

Soret and Dufour Effects on Steady MHD Natural Convection Flow Past a Semi-Infinite Moving Vertical Plate in a Porous Medium with Viscous Dissipation in the Presence of a Chemical Reaction

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1. Introduction

Transportation of heat through porous media has been a subject of many studies due to the increasing need for a better understanding of the associated transport processes. There are numerous practical applications which can be modeled or can be approximated as transport through porous media such as grain storage, packed sphere beds, high performance insulation for buildings, migration of moisture through the air contained in fibrous insulations, heat exchange between soil and atmosphere sensible heat storage beds and beds of fossil fuels and geothermal energy systems, among other areas. Double diffusive is driven by buoyancy due to temperature and concentration gradients.

Magnetohydrodynamic flows have many applications in solar physics, cosmic fluid dynamics, geophysics and in the motion of earth's core as well as in chemical engineering and electronics. Huges and Young (1996) gave an excellent summary of applications. Soret and Dufour effects become significant when species are introduced at a surface in fluid domain, with different (lower) density than the surrounding fluid. When heat and mass transfer occur simultaneously in a moving fluid, the relations between the fluxes and the driving potentials are more intricate in nature. It is now known that an energy flux can be generated not only by temperature gradients but by composition gradients as well. This type of energy flux is called the Dufour or diffusion-thermo effect. We also have mass fluxes being created by temperature gradients and this is called the Soret or thermal-diffusion. The effect of chemical reaction depends on whether the reaction is heterogenous or homogenous.

Motivated by previous works Abreu (et al. 2006) - Alam & Rahman (2006), Don & Solomonoff (1995) - Shateyi (2008) and many possible industrial and engineering applications, we aim in this chapter to analyze steady two-dimensional hydromagnetic flow of a viscous incompressible, electrically conducting and viscous dissipating fluid past a semi-infinite

moving permeable plate embedded in a porous medium in the presence of a reacting chemical species, Dufour and Soret effects.

The resultant non-dimensional ordinary differential equations are then solved numerically by the Successive Linearization Method (SLM). The effects of various significant parameters such as Hartmann, chemical reaction parameter, Soret number, Dufour number, Eckert number, permeability parameter and Grashof numbers on the velocity, temperature, concentration, are depicted in figures and then discussed.

The governing equations are transformed into a system of nonlinear ordinary differential equations by using suitable local similarity transf. This chapter is arranged into five major sections as follows. Section 1 gives an account of previous related works as well as definitions to important terms. In section 2 we give the mathematical formulation of the problem and its analysis. A brief description of the method used in this chapter is presented in section 3. In section 4 we provide the results and their discussion. Lastly the conclusion to the chapter is presented in section 5.

2. Mathematical formulation

We consider a steady two-dimensional hydromagnetic flow of a viscous incompressible, electrically conducting and viscous dissipating fluid past a semi-infinite moving permeable plate embedded in a porous medium. We assume the flow to be in the x - direction, which is taken along the semi-infinite plate and the y - axis to be normal to it. The plate is maintained at a constant temperature T_w , which is higher than the free stream temperature T_∞ of the surrounding fluid and a constant concentration C_w which is greater than the constant concentration C_∞ of the surrounding fluid. A uniform magnetic field of strength B_0 is applied normal to the plate, which produces magnetic effect in the x - direction. The fluid is assumed to be slightly conducting, so that the magnetic Reynolds number is very small and the induced magnetic field is negligible in comparison with the applied magnetic field. We also assumed that there is no applied voltage, so that electric field is absent. All the fluid properties are assumed to be constant except that of the influence of the density variation with temperature and concentration in the body force term. A first-order homogeneous chemical reaction is assumed to take place in the flow. With the usual boundary layer and Boussinesq approximations the conservation equations for the problem under consideration can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta_t(T - T_\infty) + g\beta_c(C - C_\infty) - \frac{\sigma B_0^2}{\rho} u - \frac{\mu}{\rho k^*} u, \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{Dk_t}{c_s c_p} \frac{\partial^2 C}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y} \right)^2, \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} + \frac{Dk_t}{T_m} \frac{\partial^2 T}{\partial y^2} - k_c(C - C_\infty). \quad (4)$$

The boundary conditions for the present problem are

$$\begin{aligned} u(x, 0) = U_0, \quad v(x, 0) = V_w(x), \quad T(x, 0) = T_w, \quad C(x, 0) = C_w, \\ u(x, \infty) = 0, \quad T(x, \infty) = T_\infty, \quad C(x, \infty) = C_\infty, \end{aligned} \quad (5)$$

where U_0 is the uniform velocity of the plate and $V_w(x)$ is the suction velocity at the plate. u, v are the velocity components in the x, y directions, respectively, T and C are the fluid temperature and concentration respectively. ν is the kinematic viscosity, μ is the dynamic viscosity, g is the gravitational force due to acceleration, ρ is the density, β_t is the volumetric coefficient of thermal expansion, β_c is the volumetric coefficient of expansion with concentration, α is the thermal diffusivity, B_0 is the magnetic field of constant strength, D is the coefficient of mass diffusivity, c_p is the specific heat at constant pressure, T_m is the mean fluid temperature, k_t is the thermal diffusion ratio, k^* is the permeability, σ is the electrical conductivity of the fluid, k_c is the chemical reaction parameter and c_s is the concentration susceptibility.

It is well known that boundary layer flows have a predominant flow direction and boundary layer thickness is small compared to a typical length in the main flow direction. Boundary layer thickness usually increases with increasing downstream distance, the basic equations are transformed, as such, in order to make the transformed boundary layer thickness a slowly varying function of x , with this objective, the governing partial differential equations (2) - (4) are transformed by means of the following non-dimensional quantities

$$\eta = y\sqrt{\frac{U_0}{2\nu x}}, \psi = \sqrt{\nu x U_0} f(\eta), T = T_\infty + (T_w - T_\infty)\theta(\eta), C = C_\infty + (C_w - C_\infty)\phi(\eta), \quad (6)$$

where $\psi(x, y)$ is the physical stream function, defined as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$, so that the continuity equation is automatically satisfied, θ is the non-dimensional temperature function, ϕ is the non-dimensional concentration, $f(\eta)$ is the dimensionless stream function and η is the similarity variable.

Upon substituting the above transformation (6) into the governing equations (2) - (4) we get the following non-dimensional form

$$f''' + ff'' - (f')^2 + Gr\theta + Gm\phi - (M + \Omega)f' = 0, \quad (7)$$

$$\frac{1}{Pr}\theta'' + f\theta' + Du\phi'' + Ec f'^2 = 0, \quad (8)$$

$$\frac{1}{Sc}\phi'' + f\phi' + Sr\theta'' - \gamma\phi = 0, \quad (9)$$

where the primes denote differentiation with respect to η . $M = \frac{2\sigma B_0^2 x}{\rho U_0}$ is the magnetic parameter, $Pr = \frac{\nu\rho c_p}{\alpha}$ is the Prandtl number, $Sc = \frac{\nu}{D}$ is the Schmidt number, $Sr = \frac{Dk_t(T_w - T_\infty)}{\nu T_m(C_w - C_\infty)}$ is the Soret number, $Du = \frac{Dk_t(C_w - C_\infty)}{\nu T_m(T_w - T_\infty)}$ is the Dufour number, $Gr = \frac{g\beta_t(T_w - T_\infty)2x}{U_0^2}$ is the local Grashof number, $Gm = \frac{g\beta_c(C_w - C_\infty)2x}{U_0^2}$ is the local modified Grashof number, $\gamma = \frac{k_c\delta^2}{\nu}$ is the chemical reaction parameter, $Ec = \frac{U_0^2}{c_p(T_w - T_\infty)}$ is the Eckert number, Ω is the permeability parameter, $Re = \frac{xU_0}{\nu}$, is the Reynolds number. In view of the similarity transformations, the boundary conditions transform into:

$$\begin{aligned} f(0) = f_w, f'(0) = 1, \theta(0) = 1, \phi(0) = 1, \\ f'(\infty) = 0, T(\infty) = 0, C(\infty) = 0, \end{aligned} \quad (10)$$

where $f_w = -V_w \sqrt{\frac{2x}{vU_0}}$ is the mass transfer coefficient such that $f_w > 0$ indicates suction and $f_w < 0$ indicates blowing at the surface.

3. Successive Linearisation Method (SLM): Nonlinear systems of BVPs

In this section we describe the basic idea behind the proposed method of successive linearisation method (SLM). We consider a general n -order non-linear system of ordinary differential equations which is represented by the non-linear boundary value problem of the form

$$\mathbf{L}[Y(x), Y'(x), Y''(x), \dots, Y^{(n)}(x)] + \mathbf{N}[Y(x), Y'(x), Y''(x), \dots, Y^{(n)}(x)] = 0, \quad (11)$$

where $Y(x)$ is a vector of unknown functions, x is an independent variable and the primes denote ordinary differentiation with respect to x . The functions \mathbf{L} and \mathbf{N} are vector functions which represent the linear and non-linear components of the governing system of equations, respectively, defined by

$$\mathbf{L} = \begin{bmatrix} L_1 \left(y_1, y_2, \dots, y_k; y'_1, y'_2, \dots, y'_k; \dots; y_1^{(n)}, y_2^{(n)}, \dots, y_k^{(n)} \right) \\ L_2 \left(y_1, y_2, \dots, y_k; y'_1, y'_2, \dots, y'_k; \dots; y_1^{(n)}, y_2^{(n)}, \dots, y_k^{(n)} \right) \\ \vdots \\ L_k \left(y_1, y_2, \dots, y_k; y'_1, y'_2, \dots, y'_k; \dots; y_1^{(n)}, y_2^{(n)}, \dots, y_k^{(n)} \right) \end{bmatrix}, \quad (12)$$

$$\mathbf{N} = \begin{bmatrix} N_1 \left(y_1, y_2, \dots, y_k; y'_1, y'_2, \dots, y'_k; \dots; y_1^{(n)}, y_2^{(n)}, \dots, y_k^{(n)} \right) \\ N_2 \left(y_1, y_2, \dots, y_k; y'_1, y'_2, \dots, y'_k; \dots; y_1^{(n)}, y_2^{(n)}, \dots, y_k^{(n)} \right) \\ \vdots \\ N_k \left(y_1, y_2, \dots, y_k; y'_1, y'_2, \dots, y'_k; \dots; y_1^{(n)}, y_2^{(n)}, \dots, y_k^{(n)} \right) \end{bmatrix}, \quad (13)$$

$$Y(x) = \begin{bmatrix} y_1(x) \\ y_2(x) \\ \vdots \\ y_k(x) \end{bmatrix}, \quad (14)$$

where y_1, y_2, \dots, y_k are the unknown functions. We define an initial guess $Y_0(x)$ of the solution of (11) as

$$Y_0(x) = \begin{bmatrix} y_{1,0}(x) \\ y_{2,0}(x) \\ \vdots \\ y_{k,0}(x) \end{bmatrix}. \quad (15)$$

For illustrative purposes, we assume that equation (11) is to be solved for $x \in [a, b]$ subject to the boundary conditions

$$Y(a) = Y_a, \quad Y(b) = Y_b \quad (16)$$

where Y_a and Y_b are given constants. As a guide to choosing the appropriate initial guess we consider functions that satisfy the governing boundary conditions of equation (11).

Define a function $Z_1(x)$ to represent the vertical difference between $Y(x)$ and the initial guess $Y_0(x)$, that is

$$Z_1(x) = Y(x) - Y_0(x), \text{ or } Y(x) = Y_0(x) + Z_1(x). \tag{17}$$

For example, the vertical displacement between the variable $y_1(x)$ and its corresponding initial guess $y_{1,0}(x)$ is $z_{1,1} = y_1(x) - y_{1,0}(x)$. This is shown in Figure 1.

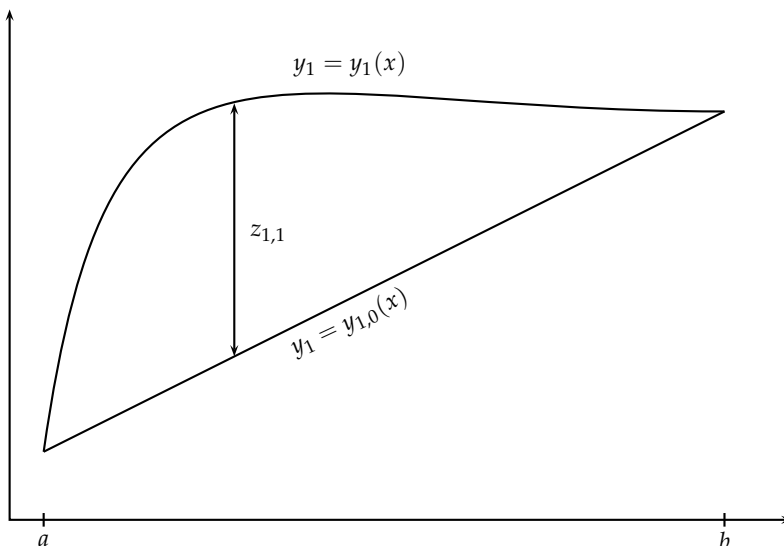


Fig. 1. Geometric representation of the function $z_{1,1}(x)$

Substituting equation (17) in (11) gives

$$\mathbf{L}[Z_1, Z_1', Z_1'', \dots, Z_1^{(n)}] + \mathbf{N}[Y_0 + Z_1, Y_0' + Z_1', Y_0'' + Z_1'', \dots, Y_0^{(n)} + Z_1^{(n)}] = -\mathcal{L}[Y_0, Y_0', Y_0'', \dots, Y_0^{(n)}]. \tag{18}$$

Since $Y_0(x)$ is an known function, solving equation (18) would yield an exact solution for $Z_1(x)$. However, since the equation is non-linear, it may not be possible to find an exact solution. We therefore look for an approximate solution which is obtained by solving the linear part of the equation assuming that Z_1 and its derivatives are small. This assumption enables us to use the Taylor series method to linearise the equation. If $Z_1(x)$ is the solution of the full equation (18) we let $Y_1(x)$ denote the solution of the linearised version of (18). Expanding (18) using Taylor series (assuming $Z_1(x) \approx Y_1(x)$) and neglecting higher order terms gives

$$\begin{aligned} \mathbf{L}[Y_1, Y_1', Y_1'', \dots, Y_1^{(n)}] &+ \left[\frac{\partial \mathbf{N}}{\partial Y_1} \right]_{(Y_0, Y_0', Y_0'', \dots, Y_0^{(n)})} Y_1 + \left[\frac{\partial \mathbf{N}}{\partial Y_1'} \right]_{(Y_0, Y_0', Y_0'', \dots, Y_0^{(n)})} Y_1' \\ &+ \left[\frac{\partial \mathbf{N}}{\partial Y_1''} \right]_{(Y_0, Y_0', Y_0'', \dots, Y_0^{(n)})} Y_1'' + \dots + \left[\frac{\partial \mathbf{N}}{\partial Y_1^{(n)}} \right]_{(Y_0, Y_0', Y_0'', \dots, Y_0^{(n)})} Y_1^{(n)} \\ &= -\mathbf{L}[Y_0, Y_0', Y_0'', \dots, Y_0^{(n)}] - \mathbf{N}[Y_0, Y_0', Y_0'', \dots, Y_0^{(n)}]. \end{aligned} \tag{19}$$

The partial derivatives inside square brackets in equation (19) represent Jacobian matrices of size $k \times k$, defined as

$$\left[\frac{\partial \mathbf{N}}{\partial Y_i} \right] = \begin{bmatrix} \frac{\partial N_1}{\partial y_{1,i}} & \frac{\partial N_1}{\partial y_{2,i}} & \dots & \frac{\partial N_1}{\partial y_{k,i}} \\ \frac{\partial N_2}{\partial y_{1,i}} & \frac{\partial N_2}{\partial y_{2,i}} & \dots & \frac{\partial N_2}{\partial y_{k,i}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial N_k}{\partial y_{1,i}} & \frac{\partial N_k}{\partial y_{2,i}} & \dots & \frac{\partial N_k}{\partial y_{k,i}} \end{bmatrix}, \quad \left[\frac{\partial \mathbf{N}}{\partial Y_i^{(p)}} \right] = \begin{bmatrix} \frac{\partial N_1}{\partial y_{1,i}^{(p)}} & \frac{\partial N_1}{\partial y_{2,i}^{(p)}} & \dots & \frac{\partial N_1}{\partial y_{k,i}^{(p)}} \\ \frac{\partial N_2}{\partial y_{1,i}^{(p)}} & \frac{\partial N_2}{\partial y_{2,i}^{(p)}} & \dots & \frac{\partial N_2}{\partial y_{k,i}^{(p)}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial N_k}{\partial y_{1,i}^{(p)}} & \frac{\partial N_k}{\partial y_{2,i}^{(p)}} & \dots & \frac{\partial N_k}{\partial y_{k,i}^{(p)}} \end{bmatrix} \quad (20)$$

where $i = 1$ and p is the order of the derivatives.

Since the right hand side of equation (19) is known and the left hand side is linear, the equation can be solved for $Y_1(x)$. Assuming that the solution of the linear part (19) is close to the solution of the equation (18), that is $Z_1(x) \approx Y_1(x)$, the current estimate (1st order) of the solution $Y(x)$ is

$$Y(x) \approx Y_0(x) + Y_1(x). \quad (21)$$

To improve on this solution, we define a slack function, $Z_2(x)$, which when added to $Y_1(x)$ gives $Z_1(x)$ (see Figure 2 for example), that is

$$Z_1(x) = Z_2(x) + Y_1(x). \quad (22)$$

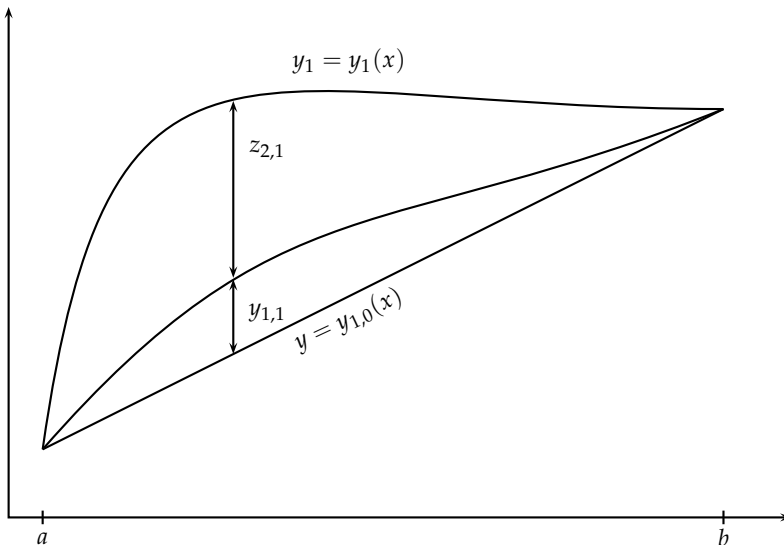


Fig. 2. Geometric representation of the functions $z_{2,1}$

Since $Y_1(x)$ is now known (as a solution of equation 19), we substitute equation (22) in equation (18) to obtain

$$\begin{aligned} \mathbf{L}[Z_2, Z_2', Z_2'', \dots, Z_2^{(n)}] + \mathbf{N}[Y_0 + Y_1 + Z_2 + Y_0' + Y_1' + Z_2', \dots, Y_0^{(n)} + Y_1^{(n)} + Z_2^{(n)}] \\ = -\mathcal{L}[Y_0 + Y_1, Y_0' + Y_1', Y_0'' + Y_1'', \dots, Y_0^{(n)} + Y_1^{(n)}]. \end{aligned} \tag{23}$$

Solving equation (23) would result in an exact solution for $Z_2(x)$. But since the equation is non-linear, it may not be possible to find an exact solution. We therefore linearise the equation using Taylor series expansion and solve the resulting linear equation. We denote the solution of the linear version of equation (23) by $Y_2(x)$, such that $Z_2(x) \approx Y_2(x)$. Setting $Z_2(x) = Y_2(x)$ and expanding equation (23), for small $Y_2(x)$ and its derivatives gives

$$\begin{aligned} \mathbf{L}[Y_2, Y_2', \dots, Y_2^{(n)}] + \left[\frac{\partial \mathbf{N}}{\partial Y_2} \right]_{(Y_0+Y_1, Y_0'+Y_1', \dots, Y_0^{(n)}+Y_1^{(n)})} Y_2 + \left[\frac{\partial \mathbf{N}}{\partial Y_2'} \right]_{(Y_0+Y_1, Y_0'+Y_1', \dots, Y_0^{(n)}+Y_1^{(n)})} Y_2' \\ + \left[\frac{\partial \mathbf{N}}{\partial Y_2''} \right]_{(Y_0+Y_1, Y_0'+Y_1', \dots, Y_0^{(n)}+Y_1^{(n)})} Y_2'' + \dots + \left[\frac{\partial \mathbf{N}}{\partial Y_2^{(n)}} \right]_{(Y_0+Y_1, Y_0'+Y_1', \dots, Y_0^{(n)}+Y_1^{(n)})} Y_2^{(n)} \\ = -\mathbf{L}[Y_0 + Y_1, Y_0' + Y_1', \dots, Y_0^{(n)} + Y_1^{(n)}] - \mathbf{N}[Y_0 + Y_1, Y_0' + Y_1', \dots, Y_0^{(n)} + Y_1^{(n)}] \end{aligned} \tag{24}$$

where the partial derivatives inside square brackets in equation (24) represent Jacobian matrices defined as in equation (20) with $i = 2$.

After solving (24), the current (2nd order) estimate of the solution $Y(x)$ is

$$Y(x) \approx Y_0(x) + Y_1(x) + Y_2(x). \tag{25}$$

Next we define $Z_3(x)$ (see Figure 3) such that

$$Z_2(x) = Z_3(x) + Y_2(x). \tag{26}$$

Equation (26) is substituted in the non-linear equation (23) and the linearisation process described above is repeated. This process is repeated for $m = 3, 4, 5, \dots, i$. In general, we have

$$Z_i(x) = Z_{i+1}(x) + Y_i(x). \tag{27}$$

Thus, $Y(x)$ is obtained as

$$Y(x) = Z_1(x) + Y_0(x), \tag{28}$$

$$= Z_2(x) + Y_1(x) + Y_0(x), \tag{29}$$

$$= Z_3(x) + Y_2(x) + Y_1(x) + Y_0(x), \tag{30}$$

⋮

$$= Z_{i+1}(x) + Y_i(x) + \dots + Y_3(x) + Y_2(x) + Y_1(x) + Y_0(x), \tag{31}$$

$$= Z_{i+1}(x) + \sum_{m=0}^i Y_m(x). \tag{32}$$

The procedure for obtaining each $Z_i(x)$ is illustrated in Figures 1, 2 and 3 respectively for $i = 1, 2, 3$.

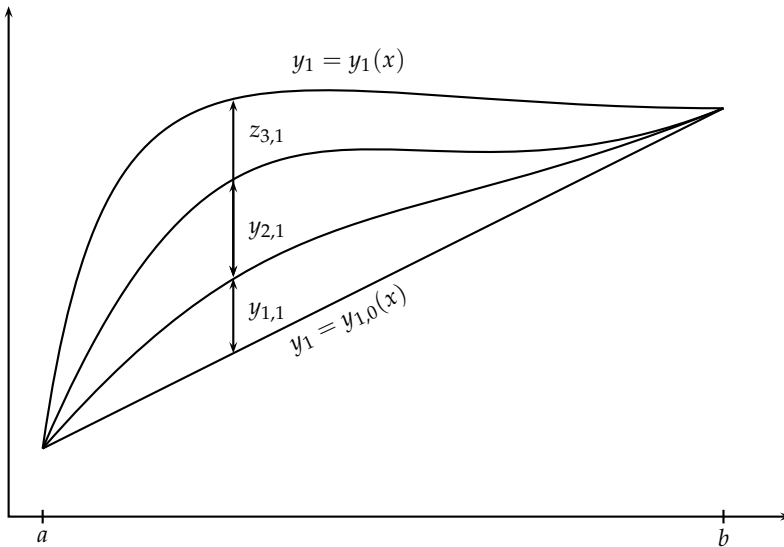


Fig. 3. Geometric representation of the functions $z_{3,1}$

We note that when i becomes large, Z_{i+1} becomes increasingly smaller. Thus, for large i , we can approximate the i th order solution of $Y(x)$ by

$$Y(x) = \sum_{m=0}^i Y_m(x) = Y_i(x) + \sum_{m=0}^{i-1} Y_m(x). \tag{33}$$

Starting from a known initial guess $Y_0(x)$, the solutions for $Y_i(x)$ can be obtained by successively linearising the governing equation (11) and solving the resulting linear equation for $Y_i(x)$ given that the previous guess $Y_{i-1}(x)$ is known. The general form of the linearised equation that is successively solved for $Y_i(x)$ is given by

$$\mathbf{L}[Y_i, Y'_i, Y''_i, \dots, Y_i^{(n)}] + \mathbf{a}_{0,i-1} Y_i^{(n)} + \mathbf{a}_{1,i-1} Y_i^{(n-1)} + \dots + \mathbf{a}_{n-1,i-1} Y'_i + \mathbf{a}_{n,i-1} Y_i = R_{i-1}(x), \tag{34}$$

where

$$\mathbf{a}_{0,i-1}(x) = \left[\frac{\partial \mathbf{N}}{\partial Y_i^{(n)}} \right] \left(\sum_{m=0}^{i-1} Y_m, \sum_{m=0}^{i-1} Y'_m, \sum_{m=0}^{i-1} Y''_m, \dots, \sum_{m=0}^{i-1} Y_m^{(n)} \right) \tag{35}$$

$$\mathbf{a}_{1,i-1}(x) = \left[\frac{\partial \mathbf{N}}{\partial Y_i^{(n-1)}} \right] \left(\sum_{m=0}^{i-1} Y_m, \sum_{m=0}^{i-1} Y'_m, \sum_{m=0}^{i-1} Y''_m, \dots, \sum_{m=0}^{i-1} Y_m^{(n)} \right) \tag{36}$$

$$\mathbf{a}_{n-1,i-1}(x) = \left[\frac{\partial \mathbf{N}}{\partial Y'_i} \right] \left(\sum_{m=0}^{i-1} Y_m, \sum_{m=0}^{i-1} Y'_m, \sum_{m=0}^{i-1} Y''_m, \dots, \sum_{m=0}^{i-1} Y_m^{(n)} \right) \tag{37}$$

$$\mathbf{a}_{n,i-1}(x) = \left[\frac{\partial \mathbf{N}}{\partial Y_i} \right] \left(\sum_{m=0}^{i-1} Y_m, \sum_{m=0}^{i-1} Y'_m, \sum_{m=0}^{i-1} Y''_m, \dots, \sum_{m=0}^{i-1} Y_m^{(n)} \right) \quad (38)$$

$$\begin{aligned} \mathbf{R}_{i-1}(x) = & -\mathbf{L} \left(\sum_{m=0}^{i-1} Y_m, \sum_{m=0}^{i-1} Y'_m, \sum_{m=0}^{i-1} Y''_m, \dots, \sum_{m=0}^{i-1} Y_m^{(n)} \right) \\ & - \mathbf{N} \left(\sum_{m=0}^{i-1} Y_m, \sum_{m=0}^{i-1} Y'_m, \sum_{m=0}^{i-1} Y''_m, \dots, \sum_{m=0}^{i-1} Y_m^{(n)} \right). \end{aligned} \quad (39)$$

4. Numerical solution

In this section we solve the governing equations (7 - 9) using the SLM method described in the last section. We begin by writing the governing equations (7 - 9) as a sum of the linear and nonlinear components as

$$-\mathbf{L}[f, f', f'', f''', \theta, \theta', \theta'', \phi, \phi', \phi''] + \mathbf{N}[f, f', f'', f''', \theta, \theta', \theta'', \phi, \phi', \phi''] = 0, \quad (40)$$

where the primes denote differentiation with respect to η and

$$\mathbf{L}[f, f', f'', f''', \theta, \theta', \theta'', \phi, \phi', \phi''] = \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix} = \begin{bmatrix} f''' - (M + \Omega)f' + Gr\theta + Gm\phi \\ \frac{1}{Pr}\theta'' + Du\phi'' \\ \frac{1}{Sc}\phi'' + Sr\theta'' - \gamma\phi \end{bmatrix} \quad (41)$$

$$\mathbf{N}[f, f', f'', f''', \theta, \theta', \theta'', \phi, \phi', \phi''] = \begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix} = \begin{bmatrix} ff'' - (f')^2 \\ f\theta' + Ec(f'')^2 \\ f\phi' \end{bmatrix}. \quad (42)$$

Using equation (34), the general equation to be solved for Y_i , where

$$Y_i = \begin{bmatrix} f \\ \theta \\ \phi \end{bmatrix}, \quad (43)$$

is

$$\mathbf{L}[Y_i, Y'_i, Y''_i, Y'''_i] + \mathbf{a}_{0,i-1}Y'''_i + \mathbf{a}_{1,i-1}Y''_i + \mathbf{a}_{2,i-1}Y'_i + \mathbf{a}_{3,i-1}Y_i = R_{i-1}(\eta), \quad (44)$$

subject to the boundary conditions

$$f_i(0) = f'_i(0) = \theta_i(0) = \phi_i(0) = f'_i(\infty) = \theta_i(\infty) = \phi_i(\infty) = 0. \quad (45)$$

where

$$\mathbf{a}_{0,i-1} = \begin{bmatrix} \frac{\partial N_1}{\partial f'''} & \frac{\partial N_1}{\partial \theta'''} & \frac{\partial N_1}{\partial \phi'''} \\ \frac{\partial N_2}{\partial f'''} & \frac{\partial N_2}{\partial \theta'''} & \frac{\partial N_2}{\partial \phi'''} \\ \frac{\partial N_3}{\partial f'''} & \frac{\partial N_3}{\partial \theta'''} & \frac{\partial N_3}{\partial \phi'''} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (46)$$

$$\mathbf{a}_{1,i-1} = \begin{bmatrix} \frac{\partial N_1}{\partial f''} & \frac{\partial N_1}{\partial \theta''} & \frac{\partial N_1}{\partial \phi''} \\ \frac{\partial N_2}{\partial f''} & \frac{\partial N_2}{\partial \theta''} & \frac{\partial N_2}{\partial \phi''} \\ \frac{\partial N_3}{\partial f''} & \frac{\partial N_3}{\partial \theta''} & \frac{\partial N_3}{\partial \phi''} \end{bmatrix} = \begin{bmatrix} \sum f_m & 0 & 0 \\ 2Ec \sum f''_m & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (47)$$

$$a_{2,i-1} = \begin{bmatrix} \frac{\partial N_1}{\partial f'} & \frac{\partial N_1}{\partial \theta'} & \frac{\partial N_1}{\partial \phi'} \\ \frac{\partial N_2}{\partial f'} & \frac{\partial N_2}{\partial \theta'} & \frac{\partial N_2}{\partial \phi'} \\ \frac{\partial N_3}{\partial f'} & \frac{\partial N_3}{\partial \theta'} & \frac{\partial N_3}{\partial \phi'} \end{bmatrix} = \begin{bmatrix} -2 \sum f'_m & 0 & 0 \\ 0 & \sum f'_m & 0 \\ 0 & 0 & \sum f'_m \end{bmatrix} \tag{48}$$

$$a_{3,i-1} = \begin{bmatrix} \frac{\partial N_1}{\partial f} & \frac{\partial N_1}{\partial \theta} & \frac{\partial N_1}{\partial \phi} \\ \frac{\partial N_2}{\partial f} & \frac{\partial N_2}{\partial \theta} & \frac{\partial N_2}{\partial \phi} \\ \frac{\partial N_3}{\partial f} & \frac{\partial N_3}{\partial \theta} & \frac{\partial N_3}{\partial \phi} \end{bmatrix} = \begin{bmatrix} \sum f''_m & 0 & 0 \\ \sum \theta'_m & 0 & 0 \\ \sum \phi'_m & 0 & 0 \end{bmatrix} \tag{49}$$

$$R_{i-1} = \begin{bmatrix} r_{1,i-1} \\ r_{2,i-1} \\ r_{3,i-1} \end{bmatrix}$$

$$r_{1,i-1} = - \left[\sum f'''_m - (M + \Omega) \sum f'_m + Gr \sum \theta_m + Gm \sum \phi_m + \sum f_m \sum f''_m - (\sum f'_m)^2 \right] \tag{50}$$

$$r_{2,i-1} = - \left[\frac{1}{Pr} \sum \theta''_m + Du \sum \phi''_m + \sum f_m \sum \theta'_m + Ec (\sum f''_m)^2 \right] \tag{51}$$

$$r_{3,i-1} = - \left[\frac{1}{Sc} \sum \phi''_m + Sr \sum \theta''_m - \gamma \sum \phi_m + \sum f_m \sum \phi'_m \right] \tag{52}$$

and the sums in equation (46 - 52) denote $\sum_{m=0}^{i-1}$. Once each solution for f_i, θ_i, ϕ_i ($i \geq 1$) has been found from iteratively solving equations (44 - 45), the approximate solutions for $f(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ are obtained as

$$f(\eta) \approx \sum_{m=0}^i f_m(\eta), \tag{53}$$

$$\theta(\eta) \approx \sum_{m=0}^i \theta_m(\eta), \tag{54}$$

$$\phi(\eta) \approx \sum_{m=0}^i \phi_m(\eta), \tag{55}$$

where i is the order of SLM approximation. Since the coefficient parameters and the right hand side of equations (44), for $i = 1, 2, 3, \dots$, are known (from previous iterations). The equation system can easily be solved using numerical methods such as finite differences, finite elements, Runge-Kutta based shooting methods or collocation methods. In this work, equations (44) are solved using the Chebyshev spectral collocation method. This method is based on approximating the unknown functions by the Chebyshev interpolating polynomials in such a way that they are collocated at the Gauss-Lobatto points defined as

$$\xi_j = \cos \frac{\pi j}{N}, \quad j = 0, 1, \dots, N, \tag{56}$$

where $N + 1$ is the number of collocation points used (see for example Canuto et al. (1988); Don & Solomonoff (1995); Trefethen (2000)). In order to implement the method, the physical

region $[0, \infty)$ is transformed into the region $[-1, 1]$ using the domain truncation technique in which the problem is solved on the interval $[0, L]$ instead of $[0, \infty)$. This leads to the mapping

$$\frac{\eta}{L} = \frac{\xi + 1}{2}, \quad -1 \leq \xi \leq 1 \tag{57}$$

where L is the scaling parameter used to invoke the boundary condition at infinity. The unknown functions f_i , θ_i and ϕ_i are approximated at the collocation points by

$$f_i(\xi) \approx \sum_{k=0}^N f_i(\xi_k) T_k(\xi_j), \tag{58}$$

$$\theta_i(\xi) \approx \sum_{k=0}^N \theta_i(\xi_k) T_k(\xi_j), \tag{59}$$

$$\phi_i(\xi) \approx \sum_{k=0}^N \phi_i(\xi_k) T_k(\xi_j), \tag{60}$$

$$j = 0, 1, \dots, N, \tag{61}$$

where T_k is the k th Chebyshev polynomial defined as

$$T_k(\xi) = \cos[k \cos^{-1}(\xi)]. \tag{62}$$

The derivatives of the variables at the collocation points are represented as

$$\frac{d^a f_i}{d\eta^a} = \sum_{k=0}^N \mathbf{D}_{kj}^a f_i(\xi_k), \quad \frac{d^a \theta_i}{d\eta^a} = \sum_{k=0}^N \mathbf{D}_{kj}^a \theta_i(\xi_k), \quad \frac{d^a \phi_i}{d\eta^a} = \sum_{k=0}^N \mathbf{D}_{kj}^a \phi_i(\xi_k), \quad j = 0, 1, \dots, N \tag{63}$$

where a is the order of differentiation and $\mathbf{D} = \frac{2}{L} \mathcal{D}$ with \mathcal{D} being the Chebyshev spectral differentiation matrix (see for example, Canuto et al. (1988); Trefethen (2000)). Substituting equations (57 - 63) in (44) - (45) leads to the matrix equation given as

$$\mathbf{M}_{i-1} \mathbf{Y}_i = \mathbf{R}_{i-1}, \tag{64}$$

subject to the boundary conditions

$$f_i(\xi_N) = \sum_{k=0}^N \mathbf{D}_{0k} f_i(\xi_k) = \sum_{k=0}^N \mathbf{D}_{Nk} f_i(\xi_k) = \theta_i(\xi_N) = \theta_i(\xi_0) = \phi_i(\xi_N) = \phi_i(\xi_0) = 0 \tag{65}$$

where

$$\mathbf{M}_{i-1} = \mathbf{A} + \mathbf{a}_{1,i-1} \bar{\mathbf{D}}^2 + \mathbf{a}_{2,i-1} \bar{\mathbf{D}} + \mathbf{a}_{3,i-1} \tag{66}$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{D}^3 - (M + \Omega) \mathbf{D} & Gr \mathbf{I} & Gm \mathbf{I} \\ \mathbf{O} & \frac{1}{Pr} \mathbf{D}^2 & Du \mathbf{D}^2 \\ \mathbf{O} & Sr \mathbf{D}^2 & \frac{1}{Sc} \mathbf{D}^2 - \gamma \mathbf{I} \end{bmatrix} \tag{67}$$

$$\bar{\mathbf{D}} = \begin{bmatrix} \mathbf{D} & \mathbf{O} & \mathbf{O} \\ \mathbf{O} & \mathbf{D} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} & \mathbf{D} \end{bmatrix} \tag{68}$$

$$\mathbf{R}_{i-1} = \begin{pmatrix} 0 \\ r_{1,i-1}(\xi_1) \\ \vdots \\ r_{1,i-1}(\xi_{N-2}) \\ 0 \\ 0 \\ \hline 0 \\ r_{2,i-1}(\xi_1) \\ \vdots \\ r_{2,i-1}(\xi_{N-2}) \\ r_{2,i-1}(\xi_{N-1}) \\ 0 \\ \hline 0 \\ r_{3,i-1}(\xi_1) \\ \vdots \\ r_{3,i-1}(\xi_{N-2}) \\ r_{3,i-1}(\xi_{N-1}) \\ 0 \end{pmatrix} \quad (78)$$

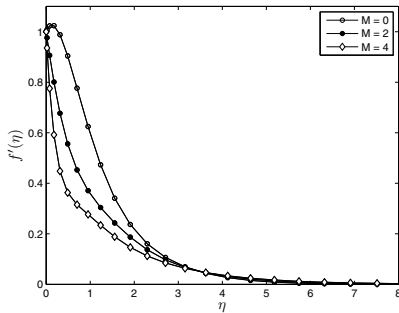
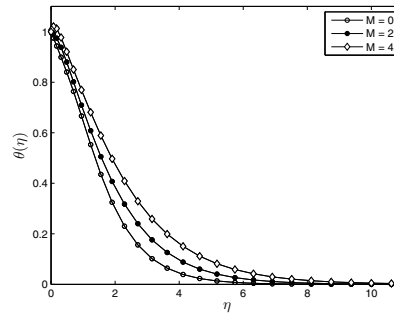
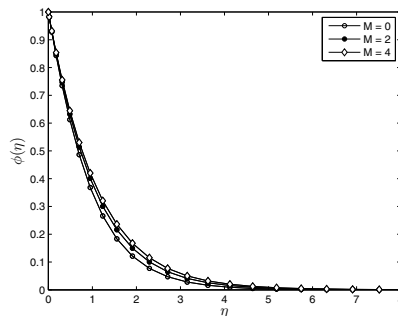
After modifying the matrix system (64) to incorporate boundary conditions, the solution is obtained as

$$\mathbf{Y}_i = \mathbf{M}_{i-1}^{-1} \mathbf{R}_{i-1}. \quad (79)$$

5. Results and discussion

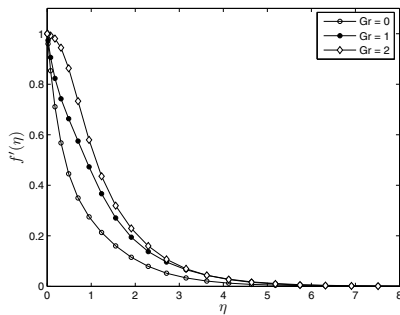
In this section we present numerical calculations for different values of M , Gr , Gm , γ , Du , and f_w for fixed values of $\Omega = 1$, and $Re = 1$ to obtain a clear insight of the physical problem. In computing the numerical results presented in this paper, unless otherwise stated, the following values of physical parameters were used: $M = 1$, $\Omega = 1$, $Gr = 1$, $Gm = 1$, $Pr = 0.71$, $Sc = 0.6$, $Sr = 0.1$, $\gamma = 1$, $Ec = 1$, $f_w = 0$, $Du = 0.1$. The numerical results are displayed graphically. The effect of the Hartmann number M on the dimensionless velocity $f'(\eta)$, temperature $\theta(\eta)$ and concentration $\phi(\eta)$ profiles are respectively represented in Figs (a), (b) and (c). It is observed in these Figs, that the velocity decreases with the increase of the magnetic parameter, the value of the temperature profiles increase with the magnetic parameter. The concentration of the fluid have a small increase with the increase of the magnetic parameter. The effects of a transverse magnetic field give rise to a resistive-type force called the Lorentz force. This force has the tendency to slow down the motion of the fluid flow and to increase its thermal boundary layer hence increasing the temperature of the fluid flow.

Figure (d), (e) and (f) depict the effects of varying the local thermal free convection (Gr) with increasing η on the dimensionless velocity, temperature and concentration. It is observed in Fig (d) that the increase of the Grashof number leads to the increase of the velocity of the fluid. This is because the increase of Gr results in the increase of temperature gradients ($T_\infty - T_\infty$), leads to the enhancement of the velocity due to the enhanced convection. From Fig (e) we observe that the effect of increasing the values of thermal free convection is to reduce the temperature profiles (θ). We also observe in Fig (f) that the concentration profiles decrease

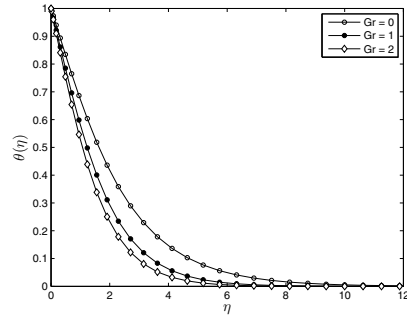
(a) Plot of $f'(\eta)$ for varying M (b) Plot of $\theta(\eta)$ for varying M (c) Plot of $\phi(\eta)$ for varying M

as the Grashof number increases. It can be clearly seen that the effect of Grashof number (Gr) is to decrease the concentration distribution as the concentration distribution species is dispersed away largely due to increased temperature gradient. The modified Grashof number Gm has the same effect as the local Grashof number (Gr) on the flow properties as depicted in Figs (g), (h) and (i).

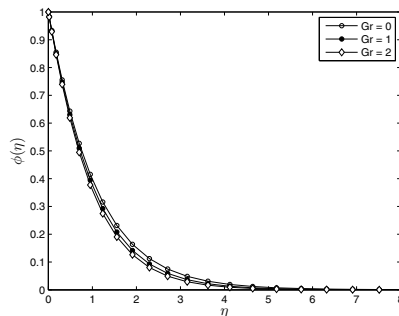
Figs (j)-(l) depict the effects of the chemical reaction parameter γ on the dimensionless velocity, temperature and concentration distributions. The effect of chemical reaction parameter is very important in the concentration field. Chemical reaction increases the rate of interfacial mass transfer. Reaction reduces the local concentration, thus increases its concentration gradient and its flux. Figs (m)-(o) show the influence of the Eckert number Ec , on the velocity, temperature and concentration profiles, respectively. By analyzing these Figs, it is clearly revealed that the effect of Eckert number is to increase both the velocity and the temperature distributions in the flow region. This is due to the fact that the heat energy is stored in liquid due to the frictional heating. Thus the effect of increasing Ec is to enhance the temperature at any point as well as the velocity. However, the Eckert number Ec has no significant effect on the concentration within the flow region. Figs (p) - (r) depict the influence of the Dufour parameter (Du) on the dimensionless velocity, temperature and concentration distributions. It can be clearly seen from Fig (p) that as the Dufour effects



(d) Plot of $f'(\eta)$ for varying Gr

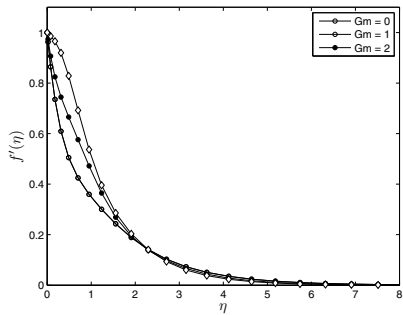


(e) Plot of $\theta(\eta)$ for varying Gr

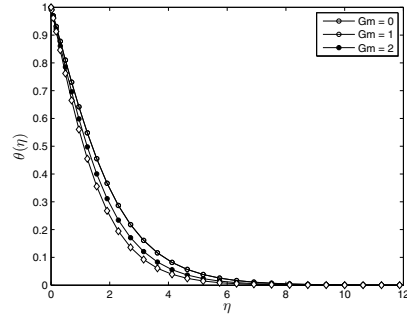


(f) Plot of $\phi(\eta)$ for varying Gr

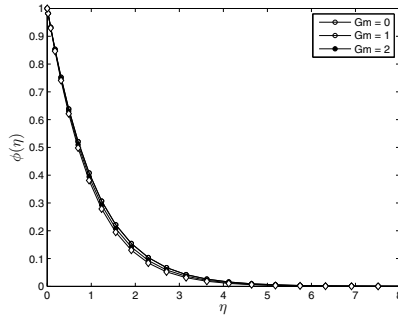
increase, the velocity of the flow increases. It is observed in Fig (q) that diffusion thermal effects greatly affect the fluid temperature. As the values of the Dufour parameter increase, the fluid temperature temperature also increases. We also observe in Fig (r) that increasing values of the Dufour parameter (Du) reduce the concentration in the fluid flow. Figs (s) - (u) show the effects the Soret parameter Sr on the dimensionless velocity, temperature and concentration distributions. We observe that the fluid velocity increases with increasing values of the Soret parameter Sr . As expected the effect of the Soret number on the temperature is quite opposite to that of the Dufour parameter. It is obvious from these Figs that increasing the Soret number (Sr) increases the boundary layer thickness for the concentration. Figs (v) - (x) depict the influence of the suction ($f_w > 0$) and injection ($f_w < 0$) on the velocity, temperature and concentration profiles. We see that the effect of suction is to reduce the velocity profiles $f'(\eta)$. While stronger suction is provided, the heated fluid is sucked through the wall where buoyancy forces act to decelerate the flow with more influence of viscosity. Sucking decelerated fluid particles through the porous wall reduce growth of the fluid boundary layer as well as thermal and concentration boundary layers. From these Figs, it is clear that the dimensionless temperature and concentration decrease due to fluid suction but they increase due to injection.



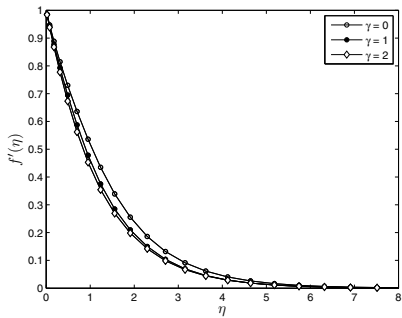
(g) Plot of $f'(\eta)$ for varying Gm



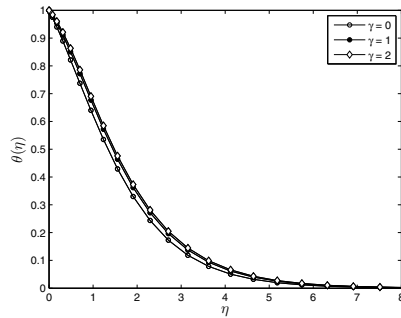
(h) Plot of $\theta(\eta)$ for varying Gm



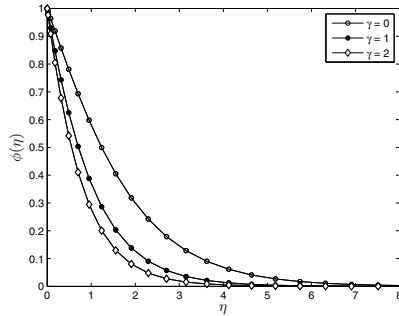
(i) Plot of $\phi(\eta)$ for varying Gm



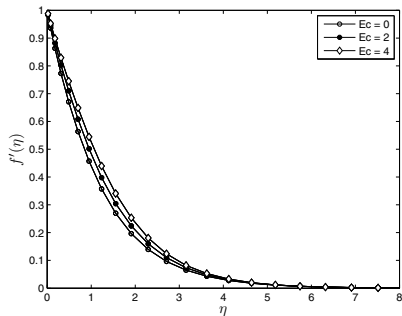
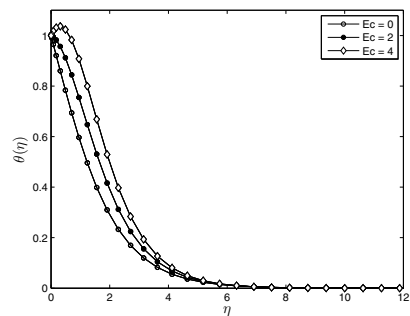
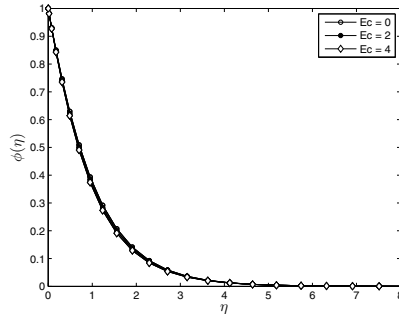
(j) Plot of $f'(\eta)$ for varying γ

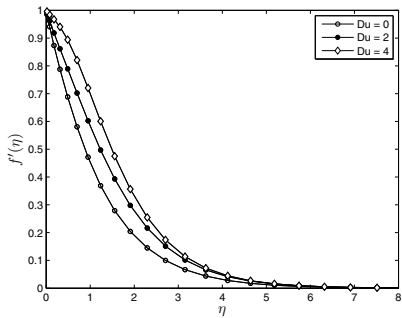


(k) Plot of $\theta(\eta)$ for varying γ

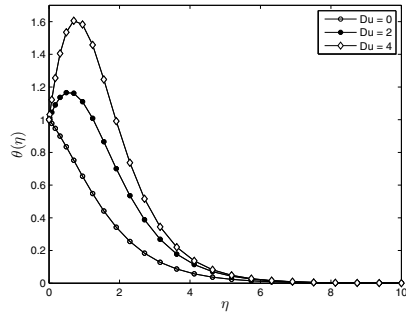


(l) Plot of $\phi(\eta)$ for varying γ

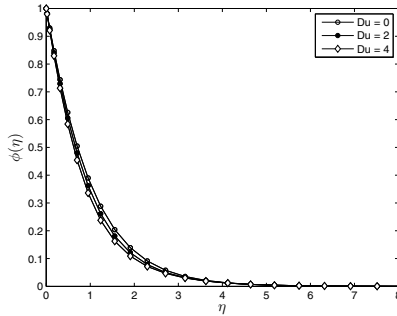
(m) Plot of $f'(\eta)$ for varying Ec (n) Plot of $\theta(\eta)$ for varying Ec (o) Plot of $\phi(\eta)$ for varying Ec



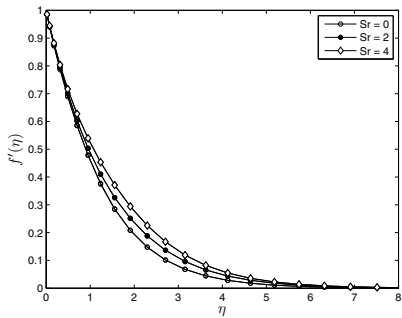
(p) Plot of $f'(\eta)$ for varying Du



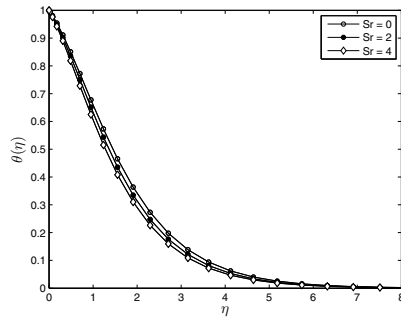
(q) Plot of $\theta(\eta)$ for varying Du



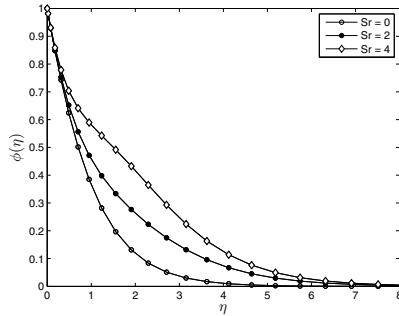
(r) Plot of $\phi(\eta)$ for varying Du



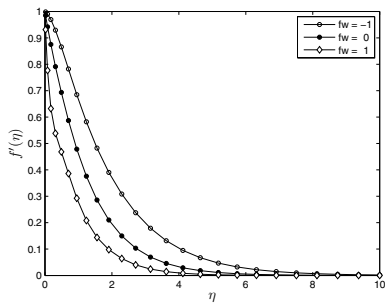
(s) Plot of $f'(\eta)$ for varying Sr



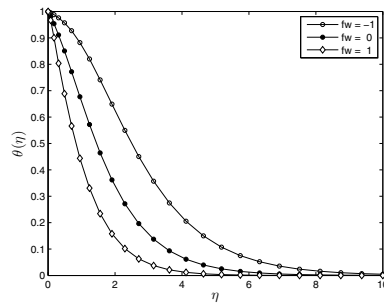
(t) Plot of $\theta(\eta)$ for varying Sr



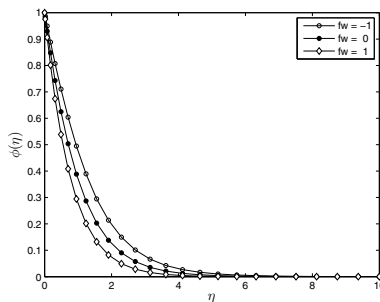
(u) Plot of $\phi(\eta)$ for varying Sr



(v) Plot of $f'(\eta)$ for varying f_w



(w) Plot of $\theta(\eta)$ for varying f_w



(x) Plot of $\phi(\eta)$ for varying f_w

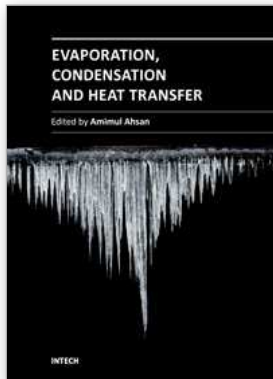
6. Conclusion

In this chapter, a new numerical technique to solve the problem of steady magnetohydrodynamic convective heat and mass transfer past a semi-infinite moving permeable vertical plate in a porous medium with Soret and Dufour effects in the presence of viscous dissipation and a chemical reaction. The non-linear momentum, energy and species boundary layer equations are transformed into ordinary differential equations using suitable local similarity equations. We then applied the successive linearization method coupled with the Chebyshev spectral collocation method. The effects of various physical parameters like the Hartmann number, Grashof numbers, chemical reaction parameter, Soret and Dufour numbers. We found out that wall suction stabilizes the fluid flow and that the boundary flow attain minimum velocity for large Hartmann numbers. In this chapter, the fluid temperature was found to increase as the Dufour parameter, magnetic strength, surface permeability increase and to decrease as the Soret effects increase. The concentration decreases as the Dufour number and chemical reaction parameter increase and decrease as the Soret effect, magnetic strength and surface permeability increase.

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Evaporation, Condensation and Heat transfer

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The theoretical analysis and modeling of heat and mass transfer rates produced in evaporation and condensation processes are significant issues in a design of wide range of industrial processes and devices. This book includes 25 advanced and revised contributions, and it covers mainly (1) evaporation and boiling, (2) condensation and cooling, (3) heat transfer and exchanger, and (4) fluid and flow. The readers of this book will appreciate the current issues of modeling on evaporation, water vapor condensation, heat transfer and exchanger, and on fluid flow in different aspects. The approaches would be applicable in various industrial purposes as well. The advanced idea and information described here will be fruitful for the readers to find a sustainable solution in an industrialized society.

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