B\nabla \cdot \rho U \right] dV , \tag{37}
$$

where A, B and C_0 are the Lagrange multipliers. Because of the different types of the constraints, A and B vary with position, while C_0 remains constant for a given viscous dissipation. The first term on the right is the entransy dissipation, the second is the constraint on prescribed viscous dissipation, the third and forth terms are the constraints on energy equation and continuum equation. The variational of Eq. (37) with respect to velocity U offers

$$
\mu \nabla^2 U + \frac{\rho c_p}{2C_0} A\nabla T + \frac{1}{2C_0} \nabla B = 0 . \tag{38}
$$

The variational of Eq. (37) with respect to temperature T is

$$
-\rho c_p U \cdot \nabla A = \nabla \cdot k\nabla A - 2\nabla \cdot k\nabla T , \tag{39}
$$

and the boundary conditions of the variable A are $A_b = 0$ for given boundary temperatures, and $(\partial A/\partial n)_b = 2(\partial T/\partial n)_b$ for given boundary heat flow rates.
Comparing the momentum equations and Eq. (38), gives the following relations

$$
B = -2C_0 P , \tag{40}
$$

$$F = C_{\Phi} A \nabla T + \rho U \cdot \nabla U \,, \tag{41}$$

where C_{Φ} is related to the viscous dissipation as

$$C_{\Phi} = \frac{\rho c_p}{2C_0} \,. \tag{42}$$

Substituting Eqs. (41) and (42) into the momentum equation gives

$$\rho U \cdot \nabla U = -\nabla P + \mu \nabla^2 U + \left(C_{\Phi} A \nabla T + \rho U \cdot \nabla U \right) \,. \tag{43}$$

This is the Euler's equation, i.e. field synergy equation, governing the fluid velocity and temperature fields with the EDE principle during a convective heat transfer process. Equation (43) essentially the momentum equation with a special additional volume force defined in Eq. (41), by which the fluid velocity pattern is adjusted to lead to an optimal temperature field with the extremum of entransy dissipation during a convective heat transfer process.

For a given C_{Φ}, solving Eqs. (39) and (43), the continuity equation and the energy equation in combination result in the optimal flow field with the extremum of entransy dissipation with a specific viscous dissipation and fixed boundary conditions. Fig. 4 shows a typical numerical result of the cross-sectional flow field (Re=400, C_{Φ}=-0.01). Compared with the fully-developed laminar convection heat transfer in a circular tube ((fRe)$_s$=64, Nu$_s$=3.66), the flow viscous dissipation is increased by 17%, the Nusselt number is increased by 313% in the case of Fig. 4. Meanwhile, the numerical analysis shows that multiple longitudinal vortex flow is the optimal flow pattern for laminar flow in tube. With the guide of this optimal flow field, the discrete double-inclined ribs tube (DDIR-tube) is designed to improve heat transfer in laminar flow. Fig. 5 is a typical cross-sectional vortex flow of a DDIR-tube and experiments have demonstrated the DDIR-tube have better heat transfer performance but lower resistance increase (Meng et al., 2005).
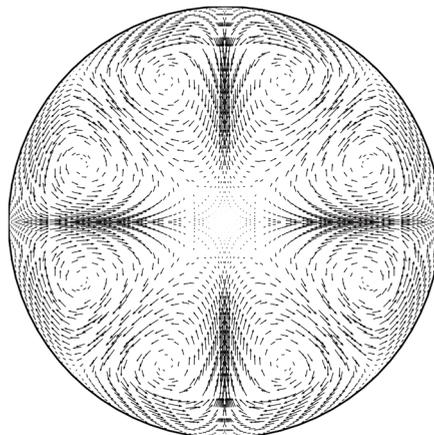

Fig. 4. Optimum flow field of laminar heat transfer in circular tube (Re=400) (Meng et al., 2005)
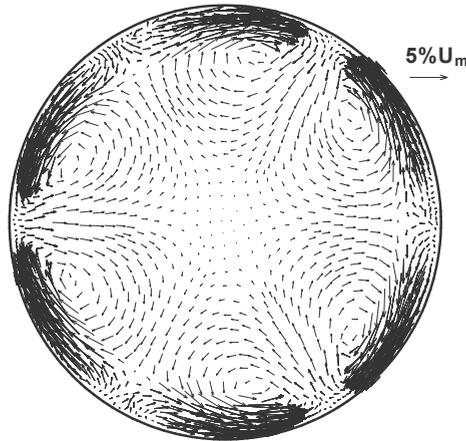
Fig. 5. Numerical solution of cross-sectional flow fields in the DDIR-tube (Meng et al., 2005)

4.4 Application to turbulent heat transfer (Chen et al., 2007)

For turbulent heat transfer, the entransy dissipation function is modified as:

$$\phi_{ht} = k_{eff}|\nabla T|^2 , \tag{44}$$

where k_{eff} is the effective thermal conductivity during turbulent heat transfer.

The optimization objective of turbulent heat transfer is to find an optimal velocity field, which has the extremum of entransy dissipation for a given decrement of the time-averaged kinetic energy, i.e. a fixed consumption of pumping power. Meanwhile, the flow is also constrained by the continuity equation and the energy equation. All these constraints can also be removed by using the Lagrange multipliers method to construct a function

$$\Pi_t = \iiint_\Omega \left[k_{eff}\nabla T \cdot \nabla T + C_0\phi_{mt} + A\left(\nabla \cdot k_{eff}\nabla T - \rho c_p U \cdot \nabla T\right) + B\nabla \cdot \rho U \right] dV . \tag{45}$$

where ϕ_{mt} is the decrement function of the mean kinetic energy

$$\phi_{mt} = (\mu + \mu_t)\left[\begin{array}{l} 2\left(\dfrac{\partial u}{\partial x}\right)^2 + 2\left(\dfrac{\partial v}{\partial y}\right)^2 + 2\left(\dfrac{\partial w}{\partial z}\right)^2 \\ + \left(\dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x}\right)^2 + \left(\dfrac{\partial u}{\partial z} + \dfrac{\partial w}{\partial x}\right)^2 + \left(\dfrac{\partial v}{\partial z} + \dfrac{\partial w}{\partial y}\right)^2 \end{array} \right]. \tag{46}$$

The decrement of the mean kinetic energy equals to the viscous dissipation by the viscous forces, plus the work of deformation of the mean motion by the turbulence stresses, which transform the mean kinetic energy to the turbulence-energy. The turbulent viscosity, μ_t, which is a function of the velocity, can be calculated using the Prandtl's mixing-length model, the Prandtl-Kolmogorov assumption or the k-ε model. However, the Prandtl-Kolmogorov assumption and the k-ε model require additional differential equations for the turbulent kinetic energy (k) and the turbulent dissipation rate (ε) to calculate the turbulent

viscosity, which severely complicate obtaining the variation of Eq. (45). Thus, a single algebraic function, which has been validated and used for indoor air flow simulation (Chen & Xu, 1998), is used to calculate the turbulent viscosity:

$$\mu_t = 0.03874 \rho |U| l \,, \tag{47}$$

where, l is the distance to the nearest wall. As with other zero-equation turbulent models, Eq. (47) is not very theoretically sound, but yields some reasonable results for turbulent flows.

The variational of Eq. (45) with respect to temperature T is:

$$-\rho c_p U \cdot \nabla A = \nabla \cdot k_{eff} \nabla A - 2\nabla \cdot k_{eff} \nabla T \,. \tag{48}$$

The variational of Eq. (45) with respect to velocity component u is:

$$\begin{aligned}
&\frac{1}{2C_0}\frac{\partial B}{\partial x} + \nabla \cdot \left(\mu_{eff}\nabla u\right) + \frac{\rho c_p A}{2C_0}\frac{\partial T}{\partial x} - \frac{\Gamma u}{2C_0 \Pr_t}\left[|\nabla T|^2 + A\nabla \cdot (\nabla T)\right] - \frac{\phi_{mt}\Gamma u}{2\mu_{eff}} \\
&- \frac{A\Gamma}{2C_0 \Pr_t}\frac{\partial T}{\partial x}\left[\frac{u}{l}\frac{\partial l}{\partial x} - \left(u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial x} + w\frac{\partial w}{\partial x}\right)|U|^{-2}u + \frac{\partial u}{\partial x}\right] \\
&- \frac{A\Gamma}{2C_0 \Pr_t}\frac{\partial T}{\partial y}\left[\frac{u}{l}\frac{\partial l}{\partial y} - \left(u\frac{\partial u}{\partial y} + v\frac{\partial v}{\partial y} + w\frac{\partial w}{\partial y}\right)|U|^{-2}u + \frac{\partial u}{\partial y}\right] \\
&- \frac{A\Gamma}{2C_0 \Pr_t}\frac{\partial T}{\partial z}\left[\frac{u}{l}\frac{\partial l}{\partial z} - \left(u\frac{\partial u}{\partial z} + v\frac{\partial v}{\partial z} + w\frac{\partial w}{\partial z}\right)|U|^{-2}u + \frac{\partial u}{\partial z}\right] \\
&+ \left(\frac{\partial \mu_{eff}}{\partial x}\frac{\partial u}{\partial x} + \frac{\partial \mu_{eff}}{\partial y}\frac{\partial v}{\partial x} + \frac{\partial \mu_{eff}}{\partial z}\frac{\partial w}{\partial x}\right) + \frac{1}{2C_0}\nabla \cdot \left(\frac{A\Gamma u}{\Pr_t}\nabla T\right) = 0
\end{aligned} \tag{49}$$

where $\Gamma = 0.03874\rho l |U|^{-1}$, $\Pr_t$ is the turbulent Prantdl number with the value of 0.9. The variationals of Eq. (45) with respect to the velocity components v and w are similar to Eq. (49). There are four unknown variables and four governing equations including Eqs. (48), (49), the continuity equation and the energy equation, so the unknown variables can be solved for a given set of boundary conditions. Meanwhile, the flow must also satisfy the momentum equation,

$$\rho U \cdot \nabla U = -\nabla P + \mu_{eff}\nabla^2 U + F \,. \tag{50}$$

Comparison with Eq. (50), Eq. (49) is actually a momentum equation with a special additional volume force, which is referred to as the field synergy equation for turbulent heat transfer. For a given set of boundary conditions, the optimal velocity field, which has a larger heat transfer rate than any other field flow, can be obtained by solving this field synergy equation.

As an example, flow in a parallel plate channel is studied to illustrate the applicability of the field synergy equation. For simplicity, a repeated segment with the height of 20 mm and the length of 2.5 mm was chosen. Water flowing between the parallel plates is assumed periodically fully developed with a Reynolds number of 20,000. The inlet water temperature is 300 K and the wall temperature is 350 K.

It is well known that for original results before optimization, the velocity vectors and the temperature gradients are nearly perpendicular to each other, leading to a small scalar product between the velocity vector and the temperature gradient, i.e. the field synergy degree (Meng et al. 2003) is relatively poor. In the computational domain, the heat transfer rate is 1782 W and the decrement of the mean kinetic energy is 4.61×10^{-3} W. The thicknesses of the laminar sublayer and the transition sublayer are 0.116 mm and 0.928 mm, respectively.

The optimized velocity and temperature fields near the upper wall for $C_0 = -1.5 \times 10^7$ are shown in Figs. 6 (a) and (b). There are several small counter-clockwise eddies near the upper wall. The distances between eddies centres is about 0.4 mm and the eddy heights perpendicular to the primary flow direction are about 0.2 mm. There are also several small clockwise eddies near the lower wall for symmetry. For this case, the heat transfer rate is 1887 W and the decrement of the mean kinetic energy is 5.65×10^{-3} W. Compared with the original results before optimization, the heat transfer rate is increased by 6 %, while the decrement of the mean kinetic energy is increased by 23 %.

For heat transfer in turbulent flow between parallel plates, the temperature gradients in the laminar sublayer are two to three orders of magnitude larger than the gradients far from the wall, which means that the thermal resistance in the laminar sublayer is the dominate resistance in turbulent flow. The conventional heat transfer enhancement viewpoint is to first reduce the dominate resistance to most effectively increase the heat transfer rate. Eddies and disturbances near the wall will increase the velocities, reduce the thermal resistances, and enhance the heat transfer. The results support that the tubes with micro fins effectively enhance turbulent heat transfer, which is different from multiple longitudinal vertex generation in laminar heat transfer, and further point out the optimal heights of the fins for different Reynolds numbers should be half of the transition sublayer of turbulent flow, which is also validated experimentally (Li et al., 2009).
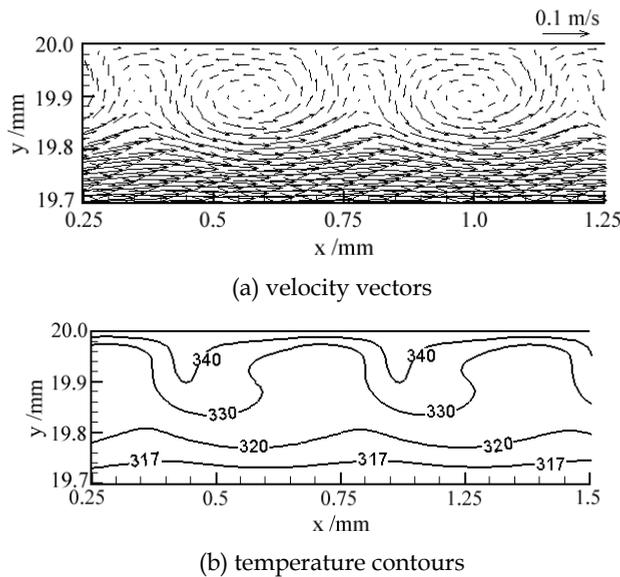

(a) velocity vectors

(b) temperature contours

Fig. 6. Optimized results near the wall for turbulent heat transfer (Chen et al., 2007)

5. Entransy dissipation-based thermal resistance for heat exchanger design

In engineering, both designing and checking heat exchanger performance are generally used such four approaches as the logarithmic mean temperature difference method (LMTD), the heat exchanger effectiveness – number of transfer units method (ε-NTU), the P-NTU method, and the ψ-P method. However, these methods are not very convenient when used in some particular situations. For example, in the LMTD method, it is inevitable to introduce a correction factor to adjust the effective temperature difference for cross-flow and multi-pass exchangers. Meanwhile, the use of LMTD method for checking heat exchanger performance has to involve tedious iterations to determine the proper outlet fluid temperatures and thereafter the value of LMTD which satisfy the requirement that heat transferred in the heat exchanger be equal to the heat carried out by the fluid. On the other hand, in the ε-NTU method, the fluid with the minimum heat capacity rate has to be first taken as the benchmark to calculate both heat exchanger effectiveness ε and NTU, so iterations are also unavoidable for the design of fluid flow rates. Besides, different types, e.g. parallel, counter-flow, cross-flow and shell-and-tube, of heat exchangers have their individual diverse and complex relations between heat exchanger effectiveness ε and NTU, which are more or less inconvenient for engineering applications. Therefore, it is necessary to develop a general criterion for the evaluation of heat exchanger performance and, more importantly, develop a common method for heat exchanger performance design and optimization.

For a heat transfer process in a parallel heat exchanger, the heat lost by the hot fluid over a differential element should be the same as that gained by the cold fluid, which both equal to the heat transferred through the elements, that is

$$dq = -m_h dh_h = m_c dh_c \,. \tag{51}$$

where, m is the mass flow rate, h is the specific enthalpy, and the subscripts h and c represent the hot and cold fluids, respectively.

Integrating Eq. (51), we will obtain the total heat transfer rate in the heat exchange from the viewpoint of energy conservation

$$Q = m_h \left(h_{h,a} - h_{h,b} \right) = m_c \left(h_{c,d} - h_{c,c} \right), \tag{52}$$

For a heat exchanger without any phase-change fluid, if the fluid specific heats are constant, Eq. (51) will be rewritten as

$$dT_h = -\frac{1}{m_h c_{p,h}} dq \,, \tag{53}$$

$$dT_c = \frac{1}{m_c c_{p,c}} dq \,, \tag{54}$$

and Eq. (52) is rewritten as

$$Q = m_h c_{p,h} \left(T_{h,a} - T_{h,b} \right) = m_c c_{p,c} \left(T_{c,d} - T_{c,c} \right). \tag{55}$$

Based on Eqs. (53) and (54), Fig. 7 gives the fluid temperature variations versus the heat transfer rate q. As shown, the shaded area is:

$$dS = T_h dq - T_c dq .$$ (56)

where, the first term on the right-hand side represents the entransy output accompanying the thermal energy dq flowing out of the hot fluid, while the second term represents the entransy input accompanying the thermal energy dq flowing into the cold fluid. Therefore, the shaded area exactly indicates the entransy dissipation rate during the heat transferred from the hot fluid to the cold one:

$$d\phi_h = (T_h - T_c) dq .$$ (57)

The total entransy dissipation in the heat exchanger is deduced by integrating Eq. (57),

$$\Phi_h = \int_0^{\Phi_h} d\phi_h = \int_0^Q (T_h - T_c) dq = \frac{(T_{h,a} - T_{c,c}) + (T_{h,b} - T_{c,d})}{2} Q = \Delta T_{AM} Q .$$ (58)

where, ΔT_{AM} is the arithmetical temperature difference between the hot and cold fluids in the heat exchanger.
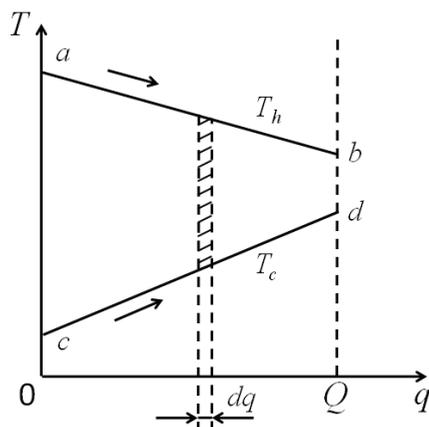

Fig. 7. Sketch of the fluid temperature variations versus the heat transfer rate in a parallel heat exchanger

Substituting Eqs. (55), (58) and the heat transfer equation

$$Q = KA \frac{(T_{h,a} - T_{c,c}) - (T_{h,b} - T_{c,d})}{\ln(T_{h,a} - T_{c,c}) - \ln(T_{h,b} - T_{c,d})}$$ (59)

into the definition of entransy dissipation-based thermal resistance, EDTR, for heat exchangers (Liu et al., 2009), $R_h = \Phi_h/Q^2 = \Delta T_{AM}/Q$, we get the formula of such thermal resistance for parallel heat exchangers:

$$R_{h,p} = \frac{\xi_p}{2} \frac{\exp(KA\xi_p) + 1}{\exp(KA\xi_p) - 1},$$ (60)

where, $\xi_p = \left(\dfrac{1}{m_h c_{p,h}} + \dfrac{1}{m_c c_{p,c}} \right)$, termed the arrangement factor for parallel heat exchangers.

Similarly, for counter-flow or TEMA E-type heat exchangers, we can also deduce the same formula of EDTR as shown in Eq. (60), except that the expression of the arrangement factor ξ is diverse, i.e. $\xi_c = \left(\dfrac{1}{m_h c_{p,h}} - \dfrac{1}{m_c c_{p,c}} \right)$ for counter-flow heat exchangers and $\xi_s = \sqrt{\dfrac{1}{\left(m_c c_{p,c} \right)^2} + \dfrac{1}{\left(m_h c_{p,h} \right)^2}}$ for TEMA E-type heat exchangers.

This general expression is convenient for us to design the heat transfer performance of heat exchangers. For example, a one shell and two tube passes TEMA E-type shell-and-tube exchanger with the heat transfer coefficient K and area A of 300 W/(m²·K) and 50 m², respectively, is used to cool the lubricating oil from the initial temperature $T_{h,in}$ = 57 °C to the desired temperature $T_{h,out}$ = 45 °C. The mass flow rate and specific heat of the oil are m_h = 10 kg/s and $c_{p,h}$ = 1.95 kJ/(kg·K), respectively. If the cooling water enters the heat exchanger at the temperature of 33 °C, what are its heat capacity rate and outlet temperature, and what is the rate of heat transfer by the exchanger.

According to the relation between EDTR and arithmetical mean temperature difference, the total heat transfer rate in the exchanger is

$$Q = \frac{\Delta T_{AM}}{R_{h,s}} = \frac{\left(T_{h,in} + T_{h,out} - T_{c,in} - T_{c,out} \right) \left(\exp \left(KA\xi_s \right) - 1 \right)}{\xi_s \left(\exp \left(KA\xi_s \right) + 1 \right)} . \tag{61}$$

Numerically solving Eq. (61) and the energy conservation equation (55) simultaneously, we can easily obtain the heat capacity rate and the outlet temperature of the cooling water are 73.8 kJ/(s·K) and 36.17 °C, respectively, and the total heat transfer rate is 234 kJ.

In this problem, neither the heat capacity rate nor the exit temperature of the cooling water are known, therefore an iterative solution is required if either the LMTD or the ε-NTU method is to be used. For instance, when using the LMTD method, the detail steps are: (1) obtain the required heat transfer rate of the exchanger Q_1 from the energy conservation equation of the oil; (2) assume a heat capacity rate of the cooling water, and then calculate its exit temperature; (3) according to the inlet and outlet temperatures of both the oil and the cooling water, obtain the logarithm mean temperature difference and the correction factor of the shell-and-tube exchanger; (4) based on the heat transfer equation, derive another heat transfer rate of the exchanger Q_2. Because the heat capacity rate of the cooling water is assumed, iteration is unavoidable to make the derived heat transfer rate Q_2 in step 4 close to the required one Q_1. Thus, it is clear that the entransy dissipation-based thermal resistance method can design heat exchanger performance conveniently.

6. Differences between entransy and entropy

Besides entransy, the concepts of entropy and entropy generation are considered to be two important functions in thermodynamic and used for estimating the irreversibility of and optimizing heat transfer based on the premise that the minimum entropy generation (MEG) will lead to the most efficient heat transfer performance. Thus, there exist two optimization

principles, and it is highly desired to investigate the physical essentials and the applicability of, as well as the differences between, them in heat transfer optimization.

The local entropy generation function induced by the heat transfer over finite temperature difference is

$$s_g = \frac{k|\nabla T|^2}{T^2} .$$
(62)

Let's reconsider the same volume-point problems as shown in Fig. 1. What we seek is also the optimal HTCM arrangement, except in this case that: 1. the optimization criterion is the minimum entropy generation; 2. the corresponding energy conservation equation should be added as a constraint, because it is not implied in the principle of minimum entropy generation when the thermal conductivity is constant (Bertola & Cafaro, 2008).

Introducing the corresponding Lagrange function

$$\Pi' = \iiint_\Omega \left[k\frac{|\nabla T|^2}{T^2} + B'k + C'\left(\nabla \cdot k\nabla T + \dot{Q}\right) \right] dV .$$
(63)

where B' and C' are also the Lagrange multipliers. The constraint of thermal conductivity is the isoperimetric condition, and consequently B' is a constant. C' is a variable related to space coordinates. The variation of Π' with respect to temperature T gives

$$-\nabla \cdot \left(k\nabla C'\right) = \frac{2k|\nabla T|^2}{T^3} + \frac{2\dot{Q}}{T^2} .$$
(64)

while the variation of Π' with respect to thermal conductivity k yields

$$\nabla C' \cdot \nabla T - \frac{\nabla T^2}{T^2} = B' = const .$$
(65)

Likewise, Eq. (65) gives the guideline for optimization based on the criterion of minimum entropy generation. That is, the HTCM be placed at the area with the extreme in absolute value of $\nabla C' \cdot \nabla T - \nabla T^2/T^2$.

Fig. 8(a) shows the distribution of HTCM based on the principle of minimum entropy generation. Comparison of Figs. 2(b) and 8(a) shows that although the distributions of HTCM are similar between the two results in most areas, the root-shape structure from the minimum entropy generation principle is not directly connected to the heat flow outlet, leaving some parts with the original material in between them so that the heat cannot be transported smoothly to the isothermal outlet boundary. Fig. 8(b) gives the optimized temperature distribution obtained by the minimum entropy generation. Because the low thermal conductivity material is adjacent to the heat outlet, the temperature gradient grows larger and thus lowers the entire heat transfer performance. The averaged temperature of the entire area is 150.8 K - 99.2 K higher than that obtained by the extremum principle of entransy dissipation. From the definition, it is easy to find that in order to decrease the entropy generation, we have to both reduce the temperature gradient, and raise the temperature, thus leading to the arrangement of HTCM showed in Fig. 8(a).

In addition, according to the principle of minimum entropy generation, the optimization objective of a steady-state heat dissipating process can be expressed as:

$$\dot{Q}\delta\left(\Delta\frac{1}{T}\right)_m = \delta\iiint\limits_\Omega \frac{k|\nabla T|^2}{T^2}dV = 0 \,.$$
(66)

where $\left(\Delta\dfrac{1}{T}\right)_m = \left(\dfrac{1}{T}-\dfrac{1}{T_0}\right)_m$, is the equivalent thermodynamics potential difference which represents the "generalized force" in the entropy picture for heat transfer. Thus, minimizing the entropy generation equals to minimizing the equivalent thermodynamics potential difference $\left(\Delta\dfrac{1}{T}\right)_m$, leading to the highest exergy transfer efficiency. That is, the minimum entropy generation principle is equivalent to the minimum exergy dissipation during a heat transfer process.

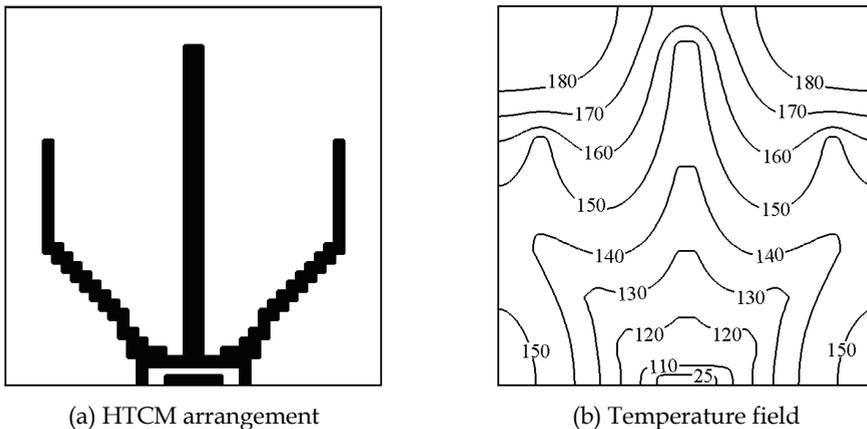(a) HTCM arrangement   (b) Temperature field

Fig. 8. Optimized results using the minimum entropy generation principle (Chen et al., 2011)

To facilitate the comparison between the two results from Figs. 3(b) and 8(b), Table 2 lists the key findings side by side, obtained respectively by the optimization criteria of the minimum entropy generation and the entransy dissipation extremum. It indubitably shows in the table that the proposed entransy based approach is more effective than the entropy based one in heat transfer optimization, for the former leads to a result with significantly reduced mean temperature than that by the latter (51.6 K vs. 150.8 K), and much lower maximum temperature (83.0 K vs. 194.9 K). Whereas the entropy based approach is preferred in exergy transfer optimization, as it results in a significantly lower equivalent thermodynamic potential (7.1×10^{-3} /K vs. 2.2×10^{-2} /K).

Besides heat conduction, we (Chen et al. 2009b) also compared the two criteria in convective heat transfer optimization. Our results indicate that both principles are applicable to convective heat transfer optimization, subject however to different objectives. The minimum entropy generation principle works better in searching for the minimum exergy dissipation during a heat-work conversion, whereas the entransy dissipation extremum principle is

more effective for processes not involving heat-work conversion, in minimizing the heat-transfer ability dissipation.

Optimization results Optimization Criterions	$\Phi_{ht}/(W\bullet K)$	$S_{gen}/(W/K)$	T_m/K	T_{max}/K	$\left(\Delta\dfrac{1}{T}\right)_m /(1/K)$
Extremum Entransy Dissipation	5.5×10^4	100.7	51.6	83.0	2.2×10^{-2}
Minimum Entropy Generation	1.58×10^5	81.7	150.8	194.9	7.1×10^{-3}

Table 2. Optimized results obtained by the optimization criterions of minimum entropy generation and entransy dissipation extremum (Chen et al., 2011)

7. Conclusion

The entransy is a parameter that is developed in recent year. It is effective in optimization of heat transfer. Entransy is an evaluation of the transport ability of heat. Both the amount of heat and the potential contribute to the entransy. Entransy will be lost during the heat transportation from a high temperature to lower one and entransy dissipation will be produces. Based on the energy conservation equation, the entransy balance equations for heat conduction and convective heat transfer are developed. The entransy dissipation extreme principles are developed, that is, the maximum entransy dissipation corresponds to the maximum heat flux for prescribed temperature difference and the minimum entransy dissipation corresponds to minimum temperature difference for prescribed heat flux. This extreme principle can be concluded into the minimum thermal resistance principle defined by entransy dissipation.

The entransy dissipation-based thermal resistance of heat exchangers is introduced as an irreversibility measurement, for parallel, counter-flow, and shell-and-tube exchangers, which has a general expression, is the function of heat capacity rates and thermal conductance, and may analyze, compare and optimize heat exchanger performance from the physical nature of heat transfer. Besides, from the relation among heat transfer rate, arithmetical mean temperature difference and EDTR, the total heat transfer rate may easily be calculated through the thermal conductance of heat exchangers and the heat capacity rates of fluids, which is convenient for heat exchanger design.

Finally, we compared the criteria of entransy dissipation extremum to that of entropy generation minimization in heat transfer optimization, the results indicates that the minimum entransy dissipation-based thermal resistance yields the maximum heat transfer efficiency when the heat transfer process is unrelated with heat-work conversion, while the minimum entropy generation leads to the highest heat-work conversion when such is involved in a thermodynamic cycle.

8. Acknowledgments

9. References

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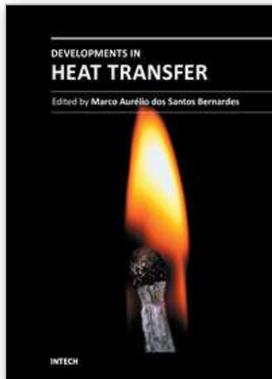
Developments in Heat Transfer

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