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Warm Inflationary Universe Models

Sergio del Campo
Pontificia Universidad Catolica de Valparaiso,
Instituto de Fisica, Curauma, Valparaiso.
Chile

1. Introduction

The most appealing cosmological model to date is the standard hot big-bang scenario. This model rests on the assumption of the cosmological principle that the universe is both homogeneous and isotropic at large scale (Peebles, 1991; 1993; 1994; Weinberg, 2008).

Even though this model could explain observational facts such that the approximately 3-K microwave background radiation (Penzias & Wilson, 1965), the primordial abundances of the light elements[1] (Alpher et al., 1948; Gamow, 1946), the Hubble expansion (Hubble, 1929; Hubble & Humason, 1931) and the present acceleration (Perlmutter et al., 1999; Riess et al., 1998), it presents some shortcomings ("puzzles") when this is traced back to very early times in the evolution of the universe. Among them we distinguish the horizon, the flatness, and the monopole problems. In dealing with these "puzzles", the standard big-bang model demands an unacceptable amount of fine-tuning concerning the initial conditions for the universe.

Inflation has been proposed as a good approach for solving most of the cosmological "puzzles" (Guth, 1981)[2]. The essential feature of any inflationary universe model proposed so far is the rapid (accelerated) but finite period of expansion that the universe underwent at very early times in its evolution.

This brief accelerated expansion serves, apart of solving most of the cosmological problems mentioned previously, to produce the seeds that, in the course of the subsequent eras of radiation and matter dominance, developed into the cosmic structures (galaxies and clusters thereof) that we observe today. In fact, the present popularity of the inflationary scenario is entirely due to its ability to generate a spectrum of density perturbations which lead to structure formation in the universe. In essence, the conclusion that all the observations of microwave background anisotropies performed so far support inflation, rests on the consistency of the anisotropies with an almost Harrison-Zel'dovich power spectrum predicted by most of the inflationary universe scenarios (Peiris et al., 2003).

The different inflationary universe model could be classified depending how the scale factor, $a(t)$, evolves with the cosmological time, t. One of the first models considered that the scale factor follows a de Sitter law of expansion, i.e. $a(t) \sim \exp Ht$, with H the Hubble "parameter". Examples of these models are "old inflaton" (Guth, 1981), "new inflation" (Albrecht & Steinhardt, 1982; Linde, 1982), "chaotic inflation" (Linde, 1983; 1986), and some corrections to this model (Cárdenas et al., 2003). Also, were described models in which the scale factor follows a power law, i.e. $a(t) \sim t^n$, with $n > 1$ (Lucchin & Matarrese, 1985). Models that present this sort of behavior are "extended inflation"

[1] For an historical review on this point, see the Alpher & Herman's article (Alpher & Herman, 1988).
[2] A complete description of inflationary scenarios can be found in the book by Linde (Linde, 1990a).

(La & Steinhardt, 1989) and its applications (Barrow & Maeda, 1990; Campuzano et al., 2006; del Campo & Vilenkin, 1989; del Campo & Herrera, 2003; 2005), "chaotic extended inflation" (Linde, 1990b), "hyperextended inflation" (Steinhardt & Accetta, 1990), which corresponds to a generalization of the extended models. Various studied of this sort of scenario have been presented in the literature (del Campo, 1991; De Felice & Trodden, 2004; Liddle & Wands, 1992). Also, there exist a particular scenario of "intermediate inflation" (Barrow, 1990; Barrow & Saich, 1990) in which the scale factor evolves as $a(t) \sim \exp At^f$, where A is constant and f is a free parameter which ranges $0 < f < 1$. In this sort of scenario, the expansion of the universe is slower than standard de Sitter inflation, but faster than power law inflation. The main motivation to study this latter kind of model becomes from string/M theory. This theory suggests that in order to have a ghost-free action high order curvature invariant corrections to the Einstein-Hilbert action must be proportional to the Gauss-Bonnet (GB) term (Boulware & Deser, 1985; 1986). This kind of theory has been applied to the study of accelerated cosmological solutions (Nojiri et al., 2005). In particular, very recently, it has been found that (Sanyal, 2007) for a dark energy model the GB interaction in four dimensions with a dynamical dilatonic scalar field coupling leads to a solution of the form a $a(t) = a_0 \exp At^f$. One of the problems that arises in these kind of models is due to the characteristic of the scalar inflaton potential, $V(\phi)$, that it does not present a minimum. The usual mechanism introduced to bring inflation to an end becomes useless. In fact, the standard mechanism is described by the stage of oscillations of the scalar field which is an essential part of the so-called reheating mechanism, where most of the matter and radiation of the universe was created, via the decay of the inflaton field, while the temperature grows in many orders of magnitude. It is at this point where the big bang universe is recovered. Here, the reheating temperature, the temperature associated to the temperature of the universe when the big bang model begins, is of particular interest. In this epoch the radiation domination begins, where there exist a number of particles of different kinds. In order to bring the intermediate inflationary period to an end it is introduced a special mechanisms of reheating via the introduction of a new scalar field, the so called curvaton field (del Campo & Herrera, 2007a; Lyth & Wands, 2002; Mollerach, 1990).
Another possible way of schematizing inflationary models is the classification scheme in term of *large-field*, *small-field* and *hybrid* models (Lyth & Riotto, 1999). In the case of large-field inflation, (where the inflaton potential, $V(\phi)$, satisfies the inequalities $V'' > 0$ and $(logV)'' < 0$, with the primes denoting the derivatives with respect to the inflaton field) we have that the scalar inflaton potential is usually taken to be a polynomial, $V(\phi) = \lambda^4(\phi/\phi_c)^n$, where λ^4 represent the vacuum energy density during inflation, ϕ_0 represents the change of the inflaton field during inflation and n is a real number, or exponential, such that $V(\phi) = \lambda^4 \exp(\phi/\phi_c)$. A typical example of this kind of model is chaotic inflation (Linde, 1983; 1986). The most appealing property that these sort of models have is they do not need special initial conditions for inflation to start (the start fine-tuning). Of course this fine-tuning has nothing to do with the fine-tuning needed during the evolution of inflation (the dynamic fine-tuning). Also, these models are interesting for their simplicity. They predict a significant amount of tensor perturbations due to the scalar inflaton field gets across the trans-Planckian distance during inflation (Lyth, 1997) (a fact that should be checked by astronomical observations). However, due to the inflaton crosses the trans-Planckian boarder, there appear some problems when one wants to calculate the trans-Planckian expectation value of the inflaton field.
There exist other type of inflationary models that do not need trans-Planckian expectation values of the inflaton field. These kind of models are part of the so-called small-field (they characterize by $V'' < 0$ and $(logV)'' < 0$). They have been discussed in the context of D-brane inflation (Baumann et al., 2007) in the supersymmetric standard model(Allahverdi et al., 2006)

and in supergravity (Lalak & Turzynski, 2008). In each of these cases some fine-tuning of the effective inflaton potential is required (see Ref. (Linde & Westphal, 2008) for recent treatment of these issues).

The third category of inflationary universe models are called hybrid inflation(in this case the inflaton potential satisfies $V'' > 0$ and $(logV)'' > 0$) (Linde, 1991; 1994). Here, are introduced two scalar fields: one of the fields is the inflaton field, ϕ, which is responsible for the slow-roll period of inflation, the other one, χ takes care of the end of inflation. In this process, inflation ends abruptly and is followed by a regime during which topological defects (like global string (Shafi & Vilenkin, 1984; Vilenkin & Everett, 1982)) could be produced. Perhaps, these topological defect might play an interesting role in giving an appropriated expression for density perturbation which is important for understanding the large scale structure in galaxy formation (Vilenkin & Shellard, 2000). One of the problems that confront hybrid inflation is related with the fine tuning needed at the beginning of inflation (only a small fraction of possible initial conditions give rise to successful inflation). This problem is solved if it is considered nonrenomalizable coupling between the two scalar fields ϕ and χ. Also, it was found that hybrid inflation is not compatible with the supersymmetric standard models. Here it is found that the gravitinos are overproduced by the inflaton decay (Kawasaki et al., 2006a;b) and thus, in this context hybrid inflation is disfavored. The solution of this problem needs to take some fine tuning.

Beside of the possible classification of the different inflationary universe scenarios presented above we may add, in general term, that there are two main competing scenarios in regard to the *slow roll* inflation: The standard inflationary model is divided into two regimes: the slow roll and reheating epochs. In the slow roll period the universe inflates and all interactions between the inflaton scalar field and any other field are typically neglected. Subsequently, a reheating period is invoked to end the brief acceleration. After reheating, the universe is filled with relativistic fluid and thus the universe is connected with the radiation big bang phase.

Warm inflation is an alternative mechanism for having successful inflation. As is well known, warm inflation[3] - as opposed to the conventional "cool" inflation (Kolb & Tuner, 1990; Liddle & Lyth, 2000) - has the attractive feature of not necessitating a reheating phase at the end of the accelerated expansion thanks to the decay of the inflaton into radiation and particles during the slow roll (Berera, 1995; 1997; Berera & Fang, 1995; del Campo et al., 2008). Thus, the temperature of the Universe does not drop dramatically and the Universe can smoothly proceed into the decelerated, radiation-dominated era essential for a successful big bang nucleosynthesis (Peebles, 1993). This scenario has further advantages, namely: (i) the slow-roll condition $\dot{\phi}^2 \ll V(\phi)$ can be satisfied for steeper potentials, (ii) the density perturbations originated by thermal fluctuations may be larger than those of quantum origin (Berera, 2000; Gupta et al, 2002; Taylor & Berera, 2000), (iii) it may provide a very interesting mechanism for baryogenesis (Brandenberger & Yamaguchi, 2003) and (iv) it may also be considered as a model, which comes from an effective high dimensional theory. Different applications of warm inflation have been presented in the literature (Cid et al., 2007; del Campo & Herrera, 2007b; 2008; Herrera et al., 2006).

Apart of the advantage described above, warm inflation was criticized on the basis that the inflaton cannot decay during the slow roll (Yokoyama & Linde, 1999). However, in recent years, it has been demonstrated that the inflaton can indeed decay during the slow-roll phase - see (Bastero-Gil & Berera, 2005; Berera & Ramos, 2005a; Hall & Moss, 2005) and references therein - whereby it now rests on solid theoretical grounds.

We should mention that in warm inflation, dissipative effects are important during inflation, so that radiation production occurs concurrently with the accelerating expansion. The

[3] For a nice review on warm inflationary scenarios see the article (Berera et al., 2009).

dissipating effect arises from a friction term which describes the processes of the scalar field dissipating into a thermal bath via its interaction with other fields. In fact, we may say that the decay of the scalar field is described by means of an interaction Lagrangian. For instance, the authors of (Berera & Ramos, 2003; 2005b; Hall et al., 2004a) take the interaction terms of the form $\frac{1}{2}\lambda^2\phi^2\chi^2$ and $g\chi\overline{\psi}\psi$ where the inflationary period presents a two-stage decay chain $\phi \to \chi \to \psi$. In this case, they reported that the damping term Γ becomes $\lambda^3 g^2\phi/256\pi^2$.
Also, warm inflation shows how thermal fluctuations during inflation may play a dominant role in producing the initial perturbations. In such models, the density fluctuations arise from thermal rather than quantum fluctuations (Berera, 2000; Berera & Fang, 1995; Hall et al., 2004b; Moss, 1985). These fluctuations have their origin in the hot radiation and influence the inflaton through a friction term in the equation of motion of the inflaton scalar field (Berera, 1996; del Campo et al., 2007c). Among the most attractive features of these models, warm inflation ends when the universe heats up to become radiation dominated; at this epoch the universe stops inflating and smoothly enters a radiation dominated big bang phase (Berera, 1995; 1997). The matter components of the universe are created by the decay of either the remaining inflationary field or the dominant radiation fluid.
In this chapter we present the warm inflationary universe scenarios in some detail. The chapter will develop recent advances on this area of continuous research, and their possible implications in the near future, specially, those related with the confrontations with new astrophysical observations, which will put strong constraints on these kind of inflationary universe models. In order to do this, our guideline has been to concentrate on resent results that seem likely still to be of general concern to those researchers that show interest in this subject. Here, we pretend to indulge in recollections of different works on this area of research that have been put forward in the literature. In this way, the intention of this chapter is to make these developments accessible to someone who is interested in understanding how the warm inflationary universe models works. Throughout this chapter we use units in which $c = \hbar = k_B = 1$.

2. Warm inflation at work

We start by considering a spatially flat Friedmann-Robertson-Walker (FRW) universe filled with a self-interacting inflaton scalar field ϕ, of energy density, $\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$ (with $V(\phi) = V$ the scalar potential), and a radiation energy density, ρ_γ.
The corresponding Friedmann equation reads

$$3H^2 = \kappa\left(\rho_\phi + \rho_\gamma\right). \tag{2.1}$$

Here, the constant κ is given by $\kappa = 8\pi G = 8\pi/m_P^2$, with m_P the Planck mass.
The dynamics of the cosmological model, for ρ_ϕ and ρ_γ in the warm inflationary scenario is described by the equations

$$\dot{\rho}_\phi + 3H\left(\rho_\phi + P_\phi\right) = -\Gamma\dot{\phi}^2, \tag{2.2}$$

and

$$\dot{\rho}_\gamma + 4H\rho_\gamma = \Gamma\dot{\phi}^2, \tag{2.3}$$

where $P_\phi = \frac{1}{2}\dot{\phi}^2 - V$ and Γ represents the dissipation coefficient and it is responsible of the decay of the scalar field into radiation during the inflationary era. Γ can be assumed to be a constant or a function of the scaler field ϕ, or the temperature T, or both (Berera, 1995; 1997). On the other hand, Γ must satisfy $\Gamma > 0$ in agreement with the Second Law of Thermodynamics. Dots mean derivatives with respect to the cosmological time.

During the inflationary epoch the energy density associated to the scalar field dominates over the energy density associated to the radiation field (Berera, 2000; Hall et al., 2004b; Moss, 1985) i.e. $\rho_\phi > \rho_\gamma$, the Friedmann equation (2.1) reduces to

$$H^2 \approx \frac{\kappa}{3}\,\rho_\phi, \tag{2.4}$$

and from Eqs. (2.2) and (2.4), we can write

$$\dot{\phi}^2 = -\frac{2\,\dot{H}}{\kappa\,(1+Q)}, \tag{2.5}$$

where Q is the rate defined as

$$Q = \frac{\Gamma}{3H}. \tag{2.6}$$

For the strong (weak) dissipation regime, we have $Q \gg 1$ ($Q \ll 1$).
We also consider that during warm inflation the radiation production is quasi-stable (Berera, 2000; Hall et al., 2004b; Moss, 1985), i.e. $\dot{\rho}_\gamma \ll 4H\rho_\gamma$ and $\dot{\rho}_\gamma \ll \Gamma\dot{\phi}^2$. From Eq.(2.3) we obtained that the energy density of the radiation field becomes

$$\rho_\gamma = \frac{\Gamma\dot{\phi}^2}{4H} = -\frac{\Gamma\,\dot{H}}{2\,\kappa\,H\,(1+Q)}, \tag{2.7}$$

which could be written as $\rho_\gamma = C_\gamma T^4$, where $C_\gamma = \pi^2 g_*/30$ and g_* is the number of relativistic degrees of freedom. Here T is the temperature of the thermal bath.
From Eqs.(2.5) and (2.7) we get that

$$T = \left[-\frac{\Gamma\,\dot{H}}{2\,\kappa\;C_\gamma H\,(1+Q)}\right]^{1/4}. \tag{2.8}$$

From first principles in quantum field theory the dissipation coefficient Γ is computed for models in cases of low-temperature regimes (Moss & Xiong, 2006) (see also Ref. Berera & Ramos (2001)). Here, was developed the dissipation coefficients in supersymmetric models which have an inflaton together with multiplets of heavy and light fields. In this approach, it was used an interacting supersymmetric theory, which has three superfields Φ, X and Y with a superpotential, $W = \frac{1}{\sqrt{2}}g\Phi X^2 - \frac{1}{\sqrt{2}}hXY^2$. The scalar components of the superfields are ϕ, χ and y respectively [4]. In the low -temperature regime, i.e. where their masses satisfy $m_\chi, m_\psi > T > H$, the dissipation coefficient, when χ and y are singlets, becomes (Moss & Xiong, 2006)

$$\Gamma \simeq 0.64\,g^2\,h^4\left(\frac{g\,\phi}{m_\chi}\right)^4\,\frac{T^3}{m_\chi^2}. \tag{2.9}$$

This latter equation can be rewritten as

$$\Gamma \simeq C_\phi\,\frac{T^3}{\phi^2}, \tag{2.10}$$

[4] This potential could be easily modified to produce Hybrid inflation (Moss & Xiong, 2006).

where $C_\phi = 0.64\, h^4\, \mathcal{N}$. Here $\mathcal{N} = \mathcal{N}_\chi \mathcal{N}^2_{decay}$, where $\mathcal{N}_\chi$ is the multiplicity of the X superfield and $\mathcal{N}_{decay}$ is the number of decay channels available in X's decay (Bueno Sanchez et al., 2008; Moss & Xiong, 2006).
From Eq.(2.8) the above equation becomes

$$\Gamma^{1/4}\,(1+Q)^{3/4} \simeq \left[\frac{-2\,\dot{H}}{9\,\kappa\, C_\gamma\, H}\right]^{3/4}\,\frac{C_\phi}{\phi^2}, \tag{2.11}$$

which determines the dissipation coefficient in the strong (or weak) dissipative regime in terms of scalar field ϕ and the parameters of the model.
In general the scalar potential can be obtained from Eqs.(2.1) and (2.7)

$$V(\phi) = \frac{1}{\kappa}\left[3\,H^2 + \frac{\dot{H}}{(1+Q)}\,\left(1+\frac{3}{2}\,Q\right)\right], \tag{2.12}$$

which could be expressed explicitly in terms of the scalar field, ϕ, by using Eqs.(2.5) and (2.11), in the weak (or strong) dissipative regime.

3. The inclusion of viscous pressure

Usually, for the sake of simplicity, in studying the dynamics of warm inflation the particles created in the decay of the inflaton are treated as radiation thereby ignoring altogether the existence of particles with mass in the fluid thus generated. However, the very existence of these particles necessarily alters the dynamics as they modify the fluid pressure in two important ways: (i) its hydrodynamic, equilibrium, pressure is no longer $p_\gamma = \rho_\gamma/3$, with ρ_γ the energy density of the radiation fluid, but the slightly more general expression $p = (\gamma - 1)\rho$ where the adiabatic index, γ, is bounded by $1 \leq \gamma \leq 2$. (ii) It naturally arises a non-equilibrium, viscous, pressure Π, via two different mechanisms: (a) the inter-particle interactions (Huang, 1987), and (b) the decay of particles within the fluid (Zeldovich, 1970).
Concerning the latter mechanism, it is well known that the decay of particles within a fluid can be formally described by a bulk viscous pressure, Π. This is so because the decay is an entropy-producing scalar phenomenon linked to the spontaneous widening of the phase space and the bulk viscous pressure is also an scalar entropy-producing agent. In the case of warm inflation it has been proposed that the inflaton can excite a heavy field and trigger the decay of the latter into light fields (Berera & Ramos, 2003; 2005a).
Recently, a detailed analysis of the dynamics of warm inflation with viscous pressure showed that when $\Pi \neq 0$ the inflationary region takes a larger portion of the phase space associated to the autonomous system of differential equations than otherwise (Mimoso et al., 2006). It then follows that the viscous pressure facilitates inflation and lends support to the warm inflationary scenario.
For the viscous pressure we shall assume the usual fluid dynamics expression $\Pi = -3\zeta H$ (Huang, 1987), where ζ denotes the phenomenological coefficient of bulk viscosity and H the Hubble function. This coefficient is a positive-definite quantity (a restriction imposed by the second law of thermodynamics) and in general it is expected to depend on the energy density of the fluid. We shall resort to the WMAP data to restrict the aforesaid coefficient. In this case Eq.(2.3) becomes

$$\dot{\rho} + 3H(\rho + p + \Pi) = \dot{\rho} + 3H(\gamma\rho + \Pi) = \Gamma\dot{\phi}^2. \tag{3.1}$$

In this section we shall restrict our analysis to the strong (or high) dissipation regime, i.e., $Q \gg 1$. The reason for this limitation is the following. Outside this regime radiation and particles produced both by the decay of the inflaton and the decay of the heavy fields will be much dispersed by the inflationary expansion, whence they will have little chance to interact and give rise to a non-negligible bulk viscosity. Likewise, because a much lower number of heavy fields will be excited the number of decays of heavy fields into lighter ones will diminish accordingly. (The weak dissipation regime ($R \leq 1$) has been considered by Berera and Fang (Berera & Fang, 1995) and Moss (Moss, 1985). Further, if R is not big, the fluid will be largely diluted and the mean free path of the particles will become comparable or even larger than the Hubble horizon. Hence, the regime will no longer be hydrodynamic but Knudsen's and the hydrodynamic expression $\Pi = -3\zeta H$ we are using for the viscous pressure will become invalid.

3.1 Scalar and tensor perturbations in presence of viscosity

We introduce the dimensionless slow-roll parameters ε and η (Kolb & Tuner, 1990; Linde, 1990b; Lyth, 2000), as a function of the inflaton scalar potential, $V(\phi)$ and its two first derivatives, $V_{,\phi} = dV(\phi)/d\phi$ and $V_{,\phi\phi} = d^2V(\phi)/d\phi^2$,

$$\varepsilon \equiv -\frac{\dot{H}}{H^2} = \frac{1}{2(1+Q)}\left[\frac{V_{,\phi}}{V}\right]^2 , \tag{3.2}$$

and

$$\eta \equiv -\frac{\ddot{H}}{H\dot{H}} \simeq \frac{1}{(1+Q)}\left[\frac{V_{,\phi\phi}}{V} - \frac{1}{2}\left(\frac{V_{,\phi}}{V}\right)^2\right]. \tag{3.3}$$

In order to find scalar (density) and tensor (gravitational) perturbations we take the perturbed FRW metric in the longitudinal gauge which is given by

$$ds^2 = (1+2\Phi)\,dt^2 - a(t)^2(1-2\Psi)\,\delta_{ij}\,dx^i dx^j, \tag{3.4}$$

where the functions $\Phi = \Phi(t, \mathbf{x})$ and $\Psi = \Psi(t, \mathbf{x})$ denote the gauge-invariant variables of Bardeen (Bardeen, 1980). Introducing the Fourier components $e^{i\mathbf{kx}}$, with k the wave number, the following set of equations, in the momentum space, follow from the perturbed Einstein field equations -to simplify the writing we omit the subscript k-

$$\Phi = \Psi, \tag{3.5}$$

$$\dot{\Phi} + H\Phi = \frac{1}{2}\left[-\frac{(\gamma\rho+\Pi)\,a\,v}{k} + \dot{\phi}\,\delta\phi\right], \tag{3.6}$$

$$\ddot{(\delta\phi)} + [3H+\Gamma]\,\dot{(\delta\phi)} + \left[\frac{k^2}{a^2} + V_{,\phi\phi} + \dot{\phi}\Gamma_{,\phi}\right]\,\delta\phi \;= 4\dot{\phi}\,\dot{\Phi} - \left[\dot{\phi}\,\Gamma + 2V_{,\phi}\right]\,\Phi, \tag{3.7}$$

$$\dot{(\delta\rho)} + 3\gamma H\delta\rho + ka(\gamma\rho+\Pi)v + 3(\gamma\rho+\Pi)\dot{\Phi} - \dot{\phi}^2\Gamma_{,\phi}\delta\phi - \Gamma\dot{\phi}[2\dot{(\delta\phi)} + \dot{\phi}\Phi] = 0, \tag{3.8}$$

and

$$\dot{v} + 4Hv + \frac{k}{a}\left[\Phi + \frac{\delta p}{(\rho+p)} + \frac{\Gamma\dot{\phi}}{(\rho+p)}\delta\phi\right] = 0\,, \tag{3.9}$$

where

$$\delta p = (\gamma - 1)\delta\rho + \delta\Pi\,, \qquad \delta\Pi = \Pi\left[\frac{\zeta_{,\rho}}{\zeta}\,\delta\rho + \Phi + \frac{\dot{\Phi}}{H}\right], \tag{3.10}$$

and the quantity v arises upon splitting the velocity field as $\delta u_j = -\frac{iak_j}{k}\, v\, e^{i\mathbf{kx}}$ $(j = 1, 2, 3)$ (Bardeen, 1980).
Since the inflaton and the matter-radiation fluid interact with each other isocurvature (i.e., entropy) perturbations emerge alongside the adiabatic ones. This occurs because warm inflation can be understood as an inflationary model with two basics fields (Oliveira, 2002; Starobinski & Yokoyama, 1995; Starobinski & Tsujikawa, 2001). In this context, dissipative effects themselves can produce a variety of spectral ranging from red to blue (Berera, 2000; Hall et al., 2004a; Oliveira, 2002), thus producing the running blue to red spectral suggested by WMAP data (Hinshaw et al., 2009; Komatsu et al., 2009; 2011; Larson et al., 2011; Spergel et al., 2007).
When looking for non-decreasing adiabatic and isocurvature modes on large scales, $k \ll aH$ (which depend only weakly on time), it is permissible to neglect $\dot{\Phi}$ and those terms with two-times derivatives. This together with the slow-roll approximation, the above equations simplify enough so we can find solutions in such a way that expressions for the corresponding scalar and tensor perturbations could be written down.
Here, the density perturbation becomes given by the expression[5]

$$\delta_H^2 \approx \frac{2}{25\,\pi^2}\exp[-2\widetilde{\Im}(\phi)]\left[\frac{T_r}{\widetilde{\varepsilon}\,Q^{1/2}\,V^{3/2}}\right], \tag{3.11}$$

where $\widetilde{\varepsilon} \approx \frac{1}{2Q}\left[\frac{V_{,\phi}}{V}\right]^2$ denotes the dimensionless slow-roll parameter in the high dissipation phase, i.e. $\widetilde{\varepsilon} = \varepsilon(Q \gg 1)$, T_r stands for the temperature of the thermal bath and the function $\widetilde{\Im}(\phi)$ result to be

$$\widetilde{\Im}(\phi) = -\int\left\{\frac{\Gamma_{,\phi}}{\Gamma} + \frac{3}{8\,G(\phi)}\left[1 - \left((\gamma-1) + \Pi\,\frac{\zeta_{,\rho}}{\zeta}\right)\frac{\Gamma_{,\phi}V_{,\phi}}{3\gamma\Gamma\,H}\right]\,(\ln(V))_{,\phi}\right\}d\,\phi\,. \tag{3.12}$$

The scalar spectral index n_s is defined by

$$n_s - 1 = \frac{d\ln\,\delta_H^2}{d\ln k}\,, \tag{3.13}$$

which, upon using Eqs.(3.11) and (3.13), results to be given by

$$n_s \approx 1 - \left[\widetilde{\varepsilon} + 2\,\widetilde{\eta} + \left(\frac{2\widetilde{\varepsilon}}{Q}\right)^{1/2}\left[2\widetilde{\Im}_{,\phi} - \frac{Q_{,\phi}}{2R}\right]\right], \tag{3.14}$$

where

$$\widetilde{\eta} \approx \frac{1}{Q}\left[\frac{V_{,\phi\phi}}{V} - \frac{1}{2}\left(\frac{V_{,\phi}}{V}\right)^2\right] \tag{3.15}$$

[5] See Ref. (del Campo et al., 2007c) for details.

stands for the second slow-roll parameter, η, when $Q \gg 1$.
One interesting feature of the seven-year data gathered by the WMAP experiment is a significant running in the scalar spectral index $dn_s/d\ln k = \alpha_s$ (Komatsu et al., 2011). Dissipative effects can lead to a rich variety of spectral from red to blue (Berera, 2000; Hall et al., 2004a; Oliveira, 2002). From Eq.(3.14) it is seen that in our model the running of the scalar spectral index is given by

$$\alpha_s \simeq -\sqrt{\frac{2\,\widetilde{\varepsilon}}{Q}}\;[\widetilde{\varepsilon}_{,\phi} + 2\widetilde{\eta}_{,\phi}] - \frac{\widetilde{\varepsilon}}{Q}\left[\left(\frac{\widetilde{\varepsilon}_{,\phi}}{\widetilde{\varepsilon}} - \frac{Q_{,\phi}}{Q}\right)\left(2\widetilde{\Im}_{,\phi} - \frac{Q_{,\phi}}{2Q}\right) + \left(4\widetilde{\Im}_{,\phi\phi} - (\ln(Q))_{,\phi\phi}\right)\right]. \tag{3.16}$$

In models with only scalar fluctuations, the marginalized value of the derivative of the spectral index can be approximated by $dn_s/d\ln k = \alpha_s \sim -0.05$ for WMAP only (Spergel et al., 2007). In including the SN "Constitution" sample[6] of type Ia supernovae (Hicken et al., 2009), which presents a proof for the current acceleration of the universe, and the Baryonic Acoustic Oscillations (BAOs), which are the sound oscillations of the primeval baryon-photon fluid prior to the recombination epoch[7] (Eisenstein et al., 1998), WMAP-7 presented the range $-0.065 < \alpha_s < 0.010$ (Komatsu et al., 2011; Larson et al., 2011) for the running scalar spectral index α_s.
With regard to the generation of tensor perturbations during inflation gives rise to stimulated emission in the thermal background of gravitational waves (Bhattacharya et al., 2006). As a consequence, an extra temperature dependent factor, $\coth(k/2T)$, where, k and T stand for the wave number and the temperature, respectively, enters the spectrum, $A_g^2 \propto k^{n_g}$. Thus it now reads,

$$A_g^2 = 2\,\left(\frac{H}{2\pi}\right)^2 \coth\left[\frac{k}{2T}\right] \simeq \frac{V}{6\,\pi^2}\,\coth\left[\frac{k}{2T}\right], \tag{3.17}$$

the spectral index being

$$n_g = \frac{d}{d\,\ln k}\,\ln\left[\frac{A_g^2}{\coth[k/2T]}\right] = -2\,\varepsilon, \tag{3.18}$$

where we have used Eq.(3.2).
A quantity of prime interest is the tensor-scalar ratio, defined as $R(k_0) = \left.\left(\frac{A_g^2}{P_{\mathcal{R}}}\right)\right|_{k=k_0}$ where $P_{\mathcal{R}} \equiv 25\delta_H^2/4$ and k_0 is known as the pivot point. Its expression in the high dissipation limit, $R \gg 1$, follows from using Eqs. (3.11) and (3.17),

$$R(k_0) = \left.\left(\frac{A_g^2}{P_{\mathcal{R}}}\right)\right|_{k=k_0} = \frac{2}{3}\left.\left[\left(\frac{\widetilde{\varepsilon}\,r^{1/2}\,V^{5/2}}{T_r}\right)\exp[2\,\widetilde{\Im}(\phi)]\,\coth\left(\frac{k}{2T}\right)\right]\right|_{k=k_0}. \tag{3.19}$$

[6] This corresponds to an extension of the "Union" sample (Kowalski et al., 2008).
[7] Quite recently, the size of the BAO peak was detected in the large-scale correlation function clustering of approximately 44,000 luminous red galaxies from the Sloan Digital Sky Survey (SDSS) (Eisenstein et al., 2005)

In the case in which we consider a chaotic scalar potential, i.e. $V(\phi) = \frac{1}{2} m^2 \phi^2$, where $m > 0$ is a free parameter, and (as mentioned above) we restrict ourselves to study the high dissipation regime ($Q \gg 1$).
From Eq.(3.11), the scalar power spectrum results to be

$$P_{\mathcal{R}}(k_0) \approx \frac{1}{2\pi^2} \left[8\gamma\Gamma_0 V(\phi_0)^{1/2} + 2\sqrt{3}m^2(1-2\gamma) + 3\sqrt{3}\zeta_0\Gamma_0(2-3\gamma)\right]^{3/2} \left[\frac{\Gamma_0^{1/2}\, T_r}{3^{1/4} m^2\, V(\phi_0)^{3/4}}\right], \tag{3.20}$$

Likewise, Eq.(3.19) provides us with the tensor-scalar ratio

$$R(k_0) \approx \frac{2}{3} \left[8\gamma\Gamma_0 V(\phi_0)^{1/2} + 2\sqrt{3}m^2(1-2\gamma) + 3\sqrt{3}\zeta_0\Gamma_0(2-3\gamma)\right]^{-3/2} \left[\frac{3^{1/4} m^2 V(\phi_0)^{7/4}}{\Gamma_0^{1/2} T_r}\right] \coth\left(\frac{k}{2T}\right), \tag{3.21}$$

where $V(\phi_0)$ and ϕ_0 stand for the potential and the scalar field, respectively, when the perturbation, of scale $k_0 = 0.002\mathrm{Mpc}^{-1}$, was leaving the horizon.
By resorting to the WMAP three-year data, $P_{\mathcal{R}}(k_0) \simeq 2.3 \times 10^{-9}$ and $R(k_0) = 0.095$, and choosing the parameters $\gamma = 1.5$, $m = 10^{-6}\,\mathrm{m}_P$, $T \simeq T_r \simeq 0.24 \times 10^{16}$ GeV and $k_0 = 0.002\,\mathrm{Mpc}^{-1}$, it follows from Eqs. (3.20) and (3.21) that $V(\phi_0) \simeq 1.5 \times 10^{-11}\,\mathrm{m}_P^4$ and $\zeta_0 \simeq 3 \times 10^{-6}\,\mathrm{m}_P^3$. When the scale k_0 was leaving the horizon the inflaton decay rate Γ_0 is seen to be of the order of $10^{-3}\,\mathrm{m}_P$. Thus Eq. (3.16) tells us that one must augment ζ_0 by two orders of magnitude to have a running spectral index α_s close to the observed value (Spergel et al., 2007).
While cool inflation typically predicts a nearly vanishing bispectrum, and hence a small (just a few per cent) deviation from Gaussianity in density fluctuations -see e.g. (Gangui et al., 1994)-, warm inflation clearly predicts a non-vanishing bispectrum. The latter effect arises from the non-linear coupling between the the fluctuations of the inflaton and those of the radiation. This can produce a moderate non-Gaussianity (Gupta, 2006; Gupta et al, 2002) or even a stronger one -likely to be detected by the PLANCK satellite (Ade et al., 2011; PLANCK Collaboration, 2009)- if the aforesaid nonlinear coupling is extended to subhorizon scales (Moss & Xiong, 2007). Because Π implies an additional coupling between the radiation and density fluctuations it is to be expected that non-Gaussianity will be further enhanced. Perhaps, this could serve to observationally constrain Π by future experiments.
Thus, our model presents two interesting features: (i) Related to the fact that the dissipative effects plays a crucial role in producing the entropy mode, they can themselves produce a rich variety of spectral ranging from red to blue. The possibility of a spectrum which does run so is particularly interesting because it is not commonly seen in inflationary models which typically predict red spectral. (ii) The viscous pressure may tell us about how the matter-radiation component behaves during warm inflation. Specifically, it will be very interesting to know how the viscosity contributes to the large scale structure of the Universe. In this respect, we anticipate that the PLANCK mission (Ade et al., 2011; PLANCK Collaboration, 2009) will significantly enhance our understanding of the large scale structure by providing us with high quality measurements of the fundamental power spectrum over an larger wavelength range than the WMAP experiment.

3.2 Viscosity and the stability of warm inflation

Any inflationary model -whether “cold" or “warm"- must fulfill the requirement of stability[8]; that is to say, its inflationary solutions ought to be attractors in the solution space of the relevant cosmological solutions. It means, in practice, that the scalar field, ϕ, must approach an asymptotic attractor characterized by $\dot{\phi} \simeq -\frac{\partial V}{\partial \phi}(3H)^{-1}$ in cold inflation, and $\dot{\phi} \simeq -\frac{(\partial V/\partial \phi)}{3H(1+Q)}$ in warm inflation (see e.g. Liddle et al. (1994); Salopek & Bond (1990)). This ensures that the system will stay sufficiently near to the slow-roll solution for many Hubble times. Here V denotes the scalar field potential and H the Hubble expansion rate.

In the case of warm inflation the conditions for stability have been considered by de Oliveira and Ramos (Oliveira & Ramos, 1998) and, recently, more fully by Moss and Xiong (Moss & Xiong, 2008) who allowed the scalar potential and the damping rate to depend not only on the inflaton field but on the temperature of the radiation gas as well. This automatically introduces two further slow-roll parameters and renders the conditions for a successful warm inflationary scenario even less restrictive.

Here, we want to study the stability of warm inflationary solutions by considering the presence of massive particles and fields in the radiation fluid as well as the existence of a the viscous pressure, Π, associated to the resulting mixture of heavy and light particles.

The corresponding field equations are those described previously, but now we will take both the scalar potential and the damping rate as a function of the temperature, i.e. $V = V(\phi, T)$ and $\Gamma = \Gamma(\phi, T)$.

The total pressure becomes

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi, T) + (\gamma - 1)Ts + \Pi, \tag{3.22}$$

where we have included the entropy density, s, that follows from the thermodynamical relation $s = -\partial f/\partial T \simeq -V_{,T}$, when the Helmholtz free-energy, $f = (1/2)\dot{\phi}^2 + V(\phi, T) + \rho_\gamma - Ts$, is dominated by the scalar potential.

The conservation of the stress-energy can be expressed as

$$T\dot{s} + 3H(\gamma Ts + \Pi) = \Gamma\dot{\phi}^2\,. \tag{3.23}$$

Making $u = \dot{\phi}$, the slow roll equations take the form

$$u = \frac{-V_{,\phi}}{3H(1+Q)}, \qquad Ts = \frac{Qu^2 + 3H\zeta}{\gamma}, \qquad 3H^2 = V(\phi, T)\,. \tag{3.24}$$

To find the conditions for the validity of the slow roll approximation, we perform a linear stability analysis to see whether the system remains close to the slow roll solution for many Hubble times. In cold inflationary scenario, the slow roll equation is of first order in the time derivative. Choosing the inflaton field as independent variable, the conservation equations (2.1) and (3.23) can be written as first order equations in the derivative with respect to ϕ, indicated by a prime,

$$x' = F(x)\,, \tag{3.25}$$

[8] For more details on this subsection see Ref. (del Campo et al., 2010)

where

$$x = \begin{pmatrix} u \\ s \end{pmatrix}. \tag{3.26}$$

Thus, the system (2.1), (3.23) becomes

$$u' = -3H - \Gamma - V_{,\phi} u^{-1}, \tag{3.27}$$

$$s' = -3H\gamma s u^{-1} - 3H\Pi (Tu)^{-1} + T^{-1}\Gamma u. \tag{3.28}$$

Here the Hubble rate and entropy density are determined by (2.2) and $s \simeq -V_{,T}$, respectively. Taking a background $\overline{x}$ which satisfies the slow roll equations (3.24), the linearized perturbations satisfy

$$\delta x' = M(\overline{x})\delta x - \overline{x}', \tag{3.29}$$

where

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}, \tag{3.30}$$

is the matrix of first derivatives of F evaluated at the slow roll solution. Linear stability demands that its determinant be positive and its trace negative.
The matrix elements read,

$$A = \frac{H}{u}\left\{-3(1+Q) - \frac{\epsilon}{(1+Q)^2}\right\}, \tag{3.31}$$

$$B = \frac{H}{s}\left\{-c\,Q - \frac{Q}{(1+Q)^2}\epsilon + b(1+Q)\right\}, \tag{3.32}$$

$$C = \gamma\frac{Hs}{u^2}\left(6 - \frac{\epsilon}{(1+Q)^2}\right)\left\{1 + \frac{\Pi}{\gamma^2\rho_\gamma}\left(\frac{6(1+Q)^2 - 2\epsilon}{6(1+Q)^2 - \epsilon}\right)\right\}, \tag{3.33}$$

$$D = \gamma\frac{H}{u}\left(c - 4 - \frac{Q\epsilon}{\gamma^2(1+Q)^2}\right) + \frac{H\Pi}{u\gamma\rho_\gamma}\left\{c - \frac{Q\epsilon}{\gamma^2(1+Q)^2} + \frac{3\Pi}{2\gamma^2\rho_\gamma}\right\}. \tag{3.34}$$

In the strong regime ($Q \gg 1$), the determinant and trace of M assume the comparatively simple expressions

$$\det M = \frac{3\gamma Q H^2}{u^2}\left(4 - 2b + c + (c - 2b)\frac{\Pi}{\gamma^2\rho_\gamma} - \frac{3}{2}\frac{\Pi^2}{\gamma^4\rho_\gamma^2}\right), \tag{3.35}$$

and

$$\mathrm{tr}M = \frac{H}{u}\left\{-3Q + \gamma(c-4) + \frac{\Pi}{2\gamma^3\rho_\gamma}\left(2\gamma^2 c + 3\frac{\Pi}{\rho_\gamma}\right)\right\}. \tag{3.36}$$

Sufficient conditions for stability are that M varies slowly and that

$$|c| \leq \frac{4 - 3\sigma^2/2}{1+\sigma} - 2b, \qquad b \geq 0, \tag{3.37}$$

where $\sigma \equiv \frac{\Pi}{\gamma^2\rho_\gamma}$. Upon these conditions the determinant results positive and the trace negative, implying stability of the corresponding solution. Expression (3.37.1) generalizes Eq. (27) of Moss and Xiong (Moss & Xiong, 2008).

Since the chosen background is not an exact solution of the complete set of equations, the forcing term in equation (3.29) depends on $\overline{x}'$, and will be valid only if $\overline{x}'$ is small. The size of $\overline{x}'$ depends on the quantities $\dot{u}/(Hu)$ and $\dot{s}/(Hs)$. From the time derivative of (3.24.3) we obtain

$$\frac{\dot{H}}{H^2} = -\frac{\epsilon}{1+Q}\,. \tag{3.38}$$

Combining this with the other slow-roll equations, (3.24.1) and (3.24.2), we get

$$\frac{\dot{u}}{Hu} = \frac{1}{\Delta}\left[-\frac{c[A(1+Q)-BQ]-4}{1+Q}\epsilon + \frac{4Q}{1+Q}\beta + (Ac-4)\eta - \frac{3(1+Q)c}{1-f}b\right], \tag{3.39}$$

and

$$\frac{\dot{s}}{Hs} = \frac{3}{\Delta}\left[\frac{A(3+Q)-B(1+Q)}{1+Q}\epsilon + \frac{Q-1}{1+Q}A\beta \right.$$
$$\left. -2A\eta - \frac{(1+Q)[Ac(Q-1)+Q+1]c}{(1-f)Q}b\right], \tag{3.40}$$

where

$$\Delta = 4(1+Q) + Ac(Q-1)\,, \qquad A = \frac{\rho_\gamma + \gamma^{-1}\Pi}{\rho_\gamma - \kappa\Pi}\,, \qquad B = \frac{\Pi}{\rho_\gamma - \kappa\Pi}\,, \tag{3.41}$$

$$f = -\frac{3}{2}\frac{(1+Q)^2}{Q}\frac{\zeta}{\gamma H\epsilon}\,, \qquad \kappa = \rho_\gamma\frac{\zeta_{,\rho_\gamma}}{\zeta}\,. \tag{3.42}$$

Notice that when $\Pi \to 0$, one has that $A \to 1$, $B \to 0$, $f \to 0$, and therefore the equations (3.38)-(3.40) reduce to the corresponding expressions in Ref. Moss & Xiong (2008). Obviously, the value of the parameter κ in this limit depends on the specific expression of the viscosity coefficient, ζ; but it does not alter the value of B in the said limit. In this limit, the κ parameter could take any value depending of the model. Its value does not affect the $\Pi \to 0$ limit.

The thermal fluctuations produce a power spectrum of scalar density fluctuations of the form (Moss & Xiong, 2008)

$$\mathcal{P}_s = \frac{\sqrt{\pi}}{2}\frac{H^3 T}{u^2}\sqrt{1+Q}\,. \tag{3.43}$$

Note that the power spectrum of fluctuations in inflationary models where the friction coefficient depends also on the temperature, i.e., $\Gamma = \Gamma(\phi, T)$, was considered recently in Ref. Graham & Moss (2009).

We calculate the spectral index by means of

$$n_s - 1 = \frac{\dot{\mathcal{P}}_s}{H\mathcal{P}_s}\,. \tag{3.44}$$

By virtue of the equations (3.38)-(3.40), we obtain

$$n_s - 1 = \frac{p_1\epsilon + p_2\beta + p_3\eta + p_4 b}{\Delta}\,, \tag{3.45}$$

where the p_i coefficients are given by

$$p_1 = -\frac{10(2+Q) - A(3+5c+Q) + B(1+Q+(5c/2)Q)}{1+Q}, \tag{3.46}$$

$$p_2 = \frac{A(Q-1) - 10Q}{1+Q}, \tag{3.47}$$

$$p_3 = \frac{8(1+Q) - A(2+2c+2Q+3cQ)}{1+Q}, \tag{3.48}$$

$$p_4 = \frac{3(1+Q)[1+(1+5c/2)Q]}{(1-f)Q}. \tag{3.49}$$

For $Q \gg 1$, and assuming c of order unity, the p_i coefficients reduce to

$$p_1 = -10 + A - B(1+5c/2);\; p_2 = A - 10;\; p_3 = 8 - A(2+3c);\; p_4 = \frac{3Q(1+5c/2)}{(1-f)}, \tag{3.50}$$

and $\Delta = Q(4 + Ac)$. Therefore (3.45) becomes

$$n_s - 1 = -\frac{10 - A + B(1+5c/2)}{(4+Ac)Q}\epsilon - \frac{10-A}{(4+Ac)Q}\beta + \frac{8 - A(2+3c)}{(4+Ac)Q}\eta + \frac{3(1+5c/2)}{(4+Ac)(1-f)}b. \tag{3.51}$$

The tensor modes happen to be the same as in the cold inflationary models (Moss & Xiong, 2008), i.e.,

$$\mathcal{P}_T = H^2, \tag{3.52}$$

and the corresponding spectral index is

$$n_T - 1 = -\frac{2}{1+Q}\epsilon. \tag{3.53}$$

With the help of of (3.52), (3.43) and (3.24.1) the tensor-to-scalar amplitude ratio can be written as

$$r = \frac{2\,V_{,\phi}(\phi, T)}{9\sqrt{\pi}\,H^3\,T(1+Q)^{5/2}}. \tag{3.54}$$

The recent WMAP seven-year results imply the upper-bound $r < 0.36$ (95% CL) (Larson et al., 2011) on the scalar-tensor ratio. Below, we shall make use of this bound to set constraints on the parameters of our models.

When applying the formalism of above to the specific case in which the thermodynamic potential is taken to be (Moss & Xiong, 2008)

$$V(\phi, T) = -\frac{\pi^2}{90}g_* T^4 - \frac{1}{12}m_\phi^2 T^2 + \frac{1}{2}m_\phi^2\phi^2, \tag{3.55}$$

where g_* is the effective number of thermal particles, and the damping coefficient may be written as

$$\Gamma(\phi, T) = \Gamma_0 \left(\frac{\phi}{\phi_0}\right)^m \left(\frac{T}{\tau_0}\right)^n, \tag{3.56}$$

with n and m real numbers and ϕ_0, τ_0, and Γ_0 some nonnegative constants. The damping term has a generic form given approximately by $\Gamma \sim g^4\phi^2\tau$, where g is the coupling constant (Hall et al., 2004a). From Ref. Hosoya & Sakagami (1984) the damping term, $\tau = \tau(\phi, T)$, is related to the relaxation time of the radiation and for the models with an intermediate particle decay, $\tau = \tau(\phi)$ is linked to the lifetime of the intermediate particle. Different choices of n and m have been adopted. For instance the case $n = m = 0$ was considered by Taylor and Berera (Taylor & Berera, 2000), whereas the choice $m = 2$, $n = -1$ corresponds to the damping term first calculated by Hosoya (Hosoya & Sakagami, 1984). This expression slightly differs from those in Hall et al. (2004a) and Zhang (2009), where a single index rather than two was considered.
As for the bulk viscosity coefficient we use the general expression

$$\zeta = \zeta_0 \, \rho_\gamma^\lambda, \tag{3.57}$$

where ζ_0 is a positive semi-definite constant and λ an integer that may take any of the two values: $\lambda = 1/2$, i.e., $\zeta \propto \rho_\gamma^{1/2}$ (Li et al., 2010) (see also Ref. Brevik & Gorbunova (2005)) and $\lambda = 1$, i.e., $\zeta \propto \rho_\gamma$ (del Campo et al., 2007c).

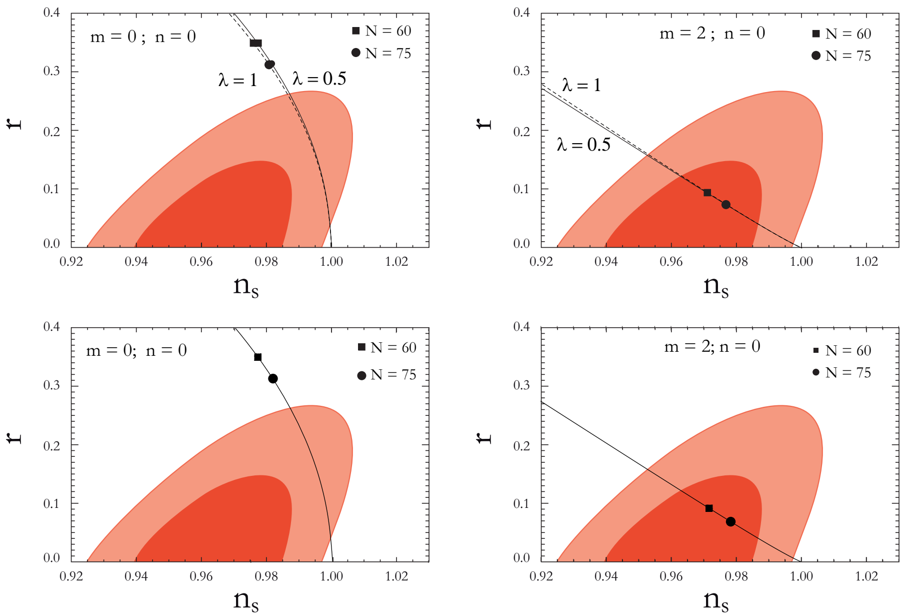

Fig. 1. Top row of panels: Plot of the tensor-scalar ratio r as a function of the spectral index n_s, for two values of the λ parameter in the case of example 1 (i.e., potential (3.55)). Bottom row: Same as the top row but assuming no viscosity ($\zeta_0 = 0$). In each panel the 68% and 95% confidence levels set by seven-year WMAP experiment are shown. The latter places severe limits on the tensor-scalar ratio (Larson et al., 2011).

Panel in Fig. 1	N	r	N	r
top left ($m = n = 0$)	60	0.351	75	0.314
top right ($m = 2, n = 0$)	60	0.094	75	0.074

Table 1. Results from first example with $\lambda = 1$ (The results for $\lambda = 1/2$ are very similar). Rows from top to bottom refers to panels of Fig. 1 from left to right.

Panel in Fig. 1	N	r	N	r
bottom left ($m = n = 0$)	60	0.350	75	0.318
bottom right ($m = 2, n = 0$)	60	0.094	75	0.074

Table 2. Results from first example with no viscosity, i.e., $\zeta_0 = 0$.

Figure 1 depicts the dependence of the tensor-scalar ratio, r, on the spectral index, n_s, for the model given by Eqs. (3.55), (3.56), and (3.57) when $\lambda = 0.5$ and when $\lambda = 1$. From Ref. (Larson et al., 2011), two-dimensional marginalized constraints (68% and 95% confidence levels) on inflationary parameters r and n_s, the spectral index of fluctuations, defined at $k_0 = 0.002$ Mpc^{-1}. The seven-year WMAP data (Larson et al., 2011) places stronger bounds on r than the five-year WMAP data (Hinshaw et al., 2009; Komatsu et al., 2009). In order to write down values that relate n_s and r, we used Eqs. (3.51) and (3.54), and the values $g_* = 100$, $\gamma = 1.5$, $\zeta_0 = (2/3)\zeta^{(1)}_{max}$, and $m_\phi = 0.75 \times 10^{-5}$, $T = 2.5 \times 10^{-6}$, $\Gamma_0 = 1.2 \times 10^{-6}$, $\tau_0 = 3.73 \times 10^{-5}$, $\phi_0 = 0.3$ for $m = 0, n = 0$; and $m_\phi = 2.5 \times 10^{-5}$, $T = 1.75 \times 10^{-6}$, $\Gamma_0 = 3.58 \times 10^{-6}$, $\tau_0 = 5.63 \times 10^{-5}$, $\phi_0 = 0.6$ for $m = 2, n = 0$, in Planck units (Hall et al., 2004a).
Figure1 suggests that the pair of indices ($m = 2$, $n = 0$), corresponding to the right panel, is preferred over the other pair of indices ($m = n = 0$), left panel. Likewise, it shows that there is little difference between choosing $\lambda = 1$ or $\lambda = 0.5$ as well as with the case of no viscosity, i.e., $\zeta_0 = 0$.
Table 2 indicates the value of the ratio r for $\lambda = 1$ and different choices of the pair of indices m and n when the number of e-folds is 60 and when it is 75. Very similar values (not shown) follow for $\lambda = 0.5$. All of them can be checked with the help of Eqs. (3.51) and (3.54).
A comparison of the results shown in both Tables indicates that only in the case of the pair ($m = n = 0$) with $N = 75$ (top and bottom left panels in Fig. 1) viscosity makes a non-negligible impact.

4. Warm inflation and non-Gaussianity

Due to the existence of a wide range of inflationary universe models it is important to discriminate between them. One of the features that can help us in this direction is the non-Gaussianity. In fact, non-Gaussian statistics (such that bispectrum) provides a powerful tool to observationally discriminate between different mechanisms for generating the curvature perturbation. But this feature not only well help us to discriminate between inflationary scenarios, but also, measurement (including an upper bound) of non-Gaussianity of primordial fluctuations is expected to have the potential to rule out many of inflationary models that have been put forward.
It has been notice that a single field, slow roll inflationary scenarios are known to produce negligible non-Gaussianity (Acquaviva et al., 2003; Maldacena, 2003), there exist now a variety of models available in the literature which may predict an observable signature. One important referent of this situation is warm inflation. The reason of this is due that warm inflation could be seen as a model which is analogous to a multi-field inflation scenario, which

is well know that can produce large non- Gaussianity which can be observed in the near future experiments such as PLANCK mission (Battefeld & Easther, 2007)
The constraint on the primordial non- Gaussianity is currently obtained from Cosmic Microwave Background measurements. WMAP sets the limit on the so-called local type of the primordial non-Gaussianity, which is parameterized by the constant dimensionless parameter f_{NL}. This parameter appears in the following expression

$$\Phi(\mathbf{x}) = \Phi_G(\mathbf{x}) + f_{NL}\left(\Phi_G^2(\mathbf{x}) - \langle\Phi_G^2(\mathbf{x})\rangle\right), \tag{4.1}$$

where Φ is Bardeeen's gauge-invariant potential, Φ_G is the Gaussian part of the potential and $\langle\ \rangle$ denotes the ensemble average. The ansatz (4.1) is known as the "local" form of non-Gaussianity[9].
The power spectrum $\mathcal{P}(k)$ of the Bardeen's gauge-invariant potential is defined by the two-point correlation function of the Fourier transform of the Bardeen's potential

$$\langle\Phi_G(\mathbf{k})\Phi_G(\mathbf{k}')\rangle = (2\pi)^3\delta^3\left(\mathbf{k}+\mathbf{k}'\right)\mathcal{P}(k), \tag{4.2}$$

where δ represents the Dirac's delta function. Similarly, The bispectrum $\mathcal{B}\left(k_1, k_2, k_3\right)$ becomes given by

$$\langle\Phi_G(\mathbf{k}_1)\Phi_G(\mathbf{k}_2)\Phi_G(\mathbf{k}_3)\rangle = (2\pi)^3\delta^3\left(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3\right)\mathcal{B}\left(k_1, k_2, k_3\right), \tag{4.3}$$

The δ^3 function in this last expression reflects translational invariance and ensures that $\mathcal{B}\left(k_1, k_2, k_3\right)$ depends on the three momenta in such a way that they form a triangle, i.e. $\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 = \mathbf{0}$. On the other hand, rotational invariance implies that the 3-spectrum function is symmetric in its arguments.
We should mentioned that the 3-point correlation function en general terms it has a very particular dependence on momenta. For instance, if it peaks when the three momenta are equal, then it is referred as *equilateral*. Now, if one of the three momenta is half of the other two, then this bispectrum is referred as *flattened*. Also, if one of the three momenta is much smaller than the other two, then we say that the bispectrum is *squeezed*. In general, the shape for the three-point spectrum could correspond to a superposition of two shapes, the *flattened* and the *equilateral* shapes, for instance (Senatore et al., 2010).
In general terms, the amount of non-Gaussianity in the bispectrum is expressed by the non-linear function f_{NL} which is given by

$$f_{NL}\left(k_1, k_2, k_3\right) = \frac{5}{6}\frac{\mathrm{B}\left(k_1, k_2, k_3\right)}{\mathcal{P}(k_1)\mathcal{P}(k_2) + \mathcal{P}(k_2)\mathcal{P}(k_3) + \mathcal{P}(k_3)\mathcal{P}(k_1)}, \tag{4.4}$$

where the numerical $5/6$ factor is introduced for convenience when compared with the results of the cosmic microwave background radiation data (Komatsu & Spergel, 2001). Models in which the function f_{NL} results to be a constant are called *local* models. This kind of models arise naturally from the non-linear evolution of density perturbations on super-Hubble scales starting from Gaussian field fluctuations during the inflationary period. Other non-Gaussian models could give different expression for the bispectrum function, specially those expression which do not result from the inflationary evolution.

[9] This is not the only well-motivated form for a non-Gaussian curvature perturbation. It could be considered a non-Gaussian part of $\Phi(x)$ which need not be correlated with the gaussian part. For instance, consider a primordial curvature perturbation of the form $\Phi(\mathbf{x}) = \Phi_G(\mathbf{x}) + FNL[\Psi_G(\mathbf{x})]$, where FNL is some arbitrary nonlinear function and the field $\Psi_G(\mathbf{x})$ is a Gaussian field which is uncorrelated with $\Phi_G(x)$ (Barnaby, 2010).

The best observational limit on the non-gaussianity at present is from the WMAP seven-year data release (Komatsu et al., 2011), which gives $-10 < f_{NL} < 74$ with 95% confidence for a constant (or local) component, when combined with Large Scale Structure (LSS) data the bound becomes somewhat stronger $-1 < f_{NL}^{local} < 65$ (Slosar et al., 2008).
A description of non-Gaussianity for different models (those could have their genesis in inflationary universe models or any other different non-inflationary one) could be made by using the so called shape function (Fergusson & Shellard, 2009). This function becomes defined as

$$S\left(k_1, k_2, k_3\right) = \frac{1}{N}\left(k_1 k_2 k_3\right)^2 \mathrm{B}\left(k_1, k_2, k_3\right), \tag{4.5}$$

where N is a normalization factor, often taken to be $N = 1/f_{NL}$.
For instance, in the case of warm inflation it results to be

$$S^{Warm}\left(k_1, k_2, k_3\right) \propto \frac{3!}{\left(k_1 k_2 k_3\right)^3} \sum_{i \neq j=1}^{3} \left(k_i k_j\right)^2 \left[k_i^2 k_j^3 - k_j^5 + \sum_{l(\neq i \neq j)=1}^{3} k_l^5\right] \tag{4.6}$$

In the Fergusson and Shellard's paper (Fergusson & Shellard, 2009) it is described an improved methods for an efficient computation of the full CMB bispectrum for any general (nonseparable) primordial bispectrum, where was incorporated the flat sky approximation and a cubic interpolation. Following this approach, they have found a range for the non-linear parameter related to warm inflation

$$-107 < f_{NL}^{Warm} < 11. \tag{4.7}$$

Very recently it has been reported that for warm inflation in the strong regime the total bispectrum corresponds to a sum of two terms (Moss & Yeomans, 2011)

$$B = \frac{6}{5} f_{NL}^{local} \sum_{cycli} \mathcal{P}(k_1)\mathcal{P}(k_2) - \frac{6}{5} f_{NL}^{adv} \sum_{cycli} \left(k_1^{-2} + k_2^{-2}\right) \mathbf{k}_1 \cdot \mathbf{k}_2 \mathcal{P}(k_1)\mathcal{P}(k_2) \tag{4.8}$$

where f_{NL}^{Adv} represents the fluid's bulk motion (advection) terms. Here, in the case of equilateral triangles it is obtained that $f_{NL} = f_{NL}^{Local} + f_{NL}^{Adv}$.
It was found that the standard deviation of the parameter f_{NL}^{adv} is around 5 times larger than the standard deviation in the estimator f_{NL}^{Local}. For PLANCK (PLANCK Collaboration, 2009), the detection limit for f_{NL}^{Local} is expected to be around 5 - 10, depending on how successfully the backgrounds can be removed. This would imply that PLANCK would only be able to detect the presence of f_{NL}^{adv} if the value was at least 25. Certainly, the detection of the f_{NL}^{adv} contribution will demand an effort where new experiment of higher resolution need to be developed. This is an issue that has to be solved by implementing appropriated futures missions.

5. Comments and remarks

In this chapter we have considered a warm inflationary universe models. We have studied this scenario in which a viscous pressure is present in the matter-radiation fluid. We investigated the corresponding scalar and tensor perturbations. The contributions of the adiabatic and entropy modes were described explicitly. Specifically, a general relation for the density perturbations, Eq.(3.41), the tensor perturbations, Eq. (3.17), and the tensor-scalar ratio -as well as the dissipation parameter- are modified by a temperature dependent factor.

We have described various aspects of warm inflationary universe models when viscosity is taken into account. This feature is a very general characteristic in multiparticle and entropy producing systems and, in the context of warm inflation, it is of special significance when the rate of particle production and/or interaction is high. In this chapter we have focused on the strong regime described by the condition that $Q \gg 1$.

On the other hand, we have seen that one important fact of warm inflation in presence of viscosity is its stability. This feature becomes expressed by the inequalities given by (3.37). Upon these conditions the determinant (expressed by Eq. (3.35)) results positive and the trace (expressed by Eq. (3.36)) negative, implying stability of the corresponding solution.

The general expression for the spectral index, n_s, expressed by Eq. (3.45), depends explicitly on viscosity through the four p_i coefficients (see Eqs. (3.50)). The latter do not depend on the slow-roll parameters $(\epsilon, \beta, \eta, \text{and } b)$, as shown by equations (3.46)-(3.49).

In order to further ensure the stability of the warm viscous inflation, the slow-roll parameters must satisfy the following conditions

$$\epsilon \ll 1 + Q, \quad |\beta| \ll 1 + Q, \quad |\eta| \ll 1 + Q,$$

as well as the condition on the slow-roll parameter that describes the temperature dependence of the potential, namely,

$$|b| \ll \frac{(1-f)Q}{1+Q}.$$

where f becomes given by $f \approx -\frac{3}{2}\frac{Q\zeta}{\gamma H\epsilon}$ in the strong regime.

These conditions give the necessary and sufficient condition for the existence of stable slow-roll solutions. Under these conditions, we got the same stability range obtained in the no-viscous case, so long as $\sigma = -8/3$. In this sense, the range of the slow-roll parameter c decreases when $-8/3 < \sigma < 0$, and increases when $\sigma < -8/3$.

To bring in some explicit results we have taken the constraint $n_s - r$ plane to first-order in the slow roll approximation. For the potential (Eq. (3.55)) we obtained that, when $\lambda = 0.5$ and $\lambda = 1$, the model is consistent with the WMAP seven year data for the pair of indices ($m = 2$, $n = 0$), see Fig. 1.

Note that in subsection 3.2 we did not address the case, in which we have that the coefficient of dissipation, Γ, does not depend on the inflaton field and the temperature. In this case a more detailed and laborious calculation for the density perturbation would be necessary in order to check the validity of expression (3.43). This is an issue that deserve further study.

The observational bound on the $|f_{NL}|$ parameter, which gives a limit on non-Gaussianity, comes from the WMAP seven-year data release (Komatsu et al., 2011), which combined with LSS data it becomes $-1 < f_{NL}^{local} < 65$. In this respect the PLANCK satellite observations have a predicted sensitivity limit of around $|f_{NL}| \sim 5$ (Komatsu & Spergel, 2001). The prediction of warm inflation lies well above the PLANCK threshold, with a specific angular dependence, should provide a means to test warm inflation observationally.

Finally, in general terms, when we count with a more precise set of data about the detection of deviations from a Gaussian distribution will allow us to check the predictions from warm inflationary universe scenario, or any another specific theoretical model.

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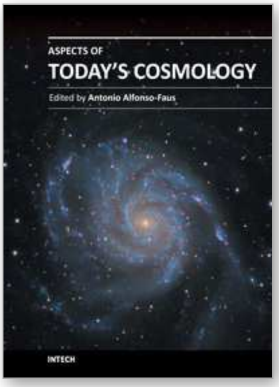

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This book presents some aspects of the cosmological scientific odyssey that started last century. The chapters vary with different particular works, giving a versatile picture. It is the result of the work of many scientists in the field of cosmology, in accordance with their expertise and particular interests. Is a collection of different research papers produced by important scientists in the field of cosmology. A sample of the great deal of efforts made by the scientific community, trying to understand our universe. And it has many challenging subjects, like the possible doomsday to be confirmed by the next decade of experimentation. May be we are now half way in the life of the universe. Many more challenging subjects are not present here: they will be the result of further future work. Among them, we have the possibility of cyclic universes, and the evidence for the existence of a previous universe.

InTech Europe
University Campus STeP Ri
Slavka Krautzeka 83/A
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Phone: +385 (51) 770 447
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InTech China
Unit 405, Office Block, Hotel Equatorial Shanghai
No.65, Yan An Road (West), Shanghai, 200040, China
中国上海市延安西路65号上海国际贵都大饭店办公楼405单元
Phone: +86-21-62489820
Fax: +86-21-62489821