

Variable Ballast Mechanism for Depth Positioning of a Spherical Underwater Robot Vehicle

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1. Introduction

A spherical shape of a submerged body with closed frame provides uniform drag at all direction along its surface. In this chapter, the shape of a spherical URV that is used in this book-chapter is presented. The vertical motion equation is also derived. The forces that affect the dynamics of the system are also described. In order to control vertical motion due to control depth position of the URV, a variable ballast mechanism is used. This mechanism controls the weight of URV's body. This chapter also presents detail mechanism of the variable ballast system and describes detail of the used parts and design of the mechanism. Kinematic and dynamic model of the variable ballast system are also derived.

2. Design of URV

The shape of spherical URV used in this book is shown in Figure 1. As a sphere body, the location of center of buoyancy (COB) of URV's body is at the center of sphere or the intersection point between vertical and horizontal diameter. The variable ballast tank is located at the top inside the hull. Location of the tank is adjusted so that the position of center of mass (COM) is aligned vertically with COB. Mechanism of the variable ballast and detail of its parts are explained in section 4.3. At the upper side of the hull above the tank, there are some holes as the way of water to enter into and exit from the tank. The space below the movable plate inside the hull is waterproofed so that the water can not enter this space.

In order to make the URV stable in equilibrium condition, the hull of URV must be designed with bottom heavy that is the center of mass is located at under of the equator or at underside hemisphere. To make the hull in bottom heavy, fixed ballast is located at the bottom of the hull. The location of COM of the hull must be aligned vertically with COB of the hull thus in equilibrium condition, the position of the ballast tank is at the top of the URV's hull exactly. This condition is important when the URV is provided with horizontal propulsion in order to give ability to the URV to move in horizontal plane.

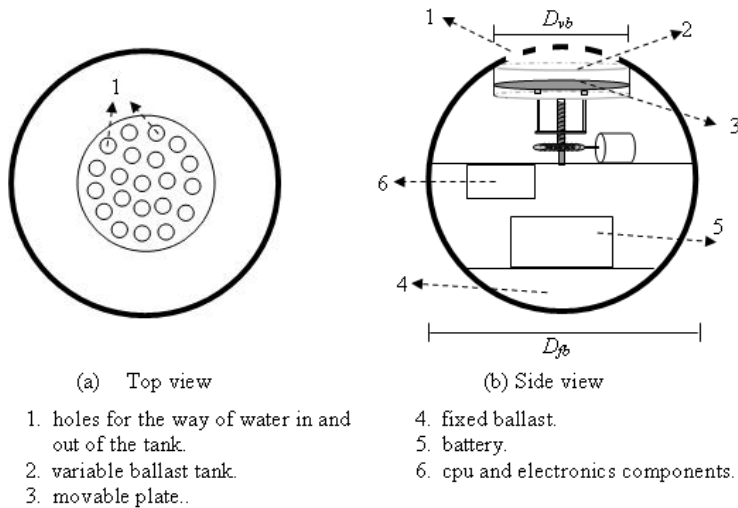


Fig. 1. Shape of spherical URV and its parts

3. Vertical motion equations

Since the URV moves in vertical plane without any propeller, so it just depends on the gravitational force, buoyant force and other forces that appear because of its motion.

By assuming there are no external forces that can disturb the motion of URV, the forces acting on the URV can be shown in Figure 2. Let F_b is buoyant force, W is gravitational force, F_D is drag force, f_a is force that appear because of the availability of the acceleration, m_a is added mass, and m_t is total mass of the URV's body which is constant then forces equation act at URV are given as (Rajput, 2003)

$$W = F_b + F_D \tag{1}$$

and,

$$W = m_t g , \tag{2.a}$$

$$F_b = \rho_w V_{fb} g , \tag{2.b}$$

$$F_D = \text{sign}(v) \frac{1}{2} C_D A_{fb} \rho_w v^2 . \tag{2.c}$$

The direction of gravitational force and buoyant force are opposite to each other when W is downward and F_b is upward. From Eq. 2.c, it can be seen that the direction of the drag force depends upon the direction of the velocity. If the URV moves downward, the velocity is positive so that the drag force is positive and its direction is upward. The drag force and velocity are negative if the URV moves upward.

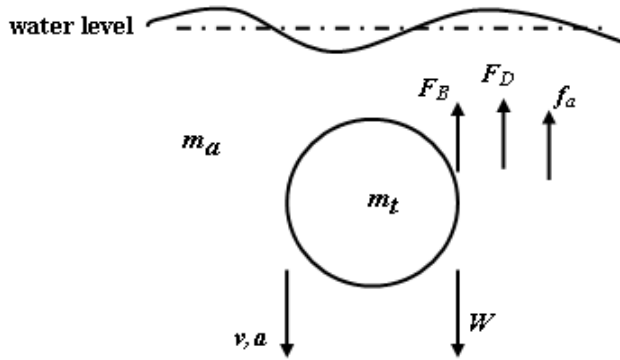


Fig. 2. Forces acting at URV's body

Substituting Eq. 2 into Eq. 1, then the force equation can be represented as

$$m_t g = \rho_w V_{\beta} g + \text{sign}(v) \frac{1}{2} C_D A_{\beta} \rho_w v^2 . \tag{3}$$

Since dimension of URV, V_{β} and A_{β} , are constant, then the velocity v , is also constant. This velocity is known as *terminal velocity*, which is expressed as

$$v = \text{sign}(m_t g - \rho_w V_{\beta} g) \sqrt{\frac{2(m_t g - \rho_w V_{\beta} g)}{C_D A_{\beta} \rho_w}} , \tag{4.a}$$

$$|v| = \sqrt{\frac{2(m_t g - \rho_w V_{\beta} g)}{C_D A_{\beta} \rho_w}} . \tag{4.b}$$

From Eq. 4.a, it can be seen that the vertical motion of the URV depends on the gravitational force and the buoyant force. If $W > F_B$, then the URV moves downward and it will move upward if $W < F_B$. If $W = F_B$, the URV will stay at its position. Since volume of URV's hull, V_{β} , g , and ρ_w are constant, the buoyant force is also constant. So, the motion of URV depends on the total mass of URV's body, m_t . By controlling m_t , the vertical motion of the URV can be controlled.

Since equilibrium condition is occurred when $W = F_B$, then $v = 0$ and $m_t = m_s$ which is initial total mass of the URV. If the total mass changes as much as Δm from the initial total mass, m_s , then the total mass of URV is expressed as

$$m_t = m_s + \Delta m . \tag{5}$$

Thus, the associated velocity will be change. The change of the velocity depends upon whether Δm is a variable or simply a constant. If Δm is a variable, the acceleration, a , occurs. This acceleration, besides accelerates mass of URV itself, m_t , also accelerates mass of surrounding water which is known as *added mass*, m_a .

Due to this acceleration, the force f_a will occur and it is expressed as

$$f_a = (m_s + \Delta m + m_a)a. \quad (6)$$

Considering this last force, f_a , Eq. 1 can be rewritten as

$$W = F_B + F_D + f_a. \quad (7)$$

Substituting Eq. 2 and Eq. 6 into Eq. 7, yields

$$m_s g + \Delta m g = F_B + \text{sign}(v) \frac{1}{2} C_D A_{\beta} \rho_w v^2 + (m_s + \Delta m + m_a)a. \quad (8)$$

Recalling equilibrium condition,

$$\begin{aligned} W &= F_B, \\ m_t &= m_s, \\ v &= 0, \\ a &= 0. \end{aligned} \quad (9)$$

From Eq. 5 obviously we have $\Delta m = 0$.

By substituting Δm and Eq. 9 into Eq. 8, yields

$$m_s g = F_B. \quad (10)$$

Since m_s and F_B are constant, then by the change of Δm , Eq. 8 becomes

$$\Delta m g = \text{sign}(v) \frac{1}{2} C_D A_{\beta} \rho_w v^2 + (m_s + \Delta m + m_a)a. \quad (11)$$

Since $\Delta m g = \Delta W$, then Eq. 11 is written as

$$\Delta W = \text{sign}(v) \frac{1}{2} C_D A_{\beta} \rho_w v^2 + (m_s + \frac{\Delta W}{g} + m_a)a. \quad (12)$$

By solving for the acceleration, a , the dynamic equation for vertical motion is given as (Xu and Smith, 1994)

$$a = \frac{\Delta W}{(m_s + m_a + \frac{\Delta W}{g})} - \frac{\text{sign}(v) C_D A_{\beta} \rho_w v^2}{2(m_s + m_a + \frac{\Delta W}{g})}. \quad (13)$$

And, if the depth position of the URV can be measured as z , then by differentiating z respect to time t , the velocity of URV in vertical plane can be expressed as

$$v = \dot{z}. \quad (14)$$

4. Variable ballast system

The common design of variable ballast uses tank as chamber for controlling amount of water in URV's body in order to control buoyancy/weight of the URV. The space or volume of the used tank is fixed so that if the amount of water in the tank is not full, there will be a space which is not filled by water. This condition can make water move freely around the space of the tank if the tilt of URV is unstable such as illustrated in Figure 3(a). This motion can produce a moment that can disturb the stability of the URV. If tilt of URV's body is change (as shown in Figure 3(b)) the center of mass, C_M , will also change. This condition sometime is undesired. Therefore, a variable-ballast with variably volume of chamber of the tank is designed in this book, in order to make water always fulfill the space in the tank but variably in term of volume.

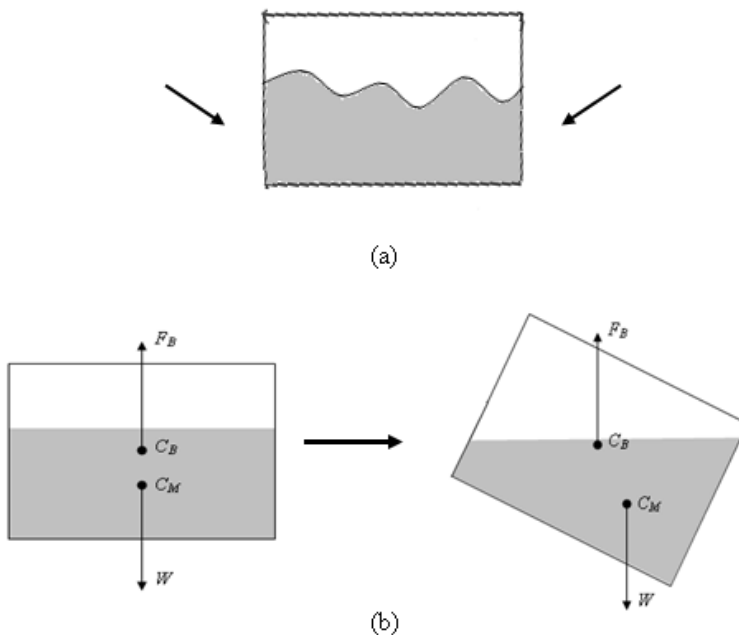


Fig. 3.(a) Surface of water in ballast tank when the URV's body is shaking; (b) Position center of mass and center of buoyancy of water in the tank when tilt is change.

4.1 Variable ballast design

In order to make water always fill space in the ballast tank, even if the volume of water is different, then the volume of the tank itself must be adjustable which is illustrated in Figure 4. The shape of the variable ballast's tank is cylinder which is opened at the top side. This part is connected directly to the water environment therefore water can always fulfill the space in the tank (as shown in Figure 1(b)).

To make variably volume of the tank, a movable plate is located at the bottom of the tank. The space below the movable plate is waterproofed, so that water can not enter this space. If

the movable plate is moving upward, the space of the tank will be decreased as well as the volume of water in the ballast tank. If the movable plate is moving in opposite, downward, the space of the tank will be increased and also the volume of water in the ballast tank. Therefore, in any volume of water in the ballast tank there is no empty space in the ballast tank that is not filled by water.

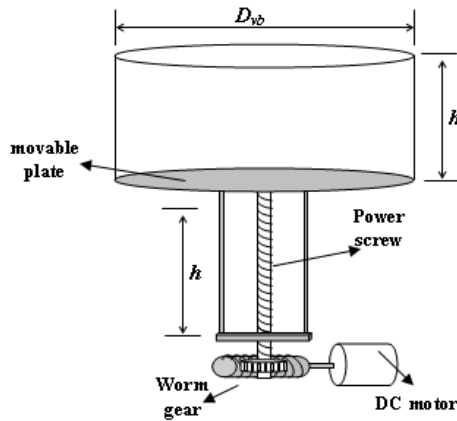


Fig. 4. Mechanism of variable ballast system

In order to change position of the movable plate, a DC motor is used to drive the movable plate through power screw and worm gear coupling. This variable ballast mechanism is analyzed in this chapter. The analyzing involves kinematics analysis and dynamics analysis.

4.2 Kinematics analysis

The movable plate of variable ballast system is coupled to the nut of power screw. This nut can be moved up and down by turning the screw.

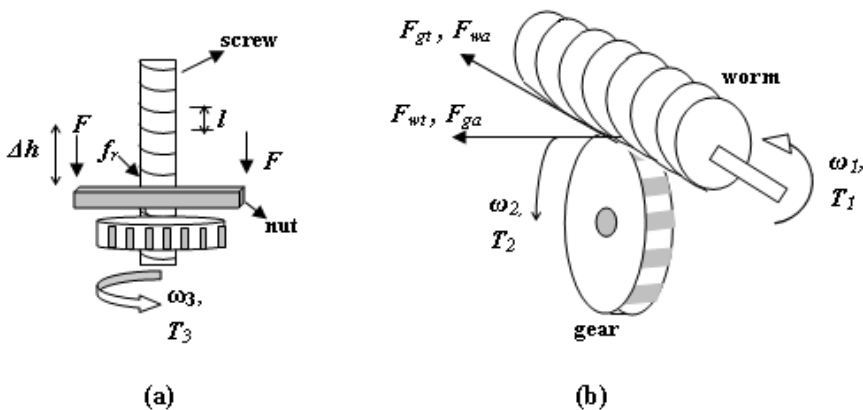


Fig. 5. (a) Power Screw; (b) Worm gear

So, the screw converts the rotation motion into linear (vertical) motion. This coupling can be seen in Figure 5(a). Based on Figure 5(a), l is lead of screw per revolution, Δh denotes change of nut position, ω_3 is angular velocity of screw. If Δt is the time needed by screw to change nut position at Δh regarding angular velocity ω_3 , then their relation can be written as

$$\frac{\Delta h}{\Delta t} = \frac{l\omega_3}{2\pi} \tag{15}$$

To turn the power screw, a DC motor is used and coupled with worm gear as illustrated in Figure 5(b). The worm has number of thread per revolution equal to N_w , and the gear has number of teeth equal to N_g . If the worm is coupled directly to the motor which turns in velocity ω_m , then the gear will turn in velocity ω_2 which is expressed as

$$\omega_2 = \frac{N_w}{N_g} \cdot \omega_m \tag{16}$$

As shown in Figure 5(a), the gear and screw is ally so that its angular velocity is the same,

$$\omega_2 = \omega_3 \tag{17}$$

By substituting Eq. 17 and Eq. 16 into Eq. 15, the change of nut position can be rewritten as

$$\begin{aligned} \frac{\Delta h}{\Delta t} &= \frac{lN_w}{N_g} \omega_m, \\ \Delta \dot{h} &= \frac{lN_w}{N_g} \omega_m, \\ \omega_m &= \frac{N_g}{lN_w} \frac{2\pi \Delta \dot{h}}{2\pi} \end{aligned} \tag{18}$$

4.3 Dynamics analysis

The dynamic of variable ballast mechanism is analyzed by considering torques and forces acting in the system. The forces and torques involved in the mechanism come from internal mechanism those are from the DC motor and the transmission system, and also come from external that is from the surrounding as hydrostatic pressure.

4.3.1 Power screw

As illustrated in Figure 5(a), T_3 is input torque that is required to operate the screw to move the nut which is coupled with movable plate, can be expressed as

$$T_3 = T_F + T_{fr} \tag{19}$$

where T_F is torque required to overcome force F , and T_{fr} is torque required to overcome friction between screw and nut. To evaluate these terms, the equilibrium conditions are applied such as illustrated in Figure 6.

Figure 6(a) illustrates coupling between nut and screw and also its parameters that must be considered. There is an additional useful geometric relationship between lead angle, α , and lead, l . Suppose the triangular segment of a plane wrapped around the screw is considered in such a way that slanted edge lies along the helix and follows it for one revolution, obviously we have

$$\tan \alpha = \frac{l}{\pi d_m} . \quad (20)$$

Figure 6(b) illustrates a force P which is applied at a mean radius r_m which causes the load to be raised. The reactive forces act at point O on the screw thread surface. The reactive force F_n acting normal to the surface has the following components: OD = f_r , which is the friction force opposing movement up the thread surface OA = is equal and opposite to the force being lifted. (F) OB = is the vector sum of OD and OA and forms an angle θ_n with vector F_n Summing the forces in the vertical direction results in

$$F_n \cos \theta_n \cos \alpha = F + f_r \sin \alpha . \quad (21)$$

If coefficient friction of screw surface is μ_s , then friction force is expressed as

$$f_r = \mu_s F_n . \quad (22)$$

By substituting Eq. 22 into Eq. 21, yields

$$F_n = \frac{F}{\cos \theta_n \cos \alpha - \mu_s \sin \alpha} . \quad (23)$$

By considering forces in horizontal direction, obviously we have

$$P = f_r \cos \alpha + F_n \cos \theta_n \sin \alpha , \quad (24)$$

and by substituting Eq. 22 into Eq. 24, yields

$$P = F_n (\mu_s \cos \alpha + \cos \theta_n \sin \alpha) . \quad (25)$$

By equating F_n at Eq. 25 and Eq. 23, force P applied on screw in order to lift force F can be expressed as

$$P = F \left(\frac{\mu_s \cos \alpha + \cos \theta_n \sin \alpha}{\cos \theta_n \cos \alpha - \mu_s \sin \alpha} \right) . \quad (26)$$

By analyzing again Figure 6(b), it also can be concluded that:

$$BC = AE = OA \cdot \tan \theta = OB \cdot \cos \alpha \tan \theta ,$$

$$\tan \theta_n = \frac{BC}{OB} = \cos \alpha \tan \theta . \quad (27)$$

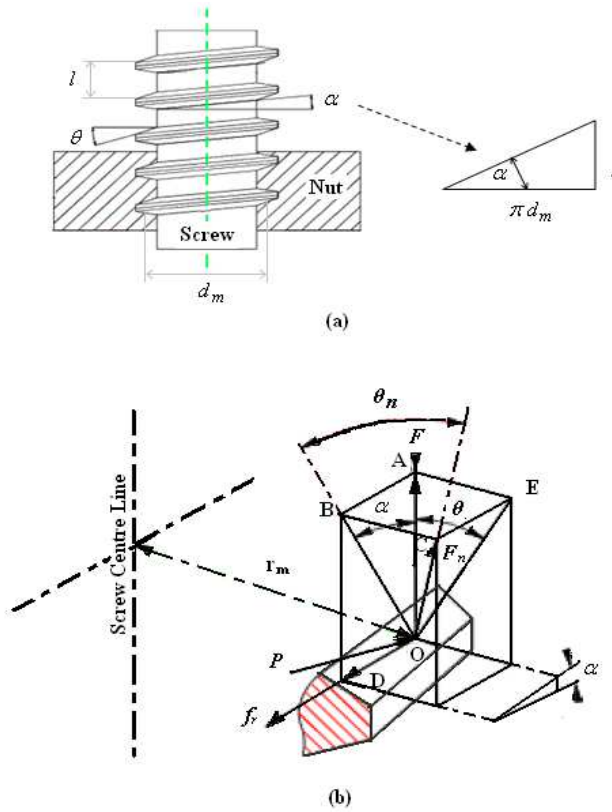


Fig. 6. (a) Screw and nut coupling; (b) Detail of forces working in the power screw (roymech.co.uk, 2008)

If lead angle α is small, then $\cos \alpha \approx 1$, so we have

$$\begin{aligned} \tan \theta_n &\approx \tan \theta, \\ \theta_n &\approx \theta. \end{aligned} \tag{28}$$

Substituting Eq. 28 into Eq. 26, yields

$$\begin{aligned} P &= F \left(\frac{\mu_s \cos \alpha + \cos \theta \sin \alpha}{\cos \theta \cos \alpha - \mu_s \sin \alpha} \right), \\ P &= F \left(\frac{\mu_s + \cos \theta \tan \alpha}{\cos \theta - \mu_s \tan \alpha} \right). \end{aligned} \tag{29}$$

Again, by substituting Eq. 20 into Eq. 29, force P applied on screw to lift load F can be rewritten as

$$P = F \left(\frac{\pi \mu_s d_m + l \cos \theta}{\pi d_m \cos \theta - \mu_s l} \right). \quad (30)$$

In order to lift load F , a torque, T_{3U} , must be applied to the screw. If the screw has mean diameter d_m , then the torque applied to the screw can be expressed as

$$T_{3U} = P \frac{d_m}{2}. \quad (31)$$

Substituting Eq. 30 into Eq. 31, the applied torque required to lift load F can be expressed as

$$T_{3U} = \frac{d_m}{2} F \left(\frac{\pi \mu_s d_m + l \cos \theta}{\pi d_m \cos \theta - \mu_s l} \right). \quad (32)$$

In order to lower load F , a torque must be applied to the screw in reverse direction with T_{3U} and it is named as T_{3L} . Applying torque in reverse direction will also deliver force P in reverse direction. By using same procedure in deriving T_{3U} , the torque required to lower load F can be expressed as

$$T_{3L} = \frac{d_m}{2} F \left(\frac{\pi \cdot \mu_s \cdot d_m - l \cdot \cos \theta}{\pi \cdot d_m \cos \theta + \mu_s \cdot l} \right). \quad (33)$$

4.3.2 Worm-gear set

Conceptually in worm-gear set, the worm can be analogous to the screw power, and worm gear or gear can be analogous to the nut, see Figure 7. The forces resolution for power screw may therefore be directly applied to the case of a worm by observing that screw lead angle α is equivalent to worm lead angle λ_w , and power screw normal angle θ_n is equivalent to normal pressure angle ϕ_n for the worm gear. Illustration of these forces, based on Figure 6(b), can be seen in Figure 8.



Fig. 7. Worm gear (www-mdp.eng.cam.ac.uk, 2008).

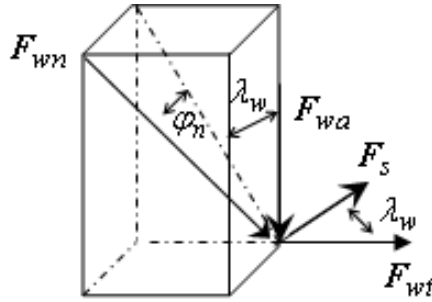


Fig. 8. Detail of forces on worm gear.

If worm has lead l_w per revolution and diameter d_w , then worm lead angle λ_w can be determined by

$$\tan \lambda_w = \frac{l_w}{\pi d_w} . \tag{34}$$

Based on Fig. 8, by summing the forces in vertical direction obviously results in

$$F_{wa} = F_{wn} \cos \varphi_n \cos \lambda_w - F_s \sin \lambda_w , \tag{35}$$

where F_{wa} is axial force of worm, F_{wn} is reactive force on worm, and F_s friction force of worm. If coefficient friction of worm surface is μ_w , then the friction force F_s is expressed as

$$F_s = \mu_w F_{wn} . \tag{36}$$

If Eq. 36 is substituted into Eq. 35, then the reactive force F_{wn} is written as

$$F_{wn} = \frac{F_{wa}}{\cos \varphi_n \cos \lambda_w - \mu_w \sin \lambda_w} . \tag{37}$$

If forces in horizontal direction are considered, then by summing of these forces will result

$$F_{wt} = F_{wn} \cos \varphi_n \sin \lambda_w + F_s \cos \lambda_w , \tag{38}$$

where F_{wt} is tangential force of worm.

By substituting Eq. 37 into Eq. 38, the tangential force of worm can be expressed as (Collins, 2003).

$$F_{wt} = F_{wa} \left(\frac{\cos \varphi_n \sin \lambda_w + \mu_w \cos \lambda_w}{\cos \varphi_n \cos \lambda_w - \mu_w \sin \lambda_w} \right) ,$$

$$F_{wt} = F_{wa} \left(\frac{\cos \varphi_n \tan \lambda_w + \mu_w}{\cos \varphi_n - \mu_w \tan \lambda_w} \right) . \tag{39}$$

Then, by substituting Eq. 34 into Eq. 39, yields

$$F_{wt} = F_{wa} \left(\frac{l_w \cos \phi_n + \pi \mu_w d_w}{\pi d_w \cos \phi_n - \mu_w l_w} \right),$$

$$F_{wa} = F_{wt} \left(\frac{\pi d_w \cos \phi_n - \mu_w l_w}{l_w \cos \phi_n + \pi \mu_w d_w} \right). \quad (40)$$

From Figure 5(b), it is shown the relation of forces working at gear and worm. Forces on gear are related by equilibrium to forces on the worm as

$$F_{gt} = F_{wa}, \quad (41.a)$$

$$F_{ga} = F_{wt}, \quad (41.b)$$

where F_{gt} and F_{ga} are tangential and axial force working at gear respectively. If T_2 is torque applied on gear with diameter d_g , then this torque T_2 is expressed as

$$T_2 = \frac{d_g}{2} F_{gt}. \quad (42)$$

By equating Eq. 40 and Eq. 41.a and substitute into Eq. 42, yields

$$T_2 = \frac{d_g F_{wt}}{2} \left(\frac{\pi d_w \cos \phi_n - \mu_w l_w}{l_w \cos \phi_n + \pi \mu_w d_w} \right). \quad (43)$$

To actuate this mechanism, the worm is coupled directly to the shaft of a DC motor. If T_m is motor torque applied on worm to result tangential force F_{wt} which is expressed as

$$F_{wt} = \frac{2}{d_w} T_m, \quad (44)$$

then by substituting Eq. 44 into Eq. 43, torque applied on gear is expressed as

$$T_2 = \frac{d_g T_m}{d_w} \left(\frac{\pi d_w \cos \phi_n - \mu_w l_w}{l_w \cos \phi_n + \pi \mu_w d_w} \right). \quad (45)$$

Reviewing Figure 5 again, obviously can be seen that the gear and power screw are allied in same shaft so that torque required to actuate the gear, T_2 , will be equal to the torque required to turn power screw, T_3 . Since $T_3 = T_2$, then $T_{3U} = T_{2U}$ and T_{3U} is torque needed by screw to lift up the load F . In order to produce torque T_{3U} on the screw or T_{2U} on the gear, the DC motor must produce torque T_{mU} . If $T_3 = T_{3U}$, $T_2 = T_{2U}$, and $T_m = T_{mU}$ then by equating Eq. 32 and Eq. 45 yields

$$\frac{d_g T_{mU}}{d_w} \left(\frac{\pi d_w \cos \phi_n - \mu_w l_w}{l_w \cos \phi_n + \pi \mu_w d_w} \right) = \frac{d_m}{2} F \left(\frac{\pi \mu_s d_m + l \cos \theta}{\pi d_m \cos \theta - \mu_s l} \right),$$

$$T_{mU} = \frac{d_m d_w F}{2d_g} \left(\frac{\pi \mu_s d_m + l \cos \theta}{\pi d_m \cos \theta - \mu_s l} \right) \left(\frac{l_w \cos \varphi_n + \pi \mu_w d_w}{\pi d_w \cos \varphi_n - \mu_w l_w} \right). \quad (46)$$

By using the same analogy for calculating T_{mU} , then the torque of the motor required to lower the load F which is known as T_{mL} , can be expressed as

$$T_{mL} = \frac{d_m d_w F}{2d_g} \left(\frac{\pi \mu_s d_m - l \cos \theta}{\pi d_m \cos \theta + \mu_s l} \right) \left(\frac{l_w \cos \varphi_n + \pi \mu_w d_w}{\pi d_w \cos \varphi_n - \mu_w l_w} \right). \quad (47)$$

From Eq. 46 and Eq. 47, it can be shown that many coefficients, which are constant, are involved in the equation, so that if the constants are simplified then we have

$$\begin{aligned} \frac{d_m d_w}{2d_g} &= k_{pr}, \\ \left(\frac{\pi \mu_s d_m + l \cos \theta}{\pi d_m \cos \theta - \mu_s l} \right) &= k_{TU}, \\ \left(\frac{\pi \mu_s d_m - l \cos \theta}{\pi d_m \cos \theta + \mu_s l} \right) &= k_{TL}, \\ \left(\frac{l_w \cos \varphi_n + \pi \mu_w d_w}{\pi d_w \cos \varphi_n - \mu_w l_w} \right) &= k_{wg}, \end{aligned} \quad (48)$$

where k_{pr} is coefficient of power transmission ratio between worm gear set and power screw, k_{TU} and k_{TL} are coefficient of power screw in lifting and lowering load mechanism respectively, and k_{wg} is coefficient of worm gear set. Hence, Eq. 46 and Eq. 47 can be simplified into

$$\begin{aligned} T_{mU} &= k_{pr} k_{TU} k_{wg} F \\ T_{mL} &= k_{pr} k_{TL} k_{wg} F \end{aligned} \quad (49)$$

or it can be written as

$$T_m = k_m F, \quad (50)$$

where $k_m = k_{mU} = k_{pr} k_{TU} k_{wg} \Rightarrow T_m = T_{mU}$ and $k_m = k_{mL} = k_{pr} k_{TL} k_{wg} \Rightarrow T_m = T_{mL}$.

From Eq. 50, it can be seen that torque T_m is the input, and F is the output. Although not explicitly stated, it does not mean that if $T_m = 0$ then F must be zero. Because of friction, a certain value of F must be reached to make it *self-locking*, before power screw start rotating and allow the load lift or lower, and it's called *overhauling*. To guarantee the screw will be self-locking, a condition based on the geometric parameter and coefficient of friction must be fulfilled (Collins, 2003).

4.3.3 External forces analysis

Torque and force which are provided by the motor and its mechanics system are used to overcome the load F in order to control amount of water in the ballast tank. Load F itself is total force working on the movable plate of variable ballast system which is coming from inside and outside of URV's hull. The illustration of forces working on the movable plate is shown in Figure 1.

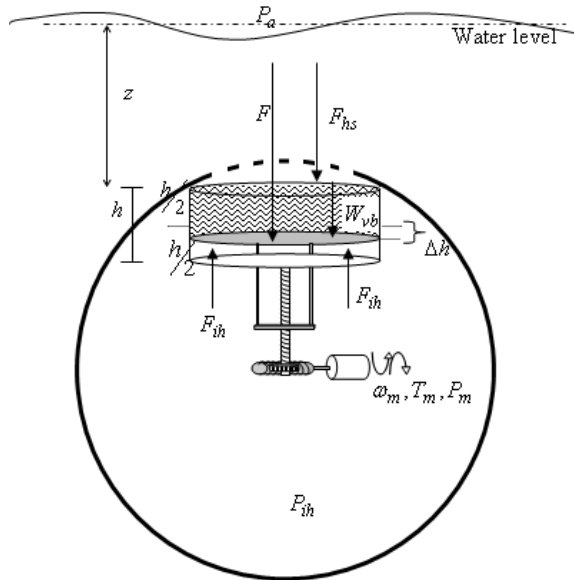


Fig. 1. External forces working on variable ballast system.

If Figure 1 is analyzed, the force coming from inside hull, F_{ih} , is caused by the change of air pressure inside the hull, P_{ih} , due to the change of space inside the hull. As explained before, the variable ballast mechanism is used to control weight of URV by controlling volume of water in ballast tank. To control volume of the water, a mechanism like piston is designed. In this mechanism, a movable plate which is base of space in the tank that can be filled by water is used. By controlling position of the movable plate which is height of the tank, the volume of water in the tank can be controlled. Since space under movable plate is impermeable, by the change of position of movable plate, the volume of space inside URV's hull is also change. This change impacts to the air pressure inside the hull.

As known that relation between pressure and volume, V_{ih} , in closed space is expressed as (Moran and Shapiro, 1998)

$$P_{ih} V_{ih} = \text{constant} . \tag{51}$$

So, if the volume of air inside URV's hull is changed, then its pressure is also changed. In initial condition or in equilibrium condition, the pressure inside the hull is equal to the pressure of the air at water surface, P_a . By assuming the air pressure at water surface and

temperature inside URV's hull are constant then since the volume of space inside the hull is constant, the pressure inside the hull is also constant. If volume of the space inside the hull is changed because of the change of position of movable plate, the pressure P_{ih} is also change and will cause a force act at movable plate surface, known as F_{ih} . The relation of P_a , P_{ih} , and F_{ih} is expressed as

$$F_{ih} = (P_{ih} - P_a) A_{vb}, \tag{52}$$

where A_{vb} is projected area of movable plate which is base of variable ballast tank.

In initial condition, where $P_{ih} = P_a$, volume of the air or empty space inside URV's hull is V_{ih} , and the position of movable plate is at the middle of full height of the tank, so if the maximum height of the tank is h then the position of movable plate is at $0.5h$ from the top of the tank, which is known as initial position. At this position, Δh , which is the change of movable plate position, is equal to zero ($\Delta h = 0$). So that if position of movable plate is upper than initial position ($< 0.5h$) then $\Delta h < 0$ and if lower than initial position then $\Delta h > 0$. By the change of Δh , the volume of the air or space inside the hull will change as ΔV . The relation is expressed as

$$\Delta V = \Delta h A_{vb}. \tag{53}$$

From Eq. 53, it is seen that the change of volume of the air inside the hull is equal to the change of volume of water in the ballast tank.

By the change of volume of the air, the pressure is also change from its initial condition and expressed as

$$P_a V_{ih} = P_{ih} (V_{ih} - \Delta V),$$

$$P_{ih} = \frac{P_a V_{ih}}{V_{ih} - \Delta V}. \tag{54}$$

By substituting Eq. 53 into Eq. 54 and substitute the result into Eq. 52, then force acts on movable plate's surface due to change of volume of the air inside hull is expressed as

$$F_{ih} = \frac{\Delta h (A_{vb})^2 P_a}{V_{ih} - \Delta h A_{vb}}. \tag{55}$$

From Figure 1, it can be shown that load F is resultant of F_{hs} , ΔW , and F_{ih} , and the relation is expressed as

$$F = W_{vb} + F_{hs} - F_{ih}, \tag{56}$$

where W_{vb} is weight of the water in the ballast tank and F_{hs} is hydrostatic force on surface of movable plate. Weight of water in the ballast tank depends on volume of the water in this tank. In equilibrium or initial condition, the volume of water is half of maximum volume of the tank, $W_{vb} = W_{bs}$. By the change of position of movable plate in Δh , the weight of water in ballast is also will change in ΔW from initial weight. So that at any condition, W_{vb} can be expressed as

$$W_{vb} = W_{bs} + \Delta W . \quad (57)$$

Hydrostatic force, F_{hs} , is force acting on surface of immersed body caused by the height of liquid (water) above of it, or in other word it can be said that hydrostatic force is weight of the liquid above immersed surface. In this system, the height of liquid above is equal to the depth position of the URV. If depth position of URV is measured form water surface to top part of URV's body known as z , then F_{hs} acting on surface of movable plate is expressed as (Rajput, 2003)

$$F_{hs} = \rho_w g A_{vb} z , \quad (58)$$

where ρ_w and g are density of water and gravitational acceleration respectively.

Substituting Eq. 55, Eq. 57, and Eq. 58 into Eq. 56, load F can be expressed as

$$F = W_{bs} + \Delta W + \rho_w g A_{vb} z - \frac{\Delta h (A_{vb})^2 P_a}{V_{ih} - \Delta h A_{vb}} , \quad (59)$$

and the change of water in the ballast tank is expressed as

$$\begin{aligned} \Delta W &= \Delta h A_{vb} \rho_w g , \\ \Delta h A_{vb} &= \frac{\Delta W}{\rho_w g} . \end{aligned} \quad (60)$$

Substituting Eq. 60 into Eq. 59, then the load F can be rewritten as

$$F = W_{bs} + \Delta W + \rho_w g A_{vb} z - \frac{\Delta W A_{vb} P_a}{\rho_w g V_{ih} - \Delta W} . \quad (61)$$

Recalling Eq. 50 and substitutes Eq. 61 into this equation, then torque of the motor that is required to change position of movable plate in order to control amount of water in ballast tank is expressed as

$$T_m = k_m \left(W_{bs} + \Delta W + \rho_w g A_{vb} z - \frac{\Delta W A_{vb} P_a}{\rho_w g V_{ih} - \Delta W} \right) . \quad (62)$$

In order to change position of the movable plate, the DC motor must provide power P_m and rotates at angular velocity ω_m in order to produce torque at T_m , and can be expressed as

$$P_m = T_m \omega_m . \quad (63)$$

By substituting Eq. 18 and Eq. 62 into Eq. 63, obviously yields

$$P_m = k_m \left(W_{bs} + \Delta W + \rho_w g A_{vb} z - \frac{\Delta W A_{vb} P_a}{\rho_w g V_{ih} - \Delta W} \right) \frac{N_g 2\pi \Delta h}{IN_w} . \quad (64)$$

If $\frac{N_g 2\pi}{lN_w} = k_{gc}$ is known as coefficient of velocity reduction of power screw and worm gear couple, then Eq. 64 can be rewritten as

$$P_m = k_m k_{gc} \Delta h \left(W_{bs} + \Delta W + \rho_w g A_{vb} z - \frac{\Delta W A_{vb} P_a}{\rho_w g V_{ih} - \Delta W} \right). \tag{65}$$

From Eq. 60, if A_{vb} , ρ_w , and g are simply constant, then by differentiating this equation results

$$\Delta \dot{h} = \frac{\Delta \dot{W}}{\rho_w g A_{vb}}, \tag{66}$$

where $\Delta \dot{h}$ is rate change of position of movable plate and $\Delta \dot{W}$ is rate change of weight of water in the ballast tank.

By substituting Eq. 66 into Eq. 65 and solving $\Delta \dot{W}$, then the rate change of weight of water in the ballast tank is obviously expressed as

$$\Delta \dot{W} = \frac{\rho_w g A_{vb} P_m}{k_m k_{gc} \left(W_{bs} + \Delta W + \rho_w g A_{vb} z - \frac{\Delta W A_{vb} P_a}{\rho_w g V_{ih} - \Delta W} \right)}. \tag{67}$$

5. Simulation of open loop system

In order to know responses of the system resulted from the modeling process, the simulation using Simulink in MATLAB is built. The Simulink model is built in block by block system based on the mathematic models. Some input models are tested in order to analyze responses of the model.

5.1 Simulink model

From Eq. 67, Simulink model of rate change of weight in the ballast tank, $\Delta \dot{W}$, is shown in Figure 2. From Figure 2, it can be seen that value of k_m depends on power input, P_m , which results torque T_m to change amount of water in the ballast tank. If $P_m > 0$ then $T_m = T_{mL}$ therefore $k_m = k_{mL}$, otherwise $k_m = k_{mU}$.

The rate change of weight in the ballast tank, $\Delta \dot{W}$, has saturation values ($\pm \Delta \dot{W}_{sat}$). This saturation value depends on the maximum angular velocity of the DC motor that drives this mechanism both in counterclockwise and clockwise direction. Then, the change of weight in the ballast tank, ΔW , can be obtained by integrating $\Delta \dot{W}$ which is shown in Figure 3. ΔW also has saturation values ($\pm \Delta W_{sat}$) which depends upon the maximum volume of the ballast tank.

Output of this model is acceleration of the URV. In order to obtain velocity of URV's vertical motion, v , an integration block diagram is used. In order to get the depth position of the URV, this velocity is integrated. The Simulink model is shown in Figure 5. The condition of depth position and velocity are depth position always be positive ($z \geq 0$) and for $z = 0 \Rightarrow v \geq 0$.

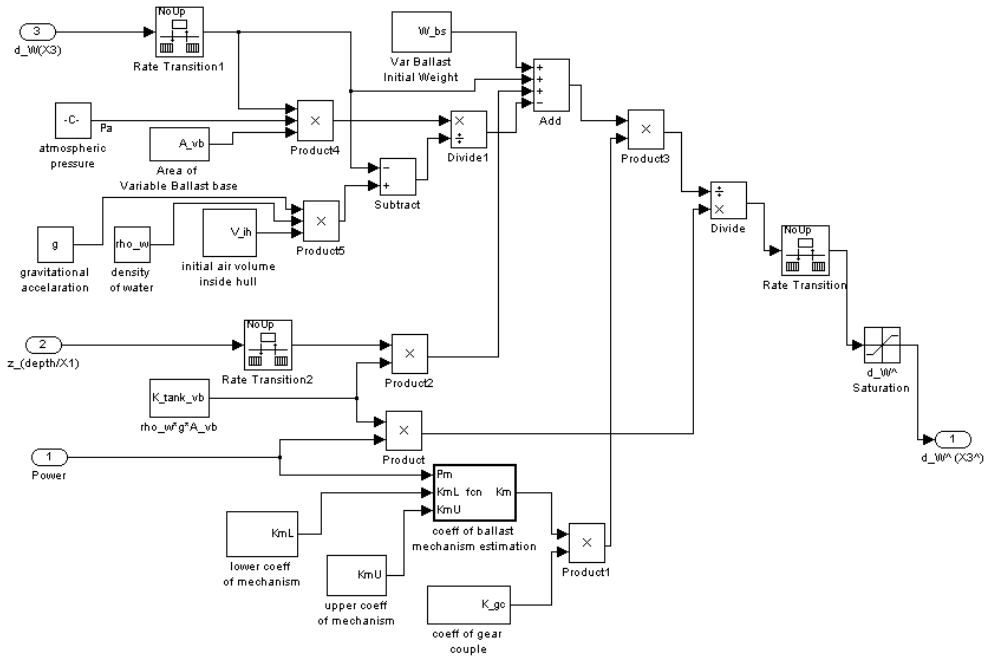


Fig. 2. Model for rate change of weight in the ballast tank.

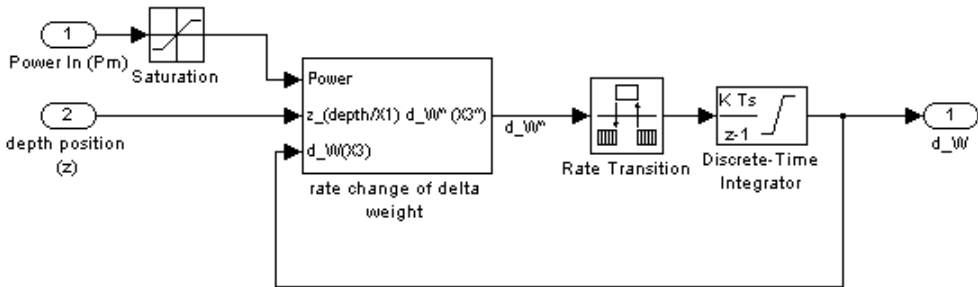


Fig. 3. Model for the change of weight in the ballast tank.

The Simulink model of URV motion in vertical plane which is taken from Eq. 13, is shown in Figure 4. By combining all models, obviously Simulink model for depth positioning of the spherical URV is shown in Figure 6.

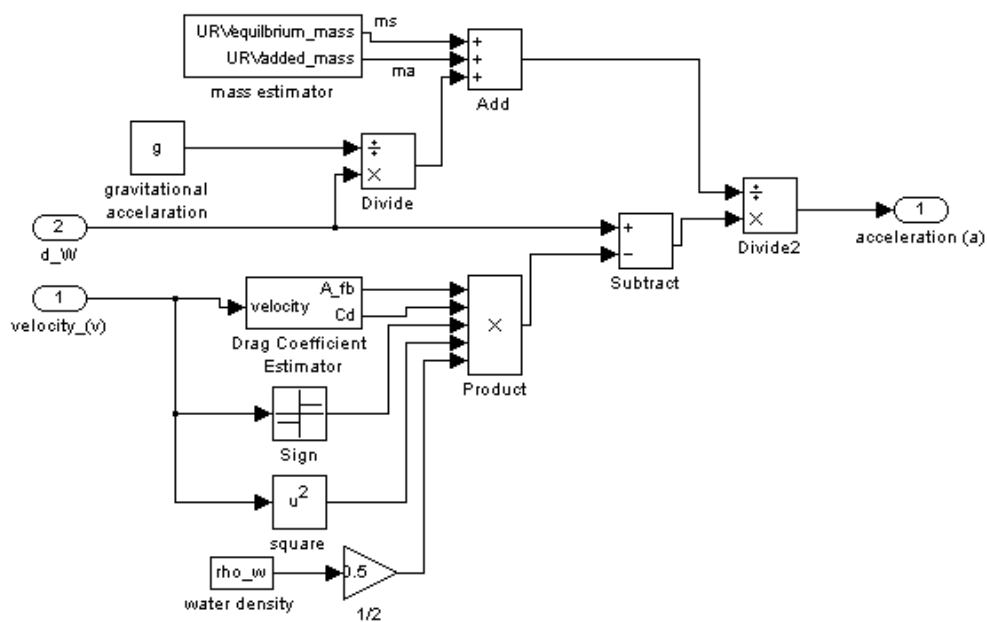


Fig. 4. Model for vertical motion acceleration of URV.

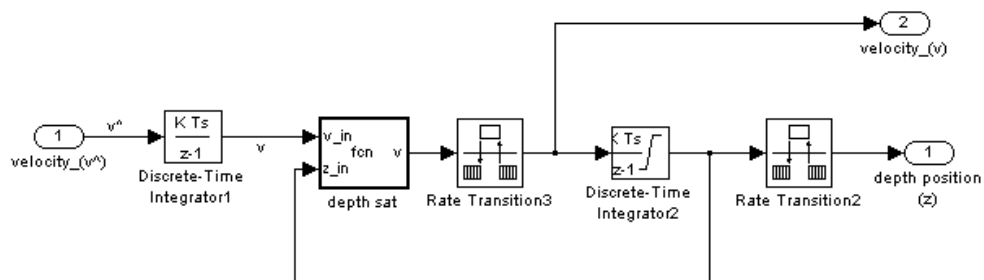


Fig. 5. Model for velocity and depth position of URV.

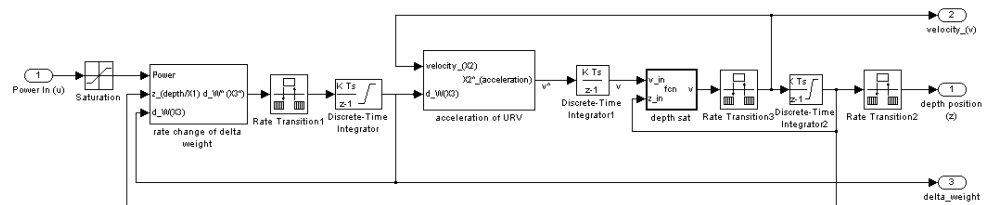


Fig. 6. Model for depth positioning of a spherical URV.

From Figure 6, it can be shown that the power input, P_m , has saturation values that is $\pm P_m$ max. This power depends upon the power provided by the DC motor.

5.2 Simulation result

The dynamic model for depth positioning of the spherical URV involves many constants and parameters both for URV and its ambient. The assumption of these constants and parameters used in the simulation are presented in Table 1.

Parameters	Symbols	Values
Ambient parameters:		
Atmospheric pressure at water surface	P_a	1 atm
Density of water	ρ_w	998 kg/m ³
Dynamic viscosity of water	μ	10 ⁻³ Ns/m ²
Gravitational acceleration	g	9.81 m/s ²
URV's hull:		
Initial mass of URV	m_s	22.39 kg
Added mass of URV	m_a	11.2 kg
Diameter of URV	D_{fb}	0.35 m
Projected area of URV	A_{fb}	0.09616 m ²
Initial volume of empty space inside URV's hull	V_{ih}	50 % of V_{fb}
Variable ballast system:		
Diameter of variable ballast tank	D_{vb}	0.18 m
Projected area of base of variable ballast tank	A_{vb}	0.0254 m ²
Maximum height of variable ballast tank	h	0.08 m
Initial weight of water in ballast tank	W_{bs}	9.96 N
Transmission ratio of worm gear and power screw	k_{sc}	8.164x10 ⁴ rad/m
Coefficient of worm gear and power screw couple for downward moving	k_{mL}	4.601x10 ⁻⁵
Coefficient of worm gear and power screw couple for upward moving	k_{mU}	1.122x10 ⁻⁴
Power saturation resulted by DC motor	$\pm P_{m_max}$	± 100 Watt
Angular velocity saturation of DC motor	$\pm \omega_{m_max}$	± 157 rad/s

Table 1. Parameters of URV and the ambient.

Some types of input are tested to analyze response of the model. The first input tested in the open loop simulation is a single pulse input. The responses of the model are shown in Figure 7. The given power input from the DC motor, P_m , is a single pulse with amplitude at 50 Watt. The origin position of URV is at 0 meter from surface. Since this input is applied, the URV is descent from surface.

From Figure 7, it is also seen when the power is applied to the motor as positive value, the weight change, ΔW , is increased and reaches saturation around 10N, as maximum value of ΔW . When the power is reset to zero, ΔW still remain at the last value, and the URV still moves with velocity v , which is proportional to ΔW . The depth position of URV, z , will increase since the velocity is available as positive.

If the velocity is negative, the URV is ascending as shown in Figure 8. The increment of ΔW depends on the total power given by the motor to actuate the variable ballast and also depends on the depth position of the URV. If the power given is small then the increment of ΔW is also small, and since the power is available, ΔW will keep increasing till reach saturation. If same power is given to the system but in different depth positions, the increment of ΔW at deeper position is lower than shallower position. This is caused by the availability of hydrostatic force.

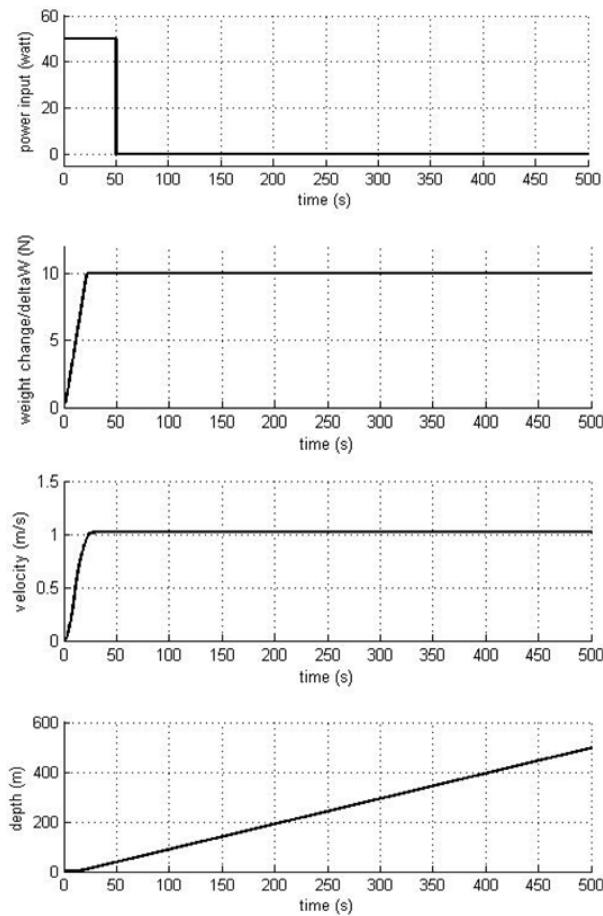


Fig. 7. Response of the system for single pulse input.

When the power is zero, ΔW remains at its last value as well as velocity v . The velocity will remain constant until ΔW change and velocity in this condition is known as *terminal velocity*. This is the advantage of using variable ballast as vertical motion actuator, even the zero power is given to the actuator, the URV still moves therefore it will save the power usage. If $\Delta W = 0$, then the zero velocity occurs. The depth position of URV, z , will remain at its last position, and this condition is called *equilibrium point*. The equilibrium point occurs at any depth position since v and ΔW is equal to zero.

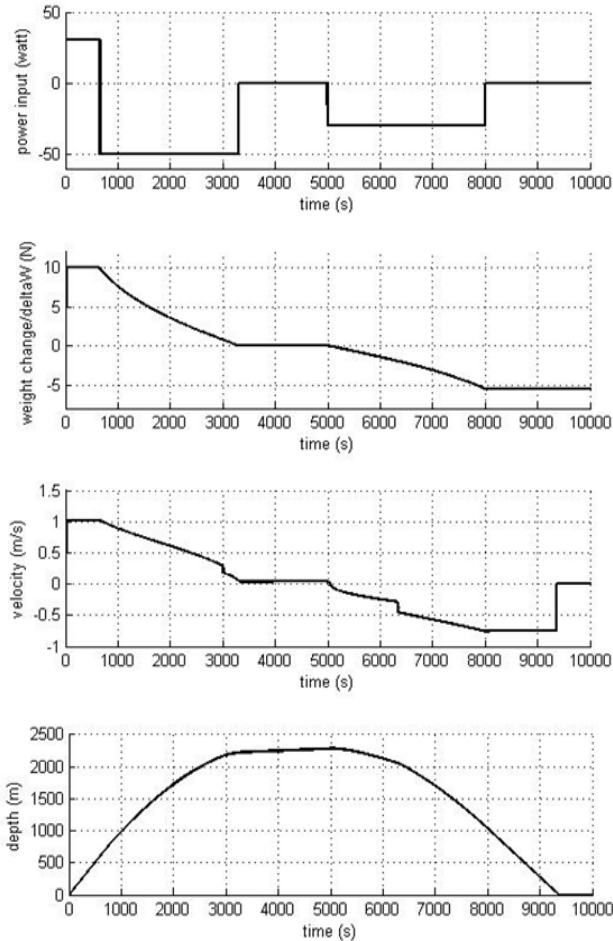


Fig. 8. Response of the system for pulse input in different amplitude.

If ramp input is applied, then the response of the system is shown in Figure 9. By looking to the response, obviously the nonlinearity of the weight change of the URV's body and the velocity in vertical motion are shown. By the increment of P_m , v and ΔW also increase until both of these reach saturation. Depth position, z , keeps increasing since $v > 0$.

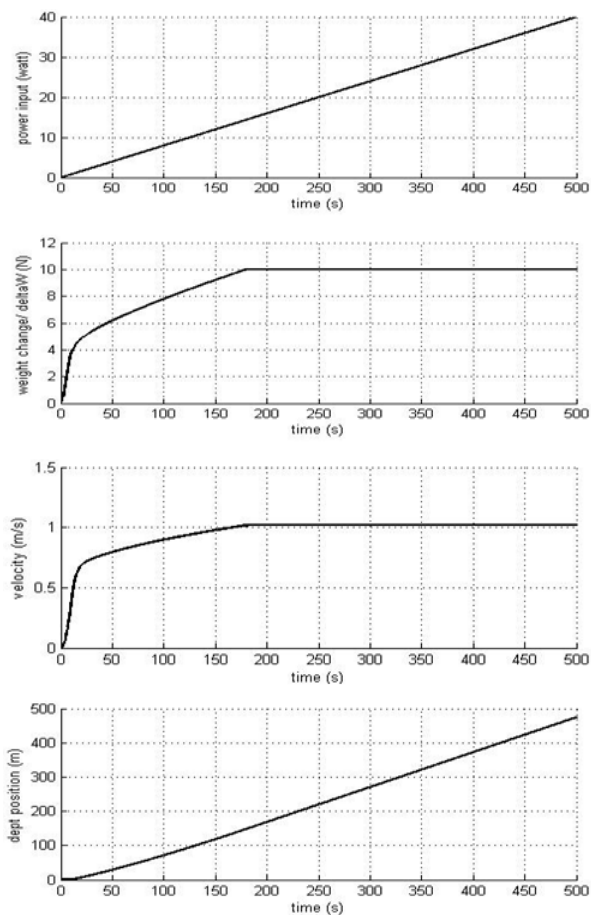


Fig. 9. Response of the system for ramp input.

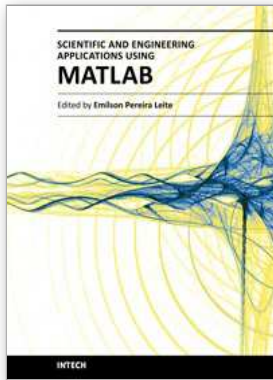
6. Conclusion

The dynamic model of depth positioning of a spherical URV is obtained by considering the physical laws involve in this system. The dynamic model presented in this chapter is nonlinear. Some inputs are tested to see the responses and characteristics of this system. In the next chapter, the control systems will be designed in order to control the depth position of this spherical URV.

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The purpose of this book is to present 10 scientific and engineering works whose numerical and graphical analysis were all constructed using the power of MATLAB® tools. The first five chapters of this book show applications in seismology, meteorology and natural environment. Chapters 6 and 7 focus on modeling and simulation of Water Distribution Networks. Simulation was also applied to study wide area protection for interconnected power grids (Chapter 8) and performance of conical antennas (Chapter 9). The last chapter deals with depth positioning of underwater robot vehicles. Therefore, this book is a collection of interesting examples of where this computational package can be applied.

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