

Evolutionary Enhanced Level Set Method for Structural Topology Optimization

Haipeng Jia¹, Chundong Jiang¹, Lihui Du², Bo Liu¹ and Chunbo Jiang²

¹*Department of Mechanical Engineering, Hebei University of Technology, Tianjin 300130*

²*State Key Laboratory of Hydrosience and Engineering of, Tsinghua University, Beijing 100084, P. R. China*

1. Introduction

During the last 20 years, structural optimization has become one of the most important topics of engineering applications. Design optimization of structure has been an interesting area of research in the field of engineering design for its ability to shorten the design cycle and to enhance product quality. Significant research activity has occurred in the area of structural optimization in the last decade. Especially for topology optimization of structure, many new theoretical, algorithmic, and computational contributions have resulted by researchers and engineers. Topology optimization is a powerful tool for global and multi-scale design of macrostructures, microstructures, and the cell of prescribed composite materials.

The population based evolutionary algorithms have emerged as a powerful mechanism for finding optimum solutions of complex optimization problems in engineering during the last two decades. Evolutionary computation is the study of computational systems which use ideas and get inspiration from natural evolution and adaptation [1]. The thinking has wide application in various engineering fields, such as computer science, artificial intelligence, operations research. Genetic algorithm is another kind of bio-inspired optimization method and it is playing an increasingly important role in studies of complex adaptive systems. Its application ranges from adaptive agents in economic theory to the use of machine learning techniques in the design of complex devices and structures, such as aircraft turbines and integrated circuits [2].

Optimization of structures can be classified into three categories: sizing, shaping, and topology optimization. In the topology optimization, it is concerned with the structure members and connectivity between members. In general, it is easily represented by discrete variables rather than by those used for continuous optimization problems. Topology optimization is the most difficult and complex among three categories and it is special useful in developing innovative conceptual designs. Structural optimization, in particular the topology optimization, has been identified as one of the most challenging tasks in structural design. Various techniques and approaches have been established during the last two decades. Topology optimization usually referred to as layout optimization or general shape optimization [3]. It lets engineers get the optimal topology of structure or new configurations during product design phase, as they are implementing the design of the size

and shape of structure. In the last two decades, topology optimization has been becoming increasingly popular in industrial applications [4-6]. For in many cases, tremendous cost savings have been achieved due to the impact of this design tool in the early stage of the design procedure. However, because of the complicity of the mathematical formulation and the difficulties in solving it, topology optimization is considered as one of the most challenging research field.

In order to improve the efficiency in global optimization search the topology of engineering problems, many heuristic algorithms [7, 8, 9] have been developed, such as evolutionary algorithm [8, 9], genetic algorithms[10, 11], ant algorithm, simulated annealing algorithm,. In recent years, many biologically inspired methods come to be used in topology optimization of structure. Evolutionary algorithms are a popular and robust strategy for structure optimization. Especially the evolutionary structure optimization (ESO) has been applied widely in solving structural topology optimization problems [12, 13, 14]. These methods have special characteristics such as parallel computing and globally optimum searching. Based on hole image interpretation techniques, Lin et al. [7] gave two-stage artificial neural networks for topology and shape optimization, which contains improved template variety and recognition reliability. Salami and Hendtlass [8] proposed a "fast evolutionary algorithm" that does not evaluate all new individuals, in which fitness and associated reliability value are assigned to each new individual that is evaluated using the true fitness function only if the reliability value is below a threshold. In 1992, Xie and Steven [9, 12, 13] proposed the ESO and bidirectional ESO (BESO) approach [13] for topology optimization and applied to the optimization of structures successfully. During implementation of the method, elements are gradually removed from the structure by altering the material properties. The removal or additional criterion is based on the comparison of the Von Mises stress, principal stress or the deformation energy of the candidate element. Considering the low efficiency of the usage of the material in the low value of stress or strain energy, the element can be removed from the structure, or added in high value region for the need of material. For its simplicity in implementation and convenience in coping with the local buckling, displacement constraint and local stress constraint, ESO algorithm has wide applications in the dynamic modification, topology optimization thermal-structure coupling problems with different criteria, for further please see references [14, 15, 16]. Mariano Victoria etc. [17] gives the isolines topology design algorithm, and in essence it is a variant of Evolutionary Structure Topology optimization approach. Based on the thinking of perfect state of harmony in musical processing, Lee and Z. W. Geem [18] give the implementation of harmony searching algorithm for structural optimization. The merit is independent of the initializing design variable and derivative information of objective and constraint function.

To get good topology of the final structure, in 1994 Eschenauer, et al. [19] put forward the Bubble Method. The main idea is first to introduce a new small circle and then implement a shape optimization by a conventional fixed topology shape optimization to get the size, shape and position of the hole. Osher and Sethian [20, 21, 23] proposed the concept of level set, it has been proven to be phenomenally successful as a numerical device, and since its appearance it has wide applications ranging from capturing multiphase fluid dynamical flows to special effects in hollywood to visualization, image processing, topology optimization of structure[24, 25], computer vision and many more. Wang and his coauthors [26, 27] Proposes level set method for structural topology optimization and many other variant of this algorithm. Wei [27] proposed piecewise constant level set method to nucleate

holes during optimization and some benchmark problems show the validity of the algorithm.

There are several advantages to this approach for topology optimization of continuum structures. Firstly, we solve the elastic strain equations on a fixed grid through a version of the immersed interface method (IIM), which avoids the complications that come from using distorted and convoluted unstructured meshes. Secondly, the Level Set Method allows us to perturb the shapes of the interface, without worrying about changes in topology, such as how many holes are required. Thirdly, the entire method carries over to three dimensions, if desired. This method attracts many interests of engineers and researchers in optimization field for the smoothness of the boundary and having no intermediate density in the final topology [24-30].

The studies all aimed at developing a robust and efficient algorithm for searching global optimum solution for engineering applications.

The demanding computational cost for engineering optimization is often very high for each iteration needs at least one finite element analysis. Because the finite element analysis for engineering model takes lots of time in finding required data for calculating the parameters of objective function in optimization problem and that of constraint function. Various mathematical programming methods have been used to solve engineering optimization problems. But these methods need calculation of the first or second order differentiation that will increase the difficulty in searching optimum solution. In another hand, the mathematical programming methods are easily to fall into local optimum for non-convexity of topology optimization problems.

The Traditional level set method algorithms for topology optimization use a Hamilton Jacobi equation to connect the evolution of the scalar function with the boundary of the topology contours. For this reason, it can hardly create new holes during evolving otherwise other measures has been taken.

This paper proposes an improved LSM algorithm. The newly modified method integrates the ESO inspired hole-inserting technique in LSM method and overcome the shortcomings of traditionally approach. Using this algorithm, new holes can be inserted at different positions during the optimization to determine the optimal topology. From the point of view of "ground structure", the proposed method of topology optimization enlarged the searching space of Level Set Method.

2. Mathematical formulation of the evolutionary optimization algorithm

Traditionally, the problem of topology optimization of structure to maximize stiffness can be specified as (1)-(4):

$$\text{Minimize:} \quad J(u) = \int_{\Omega} F(u) d\Omega \quad (1)$$

$$\text{s.t.: } a(u, v) = L(v) \quad (2)$$

$$u|_{\Gamma_d} = u_0 \forall v \in U \quad (3)$$

$$V = \int d\Omega \leq V_{\max} \quad (4)$$

Here, the design domain of the structure is represented by Ω , $J(u)$ is the objective function, $F(u)$ is specific physical or geometric type on design domain. In this paper, $F(u)$ is the compliance of structure and the objective is to find the minimum of it, let the structure be the stiffest. In terms of the energy bilinear form $a(u, v, \phi)$, $L(v)$ and $\varepsilon_{ij}(u)$ described by (5)-(7) respectively,

$$a(u, v, \phi) = \int_{\Omega} E_{ijkl} \varepsilon_{ij}(u) \varepsilon_{kl}(v) d\Omega, \quad (5)$$

$$L(v) = \int_{\Omega} p v d\Omega + \int_{\Gamma} \tau v ds, \quad (6)$$

$$\varepsilon_{ij}(u) = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right). \quad (7)$$

The purpose of the topology optimization is to optimize the objective function by layout of the material in design domain.

In the Level Set Method, the boundary of structure is described by zero level set and it can easily represent complicated surface shapes that can form holes, split to form multiple boundaries, or merge with other boundaries to form a single surface. Zero level sets are decided by the objective function such as energy of deformation, stress, eigenvalue etc., and the optimal structure can be gotten through the movement, amalgamation of the external boundary of the structure.

Compared with the homogenization method and SIMP (Solid Isotropic Material Penalty) [30] method, the LSM has some excellent aspects: no chessboard, no mesh-dependency problems, and good numerical stability.

The LSM describes the topology of structure implicitly, and the course of the topology optimization of continuum is achieved by solving the Hamilton-Jacobi equation (8).

$$\frac{\partial \phi}{\partial t} + \bar{V} \cdot \phi = 0 \quad (8)$$

Here, according to objective function how to descend, \bar{V} is chosen to let level set function change. Time variable is length that satisfies Courant-Friedrichs-Levy (CFL) condition which makes difference calculation stability. The model of optimization can be specified as (9)-(12):

$$\text{Min: } J(u, \phi) = \int_{\Omega} F(u) H(\phi) d\Omega \quad (9)$$

$$\text{s.t.: } a(u, v, \phi) = L(v, \phi) \quad (10)$$

$$u|_{\Gamma_d} = u_0 \forall v \in U \quad (11)$$

$$V = \int H(\phi) d\Omega \leq V_{\max} \quad (12)$$

In terms of the energy bilinear form $a(u, v, \phi)$, the load linear form $L(v, \phi)$, and the volume $V(\phi)$ of the structure, respectively described by (13)-(15):

$$a(u, v, \phi) = \int_{\Omega} E_{ijkl} \varepsilon_{ij}(u) \varepsilon_{kl}(v) H(\phi) d\Omega \quad (13)$$

$$L(v, \phi) = \int_{\Omega} p v H(\phi) d\Omega + \int_{\Omega} \tau v H |\nabla \phi| \delta(\phi) d\Omega \quad (14)$$

$$V(\phi) = \int_{\Omega} H(\phi) d\Omega \quad (15)$$

Where $\delta(x)$ is Dirichlet function, and $H(x)$ Heaviside function, see paper [22].

As known to all, only the moving and merging of holes can be implemented during the LSM topology optimization, no new holes can be generated through the optimization. The disadvantage of level set based topology optimization is apparent for some engineering problems. To conquer the difficulty, one method is to initialize the guess design with enough holes in order to include as more topologies as possible. To get a good result, we should comply with the following two fundamental principles to initialize the guess configuration before carrying out the optimization:

- The number of holes must be enough to include all the possible topology;
- The layout of the holes should be rationally positioned.

Cantilever beam is a benchmark problem in topology optimization. As shown in Figure 1, it has a length of 64mm and a height of 40mm, thickness of the plate $t=1$ and is subjected to a concentrated load of 80N at the middle of its free end. The objective function of the problem is the strain energy of the structure with a material volume constraint. The Young's Modulus and Poisson's Ratio of the material used in the example are 200GPa and 0.3, respectively. Parameters $\alpha = 10^{-9}$, $\Delta = 1.0$ are used in the numerical approximation of $\delta(x)$ and $H(x)$. The volume ratio is limited to 25%. A mesh including 64×40 4-node-isoparametric elements is used, and the problem is dealt with as a plane stress problem. As shown in Figure 2, the guess topology configuration is initialized with 4×6 holes in level set function.

Figure 3 gives the topology evolving procedure during optimization progress. The final result in Figure 3(f) shows that good topology can be obtained if the initialized configuration includes sufficient number of holes, see in Figure 2. It consists with the result in paper [31] in Figure 5 and that from the optimization criteria(OC) approach in Figure 6. For more detailed description on the theory and numerical computation of the level set based topology optimization, see [26].

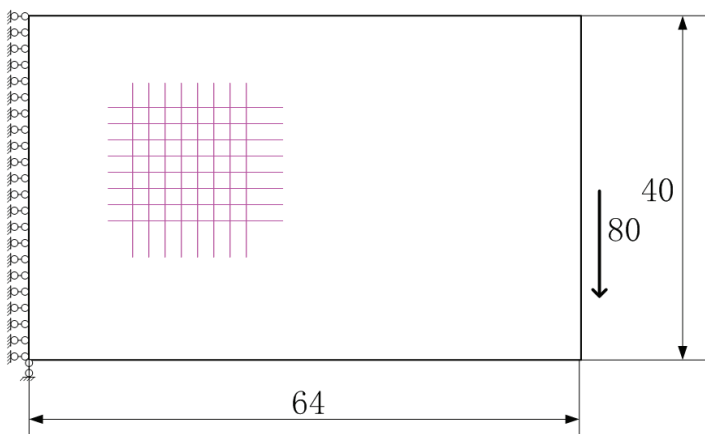


Fig. 1. Geometry parameters and boundary conditions of cantilever plate

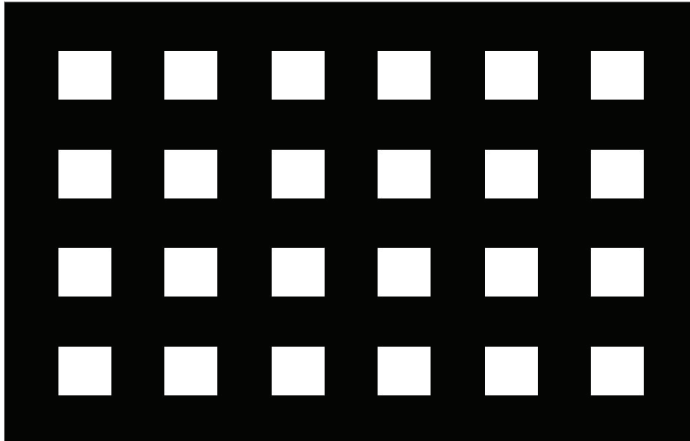


Fig. 2. Topology Initialization with evenly distributed 4X6 holes

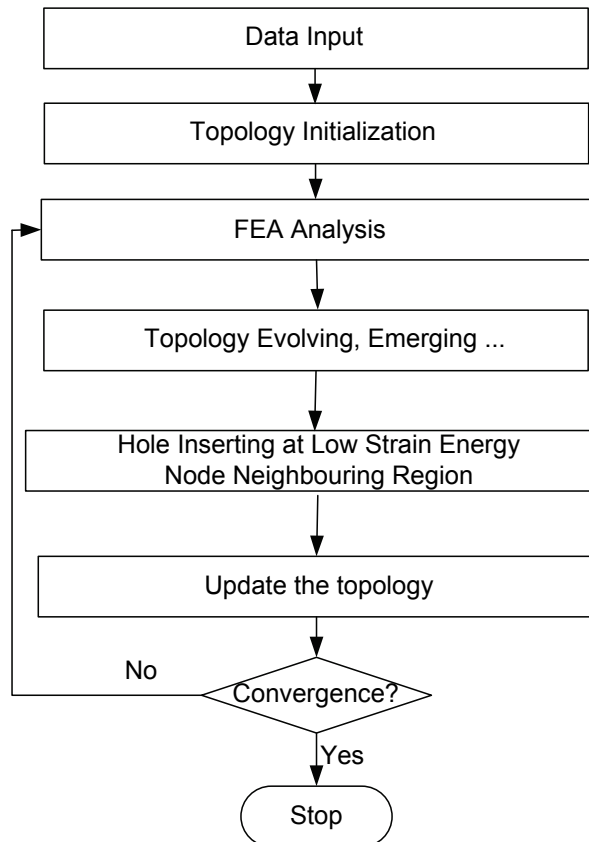


Fig. 3. Computational flow of the structural topology optimization

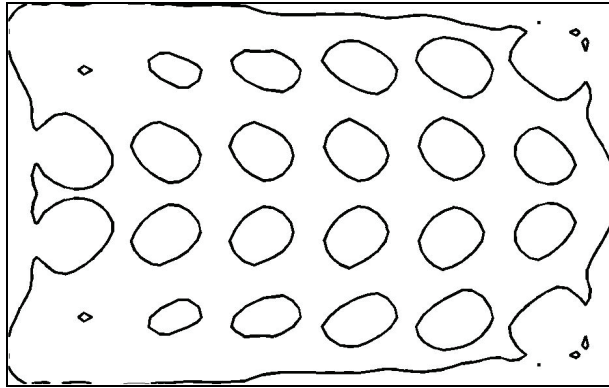


Fig. 4(a) Iteration number 15

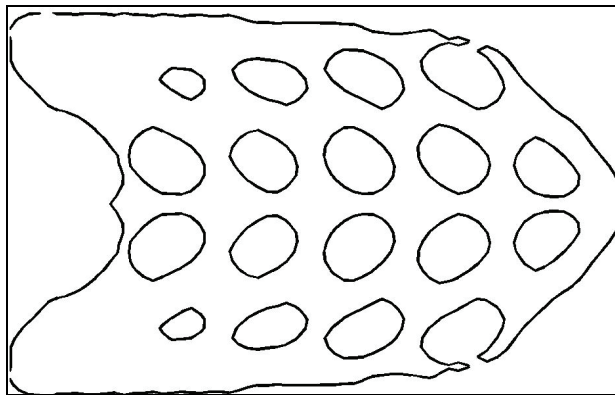


Fig. 4(b) Iteration number 30

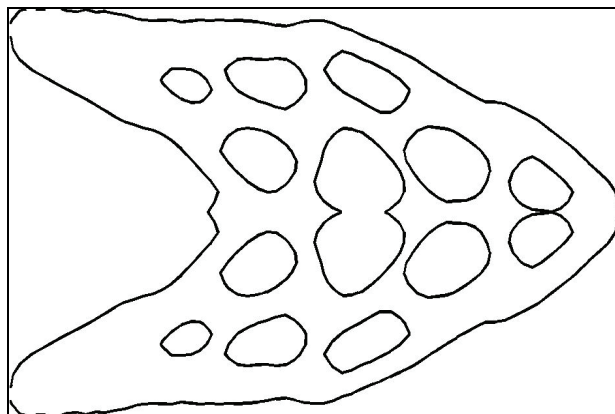


Fig. 4(c) Iteration number 45

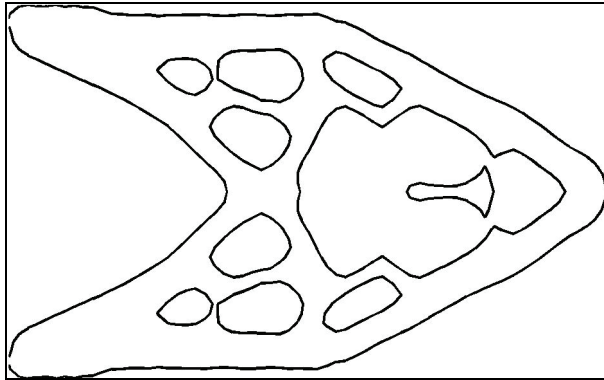


Fig. 4(d) Iteration number 60

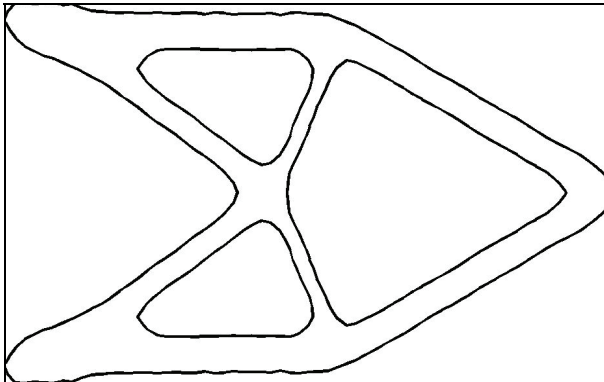


Fig. 4(e) Iteration number 75

Fig. 4. Traditional Level set method for topology optimization, initialized with uniformly distributed holes

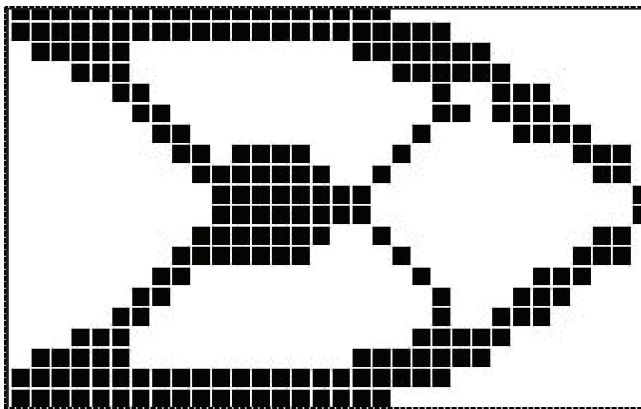


Fig. 5. Resultant topology in paper [32]

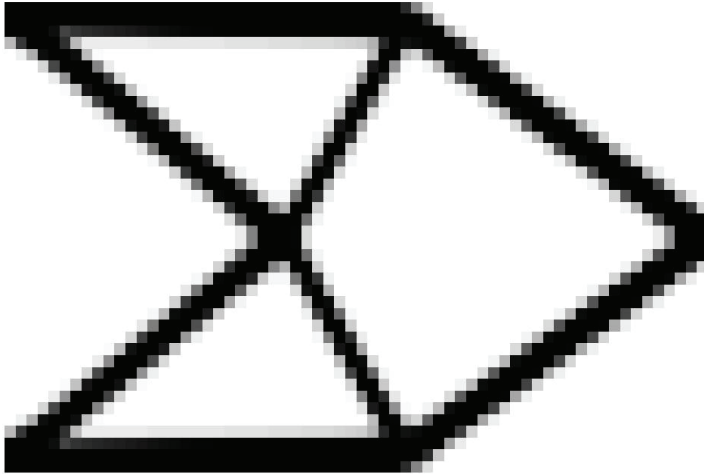


Fig. 6. Final topology through optimization criteria (OC) method

Numerical experiments show that the optimal topology depends on the initialization considerably. In fact, the final topology is only a subset of the candidate topology set of initialization. The more the topologies are included, the higher the possibility a good design can be obtained.

To illustrate the invalidity of the level set based topology optimization algorithm, let us design the topology of a cantilevered plate, a classical benchmark problem for topology optimization, from an initial guess topology with no hole. The design result indicates that the optimal topology is a two-bar-truss-like structure, which is apparently different from the real optimum topology. From this example, we can safely come to the conclusion that the optimal topology highly depends on the initial guess design, and that LSM can only find a best topology in the given topology sets in advance.

To circumvent the obstacle of independence of initialization, new criteria are needed to insert new holes at the right position during the right iteration. This is the emphasis of this paper.

The proposed method is based on the node neighboring strain energy as illustrated in Figure 7, and calculated through (16),

$$\alpha_i = \int_{\Omega^e} E_{ijkl} \varepsilon_{ij}(u) \varepsilon_{kl}(v) d\Omega \quad (16)$$

In (16), Ω^e indicate the node neighboring region as shown in Figure 8, and α_i is the Performance Index of the i -th node relative to the whole structure, this value indicate the effect of the i -th node on strain energy of whole structure when it is removed from structure. For each iteration of optimization, the algorithm finds small percentage of the lowest strain energy of all nodes within solid material region, see Figure 7. For different problems, the initial value can be changed a little to get better result accordingly.

The implementation of the proposed algorithm as follows:

Step 1. Initialization of the guess topology of the structure with signed distance function in terms of the external boundary;

- Step 2.** Solving the equilibrium equation of the structure. FEA (Finite Element Analysis) is adopted to compute the displacement field and the adjoint displacement field through the linear elastic system;
- Step 3.** Computing the sensitivity of the candidate node. The value is the strain energy of a node-neighboring region α_i ;
- Step 4.** Hole inserting, in the material region, according to the value, remove the low energy element (generally the remove rate is 2-3% of those violating the volume constraint);
- Step 5.** Evolving of the topology of the structure. Solve the level set equation to update the embedding function. Same as that of the Level Set Method.
- Step 6.** Convergence checking. If volume constraint met, then the iteration finished; or repeats Step2 - Step6 until convergence.

In Step3 and Step4, the proposed method can control the position of the inserted hole adaptively. Apart from that, the number of holes and the iteration number can be carried out individually in code implementation. For different fields of topology optimization problems, corresponding parameters should be adjusted accordingly.

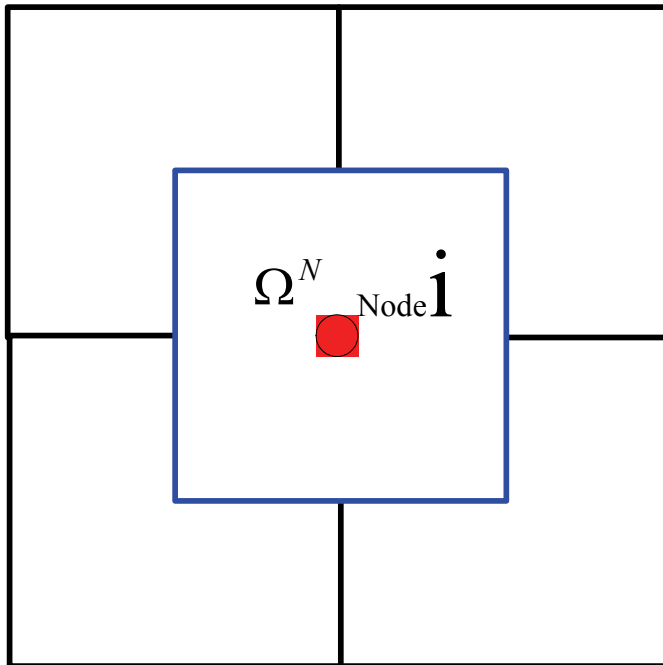


Fig. 7. Computational diagram for strain energy of node neighboring region

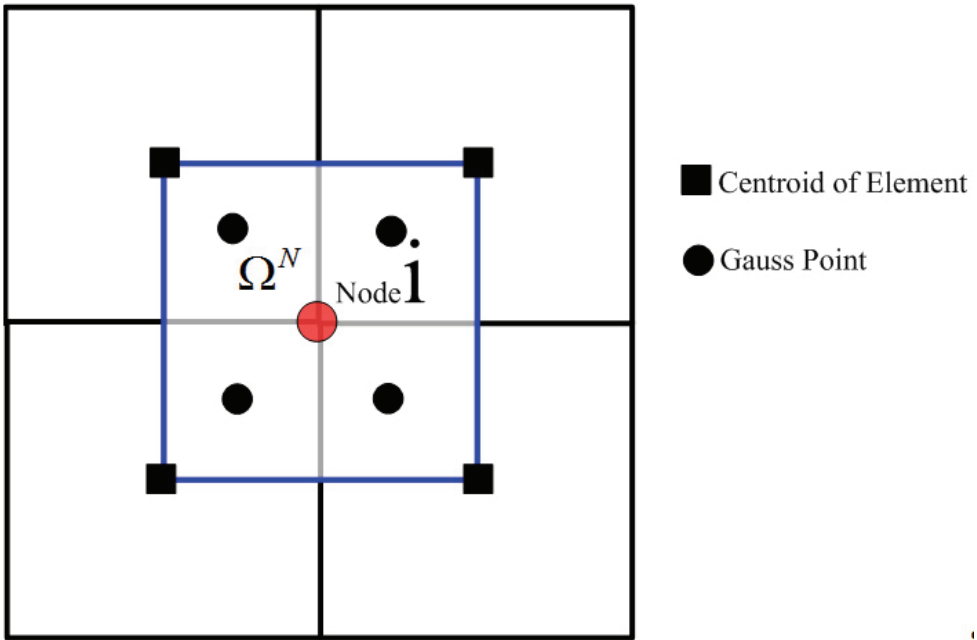


Fig. 8. Averaged strain energy of 4 neighboring gauss point

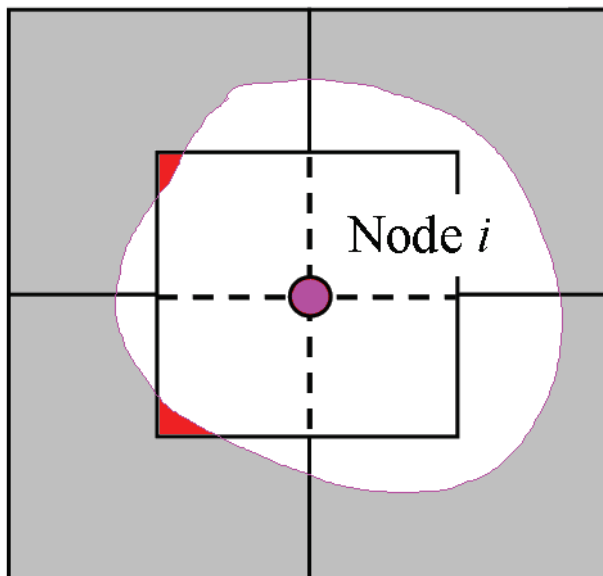


Fig. 9. Updated topology boundary after inserting a new hole during evolving optimization

3. Numerical examples

To illustrate the reliability and the validity of the nodal ESO hole-inserting LSM topology optimization method, the classical cantilever beam in Figure 1 is optimized and gets a good result. At the same time, to show the efficiency of the improved algorithm, guess topology has no hole is computed to show the characteristics.

Case 1

To solve the problem with no hole the initialization configuration has, one cannot get the optimal topology using traditional LSM algorithm easily.

According to the theory of Evolutionary Structure Optimization method, this paper gives the automated hole-inserting approach. The evolution procedure of structural topology is shown from Figure 10(a) to Figure 10(f). The topology optimization of the cantilever shows the validity of the proposed method.

Figure 11 gives the structural strain energy variation history during optimization. Figure 12 shows the iteration history of material usage within the design domain during topology evolving.

Case 2

To illustrate the efficiency of the proposed algorithm, the same benchmark problem is solved. but the initialization has initialized holes and inserting holes when impossible during the optimization iteration. Figure 13(a) until Figure 13(f) gives the key intermediate topology during optimization. Optimization history shows that the iteration number decreased from 72 to 39.

Figure 14 gives the structural strain energy variation history during optimization. Figure 15 shows the iteration history of material usage within the design domain during topology evolving.

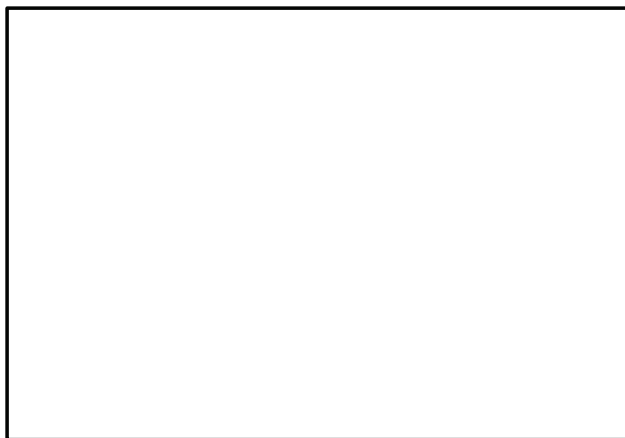


Fig. 10(a) Topology initialization without holes

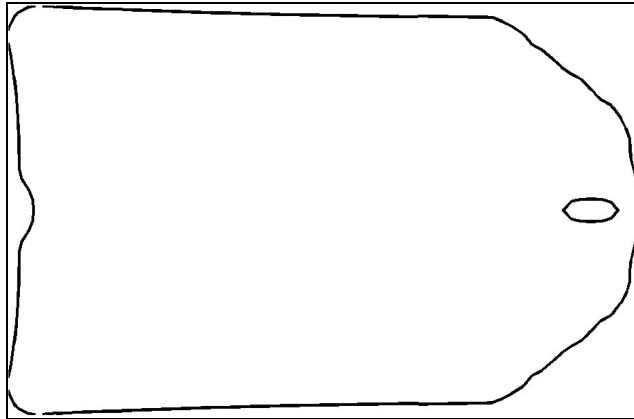


Fig. 10(b) Iteration number 14

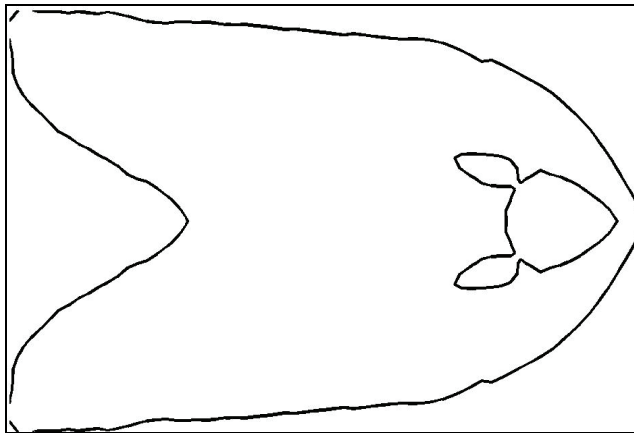


Fig. 10(c) Iteration number 30

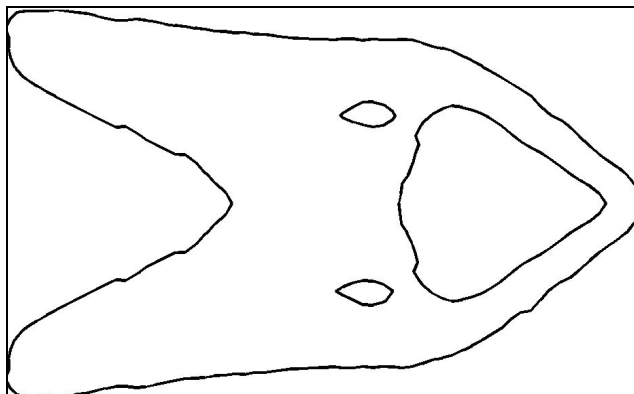


Fig. 10(d) Iteration number 46

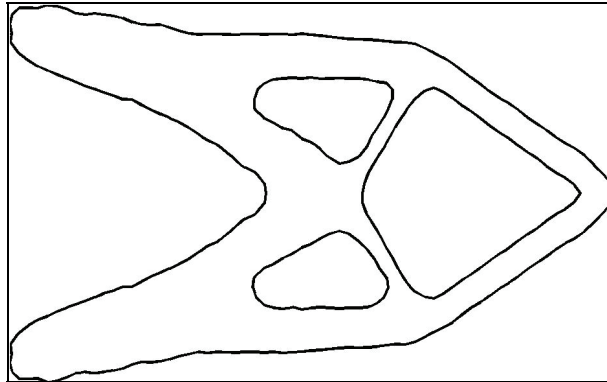


Fig. 10(e) Iteration number 61

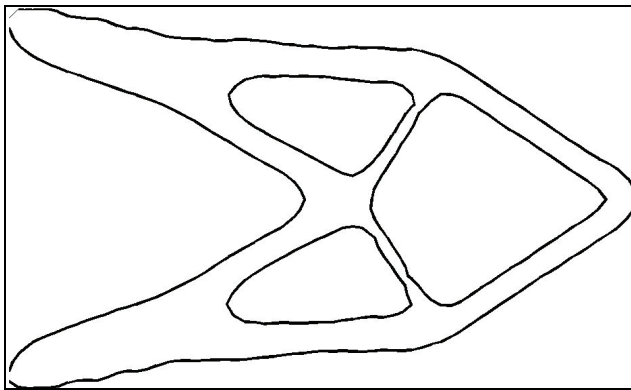


Fig. 10(f) Iteration number 72

Fig. 10. The proposed algorithm: Topology evolving process with initialization having no holes

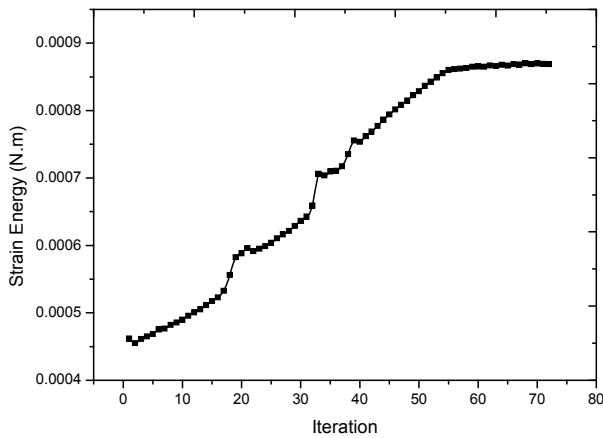


Fig. 11. Strain Energy of the structure V.S. iteration number

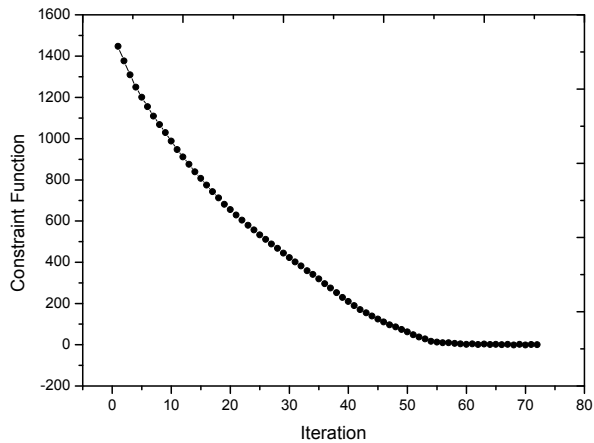


Fig. 12. Value of constraint function v.s. iteration number, the value indicted the gross material usage during the optimization

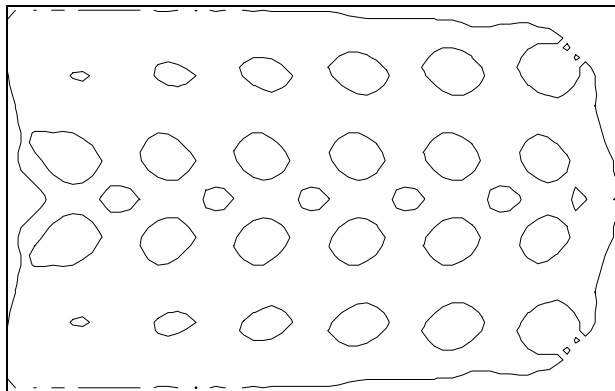


Fig. 13(a) Iteration number 6

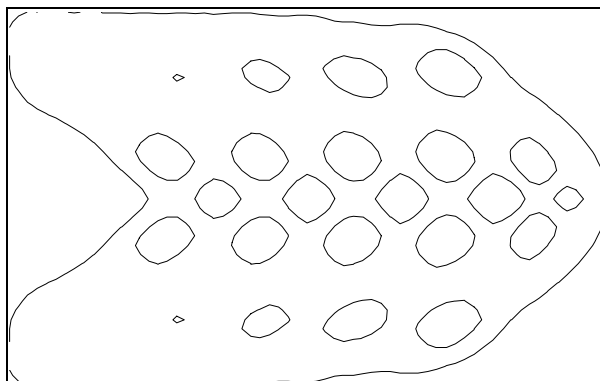


Fig. 13(b) Iteration number 13

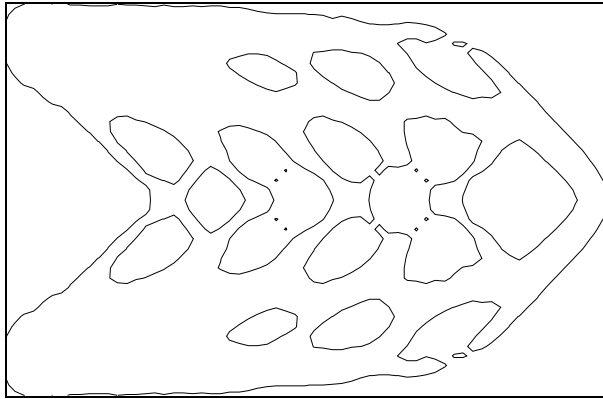


Fig. 13(c) Iteration number 20

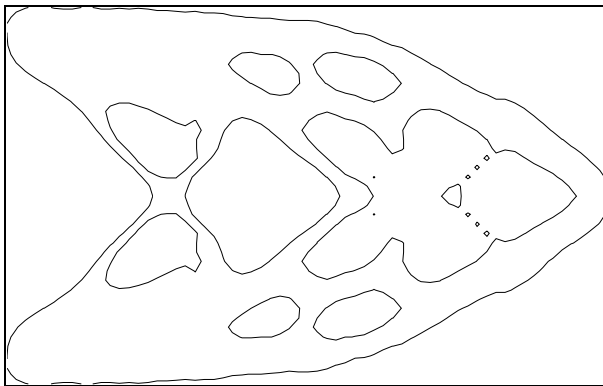


Fig. 13(d) Iteration number 27

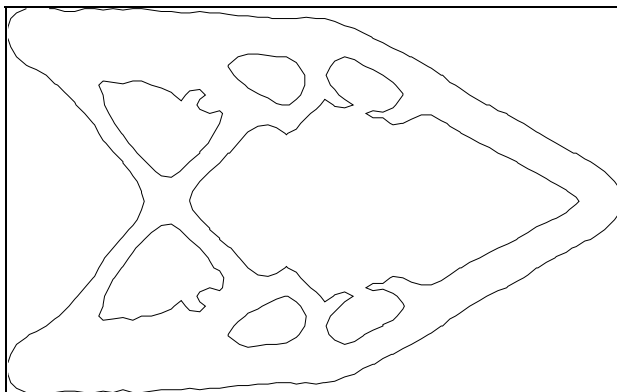


Fig. 13(e) Iteration number 33

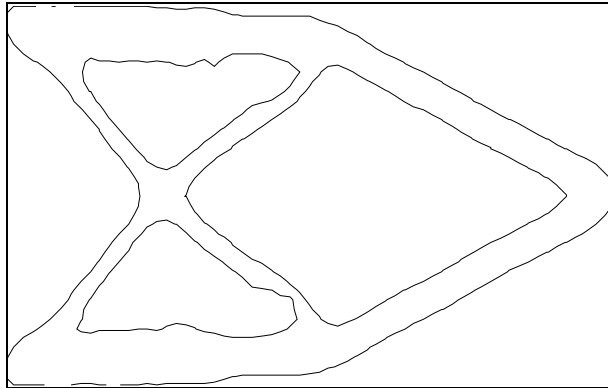


Fig. 13(f) Iteration number 39

Fig. 13. The proposed algorithm: Topology evolving process with initialization having holes

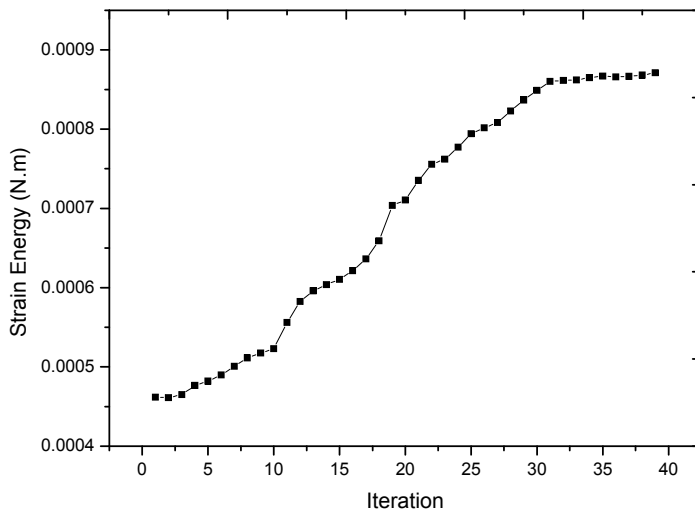


Fig. 14. Strain energy of the structure v.s. iteration number

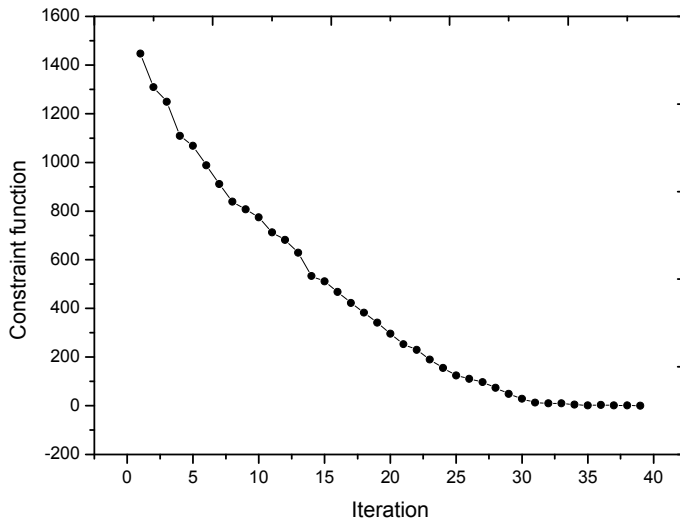


Fig. 15. Value of constraint function V.S. iteration number, the value indicted the gross material usage during the optimization

4. Conclusions

This paper proposed a LSM combined ESO hole-inserting algorithm for topology optimization. The algorithm integrated the merits of two methods and eliminated the weaknesses of conventional Level Set Method. Smooth boundary of the final topology can be gotten and need no post-processing for the manufacturability. The optimization iteration needs no explicit description of the variation of the topology, all the merits of LSM methods are kept and implemented in the new algorithm so that it makes the computation convenient and improves the efficiency accordingly. In conclusion, the nodal ESO integrated level set methods for topology optimization has the following characteristics:

1. Enlarged the optimum searching scope of the ground structure, solved the topology optimization without holes in initialization of guess configuration. which cannot get satisfied topology within proper iteration for traditional LSM method.
2. With the proposed algorithm in this paper to solve benchmark problems, the computational efficiency can be improved considerably, it can get the optimal topology in less iteration for initialization with enough initialized holes.
3. Additionally, the proposed algorithm can be used to solve other engineering problems easily if given different optimization criteria, such as local stress constraint, eigenvalue optimization and design of compliant mechanism.

Further work includes the parallelization of the genetic algorithm with the aim of reducing the iteration times as well as the extension of the proposed approach to 3D structures. Moreover, availability of the bi-direction evolutionary structure optimization algorithm the automatic mechanism of inserting and removing hole will be further implemented and integrated with the level set method to tackle more complex engineering problems.

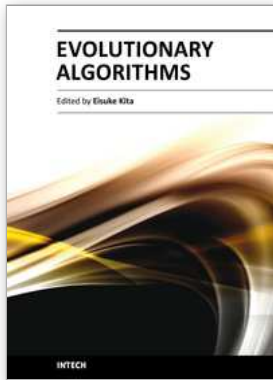
5. Acknowledgments

This work is partially supported by National Science Foundation of Hebei (grant NO.2008000087), State Key Laboratory of Hydrosience and Engineering of Tsinghua University (grant NO.2010-TC-2) and Tsinghua University Initiative Scientific Research Program (grant NO.2009THZ07060). Their financial contributions are gratefully acknowledged.

6. References

- [1] John. H. Holland. *Adaptation in Nature and Artificial System: An Introductory Analysis with Applications to Biology Control and Artificial Intelligence*. Cambridge, MA: MIT Press, 1975.
- [2] Xie Y. M., Steven G. P. *Evolutionary Structural Optimization*. Springer-Verlag, Berlin, German, 1997.
- [3] M. P. Bendsøe. Optimal shape design as a material distribution problem. *Structural and Multidisciplinary Optimization* 1989; 1:193-202.
- [4] Eschenauer H. A., Olhoff N. Topology optimization of continuum structures: a review. *Applied Mechanics Review* 2001; 54:331-390.
- [5] Bendsoe M. P., Kikuchi N. Generating optimal topologies in structural design using a homogenization method. *Computer Methods in Applied Mechanics and Engineering* 1988; 71:197-224.
- [6] Makoto Ohsaki. Simultaneous optimization of topology and geometry of a regular plane truss. *Computers & Structures* 1998; 66: 69-71.
- [7] Chyi-Yeu Lin, Shin-Hong Lin. Artificial neural network based hole image interpretation techniques for integrated topology and shape optimization. *Computer Methods in Applied Mechanics and Engineering* 2005; 194: 3817-3837.
- [8] Mehrdad Salami, Tim Hendtlass. A fast evaluation strategy for evolutionary algorithms. *Applied Soft Computing* 2003; 2(3): 156-173.
- [9] Y. M. Xie, G. P. Steven. A simple evolutionary procedure for structure optimization. *Computers and Structures* 1993; 49: 885-896.
- [10] S. Y. Wang, K. Tai, M. Y. Wang. An enhanced genetic algorithm for structural topology optimization. *International Journal for Numerical Methods in Engineering* 2006; 65:18-44.
- [11] Soon Yu Woon, Liyong Tong, Osvaldo M. Querin and Grant P. Steven, Effective optimization of continuum topologies through a multi-GA system. *Computer Methods in Applied Mechanics and Engineering* 2005; 194(30-33): 3416-3437.
- [12] X.Y. Yang, Y.M. Xie and G.P. Steven, Evolutionary methods for topology optimization of continuous structures with design dependent loads. *Computers and Structures* 2005; 83(12-13): 956-963.
- [13] X.Y. Yang, Y.M. Xie, G.P. Steven and O.M. Querin. Bi-directional evolutionary method for stiffness optimization. *American Institute of Aeronautics and Astronautics Journal* 1999. 37(11):1483-1488.
- [14] Xiaodong HUANG, Yi Min XIE and Mark Cameron BURRY. A new algorithm for bi-directional evolutionary structural optimization. *JSME International Journal Series C*. 2006. 49 (4): 1091-1099

- [15] G. P. Steven, Q. Li, Y. M. Xie. Multi-criteria optimization that minimizes maximum stress and maximizes stiffness. *Computers and Structures* 2002; 80: 2433-2448.
- [16] G. Marckmann P. Bettess and B. Peseux. Self designing structures: a new evolutionary rule for thickness distribution in 2D problems. *Communications in Numerical Methods in Engineering* 2002; 18:743-755.
- [17] Mariano Victoria, Pascual Martí, Osvaldo M. Querin. Topology design of two-dimensional continuum structures using isolines. *Computers and Structures* 2008. 87: 101-109.
- [18] K. S. Lee, Z. W. Geem. A new structural optimization method based on the harmony search. *Computers & Structures* 2004; 82(9-10): 781-798.
- [19] H. A. Eschenauer, H. A. Kobelev. A. Schumacher. Bubble method for topology and shape optimization of structures. *Structural and Multidisciplinary Optimization* 1994; 8:142-151.
- [20] Stanley Osher, J. A. Sethian. Fronts propagating with curvature-dependent speed: algorithm based on hamilton-jacobi formulations. *Journal of Computational Physics* 1988; 79:12-49.
- [21] J. A. Sethian. *Level Set Methods and Fast Marching Methods Evolving Interfaces in Computational Geometry. Fluid Mechanics, Computer Vision, and Materials Science.* Cambridge University Press, 1999.
- [22] Stanley J. Osher, Ronald P., Fedkiw. *Level Set Methods and Dynamic Implicit Surfaces.* Springer-Verlag, 2000.
- [23] Stanley Osher, and Ronald P. Fedkiw. Level set methods: an overview and some recent results. *Journal of Computational Physics* 2001; 169: 463-502.
- [24] J. A. Sethian and Andreas Wiegmann. Structural boundary design via level set and immersed interface methods. *Journal of Computational Physics*, 2000, 163: 489-528.
- [25] Stanley J. Osher, Fadi Santosay. Level set methods for optimization problems involving geometry and constraints. *Journal of Computational Physics* 2001; 171: 272-288.
- [26] Michael Yu Wang, Xiaoming Wang, Dongming Guo. A level set method for structural topology optimization. *Computer Methods in Applied Mechanics and Engineering* 2003; 192: 227-246.
- [27] Wang SY, Wang MY. Radial basis functions and level set method for structural topology optimization. *International Journal for Numerical Methods in Engineering* 2006; 65(12):2060-2090.
- [28] Peng Wei, Michael Yu Wang. Piecewise constant level set method for structural topology optimization, *International Journal for Numerical Methods in Engineering*, 2009; 78:379-402.
- [29] G. Allaire, F. Jouve. A level-set method for vibration and multiple loads structural optimization. *Computer Methods in Applied Mechanics and Engineering* 2005; 194: 3269-3290.
- [30] A. Rietz. Sufficiency of a finite exponent in SIMP (power law) methods. *Structural and Multidisciplinary Optimization* 2001; 21:159-163.
- [31] C. Y. Lin, L. S. Chao. Automated image interpretation for integrated topology and shape optimization. *Structural and Multidisciplinary Optimization* 2000; 1:125-137.
- [32] Klaus-Jurgen Bathe, *Finite Element Procedures.* Prentice Hall, 1995.
- [33] Robert D. Cook. *Concepts and Applications of Finite Element Analysis.* John Wiley & Sons, New York, 2001.



Evolutionary Algorithms

Edited by Prof. Eisuke Kita

ISBN 978-953-307-171-8

Hard cover, 584 pages

Publisher InTech

Published online 26, April, 2011

Published in print edition April, 2011

Evolutionary algorithms are successively applied to wide optimization problems in the engineering, marketing, operations research, and social science, such as include scheduling, genetics, material selection, structural design and so on. Apart from mathematical optimization problems, evolutionary algorithms have also been used as an experimental framework within biological evolution and natural selection in the field of artificial life.

How to reference

In order to correctly reference this scholarly work, feel free to copy and paste the following:

Haipeng Jia, Chungong Jiang, Lihui Du, Bo Liu and Chunbo Jiang (2011). Evolutionary Enhanced Level Set Method for Structural Topology Optimization, Evolutionary Algorithms, Prof. Eisuke Kita (Ed.), ISBN: 978-953-307-171-8, InTech, Available from: <http://www.intechopen.com/books/evolutionary-algorithms/evolutionary-enhanced-level-set-method-for-structural-topology-optimization>

INTECH

open science | open minds

InTech Europe

University Campus STeP Ri
Slavka Krautzeka 83/A
51000 Rijeka, Croatia
Phone: +385 (51) 770 447
Fax: +385 (51) 686 166
www.intechopen.com

InTech China

Unit 405, Office Block, Hotel Equatorial Shanghai
No.65, Yan An Road (West), Shanghai, 200040, China
中国上海市延安西路65号上海国际贵都大饭店办公楼405单元
Phone: +86-21-62489820
Fax: +86-21-62489821

© 2011 The Author(s). Licensee IntechOpen. This chapter is distributed under the terms of the [Creative Commons Attribution-NonCommercial-ShareAlike-3.0 License](#), which permits use, distribution and reproduction for non-commercial purposes, provided the original is properly cited and derivative works building on this content are distributed under the same license.