

Strong Lensing Systems as Probes of Dark Energy Models and Non-Standard Theories of Gravity

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1. Introduction

Gravitational lensing is a consequence of light deflection in gravitational field. Starlight deflection by the Sun, measured as $1''.7$ at the solar limb was the first evidence in favor of General Relativity. This result is often presented in contrast with so called Newtonian prediction dated back to Soldner, but in fact it announces much more by its very existence. Namely that the nature of gravity lies in geometry of spacetime shaped by distribution of mass and energy, according to the Einstein Equations. The so called Newtonian prediction is, in some sense, internally inconsistent since photons are massless so why should they feel gravity of the Sun while passing by. In relativistic context things are clear: photon's path as a geodesic in a curved spacetime is no longer a straight line.

General Relativity provided also a framework in which cosmology has been promoted from just philosophical speculations to a solid branch of physical science. Namely, we can define the goal of cosmology as studying structure of the space-time at largest scales, indeed even its global structure. First gravitational lenses were discovered in cosmological context: as multiple images of a quasar produced by foreground galaxy or as arcs around galaxy clusters representing parts of Einstein rings. Hence it is no surprise that gravitational lensing has great potential in constraining cosmological model and/or alternative theories of gravity.

This chapter starts with an introduction to the present cosmological model and its two unsolved problems of dark matter and dark energy. Especially the accelerating expansion of the Universe is a great challenge for both physics and cosmology. In light of lacking the convincing theoretical explanation, an effective description of this phenomenon in terms of a cosmic equation of state turns out useful. The strength of modern cosmology lies in consistency across independent, often unrelated pieces of evidence. Therefore, every alternative method of restricting the cosmic equation of state is important. Strongly gravitationally lensed quasar-galaxy systems create such a new opportunity by combining stellar kinematics with lensing geometry.

The utility of strongly lensed systems (galaxy and cluster lenses) for cosmology will be discussed. Then an emphasis will be put on using strong gravitational lenses as probes of cosmic equation of state which is becoming attractive in light of ongoing surveys like SLACS based on different protocols than older searches focused on potential sources. In this

approach, which has recently entered the stage of providing first estimates on cosmological parameters and will certainly develop into a technique competitive with other methods, strongly lensed systems with known central velocity dispersions act as "standard rulers" (Einstein radius being standardized by stellar kinematics).

Gravitational lenses provide also constraints on alternative theories of gravity like MOND or conformal gravity - the state of the art results in this respect will be reviewed. At last, in the present era of advances in high energy astrophysics as proclaimed by AGILE, GLAST or MAGIC projects, the discovery of lensed high energy sources appears to be a matter of not very far future. This would open new interesting opportunities. For example energy dependent time delays between images could reveal Lorentz Invariance Violation as foreseen by some approaches to quantum gravity.

Throughout this chapter the citation will many times be just indicative and by no means exhaustive, due to space limitation. Apologies go to all the authors, whose important contributions will not therefore be cited but are hereby acknowledged.

2. Standard cosmological model and its challenges

The main paradigm of modern cosmology is that geometry of the Universe can be described as one of three possible Friedman-Robertson-Walker (FRW) solutions to the Einstein equations representing homogeneous and isotropic spacetime. The FRW line element reads:

$$ds^2 = dt^2 - a(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right], \quad (1)$$

where, the parameter $k = -1, 0, +1$ denotes the sign of curvature of constant time hypersurfaces (identifying the aforementioned three classes of solutions - open, flat or closed). Currently there exists strong evidence, coming from independent and precise experiments, that the Universe is spatially flat. For example a combined analysis of cosmic microwave background (WMAP5), baryon acoustic oscillations (BAO) and supernova data (Hinshaw et al., 2009) gives $\Omega_{tot} = 1.0050^{+0.0060}_{-0.0061}$.

The energy-momentum tensor describing the matter content of the Universe is usually assumed in a hydrodynamical form:

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu} \quad (2)$$

where: u^μ is the comoving four-velocity of the "cosmic fluid" ρ and p are the energy density and pressure, respectively.

Now, the Einstein equations get reduced to a set of two ordinary differential equations for the scale factor $a(t)$, historically known as Raychaudhuri and Friedman equation, respectively:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p), \quad (3)$$

$$H(t)^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{k}{a^2}. \quad (4)$$

which becomes a closed set when the equation of state $p = p(\rho)$ is specified. Usually it is assumed to be barotropic of the form $p = w\rho$. This is a very general and convenient form, containing the physically most important cases of pressureless matter ($w = 0$), radiation ($w = 1/3$) or cosmological constant ($w = -1$) but also allowing to effectively capture other

non-standard components, e.g. such as scalar fields. Energy density of the matter content of the Universe evolves according to:

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0, \tag{5}$$

The Friedman equation allows for a very convenient parametrization of the expansion rate $H(t)$:

$$H(t)^2 = H_0^2[\Omega_m a(t)^{-3} + \Omega_r a(t)^{-4} + \Omega_X a(t)^{-3(1+w_X)} + \Omega_k a(t)^{-2}]. \tag{6}$$

where $\Omega_i, i \in \{m, r, X, k\}$ denote present energy density¹ of respective components (matter, radiation, other non-standard barotropic component "X" e.g. for cosmological constant we have $w_\Lambda = -1$ and Ω_Λ is just a constant term). The last term is the so called curvature term. Two Einstein equations (3),(4) describe the expansion rate and its acceleration. Their present values are known as the Hubble constant H_0 and the deceleration² parameter q_0 .

The only gravitational degree of freedom, in the FRW cosmology, is the scale factor $a(t)$ depending on cosmic time t and responsible for temporal changes of spatial length-scales (known as cosmic expansion). Unfortunately it is not directly observable. However, there is a unique correspondence between $a(t)$ and redshift z which is an observable quantity. Namely, $a(z) = (1 + z)^{-1}$. As we mentioned above, the expansion rate $H = \frac{\dot{a}}{a}$ is determined by some set of parameters like H_0, Ω_m, Ω_r or Ω_X (if other components "X" are considered) and the equation of state parameter w_X . We will use a shorthand notation of \mathbf{p} for such parameters. Technically speaking, testing cosmological models means to determine parameters \mathbf{p} from observable quantities measured on samples of extragalactic objects lying far enough to feel the large-scale geometry of space-time. This specific goal of cosmology is currently called cosmography.

It is quite obvious that one very direct approach could be to test the distance – redshift relation $D(z)$ (called the Hubble diagram when plotted) whenever there is possibility to determine distances and redshifts independently. However as a consequence of non-euclidean geometry assumed, one distinguishes three types of distances in cosmology:

(i) comoving distance:

$$r(z; \mathbf{p}) = c \int_0^z \frac{dz'}{H(z'; \mathbf{p})} = \frac{c}{H_0} \bar{r}(z; \mathbf{p}) \tag{7}$$

where $\bar{r}(z)$ we denotes a reduced (dimensionless) comoving distance, i.e. a comoving distance expressed as a fraction of the Hubble horizon $d_H = c/H_0$,

(ii) angular diameter distance:

$$D_A(z; \mathbf{p}) = \frac{1}{1+z} r(z; \mathbf{p}) \tag{8}$$

(iii) luminosity distance:

$$D_L(z; \mathbf{p}) = (1+z)r(z; \mathbf{p}) \tag{9}$$

Angular diameter distance should be used for standard rulers — the objects whose size is known a priori. It is also used in gravitational lensing theory (because gravitational lensing deals with light deflection i.e. essentially with angles). The luminosity distance is a measure invoked while using standard candles (in cosmological context: SNIa or gamma ray bursts).

¹ As a fraction of critical density.

² Strictly speaking $q_0 = -H_0^{-2} \left(\frac{\dot{a}}{a}\right)_0$ and is called deceleration since for known forms of matter $\rho + 3p > 0$ and expansion of the Universe should decelerate.

Both distance measures are related to each other by a $(1+z)^2$ factor (see above) which is known as Etherington reciprocity relation.

2.1 Dark matter problem

Cosmology as it evolved, became able to pose meaningful physical questions concerning processes which occurred in the early Universe (baryogenesis, nucleosynthesis, radiation domination and recombination epochs etc.) and study the large scale structure (LSS) formation. In early days of physical cosmology (in the 70-ties of the XX century) observational capabilities of those days defined the goal of cosmology as the "quest for two numbers H_0 and q_0 " (Sandage, 1970). In the present era of precise observations of cosmic microwave background (CMB) anisotropies (starting with COBE, BOOMERANG, MAXIMA, to WMAP and the present PLANCK mission), LSS studies from deep surveys (like 2dFRGS, SDSS, 2MASS) the improved light elements (^2D , ^3He , ^4He , ^7Li) abundances assessments, when the connection between cosmology and particle physics became more intimate than ever before, our knowledge of physics and evolution of the Universe is captured in more than 20 parameters determined with high precision e.g. from WMAP data (Spergel et al., 2003).

One of, then and now important, such parameters is the matter density Ω_m . This is the point at which the problem of dark matter comes into play in a very consistent way. Namely, all present matter density determinations coherently point toward $\Omega_m = 0.25 - 0.30$. However, probes such like CMB anisotropies (Dunkley et al., 2009), baryon acoustic oscillations (BAO) (Percival et al., 2007) or supernovae Ia provide fits on Ω_m entangled with H_0 . The only method sensitive exclusively to matter density comes from studying peculiar velocities of galaxies. The analysis of Feldman et al. (2003) gave $\Omega_m = 0.30^{+0.17}_{-0.07}$. Later Mohayaee & Tully (2005) applied orbit retracing methods to motions in the local supercluster and obtained $\Omega_m = 0.22 \pm 0.02$, which is also consistent with the range reported above. On the other hand, primordial nucleosynthesis restricts the baryonic matter content to $\Omega_b = 0.04 \pm 0.01$ assuming $h = 0.7$ (Cyburt, 2004). This means that most of gravitationally interacting matter at scales of galaxies and clusters should be of non-baryonic nature and is commonly known under the name of the dark matter.

The case for dark matter was strengthened in several independent ways. First of all, and this was historically the first time when dark matter was invoked, dynamical studies of spiral galaxies revealed flat rotation curves extending far beyond the region from which most of the galactic light is emitted. Contrary to the expectations it showed that galaxies are about 10 times more massive than their luminous components. Hence the dark matter problem is also called the missing mass problem. Elliptical galaxies do not have such good rotation tracers as the spirals, but velocity dispersions of their stars support the claim that they are more massive than luminous components. Then gravitational lensing provides independent way of proving that ellipticals contain dark matter. X-ray observations of galaxy clusters revealed the existence of hot intergalactic gas whose temperature reflects the gravitational potential by virtue of hydrostatic equilibrium. On one hand it also confirmed the reality of the "missing mass", independently supported by cluster mass determination from cluster strong lensing manifested as giant arc around the clusters.

The case of the so called bullet cluster - a structure in which smaller sub cluster undergone a collision with a larger galaxy cluster 1E 0657-56 is often reported as a smoking gun evidence for dark matter (Clowe et al., 2006). Namely the mass profiles reconstructed by weak lensing techniques (sensitive to the total mass distribution) and by X-ray data reveal a characteristic mismatch between the total mass (anchored in the main cluster and a sub-cluster, respectively)

and the intergalactic gas – stripped from clusters during the collision and left compressed in between. Similar behavior i.e. that dark matter and stellar systems passed through the collision while the intergalactic gas interacted and was stripped, has been found in the cluster MACS J0025.4-1222 (Bradač, 2008).

The notion of dark matter is independently supported, from theoretical point of view, by particle physicists. They have for a long time contemplated the extensions of the standard model of particle physics and provided a long list of potential candidates for dark matter particles, like supersymmetric particles (e.g. neutralino), axion, Kaluza-Klein gravitons etc. All this evidence is so compelling that many experiments aimed at direct detection of dark matter particles are underway. However, there exist several facts creating problems to this picture. For example the large scale structure formation simulations typically predict much more structure at small scales than is actually seen. Observational fact that large galaxies are typically surrounded by their dwarf companions qualitatively fits this picture, but their abundances are by far too low with respect to predictions. The solution might be that there are indeed clumps of dark matter around galaxies, which had not anchored enough gas for star formation. Because gravitational lensing is sensitive to mass irrespective of its nature (luminous - baryonic or dark), there is much hope that gravitational lensing studies will help to settle the issue.

2.2 Dark energy problem

One of the most important issues in modern cosmology – in fact having consequences for the fundamental physics – is the problem of currently accelerating expansion of the Universe, known as the dark energy problem. It was first convincingly demonstrated in Hubble diagrams obtained from the SN Ia data e.g. (Amanullah et al., 2010; Perlmutter et al., 1999; Riess et al., 1998) and later supported by other independent studies including: CMBR anisotropies (Hinshaw et al., 2009; Komatsu et al., 2009) or baryon acoustic oscillations imprinted in the large scale structure power spectrum (Eisenstein & Hu, 1999; Percival et al., 2007). This result was surprising, since as can be seen from the Einstein equations (3) and (4) all known forms of matter (having the trace of energy-momentum tensor positive) should decelerate the expansion.

The explanation of this phenomenon is far from obvious and presents a fundamental challenge to standard models of both particle physics and cosmology. The new physics of dark energy may lie in the nature of gravity, the quantum vacuum, or extra dimensions. Concerning the first possibility there exists an increasing body of literature, e.g. (Buchert, 2001; Wiltshire, 2007) pointing out that if one attempts to average out local sources of gravity (galaxies and clusters) in order to obtain the smooth description of the Universe, such averaging procedure, not commuting with temporal evolution, could manifest as an additional source term in the energy-momentum tensor. Within the second possibility our ideas about the quantum vacuum are expressed by either introducing cosmological constant Λ or some scalar field evolving in time (quintessence). The last possibility is to contemplate modifications to the Friedman-Robertson-Walker models arising e.g. in brane-world scenarios.

2.3 Different scenarios of dark energy

The Λ CDM model is a FRW cosmology with non-vanishing cosmological constant and pressure-less matter including the dark matter component. It is a standard reference point in modern cosmology also called the concordance model since it fits rather well to independent

data (such like CMBR data, Large Scale Structure considerations, supernovae data). There are, however, reasons why we are not fully satisfied with the concordance scenario. First, the cosmological constant suffers from the fine tuning problem: being constant, why does it start dominating at the present epoch? Next, if we imagine its origin as the quantum-mechanical energy of the vacuum, field theoretical estimates predict its value 120 orders of magnitude larger than observed — the biggest discrepancy of theoretical physics today (Weinberg, 1989). Therefore the next, popular explanation of the accelerating Universe is to assume the existence of a negative pressure component called dark energy. One can heuristically assume that this component is described by hydrodynamical energy-momentum tensor with (effective) cosmic equation of state: $p = w\rho$ where $w < -1/3$. In such case this component is called "quintessence". Usually the quintessence is attributed to some sort of a scalar field. Another scalar field invoked by cosmologists is the inflaton, which in order to accomplish its role as driving the inflation and creating particles at the reheating epoch, clearly had its own dynamics. Therefore thinking about quintessence as having origins in the evolving scalar field, would lead to a natural expectation that w coefficient should vary in time, i.e. $w = w(z)$. Bearing in mind that the scale factor $a(t)$ is a real physical degree of freedom instead of the redshift z , the parametrization of $w(z) = w_0 + w_a \frac{z}{1+z}$ developed by Chevalier & Polarski (2001) and Linder (2003) turned out to be well suited for such case. For the purpose of providing an effective description of dark energy testable with real data it is sufficient to stop here, since the numerous particular scenarios (see e.g. Caldwell & Kamionkowski (2009)) – stemming from different physical inspirations – all have common point of empirical confrontation in the w coefficient and its temporal evolution. Two more specific models deserve however a mention: the Chaplygin gas and brane-world model of Dvali, Gabadadze and Porrati. The formulae for expansion rates in these models are given in Table 1.

Model	Cosmological expansion rate $H(z)$ (the Hubble function)
Λ CDM	$H^2(z) = H_0^2 [\Omega_m (1+z)^3 + \Omega_\Lambda]$
Quintessence	$H^2(z) = H_0^2 [\Omega_m (1+z)^3 + \Omega_Q (1+z)^{3(1+w)}]$
Chevalier-Polarski-Linder	$H^2(z) = H_0^2 \left[\Omega_m (1+z)^3 + \Omega_Q (1+z)^{3(1+w_0+w_a)} \exp\left(\frac{-3w_a z}{1+z}\right) \right]$
Chaplygin Gas	$H^2(z) = H_0^2 \left[\Omega_m (1+z)^3 + \Omega_{Ch} \left(A_0 + (1 - A_0)(1+z)^{3(1+\alpha)} \right)^{\frac{1}{1+\alpha}} \right]$
Braneworld	$H^2(z) = H_0^2 \left[(\sqrt{\Omega_m(1+z)^3} + \Omega_{rc} + \sqrt{\Omega_{rc}})^2 \right]$

Table 1. Expansion rates $H(z)$ in cosmological models representative to various dark energy scenarios.

In the class of generalized Chaplygin gas models matter content of the Universe consists of pressure-less gas with energy density ρ_m representing baryonic plus cold dark matter (CDM) and of the generalized Chaplygin gas with the equation of state $p_{Ch} = -\frac{A}{\rho_{Ch}^\alpha}$ representing dark energy responsible for acceleration of the Universe. The original Chaplygin gas corresponds to $\alpha = 1$. In cosmological context it has been promoted to the role of free parameter in phenomenological approach. Values of α exponent close to zero mean that the model is equivalent to Λ CDM case. This exotic form of cosmic equation of state is inspired by some super-string theories, but at the phenomenological level it has an advantage that it smoothly interpolates the expansion history of the Universe from matter dominated to dark energy dominated regimes. Chaplygin models have been confronted with different cosmological data like supernovae (Biesiada et al., 2005), cosmic microwave background radiation anisotropies (Amendola et al., 2003) or baryonic acoustic oscillations (Wu & Yu, 20).

At last, the brane-world models belong to the class of theories which seek the solution of presently accelerating expansion of the Universe not in an exotic material component, but in modifications of gravity. Brane-world scenarios assume that our four-dimensional spacetime is embedded into 5-dimensional space and gravity in 5-dimensions is governed by the usual 5-dimensional Einstein-Hilbert action. The bulk metric induces a 4-dimensional metric on the brane. The brane induced gravity models (Dvali et al., 2000) have a 4-dimensional Einstein-Hilbert action on the brane calculated with induced metric. According to this picture, our 4-dimensional Universe is a surface (a brane) embedded into a higher dimensional bulk space-time in which gravity propagates. Therefore there exists a certain cross-over scale r_c above which an observer will detect higher dimensional effects.

As a consequence of modified gravity, the Friedman equation reads

$$H^2 + \frac{k}{a^2} = \left[\sqrt{\frac{\rho}{3M_{pl}^2} + \frac{1}{4r_c^2}} + \frac{1}{2r_c} \right]^2$$

from which the expansion rate function shown in Table 1 can be derived in flat (i.e. $k = 0$) case. In flat brane-world Universe the following relation is also valid: $\Omega_{r_c} = \frac{1}{4}(1 - \Omega_m)^2$. Cosmological models in brane-world scenarios have been widely discussed in the literature. Quite recent paper (Xu & Wang, 2010) presents one of the most comprehensive analysis of brane-world models by considering jointly the data from supernovae, gamma-ray bursts, BAO, CMB peaks, the look back times and growth functions for the large scale structure. Their results (posterior probability distributions for model parameters) obtained by using the Markov Chain Monte Carlo simulation yield $\Omega_m = 0.266^{+0.0298}_{-0.0304}$. The potential of constraining dark energy models with SNIa data, even though ever increasing, would not be sufficient if taken alone in separation from the other approaches. Indeed, the power of modern cosmology lies in building up consistency rather than in single, precise, crucial experiments. Therefore, every alternative method of restricting cosmological parameters is desired. In this spirit a number of combined analyses involving CMBR measurements, age-redshift relation, x-ray luminosities of galaxy clusters or the large scale structure considerations have been performed in the literature. It is exactly in this context that cosmography performed on strong lensing systems is becoming useful.

3. Gravitational lensing in brief

Gravitational lensing of quasars and extragalactic radio sources at high redshifts by foreground galaxies is now well established and has developed into a mature branch of both theoretical and observational astrophysics. For comprehensive review and introduction to the theory of gravitational lensing see e.g. Schneider et al. (1992) or Schneider et al. (2006).

Imagine the source, observer and some other massive object (later to be called lens) located exactly along a line. From the point of view of traditional optics the source would be obscured by the intervening object: the only light ray (or a small collimated bunch of rays) pointing toward the observer would not reach him. General relativistic phenomenon of light deflection near massive bodies changes this picture: out of all light rays emitted radially some (going close to the deflector – how close it depends on the mutual locations of source, deflector and observer) are now focused at the observer. Without General Relativity they would have missed him. Intervening massive body acts as a lens and a source behind reveals its existence as a luminous ring — the so called Einstein ring. Even the smallest misalignment of the

source, the lens and observer results typically in multiple images whose angular positions and magnification ratios allow reconstructing lensing mass distribution.

As in ordinary optics, there are two equivalent approaches to understand the phenomenon: the light rays formalism and the wavefronts formalism. From the point of view of Fermat's principle, the light travel time can be calculated as

$$t(\mathbf{x}) = \frac{1+z_l}{c} \frac{D_l D_s}{D_{ls}} \left[\frac{1}{2} (\mathbf{x} - \beta)^2 - \psi(\mathbf{x}) \right] \quad (10)$$

where: \mathbf{x} and β are positions (as projected on the celestial sphere) of the image and the source, D_l , D_s are angular diameter distances to the lens and the source located at redshifts z_l and z_s respectively (D_{ls} is the angular diameter distance between lens and source). $\psi(\mathbf{x})$ is the projected gravitational potential (i.e. the actual potential integrated along line of sight) satisfying two dimensional Poisson equation:

$$\Delta\psi = 2\kappa \quad (11)$$

where κ is the (projected) surface mass density in units of critical density $\Sigma_{cr} = c^2 D_s / (4\pi G D_l D_{ls})$. Then, the Fermat's principle states that images form at stationary points of time delay surface $\nabla t(\mathbf{x})$, which leads to the lens equation:

$$\beta = \mathbf{x} - \nabla\psi = \mathbf{x} - \alpha \quad (12)$$

The last equality is usually invoked in the light-rays formalism, where $\alpha = \frac{D_l}{D_s} \hat{\alpha}$ is scaled deflection angle. In axially symmetric lenses, for example: $\hat{\alpha}(\mathbf{x}) = \frac{4GM(x)}{c^2 x^2} \mathbf{x}$ where $M(x)$ is the mass enclosed by the circle of radius $x = |\mathbf{x}|$. The most useful notion in gravitational lensing theory is the Einstein radius ϑ_E . In circular lenses it is the radius of the circle inside which the average projected mass density is equal to critical density (cf. above). For non circular lenses this should be modified appropriately. Thus the Einstein radius defines the deflection scale of a given lens.

The lensing is called strong if source position happens to lie within the circle of a radius ϑ_E . In this case multiple images appear. In the opposite case (i.e. the light-rays from the source passing by the lens outside its Einstein radius) there are no multiple images. However even in this case light-ray bundle experiences systematic distortion which changes the shape of the lensed source. This phenomenon is called weak lensing, has its own place in cosmology (Schneider et al., 2006) and is beyond the scope of this chapter.

Since lensing galaxies are often ellipticals, the number of images is usually equal to four – the issue of image multiplicity is discussed e.g. in (Schneider et al., 1992). However, the surprisingly realistic model of the lens potential is that of a singular isothermal sphere (SIS) in which the 3-dimensional mass density has the following profile:

$$\rho = \frac{\sigma_{SIS}^2}{2\pi G r^2} \quad (13)$$

Indeed lensing by ellipticals can be modeled by its variant called singular isothermal ellipsoid (SIE). Therefore for the illustrative purposes it would be sufficient to restrict our attention to the SIS model. Other realistic and more sophisticated models are discussed in classical textbooks (Schneider et al., 1992).

The Einstein ring radius for the SIS model is:

$$\vartheta_E = 4\pi \frac{D_{ls}}{D_s} \frac{\sigma^2}{c^2} \quad (14)$$

where σ denotes one-dimensional velocity dispersion of stars in lensing galaxy. If the lensing is strong i.e. $\beta < \vartheta_E$ then two co-linear images A and B form on the opposite side of the lens, at radial distances $R_A = \vartheta_E + \beta$ and $R_B = \vartheta_E - \beta$.

Besides multiple images, another important ingredient of gravitational lensing is the time delay between lensed images of the source. This effect originates as a competition between Shapiro time delay from the gravitational field and the geometric delay due to bending the light rays and is best understood in terms of Fermat principle. In other words, the intervening mass between the source and the observer introduces an effective index of refraction, thereby increasing the light travel time. In the aforementioned SIS model, time delay between the images is:

$$\Delta t_{SIS} = \frac{1+z_l}{2c} \frac{D_l D_s}{D_{ls}} (R_A^2 - R_B^2) \quad (15)$$

which according to the above mentioned relations for SIS model can also be written as

$$\Delta t_{SIS} = \frac{2(1+z_l)}{c} \frac{D_l D_s}{D_{ls}} \vartheta_E \beta = \frac{8\pi}{H_0} \tilde{r}_l \beta \frac{\sigma^2}{c^2} \quad (16)$$

In the last equation \tilde{r}_l denotes the reduced comoving distance to the lens. The equation (15) is commonly used by gravitational lensing community because it reduces time delay problem to relative astrometry of images, whereas β is much harder to assess (it is small in order for strong lensing to occur) and Einstein ring radius is not a directly observable quantity (although image separation fairly represents the Einstein radius). However, the equation (16) is more useful from the theoretical point of view. In particular it shows explicitly that the time delay between images is created at the lens location (\tilde{r}_l factor).

The last observable derivable from strongly lensed systems is the flux ratio of images. It is the most sensitive with respect to details of mass distribution along the light-ray path (both in terms of detailed knowledge of smooth component of mass distribution as well as the graininess of the lens i.e. microlensing by stars or other clumped massive structures along the path.)

Let us close this brief introduction with the reminder that gravitational lensing, as general relativistic effect, is achromatic. It means that the locations of images and the time delay do not depend on the wavelength of light in which they are observed.

4. Cosmography with strong lensing systems.

4.1 Brief history

Cosmographic application of strong lensing systems has quite a long history. Since the discovery of the first gravitational lens the number of strongly lensed systems increased to a hundred (in the CASTLES database³) and is steadily increasing following the new surveys like SLACS (Sloan Lens ACS Survey⁴) and the opportunity for gravitational lenses being competitive to other techniques becomes real.

³ <http://www.cfa.harvard.edu/castles/>

⁴ <http://www.slacs.org/>

The first theoretical proposal of serious application of strong gravitational lensing was presented by Refsdal (1964) in his stimulating paper on the Hubble constant measurements from time delays between images. Namely if the lensed source is intrinsically variable (quasars being the main population of sources display such variability) and we are able to extract the variability pattern from the light-curve (which in practice is non-trivial task), this variability would be observed at different times in the images. Then the time-delay, as e.g. (15) depends on image locations and relative distances of the source, lens and observer. But the magnitude of this delay (temporal scale of the effect) is given by H_0^{-1} . This creates alternative possibility of measuring the Hubble constant H_0 which is, unlike the other methods, independent of the cosmic distance ladder and its calibration. The number of lenses with reliably measured time delays has accumulated slowly over decades. Several years ago there were about 10 such lenses and the observational status of the Hubble constant determination, as reviewed in details in (Schneider et al., 2006), was that time delays preferred $H_0 = 52 \pm 6 \text{ km s}^{-1} \text{ Mpc}^{-1}$ in contrast to the HST Key Project value of $H_0 = 72 \pm 8 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (Freedman et al., 2001). Later papers of Oguri (2007) and Coles (2008) announced the results ($H_0 = 68 \pm 16 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $H_0 = 71 \pm 8 \text{ km s}^{-1} \text{ Mpc}^{-1}$ respectively) now in agreement with HST value.

Other cosmological parameters, like Ω_m , Ω_Λ or w coefficient in quintessence models up to now have been assessed on samples drawn from radio survey Cosmic Lens All Sky Survey (CLASS)– Chae (2007) and SDSS Quasar Lens Search (SQLS) – Oguri et al. (2008). The approach they taken was to assess the frequency of lensed systems given a source population. This quantity depends on the cosmic volume over which sources are distributed, which makes it useful for cosmography. However, there are other factors influencing the result such as reliable lens model adopted, contribution from the line of sight contamination by intervening massive structures or source luminosity function. In the next section we will discuss another technique of using strong lenses as tools for cosmography.

4.2 Testing dark energy models: strong lensing systems as standard rulers

The first formulations of this approach can be traced back to Futamase & Yoshida (2001). Then, the idea of using strong lensing for measuring the cosmic equation of state was discussed in Biesiada (2006) and also in a later paper by Grillo et al. (2008).

The main idea is that formula for the Einstein radius in a SIS lens

$$\theta_E = 4\pi \frac{\sigma_{SIS}^2}{c^2} \frac{D_{ls}}{D_s} \quad (17)$$

depends on the cosmological model through the ratio of (angular-diameter) distances between lens and source and between observer and lens. The angular diameter distance in flat Friedmann-Robertson-Walker cosmology is given by (8).

Provided one has reliable knowledge about the lensing system: i.e. the Einstein radius θ_E (from image astrometry) and stellar velocity dispersion σ_{SIS} (form central velocity dispersion σ_0 obtained from spectroscopy) one can use it to test the background cosmology. This method is independent on the Hubble constant value (which gets canceled in the distance ratio) and is not affected by dust absorption or source evolutionary effects. It depends, however, on the reliability of lens modeling (e.g. SIS or SIE assumption) and measurements of σ_0 . Hopefully, starting with the Lens Structure and Dynamics (LSD) survey and the more recent SLACS survey spectroscopic data for central parts of lens galaxies became available allowing to assess their central velocity dispersions. There is a growing evidence for homologous structure

of late type galaxies (Koopmans et al., 2006; 2009; Treu et al., 2006) supporting reliability of SIS/SIE assumption. In particular it was shown there that inside one effective radius massive elliptical galaxies are kinematically indistinguishable from an isothermal ellipsoid.

In the method outlined above cosmological model enters not through a distance measure directly, but rather through a distance ratio

$$\mathcal{D}^{th}(z_l, z_s; \mathbf{p}) = \frac{D_s(\mathbf{p})}{D_{ls}(\mathbf{p})} = \frac{\int_0^{z_s} \frac{dz'}{h(z'; \mathbf{p})}}{\int_{z_l}^{z_s} \frac{dz'}{h(z'; \mathbf{p})}} \quad (18)$$

and respective observable counterpart reads:

$$\mathcal{D}^{obs} = \frac{4\pi\sigma_0^2}{c^2\theta_E}$$

This has certain consequences both advantageous and disadvantageous. The positive side is that the Hubble constant H_0 gets canceled, hence it does not introduce any uncertainty to the results. On the other hand we have a disadvantage that the power of estimating Ω_m is poor (which could be seen by inspection into specific formulae for $h(z; \mathbf{p})$ – see Table 1). Cosmological model parameters (coefficients in the equation of state, in particular) can be estimated by minimizing the chi-square:

$$\chi^2(\mathbf{p}) = \sum_i \frac{(\mathcal{D}_i^{obs} - \mathcal{D}_i^{th}(\mathbf{p}))^2}{\sigma_{\mathcal{D},i}^2} \quad (19)$$

where the sum is over the sample and $\sigma_{\mathcal{D},i}^2$ denotes the variance of \mathcal{D}^{obs} (contextual use of the same symbol for variances and velocity dispersions should not lead to confusion). Putting aside the issue that the observable quantity here is a distance ratio, one can see that strong lenses constitute a class of standard⁵ rulers.

The above method extensively investigated by Grillo et al. (2008) on simulated data was first used in practice to constrain various cosmological models in Biesiada et al. (2010) where Λ CDM, quintessence and CPL model were constrained. Later it was used (together with SNIa, CMB and BAO data) as a part of joint analysis in (Biesiada et al., 2011). Table 2 summarizes recent constraints on parameters of the cosmological model obtained with strong gravitational lensing.

The results obtained were generally in agreement with those obtained by other authors with different methods. In particular at the 2σ level they agree with the supernovae Ia results. One should note however that in the CPL model (quintessence with time varying equation of state) standard rulers (strong lenses in particular) display a systematic shift downwards in (w_0, w_a) plane with respect to supernovae Ia. Such shift in best fitted parameters inferred from supernovae (standard candles, sensitive to luminosity distance) and BAO or acoustic peaks (standard rulers, sensitive to angular diameter distance) has already been noticed and discussed by Lazkoz et al. (2009) and by Linder & Roberts (2008). Bearing in mind similar mutual inconsistency in the Hubble constant values inferred from lensing time delays and

⁵ They could be better called "standardizable" rulers because each lens has intrinsically different Einstein radius, but stellar kinematics, i.e. velocity dispersion allows for disentangling the effect of mass from that of distances. In a sense supernovae Ia could also be called "standardizable" not standard candles, since it is the stretch factor correction that makes them standard.

Cosmological model	Strong lenses	Lenses + BAO + R	SN Ia
Λ CDM	$\Omega_m = 0.266 \pm 0.280$	$\Omega_m = 0.273 \pm 0.018$	$\Omega_m = 0.275 \pm 0.020$
Quintessence	$\Omega_m = 0.27$ prior $w = -0.632 \pm 0.446$	$\Omega_m = 0.262 \pm 0.035$ $w = -1.066 \pm 0.188$	$\Omega_m = 0.299 \pm 0.075$ $w = -1.070 \pm 0.215$
Chevalier-Polarski-Linder	$\Omega_m = 0.27$ prior $w_0 = 0.359 \pm 1.245$ $w_a = -3.630 \pm 5.328$	$\Omega_m = 0.276 \pm 0.055$ $w_0 = -0.824 \pm 0.704$ $w_a = -0.757 \pm 2.148$	$\Omega_m = 0.228 \pm 0.156$ $w_0 = -0.993 \pm 0.207$ $w_a = 0.609 \pm 1.071$
Chaplygin Gas		$\Omega_m = 0.273 \pm 0.018$ $A = 1.000 \pm 0.001$ $\alpha = -0.040 \pm 2.260$	$\Omega_m = 0.275 \pm 0.020$ $A = 0.999 \pm 0.004$ $\alpha = 0.006 \pm 0.372$
Braneworld		$\Omega_m = 0.345 \pm 0.021$	$\Omega_m = 0.117 \pm 0.015$

Table 2. Constraints on the parameters of various cosmological model obtained with strong lensing systems. First column refers to a sample of strong lenses only (Biesiada et al., 2010), the middle refers to lenses analysed jointly with other standard rulers, whereas the last column shows fits with Union2 SNIa data (after Biesiada et al. (2011))

from the HST Key Project (Schneider et al., 2006), this result suggests the need for taking a closer look at compatibility of results derived by using angular diameter distances and luminosity distances respectively. The ideas of testing the Etherington reciprocity relation between these two distance measures have been discussed by Basset & Kuntz (2004) and by Uzan et al. (2005).

Although the sample of suitable lenses (i.e. with good measurements of Einstein radii, source and lens redshifts and central velocity dispersions) has been rather small ($n = 20$ lenses) the ongoing SLACS survey is providing new strong lensing systems which is very encouraging for further applications of the method. The strategy adopted in SLACS survey is particularly important. The earlier searches were focused on source population (quasars) seeking for close pairs or multiples and checking if they are multiple images of a single source lensed by an intervening galaxy. Therefore a high lensing probability was an important selection factor there.

Since lensing probability is proportional to the area of the Einstein ring, it means that two factors are crucial in this context. First, is the mass of the lens. This is the main reason why in vast majority of cases the lens is E/SO type galaxy. This could be understood since ellipticals being a latecomers in hierarchical structure formation are created in mergers of low-mass spiral galaxies. Hence they are more massive than spirals and the probability of their acting as lenses is higher. Second factor is the distance ratio D_{ls}/D_s . In details, this of course depends on the cosmological model, but it is maximal when the lens is located roughly half way between the source and the observer. The SLACS sample has an average D_{ls}/D_s ratio equal to 0.58 with an rms scatter 0.15 (Treu et al., 2006). Whereas for their purpose (investigating galactic dynamics with strong lenses) it was advantageous, in our context it weakens the performance of the method. Therefore having a sub-sample of lenses with the distance \mathcal{D} ratio deviating from the mean more than rms in either direction would be beneficial and in this respect SLACS survey is encouraging. Namely, the SLACS survey is focused on possible lens population (massive ellipticals) with good spectroscopic data. Using SDSS templates spectra are carefully checked for residual emission (at least three distinct common atomic transitions) coming from higher redshifts. Such candidates undergo image processing by subtracting parametrized brightness distribution typical for early type galaxies in order to reveal multiple images of the quasar. Details can be found in Bolton et al. (2006). Therefore,

besides the obvious bonus of having central velocity dispersion measured, such strategy is better suited for discovering systems with larger D_{ls}/D_s ratios which in turn can be used for testing cosmological models.

4.3 Cosmography with cluster lensing

Besides the galaxies acting as lenses, their clusters – first virialized structures in the Universe – do the same. The cores of galaxy clusters have surface densities which are typically much larger than the critical surface density Σ_{cr} for multiple image production. Therefore they are able to produce strongly lensed images of galaxies and quasars lying behind them. Such images manifest themselves as luminous arcs around clusters. Historically it was Paczyński (1987) who proposed that giant arcs might be gravitationally lensed images of background galaxies. First measurements of arcs' redshifts proved this definitely. The possibility of constraining cosmology with CSL systems has been explored in the past e.g. (Paczyński & Górski, 1981; Sereno, 2002) and still remains a fruitful, fast developing field of research. It is typical that we observe multiple sets of arcs in cluster lenses corresponding to different sources (with different redshifts) lensed by the same cluster. Hence, the abundance of arcs may provide useful cosmological constraints, in a manner similar to the statistics of multiple images in galaxy lenses. For example, Meneghetti et al. (2005) explored the statistics of arcs in various cosmological models and found that arc abundances can be used to differentiate between dark energy scenarios. More recently, Gilmore & Natarayan (2009) explored the prospect of combining cluster lensing systems as a more powerful probe of dark energy. Analogously to the method outlined in the preceding section, the locations of images in cluster lensing systems also contain useful cosmological information. Namely, the image positions depend not only on the mass distribution, but also on the angular diameter distances between the observer, lens, and source. If more than one set of images is observed, the geometrical dependence may be exploited to probe the cosmological parameters even with a single cluster lens. One of the best studied cluster lensing system is Abell 1689. The mean redshift of this cluster is $z_l = 0.184$ and it is one of the richest clusters in terms of the number density of galaxies in its core. In a recent paper by Jullo et al. (2010) this cluster was used to derive constraints on the cosmological parameters Ω_m and w . Based on images from the Advanced Camera for Surveys (ACS) this cluster is known to produce 114 multiple images from 34 unique background galaxies. This allowed Jullo et al. (2010) to use many observables like (18) from a single cluster. To be more specific instead using \mathcal{D}^{th} like in (18), they used quotients formed pairwise for background sources

$$\mathcal{D}_{cl}^{th} = \frac{\mathcal{D}^{th}(z_l, z_{s1}; \mathbf{p})}{\mathcal{D}^{th}(z_l, z_{s2}; \mathbf{p})} \quad (20)$$

where: z_l is the cluster's redshift, z_{s1} and z_{s2} are redshifts of respective pair of sources.

Applying the following criteria: demand of good spectroscopic data for images and excluding regions where mass reconstruction gets poorer, from the initial 114 images, Jullo et al. (2010) selected finally 28 images which they further used to constrain cosmological parameters to $\Omega_m = 0.25 \pm 0.05$ $w = -0.97 \pm 0.07$.

Even more promising is the idea of using a larger sample of cluster lensing systems. Such an approach has the advantage that results obtained from different lines of sight are statistically independent. As discussed by Gilmore & Natarayan (2009) competitive constraints can be obtained by combining at least 10 lenses with 5 or more image systems. We may therefore conclude that cluster strong lensing is becoming a very useful complementary tool,

particularly in probing dynamic dark energy models, which demand larger range of redshifts to be probed.

5. Constraining alternative theories of gravity with strong lensing

Despite the great successes of General Relativity, and Newtonian gravity (if taken in appropriate limits), one should be aware that the majority of its direct tests have been within the Solar system. Measurements of time delays in binary pulsar systems (Taylor et al., 1992) (Taylor et al. 1992) have also verified General Relativity, but all such tests were performed in large acceleration regimes (large as compared to accelerations e.g. experienced by stars at the outskirts of galaxies). Therefore, the possibility remains that gravity at large distances (or small accelerations) could be different. Similar reasoning aimed at explaining dark energy, underlaid, an already discussed, idea of brane-world cosmologies. In the context of dark matter (or "missing mass") problem the most famous idea of this kind was formulated by Milgrom (1983) and is known as MOND. MOND is able to explain (or at least not to be in conflict with) all kinematical properties of stellar systems only in terms of their baryonic constituents (stars and gas). More specifically, Milgrom introduced a preferred scale of acceleration $a_0 \approx 10^{-10} \text{ ms}^{-2}$ of the order of the centripetal accelerations of stars and gas clouds in the outskirts of disk galaxies, and postulated a modified form of the second law of dynamics: $m\mu(a/a_0)\mathbf{a} = \mathbf{F}$ where: $\mu(x)$ is certain function (not specified) which has appropriate asymptotic form: $\mu(x) = x$ for $x \ll 1$ and $\mu(x) = 1$ for $x \gg 1$ in order to interpolate between high acceleration (Newtonian) regime and low acceleration (MONDian) regime. In terms of it MOND relates the acceleration \mathbf{a} of a test particle to the Newtonian gravitational field $-\nabla\Phi_N$ generated by the baryonic mass density alone by

$$\mu(a/a_0)\mathbf{a} = -\nabla\Phi_N$$

Designed to solve the flat rotation curve, it also predicted a lot of phenomenological relations such like Tully-Fisher relation and is successful in explaining the shapes of rotation curves. It is amazing that single value of $a_0 = 1.2 \cdot 10^{-10} \text{ m/s}^2$ reproduces both rotation curves and Tully-Fisher relation in over a hundred galaxies. Excellent reviews of other phenomenological successes of MOND can be found in (Milgrom, 2008; Sanders & McGaugh, 2002).

On the other hand, it is observational fact that in all cases we know, the "missing mass" discovered kinematically manifests itself also in enhanced gravitational lensing power. It means that gravitational lensing could be used to test alternative theories like MOND. However, gravitational lensing is a relativistic effect, while MOND in its original form is not. Therefore there was a lot of effort devoted to construct proper relativistic extension of MOND, which culminated in formulation of TeVeS theory by Bekenstein (2004). TeVeS formulates the gravity theory in terms of three fundamental fields: tensor (the metric), vector and scalar, later it was shown that it could be reduced to just tensor-vector theory – for the state of art review on this see Bekenstein (2010).

One of first considerations of gravitational lensing in MOND were due to Mortlock & Turner (2001). They rightly identified two limiting regimes of lensing: high acceleration (so called Newtonian) where the deflection angle agrees with relativistic predictions and low acceleration (deep MOND) regime where the deflection angle approaches the constant value. Deep MOND regime is valid for light-ray impact parameters $b \gg b_0 := (k/2\pi)(GM/a_0)^{1/2}$. This result has been confirmed by Chiu et al. (2006), this time consequently in the TeVeS framework. However, they also demonstrated that in the intermediate MOND regime $b \approx b_0$ the deflection angle depends crucially on the details of the theory (e.g. specification of $\mu(x)$)

function in MOND). Besides they demonstrated that in point lens model, the difference in image amplifications is no longer unity, as in General Relativity, and that it depends on the mass of the lens. They also investigated the gravitational time delay in TeVeS which is important for interpreting differential time delays in doubly imaged variable quasars. The baryon distribution in a galaxies is no longer point-like but can be better represented by e.g. the Hernquist profile. Zhao et al. (2006) employed it to compare TeVeS predictions with a large sample of quasars doubly imaged by intervening galaxies and obtained encouraging (in favor of TeVeS) results. The weak lensing by clusters of galaxies, presents a problem for pure MOND. Especially the bullet cluster (a pictorial case for dark matter) was claimed to kill the MOND approach. After first encouraging approaches to explain it in TeVeS, it was established that it cannot be done with purely baryonic matter distribution – a collisionless component of massive 2 eV neutrinos is necessary (Angus et al., 2006).

6. Strong lensing tests of Lorentz Invariance Violation

Despite the fact that quantum gravity theory still remains elusive, it is generally expected that it will bring the picture of a space-time foam at short distances leading to Lorentz Invariance Violation (LIV) manifested e.g. by energy dependent modification of standard relativistic dispersion relation (Amelino-Camelia et al., 1998; Amelino-Camelia & Piran, 2001).

Several years ago it has been proposed to use astrophysical objects to look for energy dependent time of arrival delays. Specifically gamma ray bursts (GRBs) being highly energetic events visible from cosmological distances are the most promising sources of constraining LIV theories (Ellis et al., 2006). Among other sources the BL Lac objects like Mk 501 are considered. It is this particular object from which 20 TeV photons were reported (Amelino-Camelia & Piran, 2001). Such objects (also called blazars) have similar nature with quasars. The idea of searching for time of flight delays is tempered however by our ignorance of intrinsic delay (at source frame) in different energy channels. In (Biesiada & Piórkowska, 2009) a test based on gravitational lensing, which free from this limitation has been proposed. In the rest of this section we will follow it closely.

Let us consider a phenomenological approach to LIV theories by assuming the modified dispersion relation for photons in the form (Amelino-Camelia & Piran, 2001):

$$E^2 - p^2c^2 = \epsilon E^2 \left(\frac{E}{\xi_n E_{QG}} \right)^n \tag{21}$$

where: $\epsilon = \pm 1$ is the so called “sign parameter”, ξ_n is a dimensionless parameter associated with energy scale at which n -th order corrections to the dispersion relation become important. As a first guess (having no other suggestions) one may assume E_{QG} equal to the Planck energy, then: $\xi_1 = 1$ and $\xi_2 = 10^{-7}$. The dispersion relation (21) essentially corresponds to the power-law expansion so for practical purposes (due to smallness of expansion parameter E/E_{QG}) only the lowest terms of the expansion are relevant. The relation (21) leads to a hamiltonian

$$\mathcal{H} = \sqrt{p^2c^2 \left[1 + \epsilon \left(\frac{E}{\xi_n E_{QG}} \right)^n \right]} \tag{22}$$

from which time dependent group velocity $v(t) = \frac{\partial \mathcal{H}}{\partial p}$ can be inferred.

Then, the comoving distance travelled by photon to the Earth is

$$r(t) = \int_{t_{emission}}^{t_{detection}} v(t) dt = \int_0^z v(z') \frac{dz'}{H(z')(1+z')} \tag{23}$$

where in the last equation a standard time-redshift parametrization was taken into account. One can expect these considerations having clear cosmological context. The reason for this is simple — the modifications due to LIV theories are really tiny, so one has to look for sources located at cosmological distances (such like quasars or gamma ray bursts) which are far enough to compensate for the smallness of LIV corrections. This means that cosmological background geometry should be taken into account. Expressing group velocity in terms of redshift, we get

$$v(z) \simeq c(1+z) \left[1 + \epsilon \frac{(n+1)}{2} \left(\frac{E}{\xi_n E_{QG}} \right)^n (1+z)^n \right] \quad (24)$$

Time of flight for the photon of energy E is equal to

$$t_{LIV} = \int_0^z \left[1 + \epsilon \frac{n+1}{2} \left(\frac{E}{\xi_n E_{QG}} \right)^n (1+z')^n \right] \frac{dz'}{H(z')} \quad (25)$$

In the first term one easily recognizes the time of flight for photons in standard relativistic cosmology (i.e. without LIV). Due to very small magnitude of LIV corrections it also fairly represents the time of flight for low energy photons. Consequently, the time delay between a low energy and a high energy photon is equal to

$$\Delta t_{LIV} = \frac{n+1}{2} \left(\frac{E}{\xi_n E_{QG}} \right)^n \int_0^z \frac{(1+z')^n dz'}{H(z')} \quad (26)$$

where we restricted our attention to “infraluminal” motion of high energy photons (i.e. low energy photons arrive earlier to the observer). Generalization to “superluminal” motion is straightforward — time delays become early arrivals. The idea of observational strategy emerging from (26) is simple. One should monitor appropriate (i.e. emitting both low and high energy photons) cosmological source at different energy channels and try to detect this time delay. However there remains an indispensable uncertainty about intrinsic time delays: there is no reason for which low and high energy signal should be emitted simultaneously. The method outlined below, based on strong gravitational lensing allows to get rid of this ambiguity.

Let us now imagine a source at cosmological distance emitting low energy and high (TeV) energy photons which is gravitationally lensed by a foreground galaxy. Let us also assume that LIV type distorted dispersion relation (21) holds. The observer would also notice time delays between images, but this time it would be a combined effect of gravitational lensing and LIV. Therefore it would no longer be achromatic.

It is easy to calculate this by using the fictitious “LIV comoving distance” $r_{LIV}(z)$, namely:

$$\Delta t_{LIV,SIS} = \frac{8\pi}{H_0} \tilde{r}_{LIV}(z_l) \beta \frac{\sigma^2}{c^2} \quad (27)$$

where:

$$\tilde{r}_{LIV}(z_l) = \tilde{r}_l + H_0 \frac{n+1}{2} \left(\frac{E}{\xi_n E_{QG}} \right)^n \int_0^{z_l} \frac{(1+z')^n dz'}{H(z')} \quad (28)$$

Now we can assume that observations in low energy would essentially provide time delay between images equal to Δt_{SIS} , whereas monitoring of the same images in high energy (TeV)

channel would provide $\Delta t_{LIV,SIS}$. Restricting attention to the $n = 1$ case (because LIV effects are extremely small), we can see that these two measurements would differ by

$$\Delta t_{LIV,SIS} - \Delta t_{SIS} = \frac{8\pi}{H_0} \beta \frac{\sigma^2}{c^2} \frac{E}{E_{QG}} \int_0^z \frac{(1+z') dz'}{H(z')} \quad (29)$$

Let us make an estimate for the above LIV effect taking a real strong lensing system. To be specific, we will assume a flat FRW model with $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$ — the so called concordance model. Strong lensing system HST 14176+5226 can serve as an example. This system was discovered with the Hubble Space Telescope and further confirmed to be a gravitational lens. The lensed source is a quasar at redshift $z_s = 3.4$ whereas the lens is an elliptical galaxy having redshift $z_l = 0.809$. The lens model best fitted to the observed images gives the Einstein radius $\theta_E = 1''.489$ and $\beta = 0''.13$. Optical spectroscopy of the lensing galaxy in HST 14176+5226 system provided measurements of the velocity dispersion of $\sigma = 290 \pm 8 \text{ km/s}$ in the lensing galaxy. Substituting these data to (29) gives $\Delta t_{LIV,SIS} - \Delta t_{SIS}$ equal to $3.7 \times 10^{-9} \text{ s}$ for 5 TeV photons and $1.5 \times 10^{-8} \text{ s}$ for 20 TeV ones.

It would also be interesting to ask how the LIV effects might modify image configurations. It could be suspected that they might do so since from the Fermat's principle perspective images are located at stationary points of the wavefront travel time functional (given by equation (10)). Therefore since LIV modifies time of flight in an energy dependent way (due to modified dispersion relation) then one expects the images seen at different energies located at different positions. It is easy to see that for the SIS lens (generalizations to other mass profiles are also rather straightforward) the difference between Einstein radii for high and low energy photons $\Delta\theta_{E,LIV} := \theta_{E,LIV} - \theta_E$ would be given by formula:

$$\Delta\theta_{E,LIV} = \theta_E \frac{E}{E_{QG}} \left(\frac{I^{(1)}(z_l, z_s)}{\tilde{r}(z_l, z_s)} - \frac{I^{(1)}(z_s)}{\tilde{r}(z_s)} \right) \quad (30)$$

where: $I^{(1)}(z_1, z_2) := \int_{z_1}^{z_2} \frac{(1+z') dz'}{H(z')}$. For realistic lens configurations like HST 14176+5226 this would give negligibly small corrections of order 10^{-16} arc sec . Hence even if LIV were operating this would not be able to change macro-images position in a detectable way. However it cannot be excluded that such minute differences could become relevant while studying caustic crossing (Schneider et al., 2006) possibly leading to different magnification patterns due to microlensing at different energies.

One may ask if appropriate lensing systems (i.e. having sources emitting both low and high energy photons) exist. It is an observational fact that very high energy emission ($E > 100 \text{ GeV}$) has been detected from over a dozen of blazars which have similar nature with quasars. Quasars, on the other hand are the sources in all known strong lensing systems. It is a matter of coordinating strong lensing surveys with experiments in high energy astrophysics, such like AGILE, GLAST or MAGIC experiments and the future will certainly bring the discovery of lensed high energy source.

In conclusion, one should keep in mind the possibility of testing LIV effects by monitoring time delays between images of gravitationally lensed quasars in low and high energy channels. In standard theory (General Relativity) the result should be the same — gravitational lensing is essentially achromatic. On the other hand in the presence of LIV effects time delays loose this property — high energy photons should come at different times comparing with low energy ones. Therefore time delays between images should be

different at different energies (e.g. optical or gamma-rays and TeV photons). Because this method is differential in nature, it gets rid of the assumptions about intrinsic time delays of signals at different energies. In fact time delays between images at different energies could be established in different experiments (at unrelated observing sessions) performed on given lensing system. The only demand is that they are accurate enough (done with a sufficient temporal resolution). However, light curves of gamma-ray bursts are already sampled with milli-second resolution and AGILE experiment went down to micro-seconds.

7. Conclusions

Strong gravitational lensing as an effect rooted deeply in General Relativity has great potential in constraining many aspects of gravitational physics. First of all it is useful in studies of dark matter. It stems from the fact that gravitational lensing is sensitive to mass distribution regardless of its nature (whether they are baryonic or not). This is already a rich field being currently explored both theoretically and observationally.

In all known strong lensing systems producing multiple images, the population of sources is of cosmological nature (quasars or distant bright galaxies). In light of recent progress in modeling lensing galaxies, and considerable enrichment of observational data with reliable spectroscopic measurements allowing for determination of redshifts and central velocity dispersions, the new possibility opens up to use well studied strong lensing systems for constraining cosmological model parameters. Although in the past, there was certain scepticism about this technique it is currently proving its effectiveness and in the future – having in mind development of ongoing and planned lens surveys – it will eventually evolve into a competitive technique for cosmography. This is very important, because of the dark energy problem (i.e. the puzzle of presently accelerating Universe). Currently the only empirical way to address the issue of dark energy is by refining the cosmography.

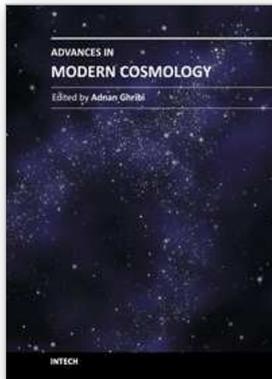
At last, since the light deflection angle by a massive bodies, which is the corner stone of gravitational lensing, is usually calculated in General Relativistic framework, strong lensing can also be used for investigating alternative theories of gravity. It could be also helpful in the context of searching the energy dependent time delays as predicted by Lorentz Invariance Violating theories. Therefore, one may hope for a bright future of strong gravitational lensing is bright and one can expect that it will bring us many interesting results especially when observing campaigns of strong lenses (besides optical or radio regimes) are extended into very high energy range.

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The twentieth century elevated our understanding of the Universe from its early stages to what it is today and what is to become of it. Cosmology is the weapon that utilizes all the scientific tools that we have created to feel less lost in the immensity of our Universe. The standard model is the theory that explains the best what we observe. Even with all the successes that this theory had, two main questions are still to be answered: What is the nature of dark matter and dark energy? This book attempts to understand these questions while giving some of the most promising advances in modern cosmology.

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