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Integrated Revenue Sharing Contracts to Coordinate a Multi-Period Three-Echelon Supply Chain

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1. Introduction

A supply chain can be defined as a network of facilities and distribution options that performs the functions of procurement of materials, transformation of these materials into intermediate and finished products, and the distribution of these finished products to customers. Different entities in a supply chain operate subject to different sets of constraints and objectives under different industrial environments. Each member of a decentralized supply chain has its own decision rights to optimize its costs or benefits. Recently, the topic of decentralized supply chain modelling and analysis has been of great interest. Most of the studies on decentralized supply chain modelling have focused on designing a mechanism to fully integrate these individualistic decisions in order to ensure that the decision outcome of an individual member of the supply chain is in accordance with the decision outcome of the entire supply chain (Cachon & Lariviere, 2001; Moinzadeh and Bassok, 1998; Tsay et al., 1999).
Perfect coordination mechanisms allow the decentralized supply chain to perform as well as a centralized one, in which all decisions are made by a single entity to maximize supply-chain-wide profits. Several types of contractual agreements which may determine incentive mechanisms to integrate a decentralized supply chain, inclunding profit sharing (Atkinson, 1979; Jeuland and Shugan, 1983), consignment (Kandel, 1996), buy-backs (Pasternack, 1985; Emmons & Gilbert, 1987), quantity-flexibility (Tsay & Lovejoy, 1999), revenue sharing (Giannoccaro & Pontrandolfo, 2004; Cachon & Lariviere, 2005; Chang & Hsueh, 2006, 2007), revenue allocation rules (Shah et al., 2001), and quantity discounts (Dolan, 1987), etc.
One of these contractual agreements, revenue sharing is a mechanism that is gaining popularity in practice and in research. Shah et al. (2001) have adopted Nash's game theory to formulate a model which explores a fair revenue allocation mechanism among the members of a multi-tier supply chain. The model provides a compromise solution of maximized revenue for each individual member of the supply chain under the inventory and production constraints. Giannoccaro & Pontrandolfo (2004) have extended the revenue sharing contract of two-tier to a three-tier supply chain model. Cachon & Lariviere (2005) have presented the revenue sharing contract concept and discussed its influence on supply chain performances. The revenue sharing contract can be described by two parameters, retail price and retailers' revenue retention ratio. Chang & Hsueh (2006, 2007) extended Giannoccaro & Pontrandolfo (2004) to explore a three-tier supply chain integration problem

with the time-varying multi-period demand and the constant price elasticity demand function. Multiple objective programming techniques are applied to determine the revenue sharing contract parameters, the purchasing price and revenue sharing ratios among the members of the supply chain. In order to heighten the incentive cooperation, equilibrium behaviors for decentralized supply chains are included and regarded as compromise benchmarks for supply chain integration.

The remainder of this chapter is organized as follows. In Section 2, two multi-period three-tier supply chain network models are presented. A equilibrium model of decentralized supply chain network is introduced first. Herein the optimality conditions of the various decision-makers are derived and formulated as a finite-dimensional variational inequality model. A multi-objectives programming model to determine the revenue sharing constract parameters is given next. In Section 3, a well-known solution algorithm, diagonalization method, is presented to solve the variation inequility model of supply chain networkequilibrium. In Section 4, a supply chain network example is provided for the demonstration. Conclusions are given in the end.

2. Model formulation

The supply chain network is composed of m manufacturers, n distributors, and o retailers. The other assumptions about the members of the supply chain network are summarized as follows:

1. To accommodate changes in demand, the product inventory within this supply chain network is stored at the manufacturers' warehouses so that the manufacturers will have sufficient inventory or production capacity to satisfy the distributors' demand in the current time period.
2. The total costs of the manufacturers have to bear are production cost, inventory cost and transportation cost. The distributors are only responsible for the product handling and purchasing costs. The retailers are directly associated with the market demand and responsible for transportation costs and purchasing cost. All the cost functions for the manufacturers, distributors, and retailers are continuous, convex, and nonlinear functions.
3. The demand function is a known function which can describe the relationship between the market demand and market price.

2.1 Notations

$d_k(t)$: The product demand of retailer k at time period t

$f_i(e)$: The production cost of manufacturer i at time period e

$\overline{f}_i(e)$: The average production cost of manufacturer i at time period e

$h_i(t)$: The inventory cost of manufacturer i at time period t

$\overline{h}_i(t)$: The average inventory cost of manufacturer i at time period t

$I_i(t)$: The inventory level of manufacturer i at time period t

$L_j(t)$: The product quantity of distributor j at time period t

$m_j(t)$: The product handling cost of distributor j at time period t

$\overline{m}_j(t)$: The average product handling cost of distributor j at time period t

$q_i(e)$: The production quantity of manufacturer i at time period e

$q_{ij}(t)$: The product quantity delivered from manufacturer i to distributor j at time period t

$q_{ijt}(e)$: The product quantity produced by manufacturer i at time period e and delivered to distributor j at time period t

$q_{jk}(t)$: The product quantity delivered from distributor j to retailer k at time period t

$s_{ij}(t)$: The transportation cost from manufacturer i to distributor j at time period t

$\bar{s}_{ij}(t)$: The average transportation cost from manufacturer i to distributor j at time period t

$s_{jk}(t)$: The transportation cost from distributor j to retailer k at time period t

$\bar{s}_{jk}(t)$: The average transportation cost from distributor j to retailer k at time period t

T_{ij} : The leading time between manufacturer i and distributor j

T_{jk} : The leading time between distributor j and retailer k

z_i, z_j, z_k : The profit for manufacturers, distributors, and retailers

z_i^*, z_j^*, z_k^* : The maximum profit for manufacturers, distributors, and retailers

z_i^E, z_j^E, z_k^E : The equilibrium profit for manufacturers, distributors, and retailers

ϕ_k^3 : The ratio of the retail revenues retained by retailer k

ϕ_{jk}^2 : The ratio of the wholesale revenue retained by distributor j, which is resulted from the transaction between distributor j and retailer k

$\rho_{ij}^1(t)$: The selling price of manufacturer i to distributor j at time period t

$\rho_j^2(t)$: The selling price of distributor j at time period t

$\rho_k^3(t)$: The selling price of retailer k at time period t

2.2 Market equilibrium model

Chang & Hsueh (2006) first focus on decision behaviours of manufacturers and then turn to decision behaviours of distributors and retailers, subsequently. A complete equilibrium model is finally constructed.

2.2.1 The manufacturers' optimality conditions

Each manufacturer's behaviour of seeking profit maximization can be expressed as follows.

$$\max \quad \pi_i = \sum_{jt} \rho_{\mathrm{ij}}^1(t) q_{ij}(t) - \sum_{e} f_i(e) - \sum_{t} h_i(t) - \sum_{jt} s_{ij}(t) \tag{1}$$

subject to

$$q_i(e) = \sum_{jt} q_{ijt}(e) \qquad \forall e \tag{2}$$

$$I_i(t) = \sum_{j,e<t} q_{ijt}(e) \qquad \forall t \tag{3}$$

$$q_{ij}(t) = \sum_{e \le t\text{-}\mathrm{T_{ij}}} q_{ij(t\text{-}\mathrm{T_{ij}})}(e) \qquad \forall j,t \tag{4}$$

$$q_{ijt}(e) \ge 0 \qquad \forall j,t,e \tag{5}$$

$$\rho_{ij}^{1}(t) \ge 0 \qquad \forall j,t \tag{6}$$

Eq. (1) designates that the profit of a manufacturer is the difference in total revenues and total costs. Eq. (2) defines that the entire volume of production of manufacturer *i* at time period *e* is equal to the sum of the quantities shipped from this manufacturer to all distributors after time period *e*. Eq. (3) defines that the entire volume of inventory at time period *t* is equal to the sum of the quantities produced by the manufacturer *i* before time period *t*. Eq. (4) defines that the volume of transaction between manufacturer *i* and distributor *j* at time period *t* is equal to the sum of the product quantity produced by manufacturer *i* for distributor *j* before time period $t - T_{ij}$. Note that the production cost $f_i(e)$ depends upon the entire volume of production at time period *e*. The inventory cost $h_i(t)$ depends upon the entire volume of inventory at time period *t*. The shared transaction cost depends upon the volume of transaction at time period *t*. Eqs. (5) and (6) are nonnegative constraints.

The manufacturers compete in a noncooperative fashion following Nash (1950, 1951). Each manufacturer will determine this optimal production quantity, inventory quantity, distribution quantity at each time period. The optimality conditions for all manufacturers simultaneously expressed as Eq. (7).

$$\frac{\partial f_i^*(e)}{\partial q_{ijt}^*(e)} + \frac{\partial h_i^*(t)}{\partial q_{ijt}^*(e)} + \frac{\partial s_{ij}^*(t)}{\partial q_{ijt}^*(e)} \begin{cases} = \rho_{ij}^{1*}(t) \ , \text{ if } q_{ijt}^*(e) > 0 \\ \ge \rho_{ij}^{1*}(t) \ , \text{ if } q_{ijt}^*(e) = 0 \end{cases} \qquad \forall i,j,t,e \tag{7}$$

2.2.2 The distributors' optimality conditions

Herein, each distributor's behavior of seeking profit maximization can be expressed as follows.

$$\max \quad \pi_j = \sum_t \rho_{\mathrm{j}}^{2}(t) \sum_k \overline{q}_{jk}(t) - \sum_{it} \rho_{\mathrm{ij}}^{1}(t - T_{ij}) q_{ij}(t - T_{ij}) - \sum_t m_j(t) \tag{8}$$

subject to

$$L_j(t) = \sum_i q_{ij}(t - T_{ij}) \qquad \forall t \tag{9}$$

$$\sum_i q_{ij}(t - T_{ij}) = \sum_k q_{jk}(t) \qquad \forall t \tag{10}$$

$$q_{ij}(t) \ge 0 \qquad \forall i,t \tag{11}$$

$$q_{jk}(t) \ge 0 \qquad \forall k,t \tag{12}$$

$$\rho_j^2(t) \geq 0 \qquad \forall t \tag{13}$$

Eq. (8) designates that the profit of a distributor is the difference in total revenues and total costs. Eq. (9) defines that the entire product quantity of distributor j at period t is equal to the sum of purchase quantity from all manufacturers at the corresponding time period $t\text{-}T_{ij}$. The handling cost $m_j(t)$ depends upon the entire product quantity at period t. Eq. (10) ensures that the received total product quantity of the distributor j from all manufacturers departing at time period $t\text{-}T_{ij}$ must be greater than or equal to the product quantity of the distributor j which can be distributed to all retailers at time period t. Eqs. (11) ~ (13) are nonnegative constraints.

Congenially, the distributors compete in a noncooperative manner, too. At each time period, each distributor will determine the optimal order quantity with each manufacturer as well as distribution quantity for each retailer. The optimality conditions for all distributors satisfy Eqs. (14)~(16).

$$\rho_{ij}^{1*}(t-T_{ij}) + \frac{\partial m_j^*(t)}{q_{ij}^*(t-T_{ij})} \begin{cases} = \gamma_j^*(t) \ , \ \text{if } q_{ij}^*(t-T_{ij}) > 0 \\ \geq \gamma_j^*(t) \ , \ \text{if } q_{ij}^*(t-T_{ij}) = 0 \end{cases} \qquad \forall i,j,t \tag{14}$$

$$\gamma_j^*(t) \begin{cases} = \rho_j^{2*}(t) \ , \ \text{if } q_{jk}^*(t) > 0 \\ \geq \rho_j^{2*}(t) \ , \ \text{if } q_{jk}^*(t) = 0 \end{cases} \qquad \forall j,k,t \tag{15}$$

$$\gamma_j(t) \geq 0 \qquad \forall j,t \tag{16}$$

Note that $\gamma_j(t)$ is the Lagrange multiplier associated with constraint (10) for distributor j at time period t.

2.2.3 The retailers' optimality conditions

On the analogy of the well-known spatial price equilibrium conditions, the equilibrium conditions for each retailer at each time period can be stated as follows:

$$\rho_j^*(t-T_{jk}) + s_{jk}^*(t) \begin{cases} = \rho_k^{3*}(t) \ , \ \text{if } q_{jk}^*(t) > 0 \\ \geq \rho_k^{3*}(t) \ , \ \text{if } q_{jk}^*(t) = 0 \end{cases} \qquad \forall j,k,t \tag{17}$$

$$d_k^*(t) \begin{cases} = \sum_j q_{jk}^*(t-T_{jk}) \ , \ \text{if } \rho_k^{3*}(t) > 0 \\ \leq \sum_j q_{jk}^*(t-T_{jk}) \ , \ \text{if } \rho_k^{3*}(t) = 0 \end{cases} \qquad \forall k,t \tag{18}$$

Eq. (17) ensures that the product will be distributed to the retailer k from distributor j at time period t, if the price charged by the distributor j for the product at time period $t-T_{jk}$ plus the transportation cost faced by retailer k at time period t doesn't exceed the price that consumers of retailer k are willing to pay for the product at time period t. Eq. (18) states that the total product quantity distributed to the retailer k from all distributors at time period

$t - T_{jk}$ is equal to the customers' demands of retailer *k* at time period *t*, if the price the consumers of retailer *k* are willing to pay for the product at time period *t* is positive.

2.2.4 Equilibrium condition of the supply chain

The equilibrium state of the multi-period supply chain is one where the time-space flows between the tiers of the supply chain network coincide and the product shipments and prices simultaneously satisfy the all optimality conditions, i.e., Eqs. (7) and (14)~(18). Furthermore, they can also be expressed as a variational inequality problem.

$$\begin{aligned}
&\sum_{ijte}\left[\frac{\partial f_i^*(e)}{\partial q_{ijt}^*(e)}+\frac{\partial h_i^*(t)}{\partial q_{ijt}^*(e)}+\frac{\partial s_{ij}^*(t)}{\partial q_{ijt}^*(e)}-\rho_{ij}^*(t)\right]\left[q_{ijt}(e)-q_{ijt}^*(e)\right]+\\
&\sum_{ijt}\left[\rho_{ij}^*(t-T_{ij})+\frac{\partial m_j^*(t)}{\partial q_{ij}^*(t-T_{ij})}-\gamma_j^*(t)\right]\left[q_{ij}(t-T_{ij})-q_{ij}^*(t-T_{ij})\right]+\\
&\sum_{jkt}\left[\gamma_j^*(t)-\rho_j^*(t)\right]\left[q_{jk}(t)-q_{jk}^*(t)\right]+\sum_{jt}\left[\sum_i q_{ij}^*(t-T_{ij})-\sum_k q_{jk}^*(t)\right]\left[\gamma_j(t)-\gamma_j^*(t)\right]+\\
&\sum_{jkt}\left[\rho_j^*(t-T_{jk})+s_{jk}^*(t)-\rho_k^*(t)\right]\left[q_{jk}(t)-q_{jk}^*(t)\right]+\sum_{kt}\left[\sum_j q_{jk}^*(t-T_{jk})-d_k^*(t)\right]\left[\rho_k(t)-\rho_k^*(t)\right]\geq 0
\end{aligned} \tag{19}$$

The equilibrium state of the multi-period supply chain is one where the time-space flows between the tiers of the supply chain network coincide and the product shipments and prices simultaneously satisfy the all optimality conditions, i.e., Eqs. (7), and (14)~(18). Since the amount of products must follow the flow conservation constraints, each product received by a retailer must come from some manufacturer by way of some distributor. Therefore, Chang & Hsueh (2007) define such a product flow as a time-dependent path flow $q_{pk}(e,t)$ where a path p is composed of a link (i,j) and a link (j,k). It means that the products are produced by manufacturer *i* at time period *e*, and then are delivered to distributor *j* and retailer *k* at time period *t*, sequentially. The equilibrium conditions of whole supply chain network can then be simplified as Eq. (18) and the following:

$$\frac{\partial f_i^*(e)+h_i^*(t)}{\partial q_{ij}(e,t)}+\frac{\partial s_{ij}^*(t)+m_j^*(t+T_{ij})}{\partial q_{ij}(t)}+s_{jk}^*(t+T_{ij})\begin{cases}=\rho_k^{3*}(t+T_{ij}+T_{jk})\ ,\ \text{if } q_{pk}^*(e,t)>0\\ \geq \rho_k^{3*}(t+T_{ij}+T_{jk})\ ,\ \text{if } q_{pk}^*(e,t)=0\end{cases}\quad \forall p,k,t,e \tag{20}$$

Let Eq. (21) stands. Equilibrium conditions (18) and (20) can be transformed into the following variational inequality formulation (22) with the constraint set Ω, i.e., (2)~(6), (9)~(13).

$$\hat{c}_{pk}(e,t)=\frac{\partial f_i(e)+h_i(t)}{\partial q_{ij}(e,t)}+\frac{\partial s_{ij}(t)+m_j(t+T_{ij})}{\partial q_{ij}(t)}+s_{jk}(t+T_{ij}) \tag{21}$$

$$\sum_{pket}\left[\hat{c}_{pk}^*(e,t)-\rho_k^{3*}(t+T_{ij}+T_{jk})\right]\left[q_{pk}(e,t)-q_{pk}^*(e,t)\right]+\sum_{kt}\left[\sum_j q_{jk}^*(t-T_{jk})-d_k^*(t)\right]\left[\rho_k(t)-\rho_k^*(t)\right]\geq 0 \tag{22}$$

The first term of Eq. (22) is a path-based variational inequiality formulation and can be equivalently transformed into a link-based VI one (Chen, 1999). Therefore, the variational inequiality model for a decentralized supply chain network can then be established as follows (Chang & Hsueh, 2007).

$$\sum_{ijet}\left[\frac{\partial f_i^*(e)+h_i^*(t)}{\partial q_{ij}(e,t)}\right]\left[q_{ij}(e,t)-q_{ij}^*(e,t)\right]+\sum_{ijt}\frac{\partial s_{ij}^*(t)+m_j^*(t+T_{ij})}{\partial q_{ij}(t)}\left[q_{ij}(t)-q_{ij}^*(t)\right]$$
$$+\sum_{jkt}s_{jk}^*(t)\left[q_{jk}(t)-q_{jk}^*(t)\right]+\sum_{kt}\left[\sum_j q_{jk}^*(t-T_{jk})-d_k^*(t)\right]\left[\rho_k^3(t)-\rho_k^{3*}(t)\right]\geq 0 \quad (23)$$

subject to: Eqs. (2)~(6) for all manufacturer *i* and Eqs. (9)~(13) for all distributor *j*.

2.3 Revenue sharing model for supply chain integration

The unique feature of revenue sharing contract is that the sellers will provide lower selling price to the buyers and the buyers will share part of the product sales revenue with the sellers. About the revenue sharing rule, Chang & Hsueh (2006) assume that the retail sales revenue can be shared within members of the third tier, second tier, and first tier of the supply chain network and the wholesale sales revenue can be shared within members of the second tier and first tier of supply chain network. In other words, excluding the portion of retail sales retained by each retailer, the remaining retail sales revenue will be returned to the distributors, and the manufacturers will receive their shares of the retail sales revenue after the distributors have retained their portion of retail sales revenue. The distributors retain their portion of wholesale sales revenue, the residual wholesale sales revenue will be returned to the manufacturers. The sales revenue resulted from selling products from manufacturers to distributors are solely retained by the manufacturers. Under such integration stipulation, the retailers' profits and distributors' profits are defined as shown in Eq. (24) and (25), respectively.

$$z_k=\sum_t \phi_k^3\rho_k^3(t)d_k(t)-\sum_{jt}s_{jk}(t)-\sum_{jt}\rho_j^2(t-T_{jk})q_{jk}(t-T_{jk}) \quad \forall k \quad (24)$$

$$z_j=\sum_{kt}\phi_{jk}^2(1-\phi_k^3)\rho_k^3(t)d_k(t)+\sum_{kt}\phi_{jk}^2\rho_j^2(t)q_{jk}(t)-\sum_{it}\rho_{ij}^1(t-T_{ij})q_{ij}(t-T_{ij})-\sum_t m_j(t) \quad \forall j \quad (25)$$

The manufacturers' profits are defined as follows.

$$z_i=\sum_{jkt}(1-\phi_{jk}^2)(1-\phi_k^3)\rho_k^3(t)d_k(t)+\sum_{jkt}(1-\phi_{jk}^2)\rho_j^2(t)q_{jk}(t)+\sum_{jt}\rho_{ij}^1(t)\sum_{e\leq t}q_{ijt}(e)-\sum_e f_i(e)$$
$$-\sum_t h_i(t)-\sum_{jt}s_{ij}(t) \quad \forall i \quad (26)$$

As a result, the profitability of members in the supply chain will differ according to the different buyers' revenue sharing ratio. Since the buyers and sellers' benefits are in conflict with each other and it is almost impossible to maximize the benefits for every member of the supply chain, only a compromised result can be achieved. Therefore, Chang & Hsueh (2006) have applied the compromise programming theory to establish an intertemporal supply chain revenue sharing model as follows:

$$\max \quad \mu = \sum_i \frac{z_i - z_i^E}{z_i^S - z_i^E} + \sum_j \frac{z_j - z_j^E}{z_j^S - z_j^E} + \sum_k \frac{z_k - z_k^E}{z_k^S - z_k^E} \tag{27}$$

subject to:

- flow conservation constraints

(2)~(6) for all i

(9)~(13) for all j

$$d_k(t) = \sum_j q_{jk}(t - T_{jk}) \quad \forall k, t \tag{28}$$

- definitional constraints

(24)~(26)

- boundary constaints

$$z_i \le z_i^s \qquad \forall i \tag{29}$$

$$z_j \le z_j^s \qquad \forall j \tag{30}$$

$$z_k \le z_k^s \qquad \forall k \tag{31}$$

$$\rho_{ij}^1(t) \le \rho_{ij}^{1^*}(t) \quad \forall i, j, t \tag{32}$$

$$\rho_j^2(t) \le \rho_j^{2^*}(t) \quad \forall j, t \tag{33}$$

$$\rho_{ij}^1(t) \le \rho_j^2(t + T_{ij}) \quad \forall i, j, t \tag{34}$$

$$\rho_j^2(t) \le \rho_k^3(t + T_{jk}) \quad \forall j, k, t \tag{35}$$

- value range constraints

$$\rho_k^3(t) \ge 0 \qquad \forall k, t \tag{36}$$

$$0 \le \phi_{jk}^2 \le 1 \qquad \forall j, k \tag{37}$$

$$0 \le \phi_k^3 \le 1 \qquad \forall k \tag{38}$$

The objective of the compromise programming model is to maximize the sum of relative distance from the negative solution $\left(z_i^E, z_j^E, z_k^E\right)$, as shown in Eq. (27). It is well known that the profit of each individual in the perfect competition market is lowest. Furthermore, in order to avoid the rejection of the revenue sharing contracts due to the fact that the compromised profit solution provide in the revenue sharing contract for each member of the supply chain is less than the profits made at market equilibrium before revenue sharing. We let the negative solution $\left(z_i^E, z_j^E, z_k^E\right)$ of the proposed compromise programming model be the equilibrium profits for manufacturers, distributors, and retailers that are obtained from the variational inequalities, Eq. (23). On the other hand, the share profits for manufacturers, distributors, and retailers are very important parameters in Eq. (27). Based on fairness doctrine, we suggest that the excess profit resulted from the supply chain integration must be shared by all the members in the supply chain.

In addition, the feasible solution is defined by flow conservation constraints, definitional constraints, boundary constraints, and value boudary constriants. Eqs. (2)~(6), (9)~(13), and (28) are flow conservation constrints. Eq. (28) limits the total market demand for retailer *k* at time *t*, which is equal to the total product quantity delivered from all distributors at time $t - T_{jk}$. Eqs. (24)~(26) define the profits of members in the supply chain.

There are three kinds of boundary constraints in this model. The first one is about the profits limits. Eqs. (29)~(31) require the profits for the manufacturers, distributors, and retailers must be less than the share profits negotiated with each member of the supply chain. They also ensure that each relative distance from the negative solution is between 0 and 1. The second one is about upper limits of selling prices. Eqs. (32)~(33) set the upper limits of selling prices be equal to the corresponding equilibrated prices. The equilibrated manufacturers' and distributors' selling prices can be obtained from the variational inequalities, Eq. (23) and estimated by using Eq. (7) and Eqs. (14), (15) respectively. The third one is to avoid a singular phenomenon, i.e. selling prices are less than prime costs, Eq. (34) requires the distributors' selling prices in each time period $t + T_{ij}$ must be greater than the manufacturers' selling prices in each time period *t*. Eq. (35) requires the retailers' selling price in each time period $t + T_{jk}$ must be greater than the distributors' selling price in each time period *t*.

Value boudary constriants includ Eqs. (36) and (37)~(38). Eq. (36) limits all retailers' selling price to be nonnegative. Eq. (37) and (38) limit each buyer's revenue sharing ratio, regardless the transaction type, to be between 0 and 1.

3. Solution algorithm

Chang & Hsueh (2007) adopted a diagonalization method to solve the variation inequility model (23). It is a well-known solution algorithm for solving the VI problem (Chen, 1999). A time-space network representation technique and a two-staged concept are utilized for solving such a problem. They are explained in detail as follows.

First, we utilize the time-space network representation technique to simplify the procedure of solution algorithm. Given a two-manufacturer two-distributor two-retailer network with three time-dependent costumers' demands, and five time periods, the time-space network can be drawn in Fig. 1. At each time period, the static network is reproduced and each manufacturer node is duplicated. In addition, one time-independent dummy origin node *O* and three time-dependent dummy destination nodes *S3*, *S4*, *S5* are created. Four types of links are present in this time-space network.

1. The bold broken line that connects dummy origin node *O* and a duplicated manufacturer node M_i' is a dummy link. Similarly, the bold broken line that connects a retailer node R_k and a dummy destination node S_t is also a dummy link. The costs of

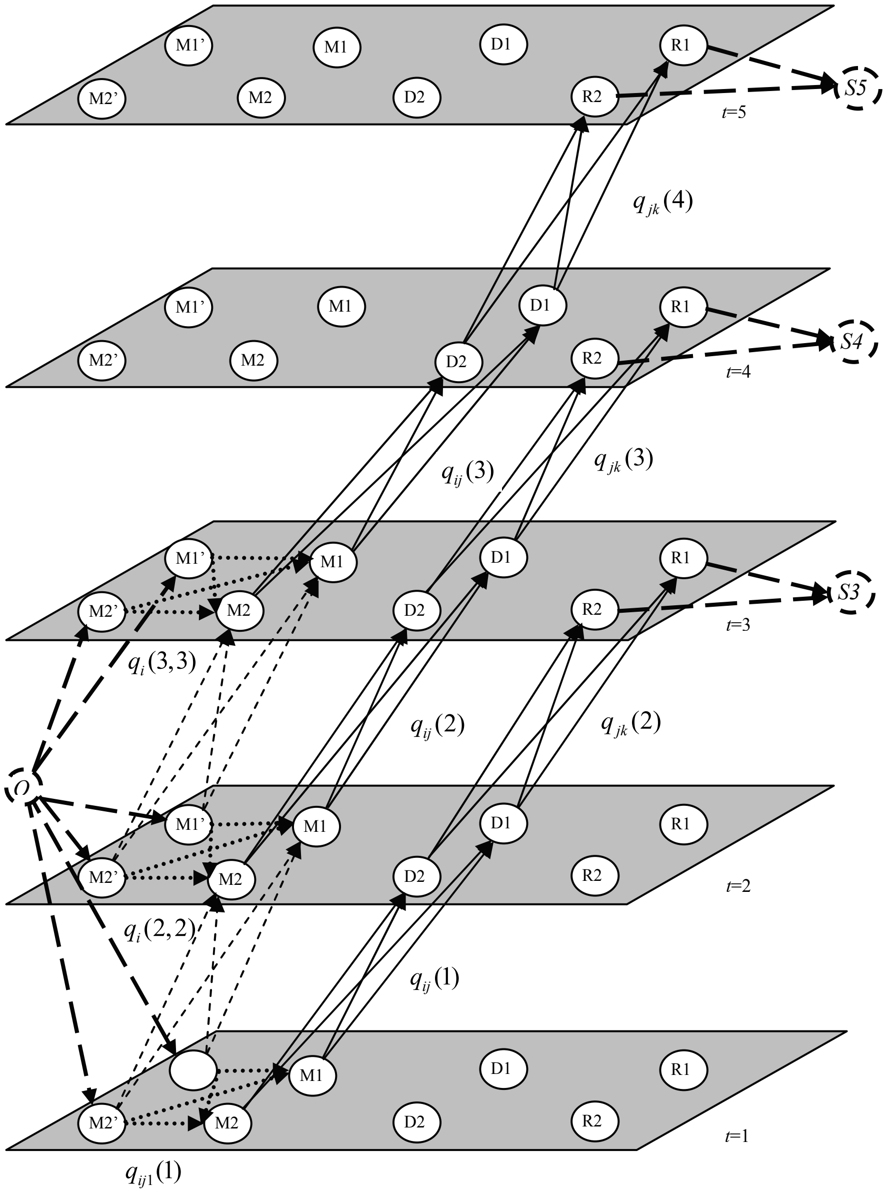

Fig. 1. Time-space network of a three-tier supply chain (Chang & Hsueh, 2007)

these dummy links are equal to zero. Having these dummy nodes and dummy links, the time-dependent costumers' demands are distributed over the supply chain from a dummy origin node to the corresponding dummy destination node.

2. The dotted line that connects a duplicated manufacturer node M_i' and a manufacturer node M_i at the same time period bears the products produced and sent out by manufacturer *i* at the same time period *e*. The cost of this link is marginal production cost $\partial f_i(e)/\partial q_i(e,e)$ where $q_i(e,e)=\sum_j q_{ij}(e,e)$.

3. The fine broken line that connects a duplicated manufacturer node M_i' and a manufacturer node M_i between diffident time periods bears the products produced by manufacturer *i* at time period *e* and sent out at following time period *t*. The cost of this link is marginal production cost $\partial f_i(e)/\partial q_i(e,t)$ and marginal inventory cost $\partial h_i(t)/\partial q_i(e,t)$ where $q_i(e,t)=\sum_j q_{ij}(e,t)$.

4. The solid line that connects a manufacturer node M_i and a distributor node D_j bears the products delivered from manufacturer *i* to distributor *j* at time period *t*. The cost of this link is marginal transportation cost $ds_{ij}(t)/dq_{ij}(t)$ and marginal handling cost $\partial m_j(t+T_{ij})/\partial q_{ij}(t)$. Furthermore, the solid line that connects a distributor node D_j and a retailer node R_k bears the product delivered from distributor *j* to retailer *k* at time period *t*. The cost of this link is transportation cost $s_{jk}(t)$.

Second, since the decision variables of the proposed variation inequiality model (23) include flow variables $\left\{q_{ijt}(e),q_{ij}(t),q_{jk}(t)\right\}$ and price variables $\rho_k^3(t)$, the flow variables and the price variables are calculate separately. The detailed steps of solution algorithm are stated as follows:

Step 0: Initialization.

Step 0.1: Set l=0.

Step 0.2: Assign the maximum demand quantity of each retailer at time period *t* to the empty time-space network in order to find an initial flow solution $\left\{q_{ij}^0(e,t),q_{ij}^0(t),q_{jk}^0(t)\right\}$.

Step 0.3: Calculate the initial price of each retailer by $\rho_k^3(t)^0=\hat{c}_{pk}(e,t)^0$, if $q_{pk}(e,\mathrm{t})^0>0$ according to the initial flow solution.

Step 1: Diagonalization operation.

Step 1.1: Set l=l+1.

Step 1.2: Fix the all retailers' price $\rho_k^3(t)^{l-1}$ and the flows for all time-space links other than on the subject time-space link at the current level, i.e., $\mathbf{q}^{l-1}\setminus q_{ij}(e,t)^{l\text{-}1}$ or $\mathbf{q}^{l-1}\setminus q_{ij}(t)^{l\text{-}1}$. Solve the following VI model (39) or (40) to find a flow solution $\left\{q_{ij}(e,t)^l,q_{ij}(t)^l,q_{jk}(t)^l\right\}$.

$$\sum_{ijet}\hat{f}_{ijet}^{*}\left(\mathbf{q}^{l-1}\setminus q_{ij}(e,t)^{l\text{-}1},q_{ij}^{*}(e,t)^{l}\right)\left[q_{ij}(e,t)-q_{ij}^{*}(e,t)^{l}\right]$$
$$+\sum_{ijet}\hat{h}_{ijet}^{*}\left(\mathbf{q}^{l-1}\setminus q_{ij}(e,t)^{l\text{-}1},q_{ij}^{*}(e,t)^{l}\right)\left[q_{ij}(e,t)-q_{ij}^{*}(e,t)^{l}\right]$$

$$+\sum_{ijt}\hat{s}_{ijt}\left(q_{ij}^{*}(t)^{l}\right)\left[q_{ij}(t)-q_{ij}^{*}(t)^{l}\right]+\sum_{ijt}\hat{m}_{j,t+T_{ij}}\left(q_{ij}^{*}(t)^{l}\right)\left[q_{ij}(t)-q_{ij}^{*}(t)^{l}\right] \tag{39}$$

$$+\sum_{jkt}s_{jk}^{*}(t)^{l}\left[q_{jk}(t)-q_{jk}^{*}(t)^{l}\right]\geq 0$$

$$\sum_{ijet}\hat{f}_{ijet}\left(\mathbf{q}^{*l}\right)\left[q_{ij}(e,t)-q_{ij}^{*}(e,t)^{l}\right]+\sum_{ijet}\hat{h}_{ijet}\left(\mathbf{q}^{*l}\right)\left[q_{ij}(e,t)-q_{ij}^{*}(e,t)^{l}\right]$$

$$+\sum_{ijt}\hat{s}_{ijt}\left(q_{ij}^{*}(t)^{l}\right)\left[q_{ij}(t)-q_{ij}^{*}(t)^{l}\right]+\sum_{ijt}\hat{m}_{j,t+T_{ij}}\left(q^{l-1}\setminus q_{ij}(t)^{l-1},q_{ij}^{*}(t)^{l}\right)\left[q_{ij}(t)-q_{ij}^{*}(t)^{l}\right] \tag{40}$$

$$+\sum_{jkt}s_{jk}^{*}(t)^{l}\left[q_{jk}(t)-q_{jk}^{*}(t)^{l}\right]\geq 0$$

Where

$$\hat{f}_{ijet}=\frac{\partial f_i(e)}{\partial q_{ij}(e,t)} \quad \forall i,j,e,t \tag{41}$$

$$\hat{h}_{ijet}=\frac{\partial h_i(t)}{\partial q_{ij}(e,t)} \quad \forall i,j,e,t \tag{42}$$

$$\hat{s}_{ijt}=\frac{\partial s_{ij}(t)}{\partial q_{ij}(e,t)}=\frac{\partial s_{ij}(t)}{\partial q_{ij}(t)}=\frac{ds_{ij}(t)}{dq_{ij}(t)} \quad \forall i,j,t \tag{43}$$

$$\hat{m}_{j,t+T_{ij}}=\frac{\partial m_j(t+T_{ij})}{\partial q_{ij}(t)} \quad \forall j,t \tag{44}$$

Step 1.3: According to the resulted flow solution $\left\{q_{ij}(e,t)^{l},q_{ij}(t)^{l},q_{jk}(t)^{l}\right\}$, calculate the corresponding price of each retailer $\rho_k^3(t)^l$ as follows.

$$\rho_k^3(t)^l=\hat{c}_{pk}(e,t)^l,\ \text{if}\ q_{pk}(e,t)^l>0 \tag{45}$$

Step 2: Convergence check.

If $\max_{ijet}\left|q_{ij}(e,t)^{l}-q_{ij}(e,t)^{l-1}\right|\leq\varepsilon$, $\max_{ijt}\left|q_{ij}(t)^{l}-q_{ij}(t)^{l-1}\right|\leq\varepsilon$, $\max_{jkt}\left|q_{jk}(t)^{l}-q_{jk}(t)^{l-1}\right|\leq\varepsilon$, and $\max_{kt}\left|\rho_k^3(t)^{l}-\rho_k^3(t)^{l-1}\right|\leq\varepsilon$, then stop; otherwise go to Step 1.1.

4. Numerical example

4.1 Input data

The numerical example of Nagurney and Toyasaki (2003) is modified and extended from one-period problem to multi-period problem. The network consists of two manufacturers, two distributors, ten retailers, and five time periods. Each of the transportation times is one period. The production cost and inventory cost functions for the manufacturers are respectively given by:

$$f_i(e) = 2.5q_i^2(e) + q_1(e)q_2(e) + 2q_i(e) \qquad \forall i; e = 1,2,3 \tag{46}$$

$$h_i(t) = \sum_{e \le t}(t-e)\left[q_{ijt}(e) + q_1(e) + 0.2q_2(e) + 0.4\right]q_{ijt}(e) \qquad \forall i; t = 1 \sim 3 \tag{47}$$

The transportation cost functions faced by the manufacturers and associated with the distributors are given by:

$$s_{ij}(t) = 0.5q_{ij}^2(t) + 3.5q_{ij}(t) \qquad \forall i; j; t = 1,2,3 \tag{48}$$

The handling cost functions of the distributors are given by:

$$m_j(t) = 0.5\left[\sum_{i=1}^{2} q_{ij}(t-1)\right]^2 \qquad \forall j, t = 2,3 \tag{49}$$

The transportation cost functions faced by the retailers and associated with the distributors are given by:

$$s_{jk}(t) = q_{jk}(t-1) + 5 \qquad \forall j; k; t = 3,4,5 \tag{50}$$

The demand functions at the demand markets are:

$$d_k(t) = -2\rho_k^3(3) - \sum_{k' \ne k} 0.1\rho_{k'}^3(t) + \delta_k(t) \tag{51}$$

where $\delta_k(t)$ is a constant of demand function of retailer *k* at time period *t*. They are listed in Table 1.

period	retailer									
	1	2	3	4	5	6	7	8	9	10
3	245	187	174	233	155	178	203	237	219	207
4	326	334	278	346	287	259	290	265	327	322
5	196	242	261	240	285	253	262	253	267	227

Table 1. Constants of demand functions

4.2 Test results

4.2.1 An equilibrated solution

The proposed diagonalization method was implemented in Visual C++ to solve the proposed network equilibrium of decentralized supply chain network. The yielded equilibrium flow patterns and selling prices are shown in Table 2. The equilibrium profits of members of supply chain network are shown in Table 3. The total profit is 3082.77. Manufacturers have most of them. Retailer 6 is not chosen. Retailer 3, 7, and 10 have negtive profit due to perfect competition.

Furthermore, the rationale of the proposed variational inequalities model and associated solution algorithm can be verified by checking if the resulting total costs of supply path satisfy the network equilibrium conditions, i.e. Eqs. (18) and (20). The check results of equilibrium condition (18) are listed in Table 4. At time period 3, products are only distributed to Retailer 1, 4, and 8. Too lower prices make no product distributed to those retailer markets. The computed equilibrium customers' demand is equal to the product quality delivered from all distributors. For example, the total amount of products is 8.06 which are delivered from distributor 1 and 2, as shown in Table 2.

time period		*t*=1	*t*=2	*t*=3	*t*=4	*t*=5
product quantity produced by manufacturer *i* at time period *e* and distributed to distributor *j* at time period *t*	1->1	*e*=1 3.48	*e*=1 1.76	*e*=1 0.00		
			e=2 6.48	*e*=2 0.00	-	-
				e=3 5.38		
	1->2	*e*=1 3.48	*e*=1 1.76	*e*=1 0.00		
			e=2 6.48	*e*=2 0.00	-	-
				e=3 5.38		
	2->1	*e*=1 3.48	*e*=1 1.76	*e*=1 0.00		
			e=2 6.48	*e*=2 0.00	-	-
				e=3 5.38		
	2->2	*e*=1 3.48	*e*=1 1.76	*e*=1 0.00		
			e=2 6.48	*e*=2 0.00	-	-
				e=3 5.38		
product quantity distributed from manufacturer *i* to distributor *j* at time period *t*	1->1	3.48	8.24	5.38	-	-
	1->2	3.48	8.24	5.38	-	-
	2->1	3.48	8.24	5.38	-	-
	2->2	3.48	8.24	5.38	-	-
product quantity distributed from distributor *j* to retailer *k* at time interval *t*	1->1	-	4.03	2.02	0.00	-
	1->2	-	0.00	4.07	0.00	-
	1->3	-	0.00	0.00	0.70	-
	1->4	-	0.96	7.14	0.00	-
	1->5	-	0.00	0.00	6.86	-
	1->6	-	0.00	0.00	0.00	-
	1->7	-	0.00	0.00	0.96	-
	1->8	-	1.98	0.00	0.00	-
	1->9	-	0.00	2.27	2.24	-
	1->10	-	0.00	0.99	0.00	-
	2->1	-	4.03	2.02	0.00	-
	2->2	-	0.00	4.07	0.00	-
	2->3	-	0.00	0.00	0.70	-
	2->4	-	0.96	7.14	0.00	-
	2->5	-	0.00	0.00	6.86	-
	2->6	-	0.00	0.00	0.00	-
	2->7	-	0.00	0.00	0.96	-
	2->8	-	1.98	0.00	0.00	-
	2->9	-	0.00	2.27	2.24	-
	2->10	-	0.00	0.99	0.00	-

time period		*t*=1	*t*=2	*t*=3	*t*=4	*t*=5
product quantity of customers of retailer *k*	1	-	-	8.06	0.00	0.00
	2	-	-	1.91	0.00	0.00
	3	-	-	0.00	3.96	0.00
	4	-	-	0.00	4.03	8.13
	5	-	-	0.00	14.29	0.00
	6	-	-	0.00	0.00	0.00
	7	-	-	4.54	1.98	0.00
	8	-	-	0.00	1.41	0.00
	9	-	-	13.72	0.00	1.92
	10	-	-	0.00	4.49	0.00
price of manufacturer *i* for distributor *j*	1->1	71.97	91.49	75.47	-	-
	1->2	71.97	91.49	75.47	-	-
	2->1	71.97	91.49	75.47	-	-
	2->2	71.97	91.49	75.47	-	-
price of distributor *j*	1	-	78.94	109.98	86.24	-
	2	-	78.94	109.98	86.24	-
price of retailer *k*	1	-	-	87.97	114.99	58.47
	2	-	-	61.69	117.04	82.68
	3	-	-	54.84	91.85	91.94
	4	-	-	84.89	120.12	81.63
	5	-	-	44.84	96.59	98.10
	6	-	-	56.95	81.85	90.42
	7	-	-	70.11	98.17	92.20
	8	-	-	85.92	85.01	88.47
	9	-	-	78.53	115.25	93.48
	10	-	-	72.21	113.97	71.63

Table 2. Equilibrated solution of supply chain network

member	equilibrated solution
manufacturer *1, 2*	1094.03
distributor *1, 2*	218.13
retailer *1*	69.02
retailer *2*	55.61
retailer *3*	-3.38
retailer *4*	148.68
retailer *5*	138.93
retailer *6*	-
retailer *7*	-0.47
retailer *8*	13.69
retailer *9*	36.49
retailer *10*	-0.12
total profit	3082.77

Table 3. Profits of members of the supply chain (equilibrated solution)

The check results of equilibrium condition (20) are summarized in Table 5 ~ Table 7. The equilibrated supply path cost is obtained by summing all relative costs along the path. For example, consider supply path *M1'*→*M1*→D1→R4 producing and sending at time period 1, the total cost is 84.89.

$$\hat{c}_{M1'\to M1\to D1\to R4}(1,1)=\frac{\partial f_1(1)}{\partial q_1(1,1)}+\frac{ds_{11}(1)}{dq_{11}(1)}+\frac{\partial m_1(2)}{\partial q_{11}(1)}+s_{14}(2)=64.98+6.98+6.97+5.96=84.89 \tag{52}$$

It is observed that the total costs of each supply path arriving at the same retailer and time period are equal to the corresponding retailer's price. For example, there are four supply paths to Retailer 4 and they arrive at the time period 3. Their total costs are 84.89. There are eight supply paths to Retailer 1 and they arrive at the time period 4. Half of them are manufactured at time period 1, but delivered to distributors till time period 2. Incured inventory costs are counted. Total costs of eight supply paths are the same and equal to the coressponding retailer price, as shown in Table 6.

retailer	t = 3			t = 4			t = 5		
	price	demand	distibuted amount	price	demand	distibuted amount	price	demand	distibuted amount
1	87.97(1)	8.06	8.06	114.99(4)	4.03	4.03	58.47	0.00	0.00
2	61.69	0.00	0.00	117.04(2)	8.13	8.13	82.68	0.00	0.00
3	54.84	0.00	0.00	91.85	0.00	0.00	91.94(4)	1.41	1.41
4	84.89(3)	1.91	1.91	120.12(1)	14.29	14.29	81.63	0.00	0.00
5	44.84	0.00	0.00	96.59	0.00	0.00	98.10(1)	13.72	13.72
6	56.95	0.00	0.00	81.85	0.00	0.00	90.42	0.00	0.00
7	70.11	0.00	0.00	98.17	0.00	0.00	92.20(3)	1.92	1.92
8	85.92(2)	3.96	3.96	85.01	0.00	0.00	88.47	0.00	0.00
9	78.53	0.00	0.00	115.25(3)	4.54	4.54	93.48(2)	4.49	4.49
10	72.21	0.00	0.00	113.97(5)	1.98	1.98	71.63	0.00	0.00

Table 4. Check results of equilibrium condition (18)

path	e = 1, t = 1	path	e = 1, t = 1	path	e = 1, t = 1
M1-D1-R1	87.97	M1-D1-R4	84.89	M1-D1-R8	85.92
M1-D2-R1	87.97	M1-D2-R4	84.89	M1-D2-R8	85.92
M2-D1-R1	87.97	M2-D1-R4	84.89	M2-D1-R8	85.92
M2-D2-R1	87.97	M2-D2-R4	84.89	M2-D2-R8	85.92
price of R1	87.97	price of R4	84.89	price of R8	85.92

Table 5. Check results of equilibrium condition (20) -- supply path costs (arrival at t = 3)

path	*e* = 1, *t* =2	*e* = 2, *t* = 2	path	*e* = 1, *t* =2	*e* = 2, *t* = 2
M1-D1-R1	114.99	114.99	M1-D1-R2	117.04	117.04
M1-D2-R1	114.99	114.99	M1-D2-R2	117.04	117.04
M2-D1-R1	114.99	114.99	M2-D1-R2	117.04	117.04
M2-D2-R1	114.99	114.99	M2-D2-R2	117.04	117.04
price of R1	114.99		price of R2	117.04	
path	*e* = 1, *t* =2	*e* = 2, *t* = 2	path	*e* = 1, *t* =2	*e* = 2, *t* = 2
M1-D1-R4	120.12	120.12	M1-D1-R9	115.25	115.25
M1-D2-R4	120.12	120.12	M1-D2-R9	115.25	115.25
M2-D1-R4	120.12	120.12	M2-D1-R9	115.25	115.25
M2-D2-R4	120.12	120.12	M2-D2-R9	115.25	115.25
price of R4	120.12		price of R9	115.25	
path	*e* = 1, *t* =2	*e* = 2, *t* = 2			
M1-D1-R10	113.97	113.97			
M1-D2-R10	113.97	113.97			
M2-D1-R10	113.97	113.97			
M2-D2-R10	113.97	113.97			
price of R10	113.97				

Table 6. Check results of equilibrium condition (20) -- supply path costs (arrival at t = 4)

path	*e* = 3, *t* =3	path	*e* = 3, *t* =3
M1-D1-R3	91.94	M1-D1-R5	98.10
M1-D2-R3	91.94	M1-D2-R5	98.10
M2-D1-R3	91.94	M2-D1-R5	98.10
M2-D2-R3	91.94	M2-D2-R5	98.10
price of R3	91.94	price of R5	98.10
path	*e* = 3, *t* =3	path	*e* = 3, *t* =3
M1-D1-R7	92.20	M1-D1-R9	93.48
M1-D2-R7	92.20	M1-D2-R9	93.48
M2-D1-R7	92.20	M2-D1-R9	93.48
M2-D2-R7	92.20	M2-D2-R9	93.48
price of R7	92.20	price of R9	93.48

Table 7. Check results of equilibrium condition (20) -- supply path costs (arrival at t = 5)

4.2.2 A compromise solution

Subsequently, the following nonlinear programming model is adopted to determine the maximal total profit of the centralized supply chain network.

$$\max \ \pi = \sum_{kt} \rho_k^3(t) d_k(t) - \sum_{jkt} s_{jk}(t) - \sum_{jt} m_j(t) - \sum_{ie} f_i(e) - \sum_{it} h_i(t) - \sum_{ijt} s_{ij}(t) \tag{53}$$

subject to: (2)~(5) for all i, (9)~(12) for all j, (28), and (36).

The LINGO 10.0 package was used to solve this model. The maximum total profit of the centralized supply chain network is 3171.81. Consequently, the increment of total profits is 89.04, compared with equilibrium profits for members of the supply chain network. Let all members of the supply chain network share the increment.

After obtaining the above data, the LINGO 10.0 package was used to solve the proposed revenue sharing compromise model for supply chain integration. Sharing rates of retail revenues for retailer *k* and sharing rates of wholesale revenues for distributor *j* are summarized in Table 8 and Table 9 respectively. The sharing rate of retail revenue for retailer *1* and retailer *10* is 77.44% and 25.12% respectively. Other retailers do not have to share their revenues with and distributors. In addition, the sharing rate of wholesale revenues for distributor *1* transacted with retailer 1 is 100% and the sharing rate of wholesale revenues for distributor *2* transacted with retailer 1 is 76.85%. Other distributors do not have to share their revenues with manufacturers. The compromise flow patterns and selling prices are shown in Table 10.

The solving results of the market equilibrium model and revenue sharing model are compared and the profits for members of the supply chain network between the equilibrium solution and the compromise solution are listed in Table 11. The total profit of the compromise solution is equal to 3171.81; that is the same as the counterpart of the centralized supply chain network.

The comparisons of retail prices and customers' demands are summarized in Table 12. Most of the retail prices of the compromise solution are greater than the counterpart of the equilibrated solution. But the market transaction volume of the compromise solution is less than the market transaction volume under market competition condition but the individual and the aggregate profits will be greater than the profits under market competition model. In other words, such an integration strategy is workable for all members within the supply chain. Parts of customers' surplus are transferred to the members of the supply chain network. Therefore, using the proposed revenue sharing model can indeed create win-win situation for all members within the supply chain.

retailer									
1	2	3	4	5	6	7	8	9	10
77.44%	0.00%	0.00%	0.00%	0.00%	-	0.00%	0.00%	0.00%	25.12%

Table 8. Sharing rates of retail revenue for retailers

distributor	retailer									
	1	2	3	4	5	6	7	8	9	10
1	100.00%	0.00%	0.00%	0.00%	0.00%	-	0.00%	0.00%	0.00%	0.00%
2	76.85%	0.00%	0.00%	0.00%	0.00%	-	0.00%	0.00%	0.00%	0.00%

Table 9. Sharing rates of wholesale revenue for distributors

time period		*t*=1	*t*=2	*t*=3	*t*=4	*t*=5
product quantity produced by manufacturer *i* at time period *e* and distributed to distributor *j* at time period *t*	1->1	*e*=1 0.00	*e*=1 0.00 *e*=2 0.00	*e*=1 0.00 *e*=2 0.00 *e*=3 13.61	-	-
	1->2	*e*=1 0.00	*e*=1 0.00 *e*=2 1.62	*e*=1 0.00 *e*=2 0.00 *e*=3 0.51	-	-
	2->1	*e*=1 0.00	*e*=1 0.00 *e*=2 0.33	*e*=1 0.00 *e*=2 0.00 *e*=3 0.00	-	-
	2->2	*e*=1 18.06	*e*=1 0.00 *e*=2 14.09	*e*=1 0.00 *e*=2 0.00 *e*=3 1.01	-	-
product quantity distributed from manufacturer *i* to distributor *j* at time period *t*	1->1	0.00	0.00	13.61	-	-
	1->2	0.00	1.62	0.51	-	-
	2->1	0.00	0.33	0.00	-	-
	2->2	18.06	14.09	1.01	-	-
product quantity distributed from distributor *j* to retailer *k* at time interval *t*	1->1	-	0.00	0.33	12.96	-
	1->2	-	0.00	0.00	0.00	-
	1->3	-	0.00	0.00	0.00	-
	1->4	-	0.00	0.00	0.09	-
	1->5	-	0.00	0.00	0.57	-
	1->6	-	0.00	0.00	0.00	-
	1->7	-	0.00	0.00	0.00	-
	1->8	-	0.00	0.00	0.00	-
	1->9	-	0.00	0.00	0.00	-
	1->10	-	0.00	0.00	0.00	-
	2->1	-	11.93	3.30	1.52	-
	2->2	-	1.08	0.00	0.00	-
	2->3	-	0.00	0.00	0.00	-
	2->4	-	1.87	0.00	0.00	-
	2->5	-	2.12	0.00	0.00	-
	2->6	-	0.00	0.00	0.00	-
	2->7	-	0.16	0.00	0.00	-
	2->8	-	0.29	0.00	0.00	-
	2->9	-	0.61	0.00	0.00	-
	2->10	-	0.00	12.41	0.00	-
product quantity of customers of retailer *k*	1	-	-	11.93	3.63	14.47
	2	-	-	1.08	0.00	0.00
	3	-	-	0.00	0.00	0.09
	4	-	-	1.87	0.00	0.00
	5	-	-	2.12	0.00	0.57

time period		t=1	t=2	t=3	t=4	t=5
	6	-	-	0.00	0.00	0.00
	7	-	-	0.16	0.00	0.00
	8	-	-	0.29	0.00	0.00
	9	-	-	0.61	0.00	0.00
	10	-	-	0.00	12.41	0.00
price of manufacturer *i* for distributor *j*	1->1	4.71	85.78	0.00	-	-
	1->2	0.00	62.66	53.96	-	-
	2->1	45.84	86.96	0.00	-	-
	2->2	0.00	81.23	53.96	-	-
price of distributor *j*	1	-	45.84	87.61	0.00	-
	2	-	0.00	81.23	53.96	-
price of retailer *k*	1	-	-	88.05	117.81	53.96
	2	-	-	63.23	123.93	85.79
	3	-	-	56.96	94.46	95.74
	4	-	-	87.03	130.25	84.74
	5	-	-	45.84	99.19	108.13
	6	-	-	-	-	-
	7	-	-	72.13	100.77	96.32
	8	-	-	89.96	87.61	91.58
	9	-	-	80.32	120.25	98.95
	10	-	-	74.32	111.08	74.74

Table 10. Compromise solution of supply chain network integration

member	equilibrated solution	compromise solution	increment
manufacturer *1*	1094.03	1100.88	6.85
manufacturer *2*	1094.03	1100.88	6.85
distributor *1*	218.13	224.98	6.85
distributor *2*	218.13	224.98	6.85
retailer *1*	69.02	75.87	6.85
retailer *2*	55.61	62.46	6.85
retailer *3*	-3.38	3.47	6.85
retailer *4*	148.68	155.53	6.85
retailer *5*	138.93	145.78	6.85
retailer *6*	-	-	-
retailer *7*	-0.47	6.38	6.85
retailer *8*	13.69	20.54	6.85
retailer *9*	36.49	43.34	6.85
retailer *10*	-0.12	6.73	6.85
total profit	3082.77	3171.81	89.04

Table 11. Comparisons of profits for members of the supply chain network

It is also found that if the manufacturers and distributors sell their products to the downstream buyers at lower prices, when the distributors share parts of their revenue with the manufacturers and the retailers share parts of their revenue with the distributors, higher profits for the manufacturers, distributors, and retailers can be achieved. For example, at time period 1, distributor 2 gets free products from manufacturer 2. Most of them are sold to retailer 1 at the free price but they are sold to the customer market at the second highest price of the time period 3. Consequently, retailer 1 are asked to share 77.44% of retail revenue to distributor 2, but distributor 2 only can have 23.15% of shared retail revenue and the other 76.85% should be shared to manufacturer 2. In addition, distributor 2 must share 76.85% of wholesale revenue to manufacturer 2.

time period	retailer	equilibrated solution		compromise solution	
		price	demand	price	demand
t = 3	1	87.97	8.06	88.05	11.93
	2	61.69	-	63.23	1.08
	3	54.84	-	56.96	-
	4	84.89	1.91	87.03	1.87
	5	44.84	-	45.84	2.12
	6	-	-	-	-
	7	70.11	-	72.13	0.16
	8	85.92	3.96	89.96	0.29
	9	78.53	-	80.32	0.61
	10	72.21	-	74.32	-
t = 4	1	114.99	4.03	117.81	3.63
	2	117.04	8.13	123.93	-
	3	91.85	-	94.46	-
	4	120.12	14.29	130.25	-
	5	96.59	-	99.19	-
	6	-	-	-	-
	7	98.17	-	100.77	-
	8	85.01	-	87.61	-
	9	115.25	4.54	120.25	-
	10	113.97	1.98	111.08	12.41
t = 5	1	58.47	-	53.96	14.47
	2	82.68	-	85.79	-
	3	91.94	1.41	95.74	0.09
	4	81.63	-	84.74	-
	5	98.10	13.72	108.13	0.57
	6	-	-	-	-
	7	92.20	1.92	96.32	-
	8	88.47	-	91.58	-
	9	93.48	4.49	98.95	-
	10	71.63	-	74.74	-
total		-	68.44	-	49.22

Table 12. Comparisons of retail prices and customers' demands

5. Conclusion

The proposed compromise programming model can determine the selling price and revenue sharing ratios among the members of the supply chain and increase the profits for manufacturers, distributors and retailers, simultaneously. The supply chain network is successfully coordinated by adopting the revenue sharing contract. The results of the supply chain network equilibrium model provide the compromise benchmarks for the supply chain network integration. This idea can be applied to explore the issue of supply chain integration by using other other negotiating coordination contracts of supply chain network. Note that developing path-based algorithms is helpful in solving the medium and large size problems, in order to increase the practicability of the proposed variational inequality model.

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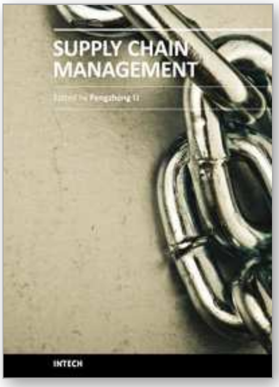

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The purpose of supply chain management is to make production system manage production process, improve customer satisfaction and reduce total work cost. With indubitable significance, supply chain management attracts extensive attention from businesses and academic scholars. Many important research findings and results had been achieved. Research work of supply chain management involves all activities and processes including planning, coordination, operation, control and optimization of the whole supply chain system. This book presents a collection of recent contributions of new methods and innovative ideas from the worldwide researchers. It is aimed at providing a helpful reference of new ideas, original results and practical experiences regarding this highly up-to-date field for researchers, scientists, engineers and students interested in supply chain management.

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