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Image Segmentation by Autoregressive Time Series Model

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1. Introduction

The objective of the image segmentation is to simplify the representation of pictures into meaningful information by partitioning into image regions. Image segmentation is a software technique to locate certain objects or boundaries within an image. There are many algorithms and techniques have been developed to solve image segmentation problems for the past 20 years, though, none of the method is a general solution. Among the best, they are neural networks segmentation, one-dimensional signal segmentation, multi-scale segmentation, model based segmentation, graphic partitioning, region growing and K-mean clustering segmentation methods. In this chapter, the newly developed Autoregressive (AR) time series model will be introduced for image segmentation.

Time series statistical models such as Autoregressive Moving Average (ARMA) were considered useful in describing the texture and contextual information of an image. To simplify the computation, a two-dimensional (2-D) Autoregressive (AR) model was used instead. The 2-D AR time series model is particularly suitable to capture the rich image pixel contextual information. This model has been applied for both rough and smooth target surfaces and performed very well for image segmentation.

In the typical statistical approach of image segmentation, there are two broad classes of segmentation procedures: The supervised and the unsupervised segmentation methods. The unsupervised segmentation procedure is the means by which pixels in the image are assigned to classes without prior knowledge of the existence or labeling of the classes. Whereas, in the supervised learning process, a teacher provides a label and cost function for each pattern in a training set and tries to minimize the sum of cost function for all patterns. Each method finds its own applications in the areas of the image analysis. The Support Vector Machine, a close cousin of classical multilayer perceptron neural networks and a newer supervised segmentation procedure, was adopted after feature extraction for single AR model image or pixel features vector extraction from multi-spectral image stack. On the other hand, the unsupervised region growing segmentation method was applied after univariate time series model was built.

For the experimental results by applying the proposed AR time series segmentation model, the USC texture data set as well as satellite digital remote sensing image data are used. The algorithms performance comparisons with other existing contextual models such as Markov Random Field model, K-means, PCA, ICA …etc can be found in reference (Ho, 2008).

2. 2-D Univariate time series image segmentation model and estimation

Time series analysis (Shumway 2000) (Wei, 1990) has been used in many applications, specifically the weather forecast, atmosphere-ocean model for global climate changes and the Dow Jones economics analysis. Interesting enough, at the University of Massachusetts ,Dartmouth School of Marine Science and Technology (SMAST) professor Joachim Gröger (Gröger 2007) can successfully trace a group of cod fishes' approximate swimming routes by time series model analysis on tidal data. In this section, we make use of time series concepts for image data mathematical formulation which is very new. To understand how, we introduce time series theory briefly. The primary objective of time series is to develop certain mathematical models that provide descriptions for sample data. We assume any image can be defined as a collection of random variables indexed according to the order they are obtained in time-space. For example, we consider a sequence of random variables $x_1, x_2, x_3,$ ...., in general, $\{x_t\}$ indexed by t is the stochastic process. The adjacent points in time are correlated. Therefore, the value of series x_t at time t depends in some fashion on the past values $x_{t-1}, x_{t-2},$ .... Suppose that we let the value of the time series at some point of time t to be denoted by x_t. A stationary time series is the one for which the probabilistic behavior of $x_{t_1}, x_{t_2},$ ....x_{t_k} is identical to that of the shifted set $x_{t_1+h}, x_{t_2+h},$ ...., x_{t_k+h}. In an image segmentation application, the 2-D image was scanned from left upper corner to right bottom as a sequence of time series pixel values. Further, to simplify the numerical calculations, we model each class of surface textures by 1st order and 2nd order Autoregressive stationary time series models. In another way of thinking, the two-dimensional Markov model is a similar mathematical model to describe. By using time series model, when the within-object interpixel correlation varies significantly from object to object, we can build effective imaging region segementation. The unsupervised Region Growing is a powerful image segmentation method for use in shape classification and analysis. Take one satellite remote sensing example to show, we present the LANDSAT 5 database in the area of Italy's Lake Mulargias image data acquired in July 1996 to be used for the computing experiments with satisfactory preliminary results (figure 1, figure 2 and figure 3). The advanced statistical techniques, such as Gaussian distributed white noise error confidence interval calculations, sampling statistics based on mean and variance properties are adopted for automatic threshold finding during Region Growing iterations. The linear regression analysis with least mean squares error estimation is implemented as a time series system optimization scheme.

2.1 Image modeling and estimation theory

The raw image data are statistically random and can be modeled with some basic ideas of time series analysis and stochastic processes. It is based on a model which treats the pixels (picture elements) of a digitized textural scene as a two-way seasonal time series. A time series is a set of observations, each one is recorded at a specific time t. The target image is well treated as a time series in which the observations are made at the fixed time intervals. Therefore, for a two dimensional image, each pixel gray tone information G(x,y) is equivalent to G(t) in the time series analysis. In the multi-spectral scanner images, the texture information is one of the important characteristics for machine vision analysis. The texture classification techniques include determination of a number of classes to which the texture region belongs. It estimates texture model parameters for a region and constructs the boundaries between different texture regions. The standard probability and the statistics concepts, such as autocovariance and sample covariance functions are considered. The techniques in parameter estimation theory to extract texture typed trend and seasonality from the observed time series are adapted for image segmentation analysis. Due to the fact that rarely is there an exact priori knowledge about the image data, a mathematical model is often used. The time series stationary Autoregressive (AR) models on both the first-order and the second-order with Region Growing algorithm are studied and the results are presented below.

2.2 First-order autoregressive image modeling and parameter estimation

In the first-order AR analysis, each gray tone pixel value of the image is estimated through its neighboring pixels. This is based on the assumption of a causal, linear time invariant remote sensing system. The gray level of the (i, j) th pixel of the remote sensing image is expressed by the equation:

$$x(i,j) = a + b * [x(i-1,j) + x(i,j-1)] + n(i,j) \tag{1}$$

where $n(i,j)$ is modeled as a Gaussian white noise random process. Let $\hat{x}(i,j)$ be an estimate of $x(i,j)$. Note that for an 100 by 100 image array, $i = 1:100$ is the row index, $j = 1:100$ is the column index of the two dimensional image example. Starting with a set of region seeds, the weighted coefficients are determined through unsupervised region growing iterative procedure to group pixels of a subregion into larger regions. The ground truth data were provided as "seeds" for initial processing. The criterion is to minimize the mean square error MSE formulated as (Seber 1977), (Papoulis 2002):

$$MSE = \sum_{i=1}^{100}\sum_{j=1}^{100} E\{[x(i,j) - \hat{x}(i,j)]^2\}$$

which will be a minimum if setting

$$\frac{\partial(MSE)}{\partial a} = 0 \qquad \frac{\partial(MSE)}{\partial b} = 0 \tag{2}$$

By ordinary linear regression calculation, we get the following optimal least squares estimator of the first order AR parameters:
Let

$$z(i,j) = x(i-1,j) + x(i,j-1)$$

Numerator $$= \sum_{i=1}^{M}\sum_{j=1}^{N} z(i,j) * x(i,j) \text{ - } M * N * E[z(i,j)] * E[x(i,j)] \tag{3}$$

Denominator $$= \sum_{i=1}^{M}\sum_{j=1}^{N} [z(i,j)]^2 - M * N * [E(z(i,j))]^2 \tag{4}$$

M = Total region growing sample row pixels number
N = Total region growing sample column pixels number

$$\hat{b} = \frac{\text{Numerator}}{\text{Denominator}} \tag{5}$$

$$\hat{a} = E(x(i,j)) - \hat{b} * E(z(i,j)) \tag{6}$$

Note that $E(x(i,j))$ denotes the expectation value.

2.3 Second-order autoregressive image modeling and parameter estimation

In the following analysis, each gray tone pixel value of the image is estimated through weighted summation of its neighborhood pixels. This is based on an assumption that a causal, linear time invariant remote sensing system can be modeled mathematically. The gray level of the (i,j) th pixel of the remote sensing image is expressed by the following formula:

$$x(i,j) = a + bx(i-1,j) + cx(i,j-1) + dx(i-1,j-1) + n(i,j) \tag{7}$$

where $n(i,j)$ is assumed a Gaussian white noise random process. Let $\hat{x}(i,j)$ be an estimate of $x(i,j)$ Note that $i = 1$ to M is the row index, $j =$ 1 to N is the column index of the two dimensional image. Starting with a set of region seeds, the weighted coefficients are determined through unsupervised region growing iterative procedure to group pixels of a subregion into larger regions. The ground truth "seeds" data were used for initialization. Again, the same as the 1st order, the criterion is to minimize the mean square error MSE (Haykin 2003) that is formulated as:

$$MSE = \sum_{i=1}^{M}\sum_{j=1}^{N} E\{[x(i,j) - \hat{x}(i,j)]^2\} \tag{8}$$

which will be a minimum if setting

$$\frac{\partial(MSE)}{\partial a} = 0; \quad \frac{\partial(MSE)}{\partial b} = 0; \quad \frac{\partial(MSE)}{\partial c} = 0; \quad \frac{\partial(MSE)}{\partial d} = 0; \tag{9}$$

By centering and scaling of the linear regression data, we can easily find the following matrix operation which relates estimated parameter matrix with covariance matrix of the sample of one image class region.

$$P * \hat{T} = Q \tag{10}$$

Let $\sum x(i,j)$ be the abbreviation of $\sum_{i=1,j=1}^{M,N} x(i,j)$ due to limited mathematical symbol spacing.

M = Total region growing sample row pixels number
N = Total region growing sample column pixels number
Then,

$$Q = \begin{bmatrix} \sum x(i,j) \\ \sum x(i-1,j)x(i,j) \\ \sum x(i,j-1)x(i,j) \\ \sum x(i-1,j-1)x(i,j) \end{bmatrix} \tag{11}$$

$$\hat{T} = \begin{bmatrix} \hat{a} \\ \hat{b} \\ \hat{c} \\ \hat{d} \end{bmatrix} = \text{The optimized parameter matrix} \tag{12}$$

$$P = \begin{bmatrix} M*N & \sum x(i-1,j) & \sum x(i,j-1) & \sum x(i-1,j-1) \\ \sum x(i-1,j) & \sum x(i-1,j)^2 & \sum x(i-1,j)x(i,j-1) & \sum x(i-1,j)x(i-1,j-1) \\ \sum x(i,j-1) & \sum x(i-1,j)x(i,j-1) & \sum x(i,j-1)^2 & \sum x(i,j-1)x(i-1,j-1) \\ \sum x(i-1,j-1) & \sum x(i-1,j-1)x(i-1,j) & \sum x(i-1,j-1)x(i,j-1) & \sum x(i-1,j-1)^2 \end{bmatrix} \tag{13}$$

In which, the estimated parameters $\hat{a}$, $\hat{b}$, $\hat{c}$, $\hat{d}$ can be found from $\hat{T}$ matrix. Also the optimal estimated pixel gray level can be calculated as

$$\hat{x}(i,j) = \hat{a} + \hat{b}x(i-1,j) + \hat{c}x(i,j-1) + \hat{d}x(i-1,j-1) \tag{14}$$

The more iterations performed to grow the region, the more accurate the estimated parameters.

Now, let

$$e(i,j) = \hat{x}(i,j) - x(i,j) \tag{15}$$

Decision making for region growing:

A two class discrimination decision can be made by:

$$\text{Set class\_label} = 1 \text{ if } e(i,j) < \text{Threshold} * \sqrt{MSE(optimal)} \tag{16}$$

$$\text{Set class\_label} = 0 \quad \text{otherwise}$$

Where $MSE(optimal)$ is the mean square error calculation of the growing region.

Based on statistical hypotheses and significance property of the sampling theory plus the Gaussian distribution nature of the pixel error populations, the 95% confidence interval threshold can be found as:

$$\text{Threshold} = \sqrt{2/\pi} + \sqrt{1-2/\pi} \tag{17}$$

To prove (17) for the threshold:

$$\text{Threshold} = \sqrt{2/\pi} + \sqrt{1-2/\pi}\,; \quad \text{let } SD = MSE(optimal) \tag{18}$$

The error Gaussian distribution function can be expressed as

$$f(e(I,J))=\frac{1}{\sqrt{2\pi}SD}\exp\left[-1/2(e(I,J)/SD)^2\right] \tag{19}$$

$$E(|e(I,J)|)= 2\int_0^\infty e(I,J)(\frac{1}{\sqrt{2\pi}SD}\exp\left[-1/2(e(I,J)/SD)^2\right])\, d(e(I,J)) \tag{20}$$

$$= (\sqrt{2/\pi})*SD \tag{21}$$

$$M_{e(I,J)} = E[|e(I,J)|] \tag{22}$$

$$(\sigma_{e(I,J)})^2 = E\{(e(I,J) - M_{e(I,J)})^2\} \tag{23}$$

$$= (SD^2) - ((\sqrt{\frac{2}{\pi}})*SD)^2 \tag{24}$$

$$= (1-\frac{2}{\pi})(SD^2) \tag{25}$$

$$\sigma_{e(I,J)} = \sqrt{(1-\frac{2}{\pi})}*SD \tag{26}$$

For 95% confidence of 1-sigma interval value

$$e(I,J) < E[|e(I,J)|] + \sqrt{(1-\frac{2}{\pi})}*(SD)= \sqrt{\frac{2}{\pi}}*(SD) + \sqrt{(1-\frac{2}{\pi})}*(SD) \tag{27}$$

$$= (\sqrt{2/\pi} + \sqrt{1-2/\pi})*(SD) \tag{28}$$

An example of experiment for explanation:

As shown in figure 1, figure 2 and figure 3, after certain iterations of Lake Mulargias (water) region growing (when we are assured that no more water pixel can be found), we stop processing. By unsupervised region growing the optimized parameter matrix $\hat{T}$ can be determined to classify the entire image.

2.4 Region growing algorithm

Region Growing, as implied by its name, is an algorithm that groups pixel subregions into bigger areas. It starts with a set of known seeds (ground truth information, for example, in figure 2) to grow regions by merging to each seed point those neighborhood pixels that have similar statistical features.

In remote sensing image analysis and segmentation application, there are just a few researchers who proposed or attempted to use region growing. As a matter of fact, its practical implementations have proven that in general, it does work well in most circumstances as far as region segmentation is concerned. In the IEEE IGARSS 2000 paper, Dr. Tilton and Lawrence (Tilton 2000) presented the interactive hierarchical segmentation to

grow a region. But, this method has some limitations and can only apply to certain scenes only. The Region Growing method we proposed is a different approach and can deal with the shortfall they have. Other useful references on the subject can be found in Reference (Haralick 1992).
A basic region growing procedure is Outlined in Table 1.

```
ITERATION_COUNT = 1; MAX_ITERATION = 100;
Border of R = lake seed (water);
Start with lake seed, calculate and estimate A, B, C, D parameters
DO  until ITERATION_COUNT > MAX_ITERATION or NOT LAKE CLASS
  Increment border of R by one pixel up, down, left, right to R'
  for each pixel p at the border of R'-R do
          if  ( |e(I,J)| < Threshold * SD ), set CLASS of p = LAKE;
          else
                    CLASS of p = OTHER;      /* CLASSIFICATION */
          Updated calculation of A,B,C,D parameters for R' region with new CLASS LABEL
  End
  ITERATION_COUNT ++ ;
End
```

Table 1. AR Model Based Region Growing Algorithm

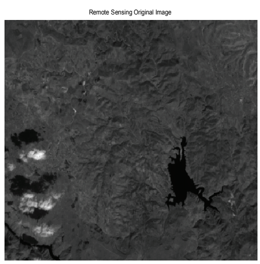


Fig. 1. Original Lake Region Remote Sensing band 5 Image

Fig. 2. Ground Truth Information

Fig. 3. Segmentation Result After Region Growing Based On AR Model

In this chapter, we have introduced a new univariate time series AR model based region growing algorithm, which attempts to extract a special property region such as water at Lake Mulagias. As a matter of fact, this new algorithm can be further used to apply to other remote sensing imagery area, such as corn field, city block, grassland, pasture … etc for scene classifications. The computer experiments show that the Time series Region Growing algorithm is efficient for the remote sensing image region segmentation problems.

A Region growing is a procedure that group pixels or subregions into larger regions based on predefined criteria (Ho 2004). By starting with a set of seed points, it forms the growing regions by appending to each seed of the neighboring pixels that have similar properties to the seed points. The process is in the iteration mode as depicted in figure 4, where (a) is the region

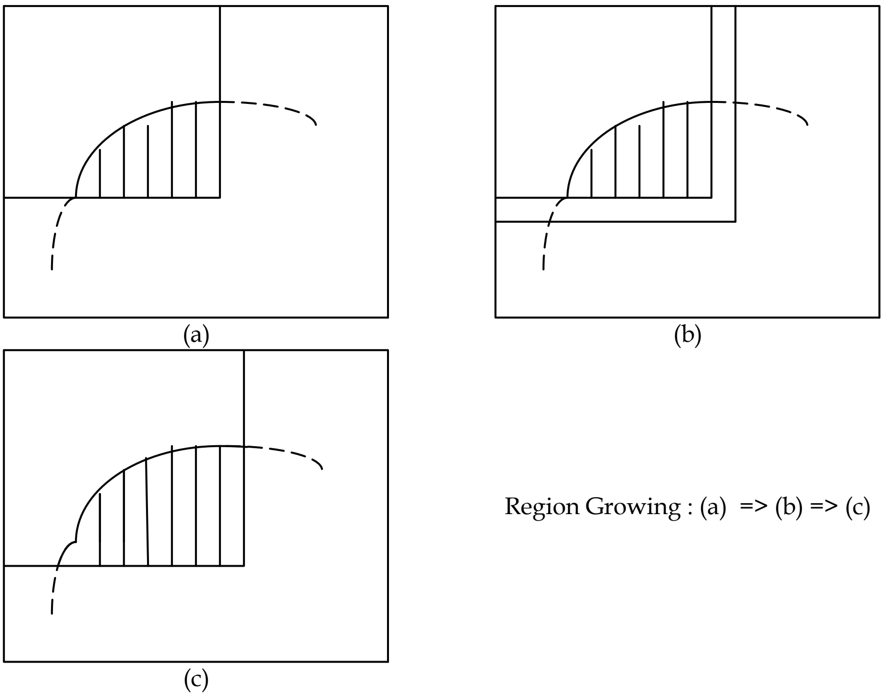


Fig. 4. Region Growing demonstration in graphics (a) => (b) => (c)

before growing, (b) is in the process of growing one pixel layer to the right and one pixel layer toward bottom. After certain decision making, the new region was formed as in (c).
There are many different ways of region growing depends on programmer's preferences. Usually, the first step of image analysis is to partition an input image that is more than one uniform region into several homogeneous subimages (Gonzalez 2004). The region of homogeneity can be determined by many different properties. In region growing method, the image segmentation is implemented by extending some starting pixels or internal small area inside regions to boundaries between texture areas. Pixels in a homogeneous region are expected to be assigned by an identical class. The boundaries will be drawn by the difference in the adjacent class regions. We also need to make sure that regions with the same texture but in different parts of an image are labeled by the same class. In the remote sensing and USC texture type data examples, the texture property for each observed window is extracted from time series 2-D AR model. Under each iteration, the unknown regions were segmented into blocks. Each internal region is expanded by comparing the properties of its surrounding blocks. As texture boundaries are reached, the size of blocks is reduced, each block might contains only one pixel at the end.
Figure 5 shows an example result on USC data set for the effectiveness of the AR time series image segementation.

(a) (b) (c)

Fig. 5. (a) Orginal 3-classes real texture image from USC data base (brickwall, grass and pigskin) (b) The segmentation result after 2-D time series multiclass SVM in black and white display (c) The Segmentation result after 2-D time series multiclass SVM in color display

2.5 Higher order AR model

The 3rd order of AR image model is described as:

$$x(i,j) = a + bx(i-1,j) + cx(i,j-1) + dx(i-1,j-1) + ex(i-2,j) + fx(i,j-2) + n(i,j) \qquad (29)$$

We have done some experiments on the same Lake Mulagias remote sensing data set as well as USC texture data set by 3rd order AR model, but the region segmentation results did not improve further. This told us that the term higher than 2nd order had no impact. The AR model order selection criteria is based on sample estimated error or residuals.

2.6 Experimental results

The following figures from Figure 6 to Figure 13 are the texture and remote sensing segmentation results:

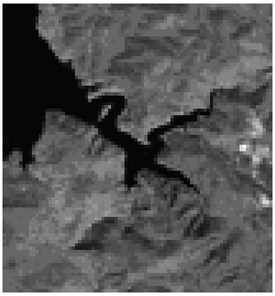


Fig. 6. Original TM remote sensing image of Italy Lake Mulagias

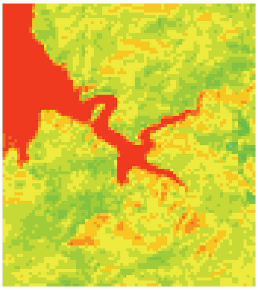


Fig. 7. Original Italy Lake Mulagias displayed in color format

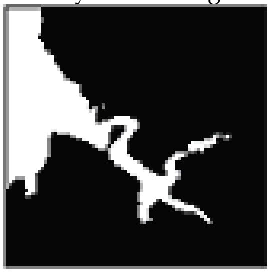


Fig. 8. Segmentation Result by 2nd Order AR in black and white

Fig. 9. Segmentation Result by 2nd Order ARMA in color format

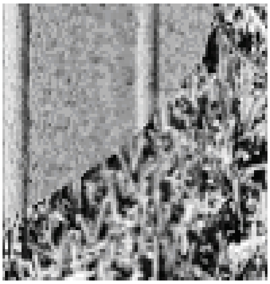


Fig. 10. Original USC brick and grass two classes natural scene

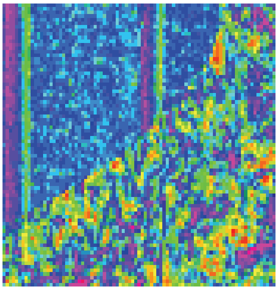


Fig. 11. Original USC brick and grass displayed in color format

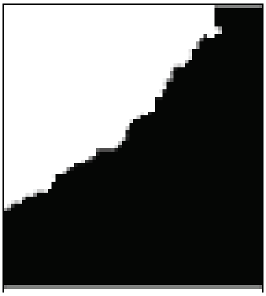
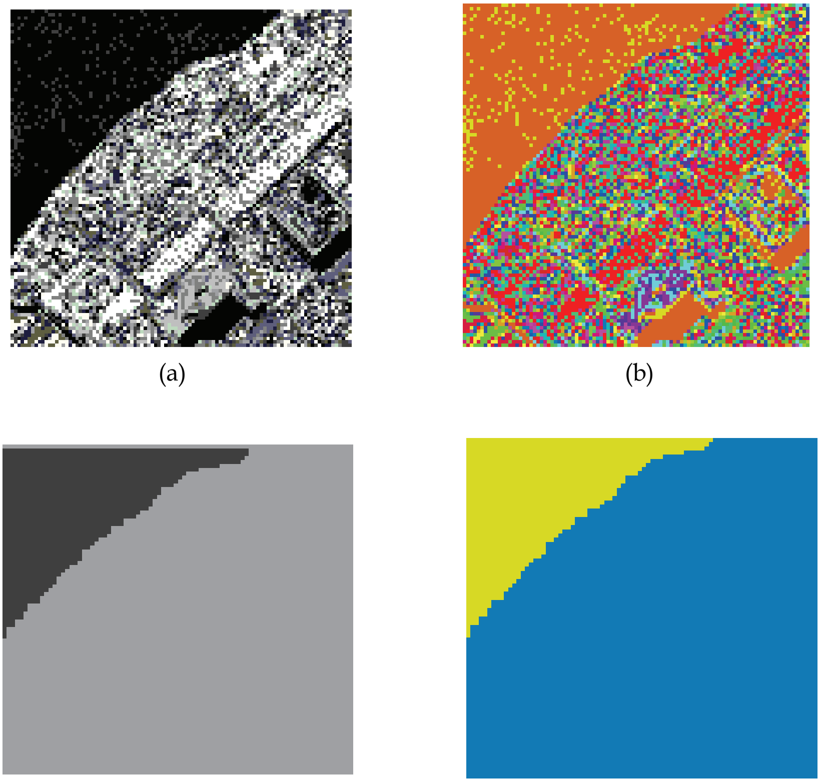


Fig. 12. Texture Segmentation Result by 2nd Order AR in black and white

Fig. 13. Texture Segmentation Result by 2nd Order AR in color format

The 2-D time series model based support vector machine (Ho, 2007) was tested with real remote sensing images from satellite and the results in figure 14 shows that this method can perform well in separating lake region from city building with the accuracy of 96.9%.



(a) (b)

(b) (d)

Fig. 14. (a) Original 2 Classes Remote Sensing Image in black and white mode
(b) Original 2 Classes Remote Sensing Image in color mode
(c) The Segmentation Result 2 classes SVM Method
(d) The Segmentation Result after 2 classes SVM Method in color display

3. Contexture image models

A problem of particular importance in remote sensing image classification is the use of contextual information. All experiments indicate that classification performance improves with proper use of contextual information. Because of interest with texture analysis, and the fact that remote sensing data is rich in texture, this problem has been studied by pattern recognition researchers for some time with a number of approaches presented.

Markov random field provides a rigorous mathematical characterization of contextual information of textures from neighboring pixels on an image. Basically the often rich textural information in remote sensing image makes Markov random field a very powerful image model. Another image model is the 2-D multivariate autoregressive time series analysis. The Markov random field model is for a single image while the 2-D multivariate autoregeessive model is for the entire stack of images under consideration and treated as a vector time series. The two image models will be briefly presented in this section.

3.1 Markov random field

Markov random field (MRF) contextual image model assumes that the texture is stochastic and stationary with conditional independence. MRF theory provides a basis for modeling contextual constraints in image processing, analysis, and interpretation (Geman, 1984). Several methods are introduced and published about 2-D image restoration or reconstruction with MRF model. Very recently a comprehensive treatment of Markov random field in remote sensing is given by Serpico and Moser. The neighborhood system of a pixel x, in an MxN image, which contains the contextual information under consideration, consists of adjacent pixels with a Euclidean distance of say *r*. and *r* takes an integer value that is denoted as the order of neighborhood system. The neighborhood system with a definitive order is shown in Figure 15 (a), where 1, 2, ⋯⋯ denote the order of neighborhood system. The second order neighborhood system has eight neighboring pixels. Therefore, it is also called the 8-neighborhood system. In a Markov random field the conditional probability density function of each pixel is dependent only on its neighborhood system So the MRF represents the local characteristics of x. The MRF can be causal or noncausal. Assume that the pixels are scanned sequentially top to down and left to right. The causal MRF depends only on the past while the noncausal or bilateral MRF depends not only on the past but also on the future as shown in Figure 15 (b). In general, the neighborhood system used in image modeling is symmetrical and sequentially ordered. Also, the noncausal Markov chain is usually selected as 2-D image model in practice.

Each observed data $\mathbf{y}$ can be regarded as the noisy version of original data $\mathbf{x}$. In other words, $\mathbf{y}=\mathbf{x}+\mathbf{n}$, where $\mathbf{n}$ is a sample of a white Gaussian noise field with variance σ_n^2. The conditional probability density function of $\mathbf{x}$ given $\mathbf{y}$ is given by:

$$p(\mathbf{x}\,|\,\mathbf{y})=\frac{1}{(2\pi\sigma_n^2)^{-MN/2}}\exp\{-\frac{1}{2\sigma_n^2}(\mathbf{y}-\mathbf{x})^T(\mathbf{y}-\mathbf{x})\} \quad (30)$$

The estimation problem is to obtain the original data $\mathbf{x}$ from the observed data $\mathbf{y}$.

As proposed by Jeng and Woods (Jeng 1991), the relationship of a pixel in a compound Gaussian Markov random field (CGMRF) and its neighborhood system is presented by the collection of binary variables $C_o = \{c^k_{m,n} : k = 1, 2, \cdots, 8;\ m = 1, 2, \cdots, M;\ n = 1, 2, \cdots, N\}$, where k is the same as the Figure 15 (b). If the neighbor pixels are independent, the associated $c^k_{m,}$ is set to be one and otherwise $c^k_{m,n}$ is equal to zero. Let $A(C_0)$ be the covariance matrix of $\mathbf{x}$, which depends on C_0. The estimation of original image $\mathbf{x}$ is obtained with observed $\mathbf{y}$ and the matrix $A(C_0)$, which should be estimated from the observed $\mathbf{y}$. In general, we have

$$p(\mathbf{x}\,|\,C_0,\mathbf{y})\propto\frac{1}{\left(2\pi\sigma_n^2\right)^{-MN/2}*Z(C_0)}\exp\left\{-\frac{1}{2\sigma_x^2}\mathbf{x}^T A(C_0)\mathbf{x}-\frac{1}{2\sigma_n^2}(\mathbf{y}-\mathbf{x})^T I(\mathbf{y}-\mathbf{x})\right\} \quad (31)$$

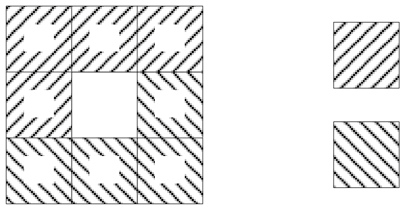
where I is the identity matrix and $Z(C_0)$ is a normalizing constant called partition function (Jeng 1991). Maximizing the equation gives us the MAP estimate of $\mathbf{x}$

$$\hat{\mathbf{x}}_{MAP} = \left(\frac{\sigma_n^2}{\sigma_x^2} A(C_0) + I \right)^{-1} \mathbf{y} = \left(kA(C_0) + I \right)^{-1} \mathbf{y} \tag{32}$$

To apply this estimation of observed image, the matrix C_0 and the variance ratio k should be estimated with the observed data.

5	4	3	4	5
4	2	1	2	4
3	1	X	1	3
4	2	1	2	4
5	4	3	4	5

(a)



(b)

Fig. 15. The neighborhood system and its order (a) Ordered neighborhood system for MRF. The order of the model relative to × is indicated by numbers; (b) The second order neighborhood system

3.2 2-D Multivariate time series analysis

The remote sensing image data from SAR (Synthetic Aperture Radar), TM (Thermal Mapper) or other sensors can be described by time series image models. To analyze and segment the images, we need to consider the correlation between neighborhood image pixels rather than independent random variables as considered by many pattern recognition researchers. The SAR and TM remote sensing data are in multi-spectral format which consists of a stack of images. There are correlations between pixels in a single image as well as correlations among image slices. The univariate Autoregressive (AR) model is not enough to describe information extracted from multi-spectral satellite sensors. The 2-D Multivariate Vector AR time series model described in this section is aimed to solve this problem.

The multivariate time series data analysis model is a generalized form of univariate models. It is also called ARV (AutoRegressive Vector) model. Each time series observation is a vector containing multiple components.

$$\text{Let } X_i = [x_{1i}, x_{2i}, x_{3i}, \ldots\ldots x_{mi}]^T \qquad -\infty \le i \le \infty \tag{33}$$

$$X_i = W + \phi_1 X_{i-1} + \phi_2 X_{i-2} + \ldots\ldots + \phi_p X_{i-p} + \varepsilon_i \tag{34}$$

X_i : m-by-1 column vector, time series observation variable

ε_i : m-by-1 column vector, multivariate white noise

$\phi_k : k = 1,2,3,.....p$ m-by-m autoregressive parameter matrix

W : m-by-1 Constant Vector (deterministic DC term)

To estimate the Multivariate AR model Parameter Matrix, we need to adopt the least squares method.

Consider an m-dimensional time series and let the Parameter Matrix be

$$B = \begin{bmatrix} W \phi_1 & \phi_2 & .... & \phi_p \end{bmatrix} \tag{35}$$

$$\text{Define } U_v = \begin{bmatrix} 1 \\ X_{v-1} \\ X_{v-2} \\ \vdots \\ X_{v-p} \end{bmatrix} \tag{36}$$

An AR(p) time series model can be expressed as the following regression model:

$$X_v = BU_v + \varepsilon_v \tag{37}$$

where $\varepsilon_v = noise$ vector with covariance matrix C $v = 1,2,.....n$ and n is the total number of samples.

Let

$$T = \sum_{v=1}^{n} U_v U_v^T \tag{38}$$

$$X = \sum_{v=1}^{n} X_v X_v^T \tag{39}$$

$$S = \sum_{v=1}^{n} X_v U_v^T \tag{40}$$

$$C = E(\varepsilon_v \varepsilon_v^T) \tag{41}$$

The Parameter Matrix $B = \begin{bmatrix} W \phi_1 & \phi_2 & .... & \phi_p \end{bmatrix}$ can be estimated as

$$\hat{B} = ST^{-1} \tag{42}$$

The error covariance matrix can be calculated as:

$$\hat{C} = \frac{1}{n - nf}(X - ST^{-1}S^T) \tag{43}$$

The matrix $\hat{C}$ can be factorized by Cholesky decomposition. The diagonal terms of this factorized Cholesky matrix are used as feature vector inputs to the pattern classifiers such as Support Vector Machine (SVM) or others.

$$\hat{C} = E^T E \tag{44}$$

where E is the upper triangular matrix.
Suppose E is an m-by-m upper triangular, lower triangular or diagonal matrix , the eigenvalues of E are the entries on the main diagonal of E. The eigenvalues of E matrix are the characteristics that E contains. The E matrix contains the major information of the estimated error covariance matrix $\hat{C}$.
The optimal estimated error covariance matrix $\hat{C}$ can be adopted to our remote sensing image classifier. When this optimal parameter matrix $\hat{B}$ is applied to the class that the data belongs, the total estimation errors are small, whereas when it is applied to any other class, the total estimation error would be much larger. Therefore, the differences in total estimation errors can be used in the classification process to distinguish among different remote sensing image classes.

4. Time series model segmentation adopted by international researchers

In this section, we like to present some evidences from international academic researchers to prove the effectiveness and efficiency out of time series models. Research scholars P. Steetal and N. Natarajan of IITM, Chennai-36 (Seetal 2010) was looking at rock fracture mapping problems that has applied to many issues related to rock mechanics. The difficult task was on fracture extraction from rock images. They found one interesting time series algorithm mathematically derived by P. Ho (Ho 2004). By using the first order autoregressive time series image segmentation model, they were able to apply and extract features on both rough and smooth fractures successfully as shown in figure 16 and figure 17. In Steetal and Natarajan computer experiments, three examples of rock fracture image segmentation results were processed by traditional edge detection methods such as Canny, Sobel and Prewitt as well as the newer time series models as described in this book chapter. The Seetal's proposed method is a minor modified version from Ho and Chen (Ho 2004). For the detailed image segmentation result comparisons, please refer to International Journal of Engineering Science and Technology Vol. 2(5), 2010 (Seetal 2010).

5. Conclusion

Image segmentation is one of the difficult tasks in image processing problems. The accuracy of the segmentation will determine a success or failure of the analysis. Therefore, decision making in the segmentation algorithm is very important. In this chapter we have established the innovative autoregressive time series model theory and presented many experimental results to support its image segmentation use. This image model indeed the effectively statistical approach to extract different properties for different objects.

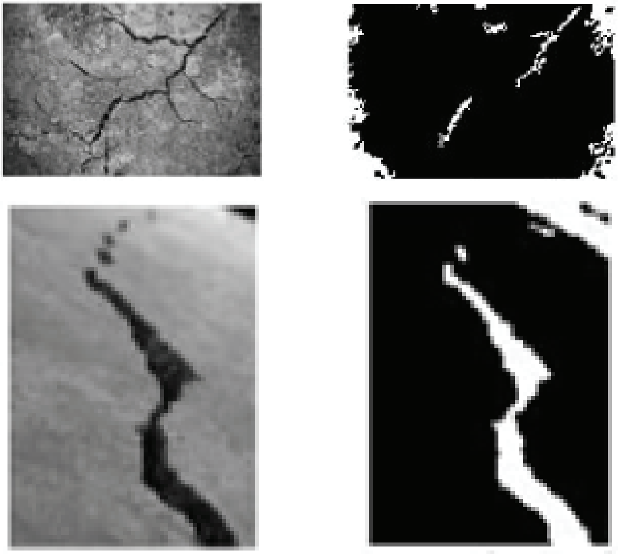


Fig. 16. Two sets of Rock fracture mapping image segmentation by AR time series model, original rock fractures are on the left column, processing results are on the right column ((Seetal 2010)

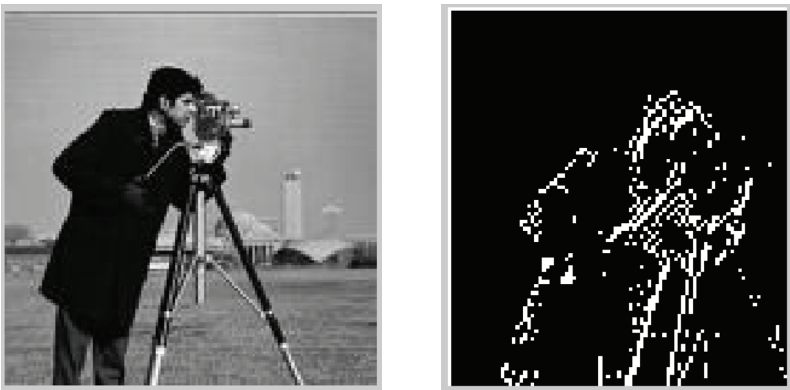


Fig. 17. Cameraman image segmentation results by AR time series methods, original Image is on the left, AR processing result is on the right (Seetal 2010)

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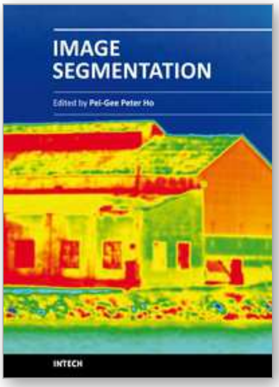

Image Segmentation
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It was estimated that 80% of the information received by human is visual. Image processing is evolving fast and continually. During the past 10 years, there has been a significant research increase in image segmentation. To study a specific object in an image, its boundary can be highlighted by an image segmentation procedure. The objective of the image segmentation is to simplify the representation of pictures into meaningful information by partitioning into image regions. Image segmentation is a technique to locate certain objects or boundaries within an image. There are many algorithms and techniques have been developed to solve image segmentation problems, the research topics in this book such as level set, active contour, AR time series image modeling, Support Vector Machines, Pixon based image segmentations, region similarity metric based technique, statistical ANN and JSEG algorithm were written in details. This book brings together many different aspects of the current research on several fields associated to digital image segmentation. Four parts allowed gathering the 27 chapters around the following topics: Survey of Image Segmentation Algorithms, Image Segmentation methods, Image Segmentation Applications and Hardware Implementation. The readers will find the contents in this book enjoyable and get many helpful ideas and overviews on their own study.

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