Revised Concepts for Cosmic Vacuum Energy and Binding Energy: Innovative Cosmology

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1. Introduction

Contemporary cosmology confronted with WMAP observations of the cosmic microwave background radiation and with distant supernova locations in the magnitude - redshift diagram obviously has to call for cosmic vacuum energy as a necessary prerequisite. Most often this vacuum energy is associated with the cosmological constant Λ, introduced by Einstein and presently experiencing a fantastic revival in form of "dark energy". Within the framework of General Relavity the term connected with Λ acts analogous to constant vacuum energy density. With a positive value, Λ describes an inflationary action on cosmic dynamics which in view of more recent cosmological data to most astronomers appears to be absolutely needed. In this article, however, we shall question this hypothesis of a constant vacuum energy density showing that it is not justifyable on physical grounds, because it claims for a physical reality that acts upon spacetime and matter dynamics without itself being acted upon by spacetime or matter.

In the past cosmic mass generation mechanisms have been formulated at different places in the literature and based on different physical concepts. A deeper study proves that these alternative theoretical forms of mass creation in the expanding universe all lead to terms in the GR field equations which can be shown to act analogously to terms arising from vacuum energy. In addition we also demonstrate that gravitational cosmic binding energy connected with structure formation acts identically to negative cosmic mass energy density, i.e. reducing the action of proper mass density. This again resembles an action of cosmic vacuum energy. Hence one is encouraged to believe that actions of cosmic vacuum energy, gravitational binding energy and mass creation are closely related to eachother, perhaps are even in some respect identical phenomena.

Based on results presented in this article we propose that the action of vacuum energy on cosmic spacetime dynamics inevitably leads to a decay of vacuum energy density. Connected with this decay is a decrease of cosmic binding energy and the appearance of new gravitating mass in the universe, identifyable with creation of newly appearing effective mass in the expanding universe. If this all is adequately taken into account by the energy-momentum tensor of the GR field equations, one is then led to non-standard cosmologies which for the first time can guarantee the conservation of the total energy both in static and expanding universes.

2. The concept of absolutely empty space

The question what means empty space , or synonymous for that - vacuum - , in fact is a very fundamental one and has already been put by mankind since the epoch of the greek natural philosophers till the present epoch of modern quantum field theoreticians. The changing opinions given in answers to this fundamental question over the changing epochs have been reviewed for example by Overduin & Fahr (2003) , but we do not want to repeat here all of these different answers that have been given in the past, but only to begin this article we want to emphasize a few fundamental aspects of our thinking of the physical constitution of empty space. Especially challenging in this respect is the possibility that empty space could be energy-charged. This we shall investigate further below in this article.

In our brief and first definition we want to denote empty space as a spacetime without any topified or localized energy representations, i.e.without energy singularities in form of point masses like baryons, leptons, darkions (i.e. dark matter particles) or photons, even without point-like quantum mechanical vacuum fluctuations. If then nevertheless it should be needed to discuss that such empty spaces could be still energy-loaded, then this energy of empty space has to be seen as a pure volume-energy, somehow connected with the magnitude of the volume or perhaps with a scalar quantity of spacetime metrics, like for instance the global curvature of this space. In a completely empty space of this virtue of course no spacepoints can be distinguished from others, and thus volume-energy or curvature, if existent, are numerically identical at all space coordinates.

Under these prerequisites it nevertheless would not be the most reasonable assumption, as many people believe, that vacuum energy density $\epsilon_{vac} = \rho_{vac}c^2$ needs to be considered as a constant quantity whatever spacetime does or is forced to do, i.e. whether it expands, collapses or stagnates. This is simply because the unit of volume is no cosmologically relevant quantity - and consequently vacuum energy density neither is. If at all, it would probably appear more reasonable to assume that the energy loading of a homologously comoving proper volume does not by its magnitude reflect the time that has passed in the cosmic evolution, i.e. perhaps that specific quantity has to be a constant. But this then, surprisingly enough, would mean that the enduring quantity, instead of the vacuum energy density ϵ_{vac}, is

$$e_{vac} = \epsilon_{vac}\sqrt{-g_3}d^3V \tag{1}$$

where g_3 is the determinant of the 3d-space metric which in case of a Robertson-Walker geometry is given by

$$g_3 = g_{11}g_{22}g_{33} = -\frac{1}{(1-Kr^2)}R^6r^4\sin^2\vartheta \tag{2}$$

with K denoting the curvature parameter, the function $R = R(t)$ determines the time-dependent scale of the universe and the differential 3-space volume element in normalized polar coordinates is given by

$$d^3V = drd\vartheta d\varphi \tag{3}$$

This then leads to the relation

$e_{vac} = \epsilon_{vac}\sqrt{R^6r^4\sin^2\vartheta/(1-Kr^2)}drd\vartheta d\varphi = \epsilon_{vac}\frac{R^3}{\sqrt{1-Kr^2}}r^2\sin\vartheta drd\vartheta d\varphi$

which shows that a postulated invariance of e_{vac} consequently and logically would lead to a variability of the vacuum energy density in the form

$$\epsilon_{vac} = \rho_{vac}c^2 \sim R(t)^{-3} \tag{4}$$

which for instance would already exclude that Einstein's cosmological constant could ever be treated as an equivalent to a vacuum energy density, since requiring the identity $\Lambda = 8\pi G\rho_{vac}/c^2$.

On the other hand the invariance of the vacuum energy per co-moving proper volume, e_{vac}, can of course only be expected with some physical sense, if this quantity does not do any work on the dynamics of the cosmic metrics, especially by physically or causally influencing the evolution of the scale factor $R(t)$ of the universe.

If on the other hand such a work is done and vacuum energy influences the dynamics of the cosmic spacetime, since it leads to a non-vanishing energy-momentum tensor, then thermodynamic requirements should be fulfilled, for example relating vacuum energy density and vacuum pressure by the standard thermodynamic relation (see Goenner (1997))

$$\frac{d}{dR}(\epsilon_{vac}R^3) = -p_{vac}\frac{d}{dR}R^3 \tag{5}$$

This equation is shown to be fulfilled by an expression of the form

$$p_{vac} = -\frac{3-n}{3}\epsilon_{vac} \tag{6}$$

if the vacuum energy density itself is represented by a scale-dependence $\epsilon_{vac} \sim R^n$. Then, however, it turns out that the above thermodynamic condition, besides for the trivial case $n = 3$ when the vacuum does not at all act as a pressure (since $p_{vac}(n = 3) = 0$), is only non-trivially fulfilled for $n \lessgtr 3$ which would still allow for $n = 0$, i.e. a constant vacuum energy density $\epsilon_{vac} \sim R^0 = const.$

A much more rigorous, but highly interesting restriction for n is, however, obtained when one recognizes that the above thermodynamic expression (5) under cosmic conditions needs to be enlarged by the work that the expanding volume does against the inner gravitational binding in this volume. In mesoscale gas dynamics (aerodynamics, meteorology etc.)this term does generally not play a role, however, on cosmic scales there is a need to take into account this term. Under cosmic perspectives binding energy is an absolutely necessary quanity to be brought into the thermodynamical energy balance. As worked out in quantitative terms by Fahr & Heyl (2007a;b) this then leads to the following completed relation

$$\frac{d}{dR}(\epsilon_{vac}R^3) = -p_{vac}\frac{d}{dR}R^3 - \frac{8\pi^2 G}{15c^4}\frac{d}{dR}[(\epsilon_{vac} + 3p_{vac})^2 R^5] \tag{7}$$

where the last term accounts for binding energy.

This completed equation, as one can easily show, is also solved by the relation of the form $p_{vac} = -\frac{3-n}{3}\epsilon_{vac}$,

$$\text{but only if: } n = 2\,!$$

meaning that the corresponding vacuum energy density must vary like

$$\epsilon_{vac} \sim R^{-2} \tag{8}$$

This thus means that, if it has to be taken into account that vacuum energy acts upon spacetime in a thermodynamical sense then the most reasonable assumption for the vacuum energy density would be to assume that it drops off with the expansion inversely proportional to the square of the cosmic scale - instead of it being a constant.

3. Philosophical perspectives of vacuum concepts and an effective vacuum-energy density

For fundamental conceptual reasons it may be necessary to explore why at all a vacuum should gravitate, since, when really being "nothing", then it should most probably not do anything. At least based on an understanding that the ancient greek atomists had, the vacuum is a complete emptiness simply offering empty places and thereby allowing atoms freely to move. One should then really not expect to have any gravitational action from such a vacuum. Aristotle, however, brought into this conceptual viewing his principle of nature's objection against emptiness ("horror vacui"). This is a new aspect realizing that empty space around matter particles is not as empty as without those particles, but is polarized by the existence or presence of real matter. This idea furtheron very much complicated the concept of vacuum making it a rather lengthy and even not yet finished story (see e.g. Barrow (2000); Fahr (2004); Wesson et al. (1996)). In the recent decades it became evident that vacuum must be energy-loaded (see e.g. Lamoreaux (2010); Streeruwitz (1975); Zeldovich (1981)) and by its energy it should hence also influence gravitational fields, even, if it is not clear in which concrete form.

Nowadays the GRT action of the vacuum is taken into account by an appropriately formulated, hydrodynamical energy-momentum tensor $T_{\mu\nu}^{vac}$, formulating the metrical source of the energy sitting in the vacuum as described by a fluid with vacuum pressure p_{vac} and equivalent vacuum mass energy density ρ_{vac}. Then with a constant vacuum energy density $\epsilon_{vac} = \rho_{vac}c^2$, as assumed in the present-day standard cosmology (Bennett et al., 2003), one obtains this tensor in the form (see e.g. Overduin and Fahr, 2001)

$$T_{\mu\nu}^{vac} = (\rho_{vac}c^2 + p_{vac})U_\mu U_\nu - p_{vac}g_{\mu\nu} = \rho_{vac}c^2 g_{\mu\nu} \tag{9}$$

where U_λ are the components of the vacuum fluid 4-velocity vector.

This term, taken together with Einstein's cosmological constant term Λ (Einstein, 1917), and placed on the right-hand side of the GRT field equations then leads to an effective cosmological constant given by

$$\Lambda_{eff} = \frac{8\pi G}{c^2}\rho_{vac} - \Lambda \tag{10}$$

The first problem always seen after Einstein (1917) is connected with the free choice one is left with concerning the numerical value of Λ. One way to obtain a first answer to that question, at least for the completely empty, i.e. matter-free space, is a rationally pragmatic and aprioristic definition, - namely an answer coming up from an apriori definition of how empty space should be constituted and should be manifesting itself. If it is rationally postulated that empty space should be free of any spacetime-curving sources, and thus free of local or global curvature, if one requires that selfparallelity of 4-vectors at parallel transports along closed wordlines in this empty space should be guaranteed, and if one expects no action of empty space on freely propagating test photons in this empty space, then as shown by Overduin & Fahr (2003) or Fahr (2004) the only viable solution is $\Lambda_{eff,0} = 0!$, meaning that the cosmological constant should be fixed such that

$$\Lambda_0 = \Lambda - \frac{8\pi G}{c^2}\rho_{vac,0} \tag{11}$$

where $\rho_{vac,0}$ denotes the equivalent mass density of the vacuum of empty, i.e. matter-free space. Once fixed in this above form, the cosmological constant cannot be different from this value Λ_0 in a matter-filled universe, simply meaning that in a matter-filled universe the effective quantity representing the action of the vacuum energy density is given by:

$$\Lambda_{eff} = \frac{8\pi G}{c^2}(\rho_{vac} - \rho_{vac,0}) \tag{12}$$

expressing the interesting fact that in matter-filled universe only the difference between the values of the vacuum energy densities $\rho_{vac,0}$ of empty space and of matter-polarized space ρ_{vac} gravitates, i.e. influences the spacetime geometry. That could give an explanation why obviously the vacuum energy calculated by field theoreticians does not gravitate by its full magnitude.

This also points to the perhaps most astonishing fact that the geometrically relevant vacuum energy density depends on the matter distributed in space, and in a homogeneous universe this can only mean that: $\rho_{vac} = \rho_{vac}(\rho)$, an idea that deeply reminds to the views already developed by Aristotle at around 400 bC.

Though this idea of the vacuum state being influenced by the presence of matter in space appears to be reasonable in view of field sources polarizing space around them by acting on sporadic quantum fluctuations and partly screening off the strength of real field sources, it stays nevertheless hard to draw any quantitative conclusions from that context. For that reason we shall try another way below to find the unknown function $\rho_{vac} = \rho_{vac}(\rho)$.

4. The standard cosmology based on five cosmic scalar quantities

Standard cosmology is based on some basic scalar quantities that are treated as 3-spacecoordinate-independent, but time-dependent. Amongst these are matter density ρ, scalar pressure p, isotropic curvature characterized by a space-independent Riemann scalar R, and the cosmological constant Λ. These basic elements can be used, if the universe is treated as homogeneously filled with matter of a space-indendent scalar pressure and carries out a homologous expansion. Then Einstein's General relativistic field equations can be condensed to a set of only two cosmologically relevant linear differential equations of second order for the scale of the universe R and its first and second derivatives with respect to time, $\dot{R}$ and $\ddot{R}$, given in the form (see e.g. Goenner (1997))

$$\left(\frac{\dot{R}(t)}{R(t)}\right)^2 = H^2(t) = \frac{8\pi G}{3}\rho(t) - \frac{kc^2}{R^2(t)} + \frac{\Lambda c^2}{3} \tag{13}$$

and:

$$\frac{\ddot{R}(t)}{R(t)} = -\frac{4\pi G}{3c^2}(3p(t) + \rho(t)c^2) + \frac{\Lambda c^2}{3} \tag{14}$$

Here $H(t) = \dot{R}/R$ is the Hubble function that depends on the contributing densities ρ, the pressure and the curvature parameter k, attaining values of $k = 0$ (uncurved space); $k = +1$ (positively curved space) or $k = -1$ (negatively curved space).

The matter density ρ nowadays in cosmology is composed of baryonic and dark matter, i.e. $\rho = \rho_b + \rho_d$, where the two quantities vary identically with cosmic time or cosmic scale. At cosmic times greater than the recombination period $t \geq t_{rec}$ the associated pressures $p_{b,d}$

usually are neglected with respect to their corresponding rest mass densities $\rho_{b,d}c^2$. Then depending on selected values for the ratios $\Omega_b = \rho_b/\rho_c$, $\Omega_d = \rho_d/\rho_c$ and $\Omega_\Lambda = \rho_\Lambda/\rho_c$, with $\rho_\Lambda = \Lambda c^2/8\pi G$ and the critical density given by $\rho_c = 3H^2/8\pi G$, one obtains a manifold of different solutions $R = R(t)$ of the above system of differential equations, each belonging to a specific set of numerical values for the five cosmologically relevant parameters: $H_0 = H(t_0)$, k, $\Omega_{b,0} = \Omega_b(t_0)$, $\Omega_{d,0} = \Omega_d(t_0)$ and $\Omega_{\Lambda,0} = \Omega_\Lambda(t_0)$. To decide which of these parameter sets best fits cosmologically relevant observational data, like the WMAP data from the "Wilkinson Microwave Anisotropy Probe" survey (Bennett et al., 2003) or the distant supernova data (Perlmutter et al., 1999), multi-parameter fit procedures have recently been carried out. As the best-fitting consensus the following set of parameters thereby has been found: $H_0 = 71 km/s/Mpc$, $k = 0$, $\Omega_{b,0} = 0,046$, $\Omega_{d,0} = 0.23$ and $\Omega_{\Lambda,0} = 0.73$. These values are nowadays taken as result of modern precision cosmology, characterizing the facts of our actual universe. Perhaps, however, a reminder to weaknesses in the basic assumptions of such a form of precision cosmology may be in place here.

One most essential ingrediente of standard cosmology is the assumption that the total, spacelike mass of the physical universe, conceivable for any spacepoint on the basis of a point-oriented spacetime metrics g_{ik}^* - irrespective of its dark or baryonic nature, is constant. This then is usually thought to imply that the corresponding matter densities $\rho_{b,d}$ in a homogeneous universe scale reversely proportional to the 3d- volume of the physical universe $V_3^* = \int^{x_\infty} d^3x \sqrt{\det^3 g^*}$, which in all cases of standard cosmology means inversely proportional to R^3.

Another essential point of standard cosmology is to assume a strict homogeneity of energy depositions in cosmic space connected with an isotropic homologous expansion of cosmic matter. Though these items seem to be cosmo-philosophically well supported by the so-called "cosmological principle" (see e.g. Stephani (1988)), one nevertheless has to recognize that the actual universe is very much different from expectations derived from this principle. In fact the actual universe is highly structured in forms of galaxies, galaxy clusters, superclusters, walls and voids (Ellis, 1983; Geller & Huchra, 1989) - perhaps one can call that a "structured homogeneity". Only on scales larger than several hundred million lightyears the universe seems to be nearly homogeneous. However if the structuring develops as function of cosmic time, then this actual universe does not expand like an equivalent one with homogeneously smeared out matter (Buchert, 2008; Wiltshire, 2007). Matter distribution had perhaps been very homogeneous, at least down to temperature fluctuations of the order of $\frac{\Delta T}{T} \approx 10^{-5}$ at the epoch of the last scattering of CMB photons when the cosmic microwave background was freezing out of cosmic matter distribution. In the cosmic eons after that phase in fact matter distribution, as evident in the appearance of the universe, must have become very inhomogeneous through gravitational growth of seed structures. Fitting a perfectly symmetrical spacetime geometry to a universe which, however, has a lumpy matter distribution up to largest scales (e.g. see Wu et al. (1999)) represents a highly questionable procedure as shown by Buchert (2001; 2005; 2008) or Wiltshire (2007) (see chapter 9.1).

Besides of the above, perhaps the even most problematic concept used in present-day standard cosmology is the application of a constant vacuum energy density $\epsilon_{vac} = \rho_{vac}c^2$. Historically and ideologically this originates from Einstein's introduction of a cosmological constant Λ (Einstein, 1917) emanating from application of the variational principle to the spacetime Lagrangian (Overduin & Fahr, 2003), appearing as such on the left, i.e. the "metrical" side of the GRT field equations, however, when transfered to the right side of these equations, is equivalent to a vacuum energy density $\rho_{vac} = c^2\Lambda/8\pi G$, also associated with a vacuum

pressure $p_{vac} = -\rho_{vac}c^2$ (e.g. see Peebles & Ratra (2003)). In this form it has experienced a great importance in the present epoch of cosmology (Bennett et al., 2003; Perlmutter et al., 1999).

The problem with this concept of a constant vacuum energy density has already been adressed in the first section of this paper and here can be enlarged to the whole universe: At the expansion of the universe, connected with the increase of the cosmic 3-space volume V^3, consequently the total vacuum energy $E_{vac} = \int \rho_{vac}c^2 dV^3 \sqrt{-g^3} \sim \int dV^3 \sqrt{-g^3}$ permanently increases. This could perhaps even be accepted, if vacuum energy is completely actionless as a cosmologically decoupled quantity with no backreaction to cosmic expansion. As we have shown before, constant vacuum energy density, however, **is** associated with a pressure $p_{vac} = -\rho_{vac}c^2$ that evidently acts on the cosmic expansion accelerating its rate. The purely geometrical increase of cosmic vacuum energy thus is untenable.

This is all the more true when matter density comoves with the cosmic scale expansion to configurations with permanently decreasing gravitational binding. Here it must appear as completely unphysical that an evolving cosmic system, at the same time, gains energy in form of increasing vacuum energy, while simultaneously it has to do work against the internal, intermaterial gravitational attractive forces. For instance for an uncurved universe (i.e. $k = 0$) and Λ put equal to zero, the first Friedmann equation (see Equ. (13)) simplifies into the form $\dot{R}^2 = (8\pi G/3)\rho R^2 = \Phi(R)$ and thus allows to identify a relevant cosmic gravitational potential $\Phi(R)$ in analogy to the one in Hamilton-Lagrangian dynamics (see Fahr & Heyl (2007a;b)). Therefore at the cosmic expansion permanently work has to be done by cosmic matter against an intermaterial force per mass which for $\rho \sim R^{-3}$ is given by
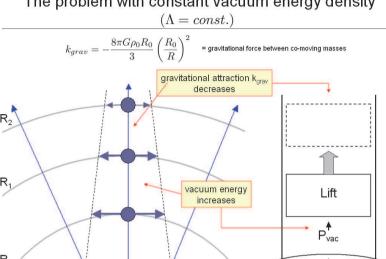
$$f(R) = -\frac{d\Phi}{dR} = \frac{8\pi G}{3}\rho_0 R_0 \left(\frac{R_0}{R}\right)^2 \tag{15}$$

Instead of loosing energy by permanently doing work $dE/dt = -\dot{R}f(R)$ against this force per time unit, - and instead of decelerating its expansion due to that, the universe may even accelerate its expansion by $\ddot{R} = f(R) + \Lambda Rc^2/3$. With the action of a constant vacuum energy density ($\Lambda = const$) this universe even accumulates more and more energy in form of vacuum energy. This shows that the concept of constant vacuum energy density implies a physically highly implausible "perpetuum mobile" principle: The vacuum permanently acts upon matter and spacetime geometry, but is itself not acted upon by these latter quantities (see Fahr & Heyl (2007a;b), and Figure 1 for illustrative purposes).

This may raise the question whether at present with the form of the standard cosmology one may have a correct basis for a successful description of the given universe and its dynamics. Thus in the ongoing part of this article we shall investigate the following four fundamental, cosmologically relevant critical points:

1. Is the mass of the universe constant?

2. What is metric-relevant cosmic mass density?

3. How is gravitational binding energy represented in the energy-momentum tensor?

4. How all of that is reflected in a variable vacuum energy density?

With the arguments given below we demonstrate that an expanding universe with constant total energy, the so-called "economic universe" (also termed as a "coasting universe) is indicated as most probable in which both cosmic mass density and cosmic vacuum energy density are decreasing according to $(1/R^2)$, R being the characteristic scale of the universe. Under these conditions the origin of the present universe from an initially pure cosmic

The problem with constant vacuum energy density
$$(\Lambda = const.)$$

$$k_{grav} = -\frac{8\pi G \rho_0 R_0}{3} \left(\frac{R_0}{R}\right)^2 \qquad \text{= gravitational force between co-moving masses}$$

gravitational attraction k_{grav} decreases

vacuum energy increases

Lift

R_2

R_1

R_0

P_{vac}

g

Fig. 1. Schematic illustration of the physical action of a constant vacuum energy density and of inter-material cosmic gravitational fields requiring work to be done, if co-moving matter is transfered to larger cosmic scales $S = R$.

vacuum state appears to be possible. This is because the incredibly huge vacuum energy density, derived by quantumfield theoreticians, in this economic universe decays during its expansion up to present-day scales to just the observationally permitted small value of the present universe, but its energy reappears in the energy density of created effective cosmic matter. It is interesting to see that very similar conclusions concerning the ratio of cosmic vacuum energy and cosmic matter density have been drawn from attempts to formulate the GRT equations in a scale-invariant, Weyl'ian form like recently tried in the Quasi-Steady-State-cosmology (QSSC) by Hoyle et al. (1993), or in conformal cosmological scalar-tensor theories by Mannheim (2000) or by Scholz (n.d.).

5. How to define the mass of the universe?

According to the famous Mach principle (Mach, 1883) inertial masses of cosmic particles are not particle-genuine quantities, but have a relational character being a functional of the spacetime constellation of other cosmic masses in the universe. Only with respect to other masses accelerations have physical relevance (see also Jammer & Bain (2000)). As a consequence, inertial particle masses, and, perhaps in the sense of the general relativistic equivalence principle, also heavy masses, should change their values when the spatial constellation of the surrounding cosmic masses changes - which is the case in an expanding universe with increase of its scale $R = R(t)$. This principle implies that inertia depends in some unclear way on the presence and distribution of other massive bodies in the universe, and has been seriously studied in its consequences (see reviews given in Barbour & Pfister (1995), or Barbour (1995),Wesson (2004),Jammer & Bain (2000)).

In the beginning even Einstein attempted to develop his GR field equations in full accordance to Mach's principle, however, in the later stages he recognized the non-Machian character of his GR theory and divorced from this principle (Holton, 1970). Experts of this field still today have controversial opinions whether or not Einstein's GR theory is "Machian" or "non-Machian". Nevertheless attempts have been made to develop an adequate form of a "relational", i.e. Machian mechanics (Goenner, 1995; Reissner, 1995). Especially the requested concrete scale-dependence of cosmic masses is unclear in its nature, though perhaps already suggested by conformal invariance requirements or general relativistic action principle arguments given by early arguments developed in Hoyle (1990; 1992); Hoyle et al. (1994a;b) along the line of the general relativistic action principle.

We study this relation a little deeper here starting from the question what at all should and could be called in a physically relevant, conceptually meaningful sense "the mass of the universe M_u" and how then it could be understood, if this quantity increases with the universal scale R? According to the most logical concept, this mass M_u should represent the spacelike sum over all masses distributed in the universe at some event of time, judged from some arbitrary cosmic vantage point, i.e. the space-like sum of all masses within the mass horizon associated to this point. One way to define such a quantity has been mathematically carried out by Fahr & Heyl (2007b) and leads to the following mathematical expression of cosmic mass

$$M_u c^2 = 4\pi\rho_0 c^2 \int_0^{R_u} \frac{\exp(\lambda(r)/2)r^2 dr}{\sqrt{1 - (\frac{H_0 r}{c})^2}} \tag{16}$$

where the function in the numerator of the integrand is given by the following metrical expression

$$\exp(\lambda(r)) = \frac{1}{1 - \frac{8\pi G}{rc^2}\rho_0 \int_0^r \frac{x^2 dx}{\sqrt{1-(\frac{H_0 x}{c})^2}}} \tag{17}$$

The reason behind this above expression is that the environment around an arbitrary vantage point is described analogous to a point in the center of a star surrounded by stellar matter distribution, the difference in this case being only that the metric in this cosmic case also is of the inner Schwarzschild form, however, with the matter density given by the cosmic density ρ_o taking into account the additional fact that matter in the surroundings of a homologously expanding universe is equipped with the Hubble dynamics of the expanding universe.

As evident from the above expression no real matter can be summed-up anymore from beyond the "local Schwarzschild infinity" (i.e. "point-associated Schwarzschild mass horizon", see Fahr & Heyl (2006)) which is at a distance

$$R_u = \frac{1}{\pi}\sqrt{\frac{c^2}{2G\rho_0}} \tag{18}$$

which, however, also means that the mass horizon distance is related to the cosmic mass density by

$$\rho_0(R_u) = \frac{c^2}{2\pi^2 G R_u^2} \tag{19}$$

and naturally leads to a point-associated mass of the universe given by Fahr & Heyl (2006)

$$M_u = \frac{3\pi c^2}{8G} R_u \qquad (20)$$

This scale dependence of cosmic mass, does not only point to the fact that Mach's relation is fulfilled for the mass of the universe in the above definition of M_u. It in addition also proves that Thirring's relation derived from a completely different context (see Mashhoon et al. (1984), and also Fahr & Zoennchen (2006)) in the form

$$M_u = \frac{3c^2 R_u}{4G} \qquad (21)$$

is also fulfilled up to the factor $(\pi/2)$.

6. Gravitational binding energy reflected in an effective mass density

In a completely different approach Fischer (1993) may be giving from a new aspect of physics an explanation for this change of cosmic mass M_u with scale R coming to conclusions very similar to the above ones. He makes an attempt to include the gravitational binding energy into the energy-momentum tensor $T_{\mu\nu}$ of the GRT field equations. Interestingly enough his derivations lead to the result, that in a positively curved universe the corresponding term for the binding, or potential energy density $T_{\mu\nu}^p$ has to be introduced into the GRT equations by

$$T_{\mu\nu}^p = -C\frac{\rho}{\Gamma} g_{\mu\nu} \qquad (22)$$

where $g_{\mu\nu}$ denotes the metric tensor, C is an appropriately defined constant which amongst other factors contains the gravitational constant G, and Γ is the actual curvature radius of the positively curved universe.

In this formulation two things are perhaps eye-catching: At first this term again contains a proportionality to the density ρ, and at second this term has a negative sign and has $g_{\mu\nu}$ as a factor, thus in the GRT field equations formally it has the same action as that term connected with the action of vacuum energy density formulated with the quantity Λ_{eff}. This points to an interesting physical connection between vacuum energy and gravitational binding energy. Obtaining its space-like components as vanishing and adding up the time-like tensor components T_{00} and T_{00}^p of cosmic matter und cosmic binding energy then shows a very surprising connection between creation of matter and binding energy given in the form

$$\hat{T}_{00} = T_{00} + T_{00}^p = (\rho - C\frac{\rho}{\Gamma}) g_{00} \qquad (23)$$

This can thus be interpreted as saying that the intermaterial, gravitational binding energy reduces the cosmologically, i.e. geometrically acting, relevant, effective cosmic matter density to $\rho^* \leq \rho$, where ρ should be called the "proper density" given in uncurved spacetimes, by the following amount

$$\rho^* = \rho(1 - C\frac{1}{\Gamma}) \qquad (24)$$

If in the course of the cosmic expansion the cosmic curvature radius Γ increases, it thus means that gravitational binding energy, and, equivalent to that, the cosmic vacuum energy should decrease, while at the same time the effective density changes in time in a Machian form with a rate

$$\dot{\rho}^* = \frac{d}{dt}[\rho(1 - C\frac{1}{\Gamma})] \tag{25}$$

It is perhaps interesting to recognize that for instance for a universe with Hoyle's "steady state requirement", i.e. with $d\rho/dt = 0!$, this then evidently would require

$$\dot{\rho}^* = \rho C\frac{1}{\Gamma^2}\dot{\Gamma} \tag{26}$$

This means a mass creation rate proportional to the matter density ρ itself which is positive for increasing cosmic curvature radius Γ. In other words: At decreasing cosmic binding energy the effective density increases by the rate $\dot{\rho}^*$ which , as will be shown further down in this paper, is identical to that one obtained by Hoyle (1948).

It is interesting to notice that an introduction of the gravitational binding energy according to the suggestion by Fischer (1993) leads to two differential equations that can be combined to

$$\ddot{S} = \frac{C\rho_c}{6\Gamma}(S_0 - S) \tag{27}$$

which leads to cosmological solutions for positively curved universes representing an oscillatory behaviour of the cosmic scale parameter R around an equilibrium value R_0 with positively valued ($R \leq R_0$) und negatively valued ($R \geq R_0$) vacuum energy densities in the successive half-phases of the oscillation. It is perhaps challenging to conjecture that the action of vacuum energy, binding energy and creation of effective matter density could be closely related to eachother and perhaps even be identical.

A similar connection between vacuum energy and mass density was also pointed out by **?** who showed that the cosmological term connected with the quantity Λ should be coupled to matter density ρ and, concretely spoken, should in fact be proportional to it.

The problem of what should be called cosmic matter density thereby is by far not a trivial one, because the "matter density" is intrinsically connected with the prevailing spacetime geometry. The latter, however, only aposteriori is obtained from solutions of the GRT field equations after putting the right mass density into the energy-momentum tensor. The usual definition of matter density as "mass per unit volume" is in fact problematic in curved spaces. Usually the density is identified with what one should call the "proper density", i.e. mass within a free-falling unit volume, i.e. within a reference system without internal tidal gravitational accelerations. Of course in the universe one finds co-moving inertial restframes, nevertheless even in such systems tidal accelerations are acting over finite dimensions of a Finite 3d-space volume, causing for a metrical distortion of unit volumes. The effect of this metrical distortion reduces the proper density ρ as has been discussed by Fahr & Heyl (2007a;b) and for the low-density limit $\rho_0 \ll \rho_c$ (with ρ_c denoting the Schwarzschild density on a scale $R_{ES}(M) = \sqrt[3]{3M/4\pi\rho_0}$ (see Einstein and Straus, 1945) given by $\rho_c = (3/4\pi)(c^2/2G)^3M^{-2}$) also leads to a reduction of the proper density given by an expression

$$\rho^* = \rho_0(1 - (\rho_0/\rho_c)^{1/3}) \tag{28}$$

7. Effective mass change as equivalence to cosmic mass generation

Early attempts to describe universes with mass creation like those presented by Hoyle (1948) show very interesting relations between this form of matter creation and the change of effective cosmic matter density. To describe a steady-state universe Hoyle (1948) introduced
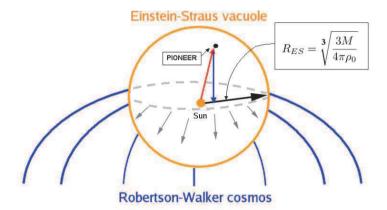
Fig. 2. Visualization of the Einstein-Straus globule surrounding a mass M within the expanding Robertson-Walker universe.

a divergence-free mass-creation tensor $C_{\mu\nu} = -3R\dot{R}\delta_{\mu\nu}/cA$ into the GR field equations, with A being a constant curvature scale. With the introduction of this term he can describe a universe with constant mass density $\rho = \rho_0 = const$, an inflationary expansion $R = R_0 \exp[c(t - t_0)/A]$, and a mass creation rate given by $\dot{\rho} = \frac{c}{A}\rho_0$. As we have recently shown (Fahr & Heyl, 2007a;b) an identical inflationary expansion is also described by an Einstein-de Sitter cosmological model of an empty universe, however, under the action of a cosmological constant Λ. This is true, if this constant Λ is related to Hoyle's creation rate by

$$\Lambda^{3/2} = \frac{8\pi G \sqrt{3}}{c^5}\dot{\rho} \tag{29}$$

This points to the fact that cosmologically analogous phenomena can be described by the action either of mass creation $\dot{\rho}$ or of a cosmological constant $\Lambda = 8\pi G\rho_{vac}/c^2$, i.e. by a vacuum energy density. It may furthermore be of interest to recognize that Hoyle's creation rate automatically leads to the fulfillment of a quasi-Machian relation between mass and radius of the universe, which has already been mentioned before, and here reappears from this context in the form

$$M_u = M_{u0} \exp[\frac{c(t - t_0)}{A}]^3 = M_{u0} \left(\frac{R(t)}{R_0}\right)^3 \tag{30}$$

The above analysis came along the early mass-creation theory published by Hoyle (1948). This early theoretical approach has, however, been consequently extended by Hoyle and his co-workers and has meanwhile been put into a larger astrophysical framework (see Hoyle et al. (1993; 1994a;b; 1997) where individual strong gravity centers in an expanding universe are considered that act as centers of mass creation called "Quasi-steady state cosmologies" (QSSC-models). Later in this paper we discuss these QSSC-models in a broader context, since these models are connected with more general scale-invariance requirements in the GRT field equations. We want, however, to emphasize already here that the above-revealed evidence (29), here derived from Hoyle's early creation theory and revealing a close relation between

mass creation rate, vacuum energy density and actual cosmic mass density, is again equally retained in these later QSSC-models as we shall show later in this paper.

8. Mass increase on local scales

According to Einstein & Straus (1945) a locally realized mass M is surrounded by a spherical shell with a radius $R_{ES}(M) = \sqrt[3]{3M/4\pi\rho_0}$. At this shell surface a steady and differentiable transition from the inner Schwarzschild metric into the outer Robertson-Walker metric of a homologously expanding universe is possible. This also implies that spacepoints on the Einstein-Straus shell are expanding with respect to the center of the shell as Robertson-Walker spacepoints do, i.e. like

$$\dot{R}_{ES}/R_{ES} = \dot{R}_0/R_0 = H_0 \tag{31}$$

with H_0 denoting the Hubble constant.

Adopting vacuum energy as being ubiquitously active in the universe one can ask, what amount of work the pressure connected with this vacuum energy does at the expansion of the local Einstein-Straus globule. For the inside of this globule this work is positively valued, and due to energy conservation reasons, it should thus lead to an increase of the energy constituted by this globule. Ascribing this energy gain to the internal mass of the globule then delivers the interesting result (Fahr & Heyl, 2007a;b)) that

$$\frac{\dot{M}}{M} = \frac{\rho_{0,vac}}{\rho_{0,mat}} H_0 \tag{32}$$

where $\rho_{0,vac}$ and $\rho_{0,mat}$ denote the densities of the present mass equivalent of the vacuum energy and of the cosmic matter. For a constant ratio of these energy densities the above relation simply expresses, - since $\dot{M}/M \sim \dot{R}/R$ - (i.e. the economical universe, see further down), a proportionality of the globular mass M, - and, if generalized to the scale R_u, of the mass M_u of the universe - , with the radius in the form

$$M/R_{ES}(M) \sim M_u/R_u = const \tag{33}$$

again as already envisioned by Mach (1883), but here proven as being valid also on local scales.

9. Why structure formation accelerates the cosmic expansion rate

Here we want to start with an easyminded exercise showing that gravitational structure formation in the universe may have the quite unexpected tendency to accelerate, like a force would do, the Hubble flow velocity, a virtue that is nowadays all over in the astrophysical literature ascribed to the action of the vacuum pressure p_{vac}. Let us assume that structure formation has developed at some epoch of cosmic evolution to some organized state such that not anymore a homogeneous matter density distribution prevails, but instead a homogeneous distribution of hierarchically organized matter distribution. From galactic number count statistics one knows that this expresses itself in observed local two-point correlation functions $\xi(l)$ expressing the probability to find another galaxy at a distance l from the local space point. For completely homogeneous matter distribution the function ξ would be constant. In cosmic reality, however, this two-point correlation probability over wide ranges of scales is shown to fall off by
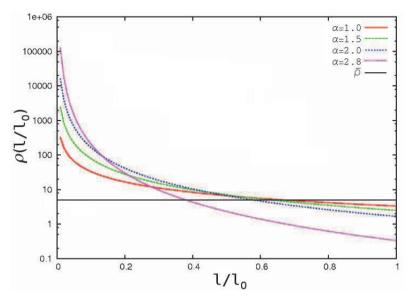
Fig. 3. Dependence of ρ for different values of α. The black solid line represents the case of a homogeneous density $\bar{\rho}$

$$\xi(l) = \xi(l_0) \cdot (\frac{l_0}{l})^{\alpha} \tag{34}$$

with the power index $\alpha \simeq 1.8$ and some inner scale l_0 typical for galaxies (see Bahcall (1988); Bahcall & Chokshi (1992)). In terms of matter density this expresses the fact that cosmic matter distribution has been organized, so that the mean density has not changed, but a density clustering has appeared at each local environment. This clustering is associated with a more pronounced gravitational binding of this organized matter, i..e. more negative potential energy has developed during the process of structuring.

To calculate the latter we start from a local density distribution corresponding to the probability function given by Eqn.(34) and write the clustered density in the form $\rho(l) = \rho_0$ $(l/l_0)^{-\alpha}$. In order to conserve the initial mass at the structuring process the central density ρ_0 has to be defined as

$$\rho_0 = \frac{3 - \alpha}{3}\bar{\rho} \cdot (l_m/l_0)^{\alpha} \tag{35}$$

with l_m as an outer integration scale. Figure 3 shows the dependence of $\rho(l)$ on the power-law index α.

Now the potential energy of this organized, clustered matter can be calculated according to Fahr and Heyl (2007b)

$$\epsilon_{pot} = G\rho_0^2 l_0^5 \int_1^{x_m} 4\pi x^2 dx x^{-\alpha} \frac{1}{x} \int_1^x 4\pi x'^2 dx' x'^{-\alpha} \tag{36}$$

where the normalized distance scale has been defined by $x = l/l_0$. Thus one obtains

$$\epsilon_{pot} = (4\pi)^2 G\rho_0^2 l_0^5 \int_1^{x_m} x dx x^{-\alpha} [\frac{1}{3 - \alpha}(x^{3-\alpha} - 1)] \tag{37}$$

which leads to

$$\epsilon_{pot} = \frac{(4\pi)^2}{3-\alpha} G\rho_0^2 l_0^5 \int_1^{x_m} dx [(x^{4-2\alpha} - x^{1-\alpha})]$$ (38)

and

$$\epsilon_{pot} = \frac{(4\pi)^2}{3-\alpha} G\rho_0^2 l_0^5 \left| \frac{x^{5-2\alpha}}{5-2\alpha} - \frac{x^{2-\alpha}}{2-\alpha} \right|_1^{x_{ma}}$$ (39)

which when taking $x_m \gg 1$ leads to

$$\epsilon_{pot} = \frac{(4\pi)^2}{3-\alpha} G\rho_0^2 l_0^5 x_m^{5-2\alpha} \left(\frac{1}{5-2\alpha} - \frac{x_m^{-3}}{2-\alpha} \right) \simeq \frac{(4\pi)^2}{(3-\alpha)(5-2\alpha)} G\rho_0^2 l_0^5 x_m^{5-2\alpha}$$ (40)

and reminding the requirement $\rho_0 = \frac{3-\alpha}{3}\bar{\rho} x_m^\alpha$ finally leads to

$$\epsilon_{pot} = \frac{(4\pi)^2(3-\alpha)}{9(5-2\alpha)} G\bar{\rho}^2 l_0^5 x_m^5$$ (41)

Now it is interesting to recognize that for $\alpha = 0$ (i.e. homogeneous matter distribution) in fact again the potential energy of a homogeneously filled sphere with radius l_m is found, namely $\epsilon_{pot}(\alpha = 0) = \frac{(4\pi)^2}{15} G\bar{\rho}^2 l_m^5$ (see Fahr and Heyl, 2007). $\epsilon_{pot}(\alpha = 0)$ serves as reference value for the potential energy in the associated re-homogenized universe.

9.1 A one-dimensional analogue

Now imagine a one-dimensional, unidirectional cosmological matter flow as an easy-minded representation of the cosmic Hubble-flow, then one should trust the validity of the following set of equations due to mass-, momentum-, and energy-flow conservation

$$\rho U = \Phi_1$$

$$\rho(\dot{U} + U\frac{d}{dz}U) = F$$

$$\rho U \left(\frac{U^2}{2} + \bar{\epsilon}_{pot} \right) = \Phi_2$$

Here Φ_1 and Φ_2 denote constant mass and energy flows, U is the flow velocity and $\bar{\epsilon}_{pot} = \epsilon_{pot}/(4\pi\rho l_m^3/3)$ denotes the potential energy per mass. F is a force per volume that we want to find, but do not know yet. Now, neglecting explicit local time-dependence (i.e. $\dot{U} = 0$) one finds from the third equation

$$\left(\frac{U^2}{2} + \bar{\epsilon}_{pot} \right) = \Phi_2/\Phi_1 = const$$ (42)

which leads to

$$\frac{d}{dz} \left(\frac{U^2}{2} + \bar{\epsilon}_{pot} \right) = \frac{F}{\rho} - \frac{d}{dz} \left[\frac{(4\pi)(3-\alpha)}{3(5-2\alpha)} G\bar{\rho} l_m^2 \right] = 0$$ (43)

Describing the ongoing of cosmic structuring purely by a change in time of the power index α, this then delivers the interesting result

$$\frac{F}{\rho} = \frac{d}{dz}\left[\frac{(4\pi)(3-\alpha)}{3(5-2\alpha)}G\bar{\rho}l_m^2\right] = -\frac{4\pi}{3}G\rho l_m^2 \frac{3-2\alpha}{(5-2\alpha)^2}\frac{d\alpha}{dz} \tag{44}$$

expressing the fact that for values $\alpha \geq 1.5$ further increase of the structuring index α manifests a positive force F that accelerates the cosmic mass flow. For us this seems the first time it has been shown that gravitational structuring in a moving cosmic flow implies an acceleration of the flow velocity, inditcating that analogously in an expanding universe this might aswell induce an acceleration of the cosmic expansion as usually ascribed to the action of vacuum-energy.

9.2 Structured universes

An independent consideration perhaps points into the same direction as derived above allowing to conclude that cosmic binding energy acts as if it would reduce the effectively gravitating matter density, hence like a form of positive vacuum energy density. It namely turns out that a structured universe expands differently from a homogenized universe with identical total mass (see Buchert (2001; 2005; 2008); Räsänen (2006); Wiltshire (2007); Zalaletdinov (1992)). Quantitatively this was especially shown by Wiltshire (2007) for a 2-phase toy-model of the universe representing the distribution of cosmic matter in form of non-homologously expanding low-density voids and high-density walls. Describing for this purpose this cosmic matter structure by so-called volume-filling factors f_v and f_w and defining the phasestructure densities by $\rho_{v,w} = \int_{V_{v,w}} d^3x\sqrt{\det^3 g}\rho(t,\vec{x})/V_{v,w}$ with $V_{v,w}$ denoting the void- and wall-volume respectively, one obtains the following relation

$$\bar{\rho}_2 = \rho_v f_v + \rho_w f_w = \rho_v f_v + \rho_w(1-f_v) \tag{45}$$

Introducing typical phase scales $R_{v,w}$ and describing their temporal variations with phase-averaged GRT field equations, one obtains the phase densities for the voids and the walls, respectively, as given by

$$\rho_v = \bar{\rho}_2(R/R_v)^3 \tag{46}$$

and:

$$\rho_w = \bar{\rho}_2(R/R_w)^3 \tag{47}$$

Reminding that the acceleration parameter, generally defined by $q = -\ddot{R}R/\dot{R}^2$, for the homogenized, above mentioned 2-phase universe turns out to be obtainable in the following form (Wiltshire, 2007)

$$\bar{q}_2(f_v) = \frac{-(1-f_v)(8f_v^3 + 39f_v^2 - 12f_v - 8)}{(4+f_v+4f_v^2)^2} \tag{48}$$

then proves that in a globally uncurved universe the structure function f_v causes a term in the GRT field equations which is analogous to that describing the action of a vacuum energy density ρ_{vac} of the value

$$\rho_{vac} = \frac{\bar{\rho}_2(1-2\bar{q}_2)}{2(\bar{q}_2+1)} \tag{49}$$

This shows that in a nearly void-dominated universe, i.e. with $f_v \simeq 1$ and $\bar{q}_2(f_v \simeq 1) = 0$, one would find a well-tuned constant expansion dynamics (i.e. a "coasting universe"; Fahr
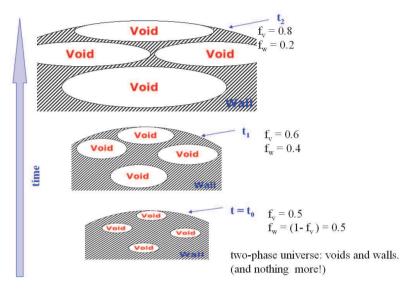
Fig. 4. Illustration of the non-homologous expansion of a two phase universe with void and wall regions having different matter densities.

& Heyl (2007a); Fahr (2006); Fahr & Heyl (2007b); Kolb (1989)) analogous to the action of a vacuum energy density given by $\rho_{vac}(f_v \simeq 1) \simeq (1/2)\bar{\rho}_2$. For phase-structures as they may come up during the non-homologous expansion of the two-phase universe (i.e. with $\dot{R}_w \leq \dot{R}_v$) characterized by a structure function $f_v \geq f_{vc} = 0.57$, where f_{vc} denotes the critical void-volume fill factor $\bar{q}_2$ changes its sign and one obtains $\bar{q}_2 \leq 0$, i.e. an accelerated expansion of the universe which is conventionally ascribed to the action of a vacuum energy $\rho_{vac}(f_v \geq f_{vc}) \geq \bar{\rho}_2/2$. In these phases, one could as well state it like that, the average density $\bar{\rho}_2$ in such a universe appears to be reduced to an effective density given by

$$\bar{\rho}_2(f_v \geq f_{vc}) = \bar{\rho}_2 - \bar{\rho}_{vac}(f_v \geq f_{vc}) = \bar{\rho}_2\left(1 - \frac{1 - 2\bar{q}_2}{2(\bar{q}_2 + 1)}\right) \tag{50}$$

This shows that in that phase of non-homologous structure evolution characterized by $f_v \geq f_{vc} = 0.57$ the average cosmic density appears to be reduced by more than 50 percent due to gravitational binding energies sitting in the wall-structured, dense matter formations.

Some caution, however, in advertizing this result too much, is perhaps in place. This is due to the fact that Wiltshire in his analysis starts out from the scalar differential equations given by Eqns. (13) and (14) and in these only treats cosmic averages of the remaining scalar quantities $R = g^{ij}R_{ij}$, denoting the Riemann scalar as contraction and the Ricci tensor R_{ij} by the metric tensor g^{ij}, and ρ. Thereby it turns out that when going back from his 2-phase universe to an averaged homogeneous replace-universe some back-reaction terms $Q = Q(\langle\langle\rho\rangle\rangle, \langle\langle R\rangle\rangle)$ are obtained, entering the two scalar differential equations of the Einstein field equations, which are left from the homogenization. A correct treatment of spacetime inhomogeneities would, however, require the calculation of 'back-reaction' terms starting from the level of nonlinear, second-order partial differential equations coming from the tensor formulation of the GRT field equations. This calculation has up to now not been carried out, and thus Wiltshire's

results should at present not be over-emphasized, but taken with some scepticism (Buchert, 2008).

10. The universe as energy-less system

Is it imaginable that the universe, enormously large and extended as it is, nevertheless does not represent huge amounts of energy, to the contrary perhaps is a system of vanishing energy. If not representing any real, countable energy, it then might be understandable that such a universe, despite its evolution, can actually even originate from nothing, since permanently constituting nothing. But how can all what we see in the universe, when added up, represent a vanishing amount of energy?

This could in fact be possible, because in physics one knows that there exist positively and negatively valued energies, so that their sum can cancel. If all the positively valued energies in the universe accumulate to E and the negatively valued energies , i.e. the gravitational binding energies in the universe, accumulate to U , then it might turn out that the sum of both, i.e. $L = E + U$, vanishes. In the following we shall show that the "$L = 0$" - universe is actually possible, if matter density and vacuum energy density vary in specific forms with the scale of the universe.

As we have shown in Fahr and Heyl (2007a/b) the total energy $E = E(R)$ of an uncurved universe can be calculated as the spacelike sum over all energies given by the following expression

$$E(R) = \int^{V^3} (\hat{\rho}c^2 + 3\hat{p}) \sqrt{-g_3} d^3 V = \frac{4\pi}{3} R^3 (\hat{\rho}c^2 + 3\hat{p}) \tag{51}$$

For a complete sum all mass densities have been subsummed by the quantity $\hat{\rho}$ which comprehends baryonic matter, dark matter and vacuum equivalent mass density, i.e. is given in the form $\hat{\rho} = \rho_b + \rho_d + \rho_{vac}$, as well all pressures constituting energy densities are subsummed by the quantity $\hat{p} = p_b + p_d + p_{vac}$. As one can see from the above expression, the total energy $E(R)$ is proportional to R^3.

In that phase of the universe which we try to energetically balance here pressures of baryonic and dark matter may be assumed to be negligible with respect to their corresponding rest mass energy densities. In addition, a polytropic relation between ρ_{vac} and p_{vac} can be used in the form

$$p_{vac} = -\frac{(3-n)}{3} \rho_{vac} c^2 \tag{52}$$

since for the most general case a scale-dependent vacuum energy density in the form $\rho_{vac} \sim R^{-n}$ must be admitted (see Fahr and Heyl, 2007b).

In a similar way one can also calculate the total gravitational binding energy $U(R)$ in this universe as the spacelike sum over the total potential energy and obtains the following expression

$$U(R) = \int_0^R 4\pi r^2 (\rho_b + \rho_d + (n-2)\rho_{vac}) \Phi(r) dr \tag{53}$$

where $\Phi(r) = -(2/3)\pi G(\rho_b + \rho_d + (n-2)\rho_{vac})r^2$ is the internal cosmic gravitational potential. This then leads to

$$U(R) = -\frac{8\pi^2 G}{15} (\rho_b + \rho_d + (n-2)\rho_{vac})^2 R^5 \tag{54}$$

Now the No-energy-requirement $L = E + U = 0$ simply leads to the following relation

$$\frac{3c^2}{2\pi G R^2} = (\rho_b + \rho_d + (n-2)\rho_{vac}) \tag{55}$$

with n being the unknown polytropic constant in the relation between vacuum pressure and vacuum mass density $p_{vac} = -\frac{(3-n)}{3}\rho_{vac}c^2$. As evident from the above relation, the requirement $L = 0$ is only fulfilled, if all mass densities in the universe scale as R^{-2}, identical to the scale-dependence already derived at different places and within different contexts presented further above in this article. The pressing question, how this mass creation could be explained, can now easily be answered on the basis of the above deduced context, namely because now vacuum energy density, different from the assumptions in the standard cosmology, is not anymore taken as constant, but turns out to be variable and decaying at the expansion of the universe with $\rho_{vac} \sim R^{-2}$ with the selfsuggesting solution $\dot{\rho}_{vac} \propto \dot{\rho}$. The most encouraging point in this view now is that the universe can start from a Planck volume V_{pl} with a Planck scale $R = r_{pl} = \sqrt{Gh/2\pi c}$ with the initial vacuum energy density of $\rho_{vac}(r_{pl}) = m_{pl}/(4\pi r_{pl}^3/3)$ (just the value calculated by field theoreticians) and then only later at our present epoch has dropped down to the accepted astrophysical values of the present universe corresponding to $\rho_{vac,0} = 0.73\rho_{c,0} \simeq 10^{-29} g/cm^3$ (see Fahr and Heyl, 2007b).

11. Discussion and outlook

We would like to finish this article reminding the readers to a series of more recent papers in which the conclusion of a scale-variability of cosmic masses, reached in this paper here, also is drawn, however, from quite independent theoretical views connected with general symmetry or invariance principles valid in a generalized form of Einstein's general relativistic field theory. The latter theory is not conformally scale-invariant as was emphasized by Hoyle (1990; 1992). Einstein's field equations can be derived from a variational principle applied to the following universal action function

$$S_{0,1} = -\sum_a \int_1^2 m_a da + \frac{1}{12}M_p^2 \int_1^2 \int R\sqrt{-g}d^4x \tag{56}$$

where the Planck mass has been defined by:

$$M_p = \frac{3ch}{4\pi G} = 1.06 \cdot 10^{-6}g \simeq 10^{19}\frac{GeV}{c^2} \tag{57}$$

Here m_a and da are the masses and worldline increments of the particles in the universe, and R and g are the Riemann scalar and the determinant of the metric tensor g_{ik}. The quantity d^4x is the differential 4D spacetime volume element. As Hoyle pointed out, if one measures the action in units of the Planck constant h, and all velocities in units of the velocity of light c, then masses attain the dimension $[1/L]$ where L is a cosmic length scale. Hoyle furtheron emphasizes in his articles - Maxwell's theory, quantum theory and Dirac's theory - they are all conformally invariant, but Einstein's theory is not.

Conformal invariance (invariance with respect to local scale-recalibrations) according to H. Weyl should also be fulfilled by the theory of general relativity. Following this conceptual view of Weyl (1961) also the field theory like GRT should fulfill conformal scale-invariance. This requirement when connected with the general request of the minimum action principle then as can be seen from Equ. (3) automatically requires that mass is created at geodetic

motions of comoving cosmic masses. To respect these theoretical prerequisites would mean that the field equations should be invariant with respect to local recalibrations of the worldline element according to:

$$da^{*2} = \Psi^2(\vec{A})g_{ik}(\vec{A})da^i da^k = L(\vec{A})^{-2}da^2 \tag{58}$$

This is now only fulfilled in connection with the cosmic action minimum, if at the same time where the above relation holds the masses in the universe do also scale by:

$$m_a^* = m_a \frac{1}{\Psi(\vec{A})} = L(\vec{A})m_a \tag{59}$$

Taking creation of matter as concequence of a scale-invariant GRT action principle Hoyle et al. (1993) have developed their Quasi-Steady-State cosmology (QSSC) deriving a scalar mass creation field $C(X)$ which is obtained as solution of a wave equation given by

$$\Box_X C(X) + \frac{1}{6}R(X)C(X) = f^{-1}\sum_{A_0} \frac{\delta_4(X - A_0)}{\sqrt{-g(A_0)}} \tag{60}$$

where $\Box_X$ is the 4-d Laplace operator, X denotes a 4-d spacetime point, $R(X)$ is the Riemannian scalar at X, and A_0 are 4-d spacetime positions of real particles in the universe. The function f is needed as a positive coupling constant. At the place of a particle A_0 one obtains the gradient components of the creation field by

$$C_i(A_0) = \left[\frac{\partial C(X)}{\partial x^i}\right]_{A_0} \tag{61}$$

and is lead to a scalar mass creation bound by the relation

$$\frac{\partial}{\partial t}\left(C_i C^i\right)_{A_0} = \dot{m}_a^2(A_0) \tag{62}$$

where m_a is the mass of the particle at A_0. As the authors analyse further down in their article (Hoyle et al., 1993) creation of field bosons can only occur in connection with massive particles at places A_0, and becomes effective only where strong gradients of the $C(X)-$ fields due to strong Riemannian scalar curvatures $R(X)$ are established in the universe, i.e. near already existing strong mass concentrations. A steady-state form of creation, like that required by Hoyle (1948), under these restricting auspices is unlikely. Mass generation in this QSSC does only happen when particles come close to cosmic mass concentrations or cosmic black holes. But from localized creation rates an average cosmic creation rate $\langle \dot{C}^2 \rangle_4$ can be derived which then instead of Eqns. (1) and (2) can be brought into the field equations of QSSC yielding the following form

$$\left(\frac{\dot{R}(t)}{R(t)}\right)^2 = \frac{8\pi G}{3}\rho(t) - \frac{kc^2}{R^2(t)} + \frac{\Lambda c^2}{3} - \frac{4\pi G}{3}f\left\langle \dot{C}^2 \right\rangle_3 \tag{63}$$

and:

$$\frac{\ddot{R}(t)}{R(t)} = -\frac{4\pi G}{3c^2}(3p(t) + \rho(t)c^2) + \frac{\Lambda c^2}{3} + \frac{8\pi}{3}Gf\left\langle \dot{C}^2 \right\rangle_3 \tag{64}$$

This system of equations has been solved by Sachs et al. (1996) in the following form

$$R_{QSSC}(t) = \exp[(\frac{t}{P})\{1 + \eta\cos\theta(t)\}]$$ (65)

where P is a constant and $\theta(t)$ is a known periodic function with a period $Q \ll P$ and $\eta \leq 1$ as a constant parameter. It turns out that the envellope of the above solution behaves like a solution of the standard cosmology, however, with a vacuum energy density given by

$$\Lambda_{QSSC} = -6\pi Gf\left\langle\dot{C}^2\right\rangle_3$$ (66)

The above demonstrates that QSSC cosmological theories, taking general-relativistic scale invariance as a serious request, will automatically lead to cosmic mass creation and to a fake form of negative vacuum energy density.

There are also recent studies by Mannheim (2001; 2003; 2006) in the literature which point into a similar direction. Mannheim (2006) investigates the logical independence of the general covariance principle, the equivalence principle and the Einstein GRT field equations and manifests several restrictions in the present-day formulation of the energy-momentum tensor which can shed light to why at present the standard cosmology is in troubles. As we do in this article here, he also argues that to solve the outstanding present-day cosmological constant problem with the enormous discrepancy of field-theoretical and astrophysically admittable vacuum energy density, it is not necessary to quench the vacuum energy term itself, but only to find out, by what amount the vacuum energy actually gravitates. His answer is going into the same direction than the one given in this article here culminating in the claim that most of the field-theoretical vacuum energy does not gravitate since it is just compensated by the action of the cosmological constant Λ leading to the fact that for empty space $\Lambda_{eff,0} = 0$!. The gravitationally relevant part of vacuum energy only is due to the matter-polarized vacuum.

To reach this conclusion he carefully checks all the ingredients of all terms on the RHS and LHS of the Einstein GRT equations. He identifies, as one of problems, the conventional formulation of the energy-momentum tensor T_{ik} based on the assumption of geodetic motions of massive, singular particles with invariant masses m which first leads to the expression

$$T^{ik} = \frac{mc}{\sqrt{-g}}\int d\tau\cdot\delta^4(x - y(\tau))\frac{dy^i}{d\tau}\frac{dy^k}{d\tau}$$ (67)

which is covariantly conserved and systematically leads to the corresponding hydrodynamical expression for T^{ik} that is generally used in present-day cosmology. This formulation is used despite the modern understanding that particles are far from being kinematic objects with invariant masses, but are thought to realize their masses dynamically by means of spontaneous symmetry breaking, and despite the fact that the standard SU(3)xSU(2)xU(1) - field unification theory ascribes the basic level of material energy representation to scalar wave fields rather than to particles. The variational principle, if applied to the scalar wave action, then leads to the following equation of motion for the scalar wave field S given by

$$S^{;\mu}_{;\mu} + \frac{\xi}{6}SR^{\mu}_{\mu} - m^2S = 0$$ (68)

This equation is very similar to the one derived by Hoyle et al. (1993), except that in the latter the mass creation is connected with the existing particle motions.

Mannheim discusses several possibilities to change Einstein's GRT equations in order to absorb the concept of dynamical masses from field theoretical considerations as discussed

above. Seeking, however, for alternatives to Einstein's GRT equations by looking for generalizations, one should always take care that in these generalizations the Einstein equations are contained as a special case. Amongst the general covariant pure metric theories of gravity the most convincing generalization, as it appears to Mannheim, is to complement the Einstein Hilbert action by additional coordinate-invariant pure metric terms which, in the Newton limit, do not perturb the validity of Newtons gravity on the scale of the solar system. Also he discusses additional macroscopic gravitational fields as a company of the metric tensor g_{ik}. Here most suggestive step would be to introduce scalar fields. As also taken up by Scholz (n.d.), the idea from H.Weyl to start from conformal gravity theories is discussed by Mannheim (2006). Weyl developing his metrical gravity theory recognized an enlarged Riemann tensor, the conformal, so-called Weyl tensor $C_{\lambda\mu\nu\kappa}$, with remarkable symmetry properties. It namely invariantly transforms under the conformal transformation $g_{\mu\nu}(x) \rightarrow \exp[2\alpha(x)]g_{\mu\nu}(x)$ as $C_{\mu\nu\kappa}^{\lambda}(x) \rightarrow C_{\mu\nu\kappa}^{\lambda}(x)$, since all derivatives of the function $\alpha(x)$ drop out identically. Due to this property the Weyl tensor manifests the same relation to conformal transformations as does the Maxwell tensor to gauge transformations. This can be used to introduce the Weyl action function

$$I_W = -\alpha_g \int d^4x \sqrt{-g} C_{\lambda\mu\nu\kappa}(x) C^{\lambda\mu\nu\kappa}(x) \tag{69}$$

which is invariant under conformal transformations. Here α_g is a dimensionless constant controling conformal cosmology by a theory-immanent effective coupling quantity, obviously replacing Newton´s gravitational constant G in Einstein´s GRT equations. This Weyl action I_W forbids interestingly enough the appearance of any fundamental integration constant like the cosmological constant Λ, as it is admitted at the application of the action-minimizing variational principle to the Einstein-Hilbert action function. The GRT field equations derived on the basis of the Weyl action I_W lead to a new energy momentum tensor of conformal cosmology given by

$$T^{\mu\nu} = T_{kin}^{\mu\nu} - \frac{1}{6}S_0^2(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R_\alpha^\alpha) - g^{\mu\nu}\lambda S_0^4 = 0 \tag{70}$$

where the first term on the RHS is the conventional energy momentum tensor of the moving matter particles which is fully compensated by a second part connected with the spacetime geometry and the scalar function S_0. In this conformal theory there is energy not just in the matter fields, but in the spacetime geometry as well. As Mannheim (2006) can show the associated generalized conformal field equation can be brought into the form

$$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R_\alpha^\alpha = \frac{6}{S_0^2}(T_{kin}^{\mu\nu} - g^{\mu\nu}\lambda S_0^4) \tag{71}$$

revealing that this conformal cosmology equation is analogous to the Einstein GRT equations with the difference of an effective dynamically induced gravitational coupling function given by $G_{eff} = -\frac{3c^2}{4\pi S_0^2}$ (see also Mannheim (1992) and the conformal analogue of Einstein's Λ given by $\bar{\Lambda} = \lambda S_0^4$. When solving the above equation for a Robertson-Walker symmetrical geometry, and introducing as conformal analogues to Einstein's GRT the quantities

$$\bar{\Omega}_m = \frac{8\pi G_{eff}\rho_m}{3c^2 H^2} \tag{72}$$

$$\bar{\Omega}_\Lambda = \frac{\bar{\Lambda}}{3cH^2} \tag{73}$$

then Mannheim (2000) obtains the following result for the acceleration parameter

$$q = \frac{1}{2}(1 + \frac{3p_m}{\rho_m})\,\bar{\Omega}_m - \bar{\Omega}_\Lambda \tag{74}$$

again demonstrating from the basis of this conformal cosmology that something analogous to vacuum energy is operating and causing an accelerated expansion but physically connected with nothing like an energy-loaded vacuum but with a scalar field S_0.

At the end of this article we would like to conclude from all what has been analysed in original studies presented in this article here and from companying literature discussed in this article, that vacuum energy density as it is treated in standard cosmology, i.e. treated as a constant quantity, does not appear to be physically justified, but a generalized representation of this term should be further discussed in cosmology which, however, is of a completely different nature and is variable in magnitude depending on geometrical properties or scalar field properties in the universe.

Although the standard model of cosmology, the ΛCDM-model celebrated big successes in the past and most of the astronomers believe in it, it seems that reality behaves a bit different. Recent investigations by Kroupa et al. (2010) have shown that ΛCDM fails, since on scales of the Local group no dark matter action can be admitted, and so the standard model is faced with a big problem. Therefore it is convenient to consider also alternative models, like the ones presented in this article in order to develop a model of the universe that reflects cosmic reality better than ΛCDM. Nevertheless these kinds of models will have to prove themselves when they are applied to modern cosmological observations like the Supernova Ia data or the anisotropies of the Cosmic Microwave Backround (CMB). However the question remains if the CMB actually represents the matter distribution for a time of about 300000 years after the big bang, or if they should be interpreted in a different way under the conditions of mass-creating models (Fahr & Zoennchen, 2009)?

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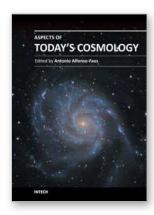
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