

The Design of a Discrete Time Model Following Control System for Nonlinear Descriptor System

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1. Introduction

This paper studies the design of a model following control system (MFCS) for nonlinear descriptor system in discrete time. In previous studies, a method of nonlinear model following control system with disturbances was proposed by Okubo, S. and also a nonlinear model following control system with unstable zero of the linear part, a nonlinear model following control system with containing inputs in nonlinear parts, and a nonlinear model following control system using stable zero assignment. In this paper, the method of MFCS will be extended to descriptor system in discrete time, and the effectiveness of the method will be verified by numerical simulation.

2. Expressions of the problem

The controlled object is described below, which is a nonlinear descriptor system in discrete time.

$$Ex(k+1) = Ax(k) + Bu(k) + B_f f(v(k)) + d(k) \quad (1)$$

$$v(k) = C_f x(k) \quad (2)$$

$$y(k) = Cx(k) + d_0(k) \quad (3)$$

The reference model is given below, which is assumed controllable and observable.

$$x_m(k+1) = A_m x_m(k) + B_m r_m(k) \quad (4)$$

$$y_m(k) = C_m x_m(k) \quad (5)$$

, where

$$x(k) \in R^n, d(k) \in R^n, u(k) \in R^\ell, y(k) \in R^\ell, y_m(k) \in R^\ell, d_0(k) \in R^\ell, f(v(k)) \in R^{\ell_f},$$

$v(k) \in R^{\ell_f}, r_m(k) \in R^{\ell_m}, x_m(k) \in R^{n_m}, y(k)$ is the available states output vector, $v(k)$ is the measurement output vector, $u(k)$ is the control input vector, $x(k)$ is the internal state vector

whose elements are available, $d(k), d_0(k)$ are bounded disturbances, $y_m(k)$ is the model output.

The basic assumptions are as follows:

1. Assume that (C, A, B) is controllable and observable, i.e.

$$\text{rank}[zE - A, B] = n, \text{rank} \begin{bmatrix} zE - A \\ C \end{bmatrix} = n.$$

2. In order to guarantee the existence and uniqueness of the solution and have exponential function mode but an impulse one for (1), the following conditions are assumed.

$$|zE - A| \neq 0, \quad \text{rank} E = \deg |zE - A| = r \leq n$$

3. Zeros of $C[zE - A]^{-1}B$ are stable.

In this system, the nonlinear function $f(v(k))$ is available and satisfies the following constraint.

$$\|f(v(k))\| \leq \alpha + \beta \|v(k)\|^\gamma,$$

where $\alpha \geq 0, \beta \geq 0, 0 \leq \gamma < 1$, $\|\cdot\|$ is Euclidean norm, disturbances $d(k), d_0(k)$ are bounded and satisfy

$$D_d(z)d(k) = 0 \tag{6}$$

$$D_d(z)d_0(k) = 0. \tag{7}$$

Here, $D_d(z)$ is a scalar characteristic polynomial of disturbances. Output error is given as

$$e(k) = y(k) - y_m(k). \tag{8}$$

The aim of the control system design is to obtain a control law which makes the output error zero and keeps the internal states be bounded.

3. Design of a nonlinear model following control system

Let z be the shift operator, Eq.(1) can be rewritten as follows.

$$C[zE - A]^{-1}B = N(z) / D(z)$$

$$C[zE - A]^{-1}B_f = N_f(z) / D(z),$$

where $D(z) = |zE - A|$, $\partial_{r_i}(N(z)) = \sigma_i$ and $\partial_{r_i}(N_f(z)) = \sigma_{f_i}$.

Then the representations of input-output equation is given as

$$D(z)y(k) = N(z)u(k) + N_f(z)f(v(k)) + w(k). \tag{9}$$

Here $w(k) = C \text{adj}[zE - A]d(k) + D(z)d_0(k)$, (C_m, A_m, B_m) is controllable and observable. Hence,

$$C_m[zI - A_m]^{-1}B_m = N_m(z) / D_m(z).$$

Then, we have

$$D_m(z)y_m(k) = N_m(z)r_m(k), \tag{10}$$

where $D_m(z) = |zI - A_m|$ and $\partial_{r_i}(N_m(z)) = \sigma_{m_i}$.

Since the disturbances satisfy Eq.(6) and Eq.(7), and $D_d(z)$ is a monic polynomial, one has

$$D_d(z)w(k) = 0. \tag{11}$$

The first step of design is that a monic and stable polynomial $T(z)$, which has the degree of $\rho(\rho \geq n_d + 2n - n_m - 1 - \sigma_i)$, is chosen. Then, $R(z)$ and $S(z)$ can be obtained from

$$T(z)D_m(z) = D_d(z)D(z)R(z) + S(z), \tag{12}$$

where the degree of each polynomial is: $\partial T(z) = \rho, \partial D_d(z) = n_d, \partial D_m(z) = n_m, \partial D(z) = n, \partial R(z) = \rho + n_m - n_d - n$ and $\partial S(z) \leq n_d + n - 1$.

From Eq.(8)~(12), the following form is obtained:

$$\begin{aligned} T(z)D_m(z)e(k) &= D_d(z)R(z)N(z)u(k) + D_d(z)R(z)N_f(z)f(v(k)) \\ &\quad + S(z)y(k) - T(z)N_m(z)r_m(z). \end{aligned}$$

The output error $e(k)$ is represented as following.

$$\begin{aligned} e(k) &= \frac{1}{T(z)D_m(z)} \{ [D_d(z)R(z)N(z) - Q(z)N_r]u(k) + Q(z)N_r u(k) \\ &\quad + D_d(z)R(z)N_f(z)f(v(k)) + S(z)y(k) - T(z)N_m(z)r_m(k) \} \end{aligned} \tag{13}$$

Suppose $\Gamma_r(N(z)) = N_r$, where $\Gamma_r(\cdot)$ is the coefficient matrix of the element with maximum of row degree, as well as $|N_r| \neq 0$. The next control law $u(k)$ can be obtained by making the right-hand side of Eq.(13) be equal to zero. Thus,

$$\begin{aligned} u(k) &= -N_r^{-1}Q^{-1}(z)\{D_d(z)R(z)N(z) - Q(z)N_r\}u(k) \\ &\quad - N_r^{-1}Q^{-1}(z)D_d(z)R(z)N_f(z)f(v(k)) - N_r^{-1}Q^{-1}(z)S(z)y(k) + u_m(k) \end{aligned} \tag{14}$$

$$u_m(k) = N_r^{-1}Q^{-1}(z)T(z)N_m(z)r_m(k). \tag{15}$$

Here, $Q(z) = \text{diag}[z^{\delta_i}]$, $\delta_i = \rho + n_m - n + \sigma_i (i = 1, 2, \dots, \ell)$, and $u(k)$ of Eq.(14) is obtained from $e(k) = 0$. The model following control system can be realized if the system internal states are bounded.

4. Proof of the bounded property of internal states

System inputs are both reference input signal $r_m(k)$ and disturbances $d(k), d_0(k)$, which are all assumed to be bounded. The bounded property can be easily proved if there is no nonlinear part $f(v(k))$. But if $f(v(k))$ exists, the bound has a relation with it.

The state space expression of $u(k)$ is

$$u(k) = -H_1\xi_1(k) - E_2y(k) - H_2\xi_2(k) - E_3f(v(k)) - H_3\xi_3(k) + u_m(k) \tag{16}$$

$$u_m(k) = E_4r_m(k) + H_4\xi_4(k). \tag{17}$$

The following must be satisfied:

$$\xi_1(k+1) = F_1\xi_1(k) + G_1u(k) \quad (18)$$

$$\xi_2(k+1) = F_2\xi_2(k) + G_2y(k) \quad (19)$$

$$\xi_3(k+1) = F_3\xi_3(k) + G_3f(v(k)) \quad (20)$$

$$\xi_4(k+1) = F_4\xi_4(k) + G_4r_m(k). \quad (21)$$

Here,

$$|zI - F_i| = |Q(z)|, \quad (i = 1, 2, 3, 4).$$

Note that there are connections between the polynomial matrices and the system matrices, as follows:

$$N_r^{-1}Q^{-1}(z)\{D_d(z)R(z)N(z) - Q(z)N_r\} = H_1(zI - F_1)^{-1}G_1 \quad (22)$$

$$N_r^{-1}Q^{-1}(z)S(z) = H_2(zI - F_2)^{-1}G_2 + E_2 \quad (23)$$

$$N_r^{-1}Q^{-1}(z)D_d(z)R(z)N_f(z) = H_3(zI - F_3)^{-1}G_3 + E_3 \quad (24)$$

$$N_r^{-1}Q^{-1}(z)T(z)N_m(z) = H_4(zI - F_4)^{-1}G_4 + E_4. \quad (25)$$

Firstly, remove $u(k)$ from Eq.(1)~(3) and Eq.(18)~(21). Then, the representation of the overall system can be obtained as follows.

$$\begin{bmatrix} E & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \begin{bmatrix} x(k+1) \\ \xi_1(k+1) \\ \xi_2(k+1) \\ \xi_3(k+1) \end{bmatrix} = \begin{bmatrix} A - BE_2C & -BH_1 & -BH_2 & -BH_3 \\ -G_1E_2C & F_1 - G_1H_1 & -G_1H_2 & -G_1H_3 \\ G_2C & 0 & F_2 & 0 \\ 0 & 0 & 0 & F_3 \end{bmatrix} \begin{bmatrix} x(k) \\ \xi_1(k) \\ \xi_2(k) \\ \xi_3(k) \end{bmatrix} + \quad (26)$$

$$+ \begin{bmatrix} BH_4 \\ G_1H_4 \\ 0 \\ 0 \end{bmatrix} \xi_4(k) + \begin{bmatrix} B_f - BE_3 \\ -G_1E_3 \\ 0 \\ G_3 \end{bmatrix} f(v(k)) + \begin{bmatrix} BE_4 \\ G_1E_4 \\ 0 \\ 0 \end{bmatrix} r_m(k) + \begin{bmatrix} d(k) - BE_2d_0(k) \\ -G_1E_2d_0(k) \\ G_2d_0(k) \\ 0 \end{bmatrix}$$

$$\xi_4(k+1) = F_4\xi_4(k) + G_4r_m(k) \quad (27)$$

$$v(k) = \begin{bmatrix} C_f & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x(k) \\ \xi_1(k) \\ \xi_2(k) \\ \xi_3(k) \end{bmatrix} \tag{28}$$

$$y(k) = \begin{bmatrix} C & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x(k) \\ \xi_1(k) \\ \xi_2(k) \\ \xi_3(k) \end{bmatrix} + d_0(k). \tag{29}$$

In Eq.(27), the $\xi_4(k)$ is bounded because $|zI - F_4| = |Q(z)|$ is a stable polynomial and $r_m(k)$ is the bounded reference input. Let $z(k), A_s, \tilde{E}, d_s(k), B_s, C_v, C_s$ be as follows respectively:

$$z(k) = \begin{bmatrix} x^T(k) & \xi_1^T(k) & \xi_2^T(k) & \xi_3^T(k) \end{bmatrix}^T, \quad A_s = \begin{bmatrix} A - BE_2C & -BH_1 & -BH_2 & -BH_3 \\ -G_1E_2C & F_1 - G_1H_1 & -G_1H_2 & -G_1H_3 \\ G_2C & 0 & F_2 & 0 \\ 0 & 0 & 0 & F_3 \end{bmatrix}$$

$$\tilde{E} = \begin{bmatrix} E & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix}, \quad d_s(k) = \begin{bmatrix} Bu_m(k) + d(k) - BE_2d_0(k) \\ G_1u_m(k) - G_1E_2d_0(k) \\ G_2d_0(k) \\ 0 \end{bmatrix}, \quad B_s = \begin{bmatrix} B_f - BE_3 \\ -G_1E_3 \\ 0 \\ G_3 \end{bmatrix}$$

$$C_v = \begin{bmatrix} C_f & 0 & 0 & 0 \end{bmatrix}, \quad C_s = \begin{bmatrix} C & 0 & 0 & 0 \end{bmatrix}.$$

With the consideration that $\xi_4(k)$ is bounded, the necessary parts to an easy proof of the bounded property are arranged as

$$\tilde{E}z(k+1) = A_s z(k) + B_s f(v(k)) + d_s(k) \tag{30}$$

$$v(k) = C_v z(k) \tag{31}$$

$$y(k) = C_s z(k) + d_0(k), \tag{32}$$

where the contents of $A_s, \tilde{E}, d_s(k), B_s, C_v, C_s$ are constant matrices, and $d_s(k)$ is bounded. Thus, the internal states are bounded if $z(k)$ can be proved to be bounded. So it needs to prove that $|z\tilde{E} - A_s|$ is a stable polynomial. The characteristic polynomial of A_s is calculated as the next equation.

From Eq.(26), $|z\tilde{E} - A_s|$ can be shown as

$$|z\tilde{E} - A_s| = \begin{vmatrix} zE - A + BE_2C & BH_1 & BH_2 & BH_3 \\ G_1E_2C & zI - F_1 + G_1H_1 & G_1H_2 & G_1H_3 \\ -G_2C & 0 & zI - F_2 & 0 \\ 0 & 0 & 0 & zI - F_3 \end{vmatrix}. \tag{33}$$

Prepare the following formulas:

$$\begin{vmatrix} X & Y \\ W & Z \end{vmatrix} = |Z| |X - YZ^{-1}W|, (|Z| \neq 0),$$

$$I - X(I + YX)^{-1}Y = (I + XY)^{-1}$$

$$|I + XY| = |I + YX|.$$

Using the above formulas, $|z\tilde{E} - A_s|$ is described as

$$\begin{aligned} & |z\tilde{E} - A_s| \\ &= |zI - F_3| |zI - F_2| |zI - F_1| \left| I + H_1 [zI - F_1]^{-1} G_1 \right| \\ & \cdot \left| zE - A + B \{ I - H_1 [zI - F_1 + G_1 H_1]^{-1} G_1 \} \{ E_2 + H_2 [zI - F_2]^{-1} G_2 \} C \right| \\ &= |Q(z)|^3 \left| I + H_1 [zI - F_1]^{-1} G_1 \right| \left| zE - A + B \{ I + H_1 [zI - F_1]^{-1} G_1 \}^{-1} \{ E_2 + H_2 [zI - F_2]^{-1} G_2 \} C \right| \quad (34) \\ &= |Q(z)|^3 |J_1| |zE - A| \left| I + B J_1^{-1} J_2 [zE - A]^{-1} \right| \\ &= |Q(z)|^3 |zE - A| \left| J_1 + J_2 [zE - A]^{-1} B \right|. \end{aligned}$$

Here

$$J_1 = I + H_1 [zI - F_1]^{-1} G_1 \quad (35)$$

$$J_2 = \{ E_2 + H_2 [zI - F_2]^{-1} G_2 \} C. \quad (36)$$

From Eq.(22),(23),(35) and Eq.(36), we have

$$J_1 = N_r^{-1} Q^{-1}(z) D_d(z) R(z) N(z) \quad (37)$$

$$J_2 = N_r^{-1} Q^{-1}(z) S(z) C. \quad (38)$$

Using $C[zE - A]^{-1} B = N(z) / D(z)$ and $D(z) = |zE - A|$, furthermore, $|z\tilde{E} - A_s|$ is shown as

$$|z\tilde{E} - A_s| = T^\ell(z) D_m^\ell(z) |Q(z)|^2 \frac{|N(z)| |N_r|^{-1}}{D^{\ell-1}(z)}$$

and $V(z)$ is the zeros polynomial of $C[zE - A]^{-1} B = N(z) / D(z) = U^{-1}(z)V(z)$ (left coprime decomposition), $|U(z)| = D(z)$, that is, $|N(z)| = D^{\ell-1}(z)|V(z)|$. So $|z\tilde{E} - A_s|$ can be rewritten as

$$|z\tilde{E} - A_s| = |N_r|^{-1} T^\ell(z) D_m^\ell(z) |Q(z)|^2 |V(z)| \quad (39)$$

As $T(z), D_m(z), |Q(z)|, |V(z)|$ are all stable polynomials, A_s is a stable system matrix. Consider the following:

$$z(k) = Q\bar{z}(k) = Q \begin{bmatrix} \bar{z}_1(k) \\ \bar{z}_2(k) \end{bmatrix}. \tag{40}$$

Using Eq.(40), one obtains

$$P\tilde{E}Q\bar{z}(k+1) = PA_sQ\bar{z}(k) + PB_s f(v(k)) + Pd_s(k).$$

Namely,

$$\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{z}(k+1) \\ \bar{z}_2(k+1) \end{bmatrix} = \begin{bmatrix} A_{s1} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \bar{z}_1(k) \\ \bar{z}_2(k) \end{bmatrix} + \begin{bmatrix} B_{s1} \\ B_{s2} \end{bmatrix} f(v(k)) + \begin{bmatrix} d_{s1}(k) \\ d_{s2}(k) \end{bmatrix} \tag{41}$$

One can rewritten Eq.(41) as

$$\bar{z}_1(k+1) = A_{s1}\bar{z}_1(k) + B_{s1}f(v(k)) + d_{s1}(k) \tag{42}$$

$$0 = \bar{z}_2(k) + B_{s2}f(v(k)) + d_{s2}(k) \tag{43}$$

, where $\bar{z}(k), Pd_s(k), PA_sQ, PB_s$ can be represented by

$$\bar{z}(k) = \begin{bmatrix} \bar{z}_1(k) \\ \bar{z}_2(k) \end{bmatrix}, Pd_s(k) = \begin{bmatrix} d_{s1}(k) \\ d_{s2}(k) \end{bmatrix}, PA_sQ = \begin{bmatrix} A_{s1} & 0 \\ 0 & I \end{bmatrix}, PB_s = \begin{bmatrix} B_{s1} \\ B_{s2} \end{bmatrix}. \tag{44}$$

Let $C_vQ = [C_{v1} \ C_{v2}]$, then

$$v(k) = C_{v1}\bar{z}_1(k) + C_{v2}\bar{z}_2(k). \tag{45}$$

From Eq.(43) and Eq.(45), we have

$$v(k) + C_{v2}B_{s2}f(v(k)) = C_{v1}\bar{z}(k) - C_{v2}d_{s2}(k). \tag{46}$$

From Eq.(46), we have

$$\frac{\partial}{\partial v^T(k)}(v(k) + C_{v2}B_{s2}f(v(k))) = I + C_{v2}B_{s2} \frac{\partial f(v(k))}{\partial v^T(k)}.$$

Existing condition of $v(k)$ is

$$\left| I + C_{v2}B_{s2} \frac{\partial f(v(k))}{\partial v^T(k)} \right| \neq 0. \tag{47}$$

From Eq.(44), we have

$$|P||z\tilde{E} - A_s||Q| = \alpha_{PQ}|z\tilde{E} - A_s| = \begin{vmatrix} zI - A_{s1} & 0 \\ 0 & -I \end{vmatrix} = \alpha_I|zI - A_{s1}|. \tag{48}$$

Here, α_{PQ} and α_I are fixed. So, from Eq.(39), A_{s1} is a stable system matrix. Consider a quadratic Lyapunov function candidate

$$V(k) = \bar{z}_1^T(k)P_s\bar{z}_1(k). \tag{49}$$

The difference of $V(k)$ along the trajectories of system Eq.(42) is given by

$$\begin{aligned}\Delta V(k) &= V(k+1) - V(k) \\ &= \bar{z}_1^T(k+1)P_s\bar{z}_1(k+1) - \bar{z}_1^T(k)P_s\bar{z}_1(k) \\ &= [A_{s1}\bar{z}_1(k) + B_{s1}f(v(k)) + d_{s1}(k)]^T P_s [A_{s1}\bar{z}_1(k) + B_{s1}f(v(k)) + d_{s1}(k)] - \bar{z}_1^T(k)P_s\bar{z}_1(k) \\ &= A_{s1}^T P_s A_{s1} - P_s = -Q_s,\end{aligned}\tag{50}$$

where Q_s and P_s are symmetric positive definite matrices defined by Eq.(51). If A_{s1} is a stable matrix, we can get a unique P_s from Eq.(51) when Q_s is given. As $d_{s1}(k)$ is bounded and $0 \leq \gamma < 1$, $\Delta V(k)$ satisfies

$$\begin{aligned}\Delta V(k) &\leq -\bar{z}_1^T(k)Q_s\bar{z}_1(k) + X_1\|\bar{z}_1(k)\| \|f(v(k))\| \\ &\quad + X_2\|\bar{z}_1(k)\| + \mu_2\|f(v(k))\|^2 + X_3\|f(v(k))\| + X_4\end{aligned}\tag{52}$$

From Eq.(40), we have

$$\|\bar{z}_1(k)\| \leq M\|z(k)\|.\tag{53}$$

Here, M is positive constant. From Eq.(52), Eq.(53), we have

$$\begin{aligned}\Delta V(k) &\leq -\mu_1\|z(k)\|^2 + X_5\|\bar{z}_1(k)\|^{1+\gamma} + X_6 \\ &\leq -\mu_c\|z(k)\|^2 + X \\ &\leq -\mu_{c1}\|\bar{z}_1(k)\|^2 + X \\ &\leq -\mu_m V(k) + X,\end{aligned}\tag{54}$$

where $0 < \mu_1 = \lambda_{\min}(Q_s), \mu_2 \geq 0$ and $0 < \mu_m < \mu_c < \min(\mu_1, 1)$. Also, $\mu_1, \mu_2, X_i (i = 1 \sim 6)$ and X are positive constants. As a result of Eq.(54), $V(k)$ is bounded:

$$V(k) \leq V(0) + X / \mu_m.\tag{55}$$

Hence, $\bar{z}_1(k)$ is bounded. From Eq.(43), $\bar{z}_2(k)$ is also bounded. Therefore, $z(k)$ is bounded. The above result is summarized as Theorem1.

[Theorem1]

In the nonlinear system

$$\begin{aligned}Ex(k+1) &= Ax(k) + Bu(k) + B_f f(v(k)) + d(k) \\ v(k) &= C_f x(k) \\ y(k) &= Cx(k) + d_0(k),\end{aligned}\tag{56}$$

where $x(k) \in R^n, u(k) \in R^\ell, y(k) \in R^\ell, v(k) \in R^{\ell_f}, d(k) \in R^n, d_0(k) \in R^\ell, f(v(k)) \in R^{\ell_f}$, $d(k)$ and $d_0(k)$ are assumed to be bounded. All the internal states are bounded and the output error $e(k) = y(k) - y_m(k)$ asymptotically converges to zero in the design of the model following control system for a nonlinear descriptor system in discrete time, if the following conditions are held:

1. Both the controlled object and the reference model are controllable and observable.
2. $|N_r| \neq 0$.
3. Zeros of $C[zE - A]^{-1}B$ are stable.
4. $\|f(v(k))\| \leq \alpha + \beta \|v(k)\|^\gamma, (\alpha \geq 0, \beta \geq 0, 0 \leq \gamma < 1)$.
5. Existing condition of $v(k)$ is $\left| I + C_{v2}B_{s2} \frac{\partial f(v(k))}{\partial v^T(k)} \right| \neq 0$.
6. $|zE - A| \neq 0$ and $rankE = \deg|zE - A| = r \leq n$.

5. Numerical simulation

An example is given as follows:

$$\begin{aligned}
 \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} x(k+1) &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.2 & -0.5 & 0.6 \end{bmatrix} x(k) + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} u(k) + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} f(v(k)) + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} d(k) \\
 v(k) &= [1 \ 1 \ 1]x(k), \\
 y(k) &= \begin{bmatrix} 0 & 0.1 & 0 \\ 0.1 & 0 & 0.1 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\
 f(v(k)) &= \frac{3v^3(k) + 4v(k) + 1}{1 + v^4(k)}.
 \end{aligned}
 \tag{57}$$

Reference model is given by

$$\begin{aligned}
 x_m(k+1) &= \begin{bmatrix} 0 & 1 \\ -0.12 & 0.7 \end{bmatrix} x_m(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r_m(k) \\
 y_m(k) &= [1 \ 0]x_m(k) \\
 r_m(k) &= \sin(k\pi / 16).
 \end{aligned}
 \tag{58}$$

In this example, disturbances $d(k)$ and $d_0(k)$ are ramp and step disturbances respectively. Then $d(k)$ and $d_0(k)$ are given as

$$\begin{aligned}
 d(k) &= 0.05(k - 85), (85 \leq k \leq 100) \\
 d_0(k) &= 1.2, (20 \leq k \leq 50)
 \end{aligned}
 \tag{59}$$

We show a result of simulation in Fig. 1. It can be concluded that the output signal follows the reference even if disturbances exit in the system.

6. Conclusion

In the responses (Fig. 1) of the discrete time model following control system for nonlinear descriptor system, the output signal follows the references even though disturbances exit in the system. The effectiveness of this method has thus been verified. The future topic is that the case of nonlinear system for $\gamma \geq 1$ will be proved and analysed.

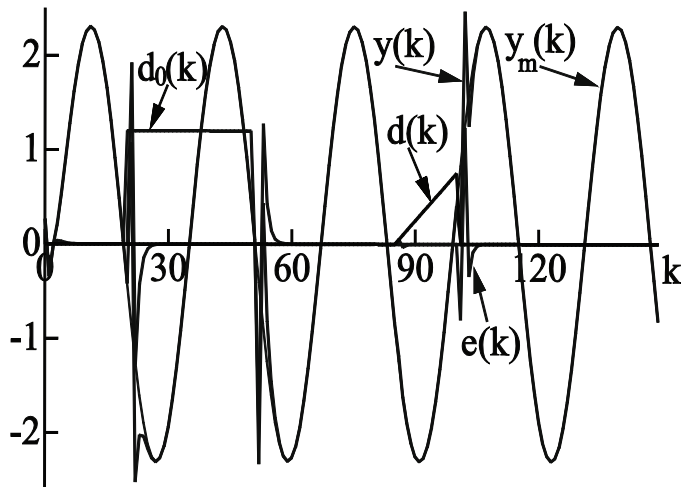
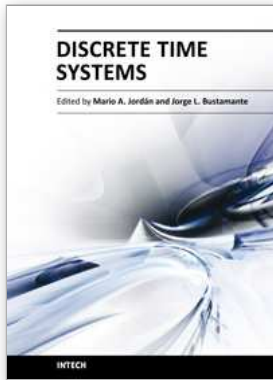


Fig. 1. Responses of the system for nonlinear descriptor system in discrete time

7. References

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Discrete-Time Systems comprehend an important and broad research field. The consolidation of digital-based computational means in the present, pushes a technological tool into the field with a tremendous impact in areas like Control, Signal Processing, Communications, System Modelling and related Applications. This book attempts to give a scope in the wide area of Discrete-Time Systems. Their contents are grouped conveniently in sections according to significant areas, namely Filtering, Fixed and Adaptive Control Systems, Stability Problems and Miscellaneous Applications. We think that the contribution of the book enlarges the field of the Discrete-Time Systems with signification in the present state-of-the-art. Despite the vertiginous advance in the field, we also believe that the topics described here allow us also to look through some main tendencies in the next years in the research area.

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