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A Polytropic Solution of the Expanding Universe – Constraining Relativistic and Non-Relativistic Matter Densities Using Astronomical Results

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1. Introduction

Important programs are being pursued currently to collect distance and recession velocity data from supernovae type Ia of several groups collecting and analyzing data (SNe Ia) (Astier et al., 2006; Hicken et al., 2009; Wood-Vasey et al., 2007) and from gamma ray bursts (GRB) (Schaefer, 2007) and most recently from other supernovae types. These events are our best hope of "standard candles" looking back in a quantitative manner towards the epochs of recombination and early galaxy formation, to nearly to the beginning of time (Leibundgut, 2008). While some optical data have been obtained from orbiting observatories, data of good quality have been collected from grounded telescopes with the hope to collect several thousand distance/velocity pairs of ever increasing quality over the next few years. Likewise, interpretation of the cosmic microwave background (CMB), which consists of signals remaining from the very primitive Universe after 13 billion years of cooling, may provide another independent source of data for estimating these parameters (Komatsu et al., 2009). Interpretation of the Sloan Digital Sky Survey (SDSS) data are being continually refined and the interpretations broadened to include analysis for the baryonic acoustic oscillations (BAO) remnant signals (Eisenstein et al., 2005). Another tool for estimation of gross Universe structure is the X-ray emissions from galaxy clusters (Vikhlinin et al., 2008) with future data collections planned to answer several cosmological questions (Vikhlinin et al., 2009). Data from dozens of studies have been used to estimate the Hubble constant, the Universe age and to support the new idea of Dark Energy (DE) assisted universe expansion (Carroll et al., 1992). Many decades ago Chandrasekhar successfully investigated the nature of white dwarf stars with gravitational fields large enough to quickly accrete considerable surrounding gas (Chandrasekhar, 1964). He predicted that as the object mass increases, Pauli instability of the dwarf star constituents is approached with a thermonuclear critical mass dependent both on the nature of the matter, the energy contained within and by the repulsive pressure exerted from the various nuclear species. When the instability limit is reached the object suffers collapse quickly followed by thermonuclear explosion which we gaze at in awe as a supernova (Chandrasekhar & Trooper, 1964). This very successful description, which has been confirmed many times by observation, is dependent on the polytropic generalizations of matter and energy. The useful relationships of Chandrasekhar demand unique constants which differ

between common matter and relativistic matter or radiation. This property allows us to easily discriminate between the two matter types and manipulate both using modeling with astronomical data such as that from SNe Ia events. We present a brief derivation of the general polytropic model in Appendix 5.1.
People are currently investigating the physics of SNe Ia explosions and other cosmological data with aim to better use improved data for solving many problems of astronomy and physics (Linden et al., 2009), but reports of calculations of neutrino abundance using these data are rare. Systematic error remains in the SNE Ia data, however, with SNe Ia luminosity being correlated to the host galaxy size and mass density (Kelly et al., 2009). We and others have recently warned of even more systematic errors entering analyses of these data, both from the viewpoint of mathematical arguments (Oztas & Smith, 2006) and a call to use proper statistical analysis of the SNe Ia data (Hartnett & Oliveira, 2007; Oztas et al., 2008). A similar situation, though not identical in nature, has recently arisen regarding the analysis of the CMB data. The CMB temperature maps published by the Wilkinson Microwave Anisotropy Probe (WMAP) team are inconsistent with the differential, time-ordered data from which the descriptive maps are reconstructed (Liu et al., 2009). When a simple correction to the analytical routine is used the resulting maps become much "smoother" losing the vast majority of temperature details (Li & Liu, 2009; Liu & Li, 2010a). Indeed, these reinterpreted maps seem featureless; consistent with extreme homogeneity of the early Universe. Since interpretation of the analysis of the WMAP group is under question, we leave comparisons between CMB observations and our analysis of SNe Ia data until that time when these issues are resolved (Roukema, 2010) and concentrate on the SNe Ia and BAO analyses. We have also recently published results from an attempt to combine SNe Ia and GRB data for analysis of cosmological parameters (Smith et al., 2010). Unfortunately the GRB data are exceptionally noisy making firm conclusions impossible.
Here we extend the use of polytropic tools for the first time to estimate ranges for the average common matter and relativistic matter densities of the Universe, as well as better estimating spacetime curvature and DE. We analyze SNe Ia data using luminary distances and associated distance errors rather than log distances and log errors. Because we use actual distances rather than the logs (not a trivial difference), we also include a data pair for the earth with no error for the first time and shall show this important data pair should be included in all analyses utilizing SNe Ia data. Using actual estimated errors for distance rather than log errors accents the differences between models with and without the cosmic constant and between results from different reports. We examine data from four recent SNe Ia collections and can roughly estimate the current, non-relativistic matter density and the order of the relativistic matter density. We obtain these values using models combining the polytropic indexes with two variations of the Friedmann-Robertson-Walker (FRW), the *standard model.* Our results from the model admitting significant spacetime but without the cosmic constant are significantly different from the results using this term, Ω_Λ, but in a flat Universe. In general, we also find the matter densities for the models not invoking the cosmic constant to be much lower than those with that term. Our calculated low matter densities agree with several estimates from Big Bang Nucleosynthesis (BBN); calculations derived from first principles (Burles et al., 2001a;b). Not surprisingly, we also find the typical values for non-relativistic matter to be at least one order of magnitude larger than the relativistic matter density. Our results are driven by the great difference in distance errors between nearby and distant SNe Ia which are not evident when modeling using log errors. The matter densities from many solutions here are near the low end of the range as predicted by others while the estimated DE and/or spacetime portions of the Universe are often larger than previous findings. We must emphasize such

results, varying from popular expectations, are not unexpected since we take care in weighing observational errors.
On the other hand our results derived from reported BAO parameters do agree with currently popular values for Ω_m and Ω_Λ. The suggestion was made over a decade ago that the SDSS data could be useful to estimate neutrino abundance (Hu et al., 1998) though reports of progress using that data have not come to our attention. Here we have expanded the range of data which can be used for such analysis by considering the BAO solutions. Our findings are extremely sensitive to total matter density and the ratio of polytropic matter species. We present a brief explanation of our analytical technique in Appendix 5.2.
We are also aware of reports suggesting the Universe may be modeled considering some components as a Chaplygin gas (Setare, 2009) but we think our approach significantly different and perhaps more rigorous. We have not performed calculations using SNe Ia data, that include the effects of Cold Dark Matter (CDM), because the properties are not understood not allowing us to estimate these polytropic constants. We suggest the SNe Ia and SDSS/BAO data are extremely useful observations, which in addition to constraining the Hubble constant, the Universe age and DE, can also be used to define the limits of non-relativistic and relativistic matter densities. This is because large spacetime is the important independent variable rather than the much more commonly tested energy, pressure, density, etc. Better constraints on these cosmological values can be made using larger SNe Ia (and SDSS) data sets, hopefully with smaller errors, which should be available in the near future after systematic errors have been corrected (Kelly et al., 2009). Likewise estimates of H_0 should await reconciliation between groups analyzing the CMB data (Li & Liu, 2009; Liu & Li, 2010a). The overall aim of all this mental anguish will hopefully lead to a much better understanding of the dynamics and parameters of our expanding Universe and hence our origin and eventual fate.

2. Expansion of the polytropic Universe

We begin with two equations of state we suggest describe the early Universe both during and after recombination, allowing $c = 1$ as usually presented

$$(\frac{\dot{a}}{a})^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} + \frac{k}{a^2} \tag{1}$$

$$\frac{\ddot{a}}{a} = \frac{-4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}. \tag{2}$$

Here ρ is the material content density, p the pressure, a the expansion factor, G the gravitational constant, Λ the cosmic constant and k the constant of integration; typically indicating spacetime curvature. For the general situation both variables include contributions from normal matter, relativistic matter and radiation. Cold dark matter (CDM) is sometimes considered to be described within ρ. We take the derivative of the first equation with respect to time and use the result to eliminate $\ddot{a}$ and arrive at the following relationship

$$\dot{\rho} = -3\frac{\dot{a}}{a}(\rho + p). \tag{3}$$

We presume adiabatic processes dominate Universe expansion and use the polytropic relationship between pressure and density from Chandrasekhar as $p = K\rho^\gamma$, where $\gamma = 1 + 1/n$, and n is the polytropic index. This relationship is a well-known equation of state with $n \to \infty$ for isothermal processes and $n = -1$ for isobaric processes. By substitution with Chandrasekhar's relationship we now have

$$\dot{\rho} = -3\frac{\dot{a}}{a}(\rho + K\rho^{\gamma}) \tag{4}$$

and with separation of variables we can then integrate both sides in a few steps

$$\int \frac{\dot{\rho}}{\rho + K\rho^{\gamma}} dt = -3 \int \frac{\dot{a}}{a} dt. \tag{5}$$

We separate the integrand on the left-hand side into parts and perform some algebra

$$\begin{aligned}
\frac{1}{\gamma - 1} \int \left(\frac{\gamma}{\rho} - \frac{1 + \gamma K \rho^{\gamma - 1}}{\rho + K\rho^{\gamma}} \right) \dot{\rho} dt &= -3 \int \frac{\dot{a}}{a} dt \\
\frac{1}{\gamma - 1} \left(\ln(\rho^{\gamma}) - \ln(\rho + K\rho^{\gamma}) \right) &= -3\ln(a) + \ln(C) \\
\ln\left(\frac{\rho^{\gamma}}{\rho + K\rho^{\gamma}} \right) &= (\gamma - 1)(-3\ln(a) + \ln(C)) \\
\frac{\rho^{\gamma}}{\rho + K\rho^{\gamma}} &= \left(\frac{C}{a^3} \right)^{\gamma - 1} \\
\frac{\rho^{\gamma - 1}}{1 + K\rho^{\gamma - 1}} &= \left(\frac{C}{a^3} \right)^{\gamma - 1}.
\end{aligned} \tag{6}$$

We can rearrange Eq. (6) in two steps

$$\rho^{\gamma - 1} = \frac{C^{\gamma - 1}}{a^{3(\gamma - 1)} - KC^{\gamma - 1}} \tag{7}$$

to arrive at a useful relationship

$$\rho = \frac{C}{(a^{3(\gamma - 1)} - KC^{\gamma - 1})^{\frac{1}{(\gamma - 1)}}}. \tag{8}$$

By presuming an expansion factor of $a_0 = 1$ in Eq. (6), we can solve for C to eliminate this term

$$\frac{\rho_0^{\gamma - 1}}{1 + K\rho_0^{\gamma - 1}} = C^{\gamma - 1} \tag{9}$$

and substituting Eq. (9) into Eq. (8) we arrive at our relationship of interest

$$\rho = \frac{\rho_0}{(a^{3(\gamma - 1)}(1 + K\rho_0^{\gamma - 1}) - K\rho_0^{\gamma - 1})^{\frac{1}{(\gamma - 1)}}}. \tag{10}$$

We presume the Universe consists of different matter species with unique values of K_i and γ_i describing each variety. We can eliminate each K_i species and use the present values for our parameters by adhering to the *cosmological principle* that expansion occurs isotropically for each species, since both non-relativistic and relativistic matter species have been dilute, except for stars, since near singularity. We also presume that energy, relativistic matter and

non-relativistic matter only weakly interact currently, except for the stars, and the contribution of radiant energy to the Universe has been tiny since recombination.
In the neo-Newtonian framework two phases of matter are important - non-relativistic matter and relativistic matter and the values we shall use for the associated constants for each are listed (Chandrasekhar, 1983) and we present brief derivations in Appendix 5.1. We estimate the relative values for the Chandrasekhar constants for relativistic and normal matter from considerations of cosmological parameters presented for solution of the SDSS data (Eisenstein et al., 2005) in our "Modeling and Results" section.

non-relativistic	relativistic
$n_{nr} = 3/2$	$n_r = 3$
$\gamma_{nr} = 5/3$	$\gamma_r = 4/3$
$K_{nr} = 0.645$	$K_r = 1.124$

With these generalizations we can separate the matter density into two different variables within the Friedmann relationship

$$(\frac{\dot{a}}{a})^2 = \frac{8\pi G}{3}(\rho_{nr} + \rho_r) + \frac{\Lambda}{3} - \frac{k}{R_0^2 a^2} \tag{11}$$

where ρ_{nr} and ρ_r are the densities of non-relativistic and relativistic matter.
For use with SNe Ia data these are redefined as normalized parameters with

$$\Omega_r = \frac{\rho_{0,r}}{\rho_c}, \quad \Omega_{nr} = \frac{\rho_{0,nr}}{\rho_c}, \quad \Omega_\Lambda = \frac{\Lambda}{3H_0^2} \quad \text{and} \quad \Omega_k = -\frac{k}{R_0^2 H_0^2} \tag{12}$$

where Ω_r and Ω_{nr} can be calculated as averages from the present range for H_0, the Hubble constant. For this work we use the critical density parameter $\rho_c = \frac{3H_0^2}{8\pi G}$ for the case of a Universe without DE($\Lambda = 0$) and so do not require a flat Universe.
We present a more general variation of the common normalization condition for the parameters of interest as

$$1 = \Omega_{nr} + \Omega_r + \Omega_\Lambda + \Omega_k. \tag{13}$$

When we substitute τ for $H_0 t$ we can simplify these equations into a usable form as derived below

$$\left(\frac{da}{d\tau}\right)^2 = a^2 \left\{ \frac{\Omega_{nr}}{\left(a^2\left(1 + K_{nr}\rho_{0,nr}^{2/3}\right) - K_{nr}\rho_{0,nr}^{2/3}\right)^{3/2}} + \frac{\Omega_r}{\left(a\left(1 + K_r\rho_{0,r}^{1/3}\right) - K_r\rho_{0,r}^{1/3}\right)^3} + \Omega_\Lambda + \frac{\Omega_k}{a^2} \right\} \tag{14}$$

$$\left(\frac{da}{d\tau}\right)^2 = \frac{1}{a}\left\{ \frac{\Omega_{nr}}{A_{nr}^{3/2}} + \frac{\Omega_r}{A_r^3} + \Omega_\Lambda a^3 + \Omega_k a \right\} \tag{15}$$

and for brevity we use $A_{nr}(a) = \left(1 + K_{nr}\rho_{0,nr}^{2/3}\right) - \frac{K_{nr}\rho_{0,nr}^{2/3}}{a^2}$ and $A_r(a) = \left(1 + K_r\rho_{0,r}^{1/3}\right) - \frac{K_r\rho_{0,r}^{1/3}}{a}$. We collect the normalized terms in a familiar form

$$d\tau = \frac{\sqrt{a}da}{\sqrt{\frac{\Omega_{nr}}{A_{nr}^{3/2}} + \frac{\Omega_r}{A_r^3} + \Omega_\Lambda a^3 + \Omega_k a}} \tag{16}$$

and presuming the null geodesic of the FRW Universe and multiplying by R_0 we get

$$\frac{dR_0 r}{dt} = \frac{R_0}{R}(1 - kr^2)^{1/2} \tag{17}$$

$$R_0\frac{dr}{dt} = \frac{1}{a(t)}(1 - kr^2)^{1/2} \tag{18}$$

and by rearranging the differential equation for the separation of variables

$$R_0\frac{dr}{(1 - kr^2)^{1/2}} = \frac{dt}{a(t)} \tag{19}$$

and introducing H_0 we can obtain a relationship between which shall begin to allow us to calculate the two forms of interesting matter in relationship with Universe expansion

$$H_0R_0\frac{dr}{(1 - kr^2)^{1/2}} = \frac{d\tau}{a(\tau)}$$

$$H_0R_0\frac{dr}{(1 + \Omega_k R_0^2 H_0^2 r^2)^{1/2}} = \frac{d\tau}{a(\tau)}. \tag{20}$$

We now reintroduce Eq.(16) for $d\tau$

$$\frac{H_0R_0dr}{\sqrt{1 + \Omega_k R_0^2 H_0^2 r^2}} = \frac{1}{a}\frac{\sqrt{a}da}{\sqrt{\frac{\Omega_{nr}}{A_{nr}^{3/2}} + \frac{\Omega_r}{A_r^3} + \Omega_\Lambda a^3 + \Omega_k a}}. \tag{21}$$

We then integrate both sides to allow for the frequency drop to respond to the Universe expansion

$$\int_0^{r_1}\frac{H_0R_0dr}{\sqrt{1 + \Omega_k R_0^2 H_0^2 r^2}} = \int_{a_1}^{1}\frac{da}{\sqrt{a}\sqrt{\frac{\Omega_{nr}}{A_{nr}^{3/2}} + \frac{\Omega_r}{A_r^3} + \Omega_\Lambda a^3 + \Omega_k a}}. \tag{22}$$

By collecting several variables into a simpler term $\sqrt{\Omega_k}R_0H_0r = y$ and introducing the redshift relation in terms of frequency decline $a = 1/(1+z) = \xi$ on the right-hand side (we shall use ξ as the ratio of observed frequency to emitted frequency) and after a few steps we get

$$D_L = \frac{c}{\xi H_0 \sqrt{|\Omega_k|}} \text{sinn} \left\{ \sqrt{|\Omega_k|} \int_{\xi_1}^{1} \frac{d\xi}{\sqrt{\xi}\sqrt{\frac{\Omega_{nr}}{A_{nr}^{3/2}} + \frac{\Omega_r}{A_r^3} + \Omega_\Lambda \xi^3 + \Omega_k \xi}} \right\} \tag{23}$$

where the integration begins with the past ξ_1 to the present 1, sinn is either *sinh* or *sin* dependent on positive or negative spacetime curvature and the speed of light is in km/s.
For a universe without DE we simply drop the $\Omega_\Lambda \xi^3$ term and allow spacetime curvature and we designate this solution as Ω_r-ST. For a flat, relativistic universe with DE we can greatly simplify the above relationship as

$$D_L = \frac{c}{\xi H_0} \int_{\xi_1}^{1} \frac{d\xi}{\sqrt{\xi}\sqrt{\frac{\Omega_{nr}}{A_{nr}^{3/2}} + \frac{\Omega_r}{A_r^3} + \Omega_\Lambda \xi^3}} \tag{24}$$

which we denote as the Ω_r-DE model.
For purposes of comparison we also fit the now famous DE relationship for a flat universe without relativistic matter in terms of frequency decline

$$D_L = \frac{c}{\xi H_0} \int_{\xi_1}^{1} \frac{d\xi}{\xi \sqrt{\frac{\Omega_m}{\xi} + \Omega_\Lambda \xi^2}} \tag{25}$$

which we designate as the *Simple*-DE model. For evaluation of the FRW model without DE we simply replace the $\Omega_\Lambda \xi^2$ term from the denominator with the $\Omega_k \xi$ term leaving us with an integral which has been solved analytically (Oztas & Smith, 2006; Peebles, 1993)

$$D_L = \frac{c}{\xi H_0 \sqrt{|\Omega_k|}} \text{sinn}[2(\text{arctanh}(\sqrt{|\Omega_k|}) - \text{arctanh}(\frac{\sqrt{|\Omega_k|}}{\sqrt{\frac{\Omega_m}{\xi} + \Omega_k}}))] \tag{26}$$

which we designate as the *Analytic*-ST model and sometimes as the spacetime model.
Notice the normalized matter density terms for Eqs.(23,24) encompass, non-relativistic, relativistic and light-energy densities. We also need to emphasize the introduction of terms to account for the two natures of matter also places a dependence of matter density on the Hubble constant and hence spacetime expansion. This can be easily justified since the range of SNe Ia data from the present to z=1.55 covers the majority of the Universe age and very large changes of matter densities. By use of the above models, solidly based on the work of Chandrasehkar, we attempt to avoid introducing additional parameters as sometimes done by *ad hoc* modification of the equation of state, for instance as a Chaplygin gas.
We are able to estimate the Chandrasekhar constants for normal and relativistic matter with principles used to estimate parameter fits with SDSS data Eisenstein et al. (2005). The densities for relativistic and normal matter species, ρ_i, define the various terms of Ω_i as

$$\Omega_i = \frac{\rho_{0,i}}{\rho_c} \quad \text{for} \quad K_i \rho_{0,i}^{\gamma-1} = K_i \rho_c^{\gamma-1} \Omega_i^{\gamma-1}. \tag{27}$$

So we can use these terms as coefficients for both species of normal and relativistic matter

$$B_{nr} = K_{nr} \rho_c^{2/3}, \qquad B_r = K_r \rho_c^{1/3}.$$

We now define the current matter densities in terms of B_i, matter densities and the expansion factor as

$$\rho_{nr} = \frac{\rho_c \Omega_{nr}}{\left(a^2 \left(1 + B_{nr}\Omega_{nr}^{2/3}\right) - B_{nr}\Omega_{nr}^{2/3}\right)^{3/2}}, \quad \rho_r = \frac{\rho_c \Omega_r}{\left(a(1 + B_{nr}\Omega_r^{2/3}) - B_r\Omega_{nr}^{1/3}\right)^3}. \tag{28}$$

We follow the lead of Eisenstein, using their A and R parameters and perform evaluations independent of H_0, to uncover the relative dependencies of B_i species on normalized matter density Ω_m. Details of our technique for evaluation are presented in Appendix 5.2. Results and relative errors with evaluations at several popular matter densities are presented below. There is a recent report indicating systematic errors of up to 10% buried in the luminosities of many SNe Ia observations. For this reason we present results considering four independent reports of SNe Ia distances and redshifts, rather than a single or combined set. Hopefully, removal of more systematic error from these data, which shall soon be made public (Kelly et al., 2009), will allow better detailed investigation than the 3 parameters used here. A recent submission claiming serious problems with interpretation of the WMAP5 presentations dissuades us from investigating this area with our polytropic model until these difficulties have been resolved (Liu & Li, 2010a;b; Moss et al., 2010; Roukema, 2010).

3. Modeling and results

Calculations are made using only recently published SNe Ia sets. We first examine the 397 data pairs of Hicken and coworkers from their Table 1 as they prefer (Hicken et al., 2009). We next present fits of the combined 288 SNe Ia treated with the MLCS2K2 Light Curve Fits from data recently compiled from several sources as presented by (Kessler et al., 2009). We also present results from the list of Kowalski and coworkers (Kowalski et al., 2008), which are 307 SNe Ia culled from the Union compilation of 414 SN Ia including data from the Supernova Legacy Survey, the ESSENCE Survey, the Hubble Space Telescope and some older data and finally, we present results from modeling with 162 SNe Ia data presented by Wood-Vasey and coworkers (Wood-Vasey et al., 2007). In addition, we add the present frequency ratio of 1 at a distance of 0 with no error, for an exacting data pair at no financial cost to all data sets (Oztas et al., 2008).

The actual luminary distances and geometric errors are extracted from the published log distance and log errors rather than use log-log estimates and we best-fit these curves using robust minimization. The fitting routines for Tables 1 through 4 with solution trace examples presented in Figure 1, allow two or three free parameters, one of which is always the Hubble constant. We prefer H_0 as a free parameter having noticed the goodness of fit to be highly dependent on the freedom of this parameter (no surprise this). The other free parameters are either Ω_{nr} or Ω_{nr} with Ω_k while the remainder of the estimates is always the spacetime curvature or relativistic matter density, for instance, $\Omega_k = 1 - \Omega_{nr}$ or $\Omega_r = 1 - \Omega_{nr} - \Omega_k$. We do not think the data firm enough to report results from models containing 4 parameters; necessary for simultaneous solution of Ω_k, Ω_Λ, Ω_{nr} and Ω_r.

Figure 1 is an illustration of the data with the curves from the fits for *Simple*-DE and *Analytic*-ST models to the complete Hicken *et al.* data. Note the very large errors associated with ancient SNe Ia distances, to the left on the graph, compared to those of more nearby explosions, at the lower right-side. Rightly so - one should expect noisy data from signals more than half the Universe age. Precise distance determination is a problem which has plagued astronomers from time immemorial (Sharaf & Sendi, 2010). These very large errors mean the data from ancient SNe Ia play a much smaller role in determining the fit parameters

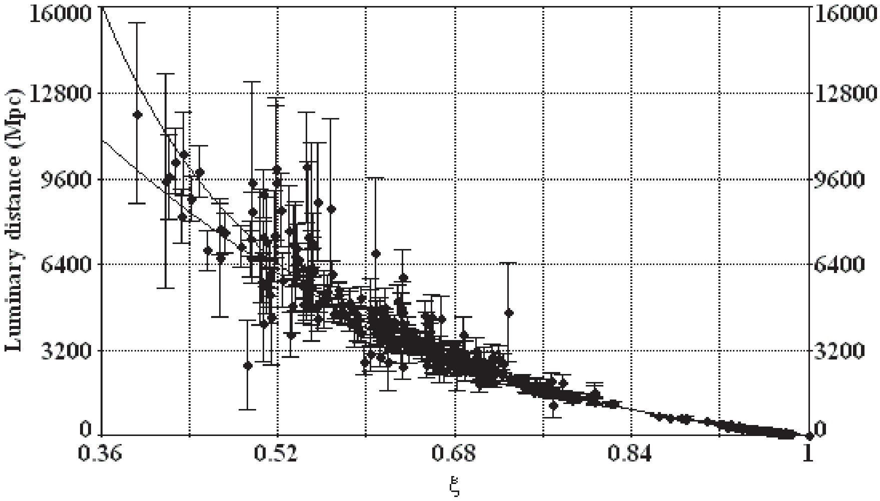

Fig. 1. Comparison of the two Standard Models using the abscissa of observed SNe Ia galaxy frequency ratios rather than redshifts with all 397 pairs plus today. The bottom curve represents the fit for the *Simple*-DE model, and the top line the fit for the *Analytic*-ST model.

Model	FP	H_0	Ω_{nr} or Ω_m	Ω_Λ or Ω_k	$\chi^2/(\text{N-FP})$
Ω_r-DE	3	68.4 ± 0.5	0.05 ± 0.03	$0.95 \pm 0.01(\Omega_\Lambda)$	1.37
Simple-DE	2	67.9 ± 0.5	$0.27 \pm 0.03(\Omega_m)$	$0.73(\Omega_\Lambda)$	1.40
Ω_r-ST	3	73.6 ± 0.6	0.034 ± 0.30	$0.966 \pm 0.01(\Omega_k)$	1.79
Analytic-ST	2	67.5 ± 0.5	$0.0003 \pm 0.06(\Omega_m)$	$0.9997(\Omega_k)$	1.92

*With N the number of data pairs (Hicken et al., 2009) after culling 11 outliers(387) and FP the number of free parameters. H_0 in km s^{-1}Mpc^{-1}.

Table 1. Results with data from 386 SNe Ia and today

than nearby supernovae. This contrasts to the more usual fitting regimes which place value on the ancient distances almost as strongly as those of nearby SNe Ia. Rather than distance errors increasing somewhat marginally between nearby SNe Ia and distant explosions as usually presented in the typical log/log plots of luminary magnitude *vs.* redshift, we present a more realistic view of error estimates which are incredibly large from distant signals. The results of this can be seen as the obvious difference between curve traces for both *standard models* in the figure. It is more usual that the traces for these two models be nearly inseparable on displayed graphs - and they do overlap at the smaller distances - lower right-side.
There are 11 data pairs of the Hicken *et al.* set (397 pairs) which reside more than 2500 km s^{-1}Mpc^{-1} from the best fit *Analytic*-ST model; $> 3\sigma$. We remove these and calculate the values from 387 data pairs with results in Table 1; the goodness of fits for three models are nicely improved by discarding these 11 pairs. This improvement is surprising considering the 11 being outliers which should least impact the robust curve fitting process. None of the models however, return excellent fits with this data as judged by a χ^2/N-FP of less than 1.35 (Hartnett & Oliveira, 2007). Noteworthy of this data reduction is that the *Simple*-DE model now presents a better fit than the *Analytic*-ST, for the reverse is true when all 398 data pairs are

Model	FP	H_0	Ω_{nr} or Ω_m	Ω_Λ or Ω_k	χ^2/(N-FP)
Ω_r-DE	3	74.2 ± 0.5	0.04 ± 1	$0.96 \pm 1(\Omega_\Lambda)$	2.24
Ω_r-ST	3	79.8 ± 0.6	0.0003 ± 0.02	$0.9997 \pm 1(\Omega_k)$	2.34
Analytic-ST	2	67.8 ± 0.5	$0.42 \pm 0.09(\Omega_m)$	$0.58(\Omega_k)$	2.66
Simple-DE	2	69.9 ± 0.7	$0.76 \pm 0.07(\Omega_m)$	$0.24(\Omega_\Lambda)$	3.05

*With N the number of data pairs (283) and FP the number of free parameters (Kessler et al., 2009). H_0 in km s^{-1}Mpc^{-1}.

Table 2. Results with data from 282 SNe Ia and today

Model	FP	H_0	Ω_{nr} or Ω_m	Ω_Λ or Ω_k	χ^2/(N-FP)
Analytic−ST	2	68.0 ± 0.5	$0.0002 \pm 0.05(\Omega_m)$	$0.9996(\Omega_k)$	1.34
Ω_r-ST	3	77.4 ± 0.6	0.0002 ± 0.03	$0.9997 \pm 1(\Omega_k)$	1.44
Ω_r-DE	3	75.4 ± 0.6	0.045 ± 0.03	$0.955 \pm 0.02(\Omega_\Lambda)$	1.49
Simple−DE	2	75.3 ± 0.7	$0.14 \pm 0.02(\Omega_m)$	$0.86(\Omega_\Lambda)$	1.50

*With N the number of data pairs (308) and FP the number of free parameters (Kowalski et al., 2008). H_0 in km s^{-1}Mpc^{-1}.

Table 3. Results with data from 307 SNe Ia and today

used. While the matter density for the *Simple*-DE model is near that expected from previous publications, ≈0.27, for instance the BAO analysis (Eisenstein et al., 2005), the matter density returned by the *Analytic*-ST model is quite low. Low matter density values are also found from the fits of both polytropic models and the Hubble constant also tends on the low side of the usual expectation for 3 of the 4 models. When we consider the more detailed polytropic model which includes the cosmic constant, Ω_r-DE, we find it fits the data better than other models, being on the border of a good fit.

When we discard our exact data pair of today on earth and analyze with only the 386 SNe Ia data we find significant differences in the goodness of fit of the two *standard models*. Exclusion of this single data pair allows the models to "drift" from the origin at x, y of exactly 1,0 and the values for χ^2/(N-FP) increase by nearly 0.1 for both the *Simple*-DE model and the *Analytic*-ST model (results not presented).

In our next analyses we use the 288 data pairs from Kessler and friends (Kessler et al., 2009) after culling 6 pair outside 2500 Mpc from the curve defined by the *Analytic*-ST fit. Even after discarding these outliers, the fits are not considered good as judged by the χ^2/(N-FP), Table 2. The values for matter densities, Ω_m, for the two *standard models* are much greater than commonly reported. This might be evidence for systematic error which is revealed by our direct plot and "hidden" in the typical log/log plots. Here we find the polytropic models to be significant improvements over the two *standard models*, again with rather low values for Ω_m and with somewhat high values for the Hubble constant, compared with expectations.

We next extend our analysis to all 307 SNe Ia data of Kowalski (Kowalski et al., 2008) and we find the modeling to yield significantly better fits as judged by lower χ^2/(N-FP) for all models, Table 3. The fit for the *Analytic*-ST model with a χ^2/(N-FP) of 1.34 suggests a good fit but again the *Simple*-DE model does not fit the data very well. The matter densities of the better fitting models are much lower than those usually published from log/log plots, even the normalized matter density, Ω_m, of the *Simple*-DE model, at 0.14, is well below the oft presented 0.25 to 0.27.

Model	DF	H_0	Ω_{nr} or Ω_m	Ω_Λ or Ω_k	$\chi^2/$(N-FP)
Simple-DE	2	66.6 ± 0.7	$0.21 \pm 0.04(\Omega_m)$	$0.79(\Omega_\Lambda)$	2.12
Ω_r-ST	3	69.0 ± 0.7	0.025 ± 0.02	$0.975 \pm 0.02(\Omega_k)$	2.35
Ω_r-DE	3	68.0 ± 0.8	0.065 ± 0.02	$0.935 \pm 0.05(\Omega_\Lambda)$	2.60
Analytic-ST	2	65.2 ± 0.7	$0.08 \pm 0.10(\Omega_m)$	$0.92(\Omega_k)$	2.61

*With N the number of data pairs (163) and FP the number of free parameters (Wood-Vasey et al., 2007). H_0 in km s^{-1}Mpc^{-1}.

Table 4. Results with data from 162 SNe Ia and today

We examine our final data set published by Wood-Vasey (Wood-Vasey et al., 2007) consisting of 60 distance SNe Ia from the ESSENCE Supernova Survey normalized with about 100 other observations. This is the only time the *standard model*, *Simple*-DE, is the the best fit, Table 4. The *Simple*-DE model also presents a value for Ω_m of 0.21, not much lower than currently popular values. Notice that all values for $\chi^2/$(N-FP) are much greater than two other analyses (Tables 1,3) and similar to the large values reported in Table 2. These, and the results from the Kessler *et al.* data may be caused by assigning consistently smaller errors to distance measurements, compared with other reports. This results in the curve fit being described primarily by the nearby SNe Ia events, with very low assigned errors, almost totally ignoring earlier SNe Ia. The curves from this data and the fits of the Kessler data are "flatter" than expected, reflecting the near linear alignments of nearby SNe Ia as graphed, resulting in large values of $\chi^2/$(N-FP). The Ω_r-ST models typically return a value for H_0 of 69 to 80 km s^{-1}Mpc^{-1} which are our only consistent results in that popular range, but favored by recent multi-parameter WMAP (6 free parameters)and BAO analyses (Komatsu et al., 2009). The *Simple-DE* model, which is most popular model currently, here presents values for H_0 of 67 to 75 using these four sets, so cannot really discriminate between the two controversial values towards each end of this range (Sandage et al., 2006).
A significant value for relativistic matter density was never found from any model with any of the the four data sets examined here; we do not present numerical estimates for Ω_r since these report much smaller than the calculated errors of Ω_{nr} for our two models. We do venture an upper bound for Ω_r of <0.001 from the present to near reionization. It seems likely this is because the portion of universal gravitation due to relativistic particles and photons is and has been $<<0.001$. The values of Ω_{nr} and Ω_m for the DE models, both presuming a flat Universe, differ widely. The more sophisticated Ω_r-DE model with a Ω_{nr} of about 0.05 or less suggests the Universe contains much less matter than previous estimates. On the other hand the *Simple*-DE model fit with an estimated Ω_m of 0.27 ± 0.03 to the culled Hicken *et al.* data, which is similar to values published using log luminary distance data (Astier et al., 2006; Davis et al., 2007) or the official 6-parameter WMAP results(Komatsu et al., 2009) but is short of a total Ω_m of 0.32 found from reanalysis of WMAPLiu et al. (2009). A similar situation is found for the two models allowing spacetime curvature, *Analytic*-ST and Ω_r-ST, where a more "typical" value for Ω_m of ≈ 0.25 becomes ≈ 0.001 with the Ω_r-ST model. The two terms introduced to account for the different natures of matter and the Hubble flow in Eqs.(23,24) allow matter densities to vary with respect to Universe expansion; these more sophisticated models suggest a much lower average matter density in the current epoch. The differences in matter densities between the two DE and two ST models are consistent with the idea that a universe with more matter requires more energy to continue expanding.
We present two Figures (2 and 3) which are the results from a series of single free parameter fits, with data of the culled Hicken *et al.* set and modeling with the two *standard models*,

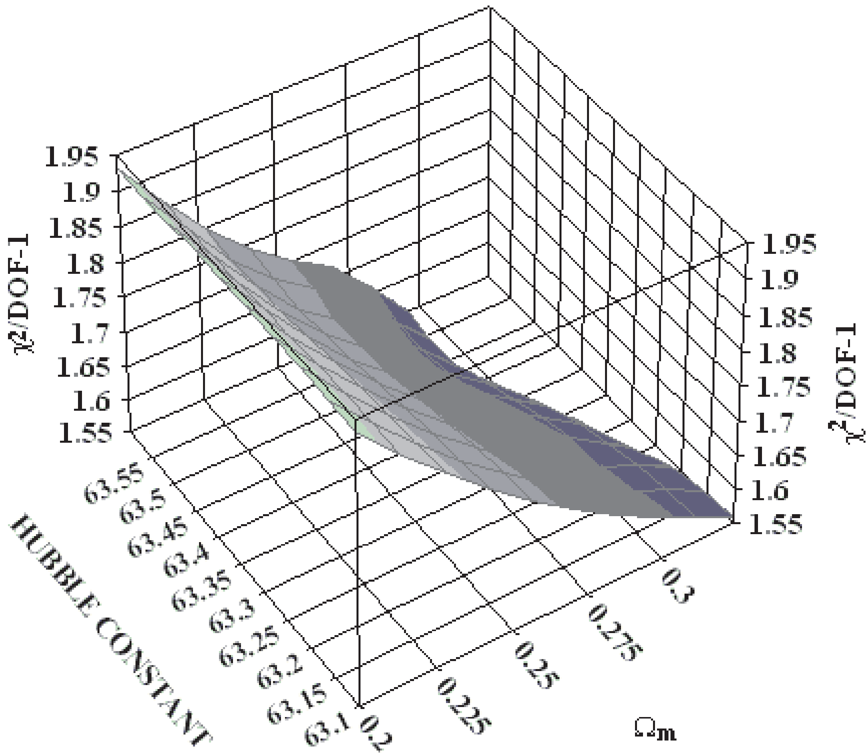

Fig. 2. Surface of Hubble constants and goodness of fits as functions of normalized matter densities with the *Simple*-DE model.

allowing Ω_m to be fixed but vary over the ranges close to values found here (Table 1) and solving for the Hubble constant and $\chi^2/(387)$ for the two *standard models*. Because Ω_m are tightly bound the fitting preference for any Ω_m at the minima are in the neighborhood but not exactly those determined *via* 2 and 3 free parameter modeling. These pseudo, three-dimensional figures display smooth, declining surfaces with increasing Ω_m for the *Simple*-DE model. On the other hand, the *Analytic*-ST model displays a fairly flat surface below Ω_m of 0.10. Notice the reduced $\chi^2/(387)$ for the *Analytic*-ST model at low values of Ω_m while the display of the *Simple*-DE fits suggests lower $\chi^2/(387)$ with increasing values of Ω_m. While the surfaces for the two models are quite different, the Hubble constants calculated for both are low and nearly invariant over the two ranges shown.

In Fig. 4 we present the results of fixing Ω_m at 0.27 and solving for the constant A_r from Eq. (24). We see the values found for this working constant are about 1/10 or less than those we derive from first principles. Unfortunately at this matter density, the goodness of fits are poor, but we can judge that the empirical value for K_r may be larger than those used by Chandrasekhar and this presentation. The figure also suggests a strong dependence of H_0 on the nature of matter density retarding Universe expansion. For instance, if Ω_m is really around 0.27, as used here, the effects of relativistic matter might be observed in SNe Ia signals.

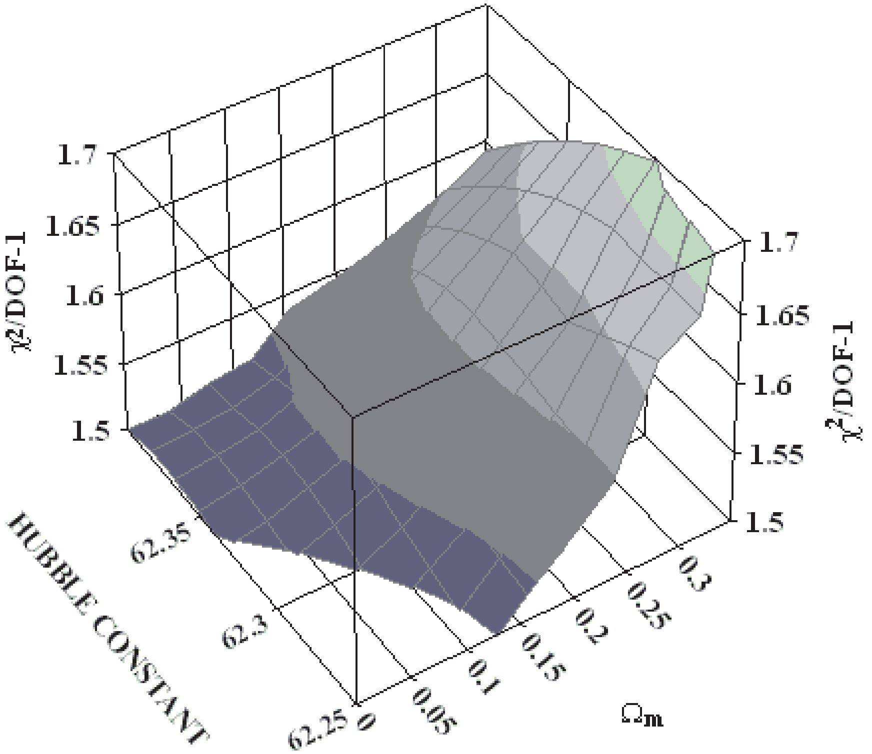

Fig. 3. Surface of Hubble constants and goodness of fits as functions of normalized matter densities with the *Analytic*-ST model

Using parameters from the Eisenstein (Eisenstein et al., 2005) investigation of cosmic BAO and presuming a flat Universe we have calculated several values for our coefficients B_{nr} and B_r of Eq. (28). We have selected three values for Ω_m from 0.273 preferred by Eisenstein to 0.32 preferred by Li and Liu Li & Liu (2009). Our results are reported in Table 5 as functions of Ω_m and x, where x is the ratio of Ω_r/Ω_m, with the sum of relativistic and non-relativistic matter being Ω_m.

In general, the evaluation errors are smallest for Ω_m of 0.273, where positive, though small, values were found for B_{nr} rather than 0. For this value of normalized matter density the relative ratios of Ω_{nr} to Ω_r might be considered of interest and perhaps even realistic. (Note the relative magnitudes of B_{nr} and B_r are not directly proportional to the weight fraction of these species in the Universe.) We also evaluate this routine at the much smaller values of Ω_m of 0.01 and 0.001 but the relative errors are about 2 orders of magnitude larger than for Ω_m of 0.273 and not worth reporting.

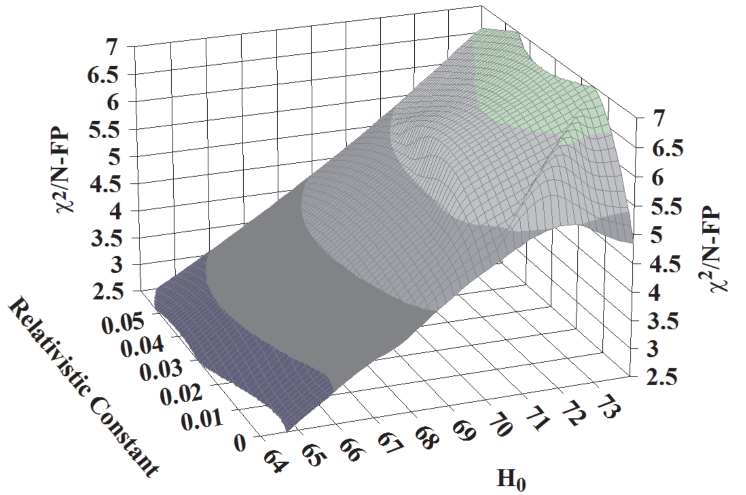

Fig. 4. Calculations of Relativistic Constant A_r from the Simple-DE model with Ω_m of 0.27.

Ω_m	0.75	0.5	0.25
0.32	$B_{nr} = 0$ $B_r = 0.326335$ $\delta = 3.3566$	$B_{nr} = 0$ $B_r = 0.980374$ $\delta = 0.1259$	$B_{nr} = 0$ $B_r = 2.26713$ $\delta = 1.7939$
0.3	$B_{nr} = 0$ $B_r = 0.294466$ $\delta = 1.2194$	$B_{nr} = 0$ $B_r = 0.916476$ $\delta = 1.5471$	$B_{nr} = 0$ $B_r = 2.144604$ $\delta = 3.0469$
0.273	$B_{nr} = 0.124236$ $B_r = 0.01$ $\delta = 0.3507$	$B_{nr} = 0.008885$ $B_r = 0.001121$ $\delta = 0.2151$	$B_{nr} = 0.004552$ $B_r = 0.003291$ $\delta = 0.2159$

δ is the relative calculation error as presented in Appendix 5.2.

Table 5. Calculation of the coefficients for Eq. 28 from BAO parameters

4. Conclusions

The data from the SDSS and SNe Ia collections are unique to science being by far the best ensembles of events stretching a large fraction of the Universe age. By using great times as a variable we can begin to answer questions which cannot be addressed by high energy experiments, perhaps even at the level of CERN. These data have been used to support several theories of spacetime and matter expansion including many models of DE (Davis et al., 2007; Sahni & Starobinsky, 2006) or the related gravitational/DE quintessence (Caresia et al., 2004; Ratra & Peebles, 2003) or even a decline of light emission frequency with local absolute time (Oztas et al., 2008). Here we begin to address the problem of the average densities of both non-relativistic and relativistic matter following the polytropic approach of Chandrasekhar applying this to variations of the FRW model using SNe Ia data and BOA results. We did attempt our own calculations of polytropic constants of a more universal nature than those of Chandrasekhar from first principles, but found these not as well suited for fitting real data as those presented by that exceptional individual and used for decades to describe SNe Ia explosions. Our results with these constants are successful to a first approximation and suggest this general approach may be useful for investigations into other, seemingly unrelated, fields.

Using the SNe Ia data from several sources and our sophisticated models we have found the preponderance of the Universe is either spacetime or dark energy. Our low values for Ω_{nr} from both regimes, might be equated with the combined densities of baryonic and CDM, though we prefer only equivalence with Ω_b for the following reasons. Our small values are consistent with results reported for another fit of SNe Ia data of a 5-dimensional model of the Universe that lumps all matter and energy into a single term but without resorting to CDM(Hartnett & Oliveira, 2007). A small Ω_{nr} is consistent with two recently published values for the baryonic density of about 0.05 for the Universe within seconds of singularity (Fields & Sakhar, 2009; Schramm , 2006) so the value will obviously decline towards our results with Universe expansion. Our small values for Ω_{nr} is also consistent with recent results from WMAP analysis where Ω_b was calculated as 0.046(Hinshaw et al., 2009). The difference in our findings and others (Burles et al., 2001a;b) from those of astronomers (Komatsu et al., 2009) for Ω_m might be thought the difference between baryonic and CDM. To solve this problem, a polytropic model which includes a term for CDM would have to be investigated, realistic polytropic constants discovered and the model fit to the astronomical data. Unfortunately for this investigation, CDM seems absent in our neighborhood of the Milky Way as attested in several un-refuted reports (Bahcall et al., 1995; Bienayme et al., 2006; Creze et al., 1998) which makes discovery and characterization extremely difficult.

In general, our individually determined estimated errors for the Hubble constant are smaller than other reports Komatsu et al. (2009). These smaller values for H_0, with smaller estimated errors, mean an older Universe which is helpful for those trying to adjust estimates of the minimum age from radioactive decay (Dauphas, 2005), globular cluster star composition (Formicola et al., 2004) and suffer the demands of very early galaxy formation (Primack, 2005). Most of the models investigated here, and with different data sets, return values for H_0 slightly larger than the estimate by A. Sandage of 62.3$\pm$6.3 km s^{-1}Mpc^{-1} from his accumulated works (Sandage et al., 2006) but also lower than the 70 to 73 km s^{-1}Mpc^{-1} currently fashionable. Since he and his coworkers have spent lifetimes evaluating the Hubble constant, and the present work, one cannot discard values in the low to mid-60s without very serious consideration. It has been well argued that values of H_0 derived from FRW modeling should only be used to estimate the lower bound of Universe age (Melia, 2009). Use of the polytropic constants require values for H_0 deep within the normalized matter parameters in the models presented here. This "constant" is therefore, of even greater importance than heretofore imagined for determination of matter densities, spacetime curvature and perhaps DE. Unfortunately, H_0 seems the least well known of any important constant today and deserves continued, intense investigation(Huchra, 2008). Unfortunate too, because many astronomers consider H_0 a nuisance parameter forgetting this determination an important reason for throwing the Hubble satellite into outer space.

The nagging and serious *cosmological coincidence* problem remains, where the expectation value for DE differs by more than the Planck constant from expectation (Carroll, 2008), with no resolution on the horizon. Serious flaws in the mathematics of the cosmic constant, which present as discontinuities, have been published (Oztas & Smith, 2006). In addition to this, it has also been shown by fundamental argument that the concept of dark energy as currently employed should not be estimated by the cosmic constant, where use of Ω_Λ fails as the origin of distant signals approach the gravitational horizon (Melia, 2009). Our analysis does not

support the superiority of the *Simple*-DE model since this does not fit the distance-frequency data any better than other models. The concept of DE itself demands more and better SNe Ia data with more analyses, for resolution of all these problems does not seem at hand (Carroll, 2008).
We suggest our very small result for Ω_r of $< 10^{-3}$ is probably the upper bound for the abundance of relativistic matter in the Universe, during epochs between the present and reionization. Estimates of the normalized relativistic matter density have been made and seem to lie between this value and about 10^{-6}, as suggested from WMAP 3 year data (Goobar et al., 2006). So the data from SNe observations are useful to establish limits for not only non-relativistic but relativistic matter. If relativistic matter consists primarily of neutrinos this is the upper bound of the current, small gravitational contribution of these particles to our Universe.
Our results following Chandrasekhar's reasoning and with his constants are moderately successful to the first approximation when applied to both SNe Ia data and BAO parameters and suggest the polytropic model is of a very general nature and might be used by investigators from other, seemingly unrelated, fields. We demonstrate that data from SNe Ia observations are useful to estimate limits not only for non-relativistic but relativistic matter; much more astronomical data are needed to better define these values. Solutions from BAO investigations may provide a particularly good method for investigating a polytropic Universe; such has already been predicted (Hu et al., 1998).
We should point out our approach approximates changing matter densities with lookback time, something simpler, *standard models* ignore. The *standard models* presume a constant matter density, which is obviously not the case when fitting data back to z≈1.5. By incorporating the Hubble constant into the matter density terms, our model corrects for changing matter densities with expansion and better fit the data. Since Chandrasekhar's insight is confirmed by daily supernova explosions across our Universe, serious consideration should be given to his polytropic approach when dealing with gravity, density and pressure in cosmology.

5. Appendix

5.1 Polytropic constants

We quickly review the classic derivation of the polytropic constants of Chandrasekhar. We begin with the usual relationships describing the heat capacities at constant volume, V, and constant pressure, P, as

$$C_V = (\frac{dQ}{dT}) \tag{29}$$

and

$$C_P = (\frac{dQ}{dT}) + R \tag{30}$$

where R is the ideal gas constant, so that $C_P - C_V = R$ and $\frac{C_P}{C_P} = \gamma$. For an ideal gas, such as monatomic H and He at high temperature in the primitive Universe these heat capacities are straightforwardly related to the gas constant by

$$C_P = \frac{5}{2}R \tag{31}$$

and

$$C_V = \frac{3}{2}R \tag{32}$$

with the ratio of Eq. (31) over (32) to be

$$\gamma = \frac{C_P}{C_V} = \frac{5}{3}. \tag{33}$$

For the adiabatic situation of a system without heat exchange, $dQ = 0$ the heat capacity at constant volume may be cast in the form

$$C_V dT + \frac{RT}{V} dV = 0 \tag{34}$$

and substituting $C_P - C_V$ for R we get

$$C_V \frac{dT}{T} + (C_P - C_V)\frac{dV}{V} = 0 \tag{35}$$

With separation of variables we can integrate the equation above to get

$$C_V log(T) + (C_P - C_V) log(V) = constant \tag{36}$$

which rearranges to the simple relationship of

$$TV^{\gamma - 1} = constant \tag{37}$$

after substitution of T with $\frac{PV}{R}$ we get

$$\frac{PV}{R} V^{\gamma - 1} = constant \tag{38}$$

which is more simply

$$PV^{\gamma} = constant. \tag{39}$$

In the case where the specific heat remains constant with a changing temperature, $\frac{dQ}{dT} = constant = c$ we use a similar argument as previously with

$$\gamma\prime = \frac{C_P - c}{C_V - c} \tag{40}$$

leading to a similar relationship with Eq. (37) as

$$PV^{\gamma\prime} = constant. \tag{41}$$

5.2 Evaluation of B_i coefficients

We briefly review several relationships presented by Eisenstein *et al.* used in their evaluation of SDSS data. We use these to evaluate numerical candidates for our B_i species and we are especially interested in their dependence on the normalized matter density, Ω_m. There might appear to be a hidden dependency of our B_{nr} and B_r on the Hubble constant, because ρ_c includes H_0, but the ratio of numerical values (for instance, dependence on Ω_r/Ω_m) are actually Hubble flow independent.

The parameter A, used for BAO evaluation, is dependent on several common cosmological parameters

$$A = D_V(0.35)\frac{\sqrt{\Omega_m H_0^2}}{0.35c} \quad (42)$$

where $D_V(z)$ is the distance to recent redshifts

$$D_V(z) = \left[D_M(z)^2 \frac{cz}{H(z)}\right]^{1/3}$$

and $H(z)$ is typically defined in terms of $E(z)$

$$E(z) = \frac{H_z^2}{H_0^2} = \left(\frac{\dot{a}}{a}\right)^2.$$

$E(z)$ at a given redshift is used in the present study as the traditional version for a flat Universe

$$E(z) = \sqrt{\Omega_m(1+z)^3 + \Omega_r(1+z)^4}$$

which is the special case of our interest. The term $D_M(z)$ is a value reflecting the redshift distance at recombination depending on the constant $D_H = c/H_0 = 3000/h$ (in Mpc) and the value for the integral of $1/E(z)$

$$D_M(z) = D_H \int_0^z \frac{dz}{E(z)}.$$

The value for R is a ratio of redshift dependents, where 0.35 is a typical present redshift and 1089 is a popular value for the redshift at recombination during the primitive Universe

$$R_{0.35} = \frac{D_V(0.35)}{D_M(1089)}. \quad (43)$$

For evaluation of relative errors we use $R_{0.35}$ of 0.0979 and A of 0.469 from Table 1 of Eisenstein *et al.* 2005 as reference values. The coefficients B_{nr} and B_r were calculated by minimizing the relative error of our $E(z)$ with respect to these two reference values as

$$\delta = \frac{|A - 0.469|}{0.469} + \frac{|R_{0.35} - 0.0979|}{0.0979}.$$

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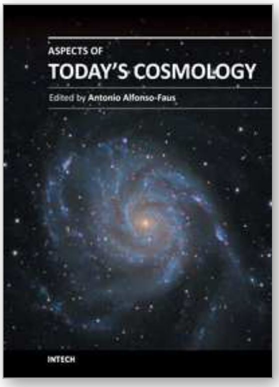

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